

GAMMA PREDICTION MODELS FOR LONG-TERM CREEP DEFORMATIONS OF PRESTRESSED CONCRETE BRIDGES

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Abstract. For long-span bridges as well as statically indeterminate frame structures it is essential to implement efficient and realistic prediction models for the long-term processes of concrete creep, shrinkage, and steel relaxation. In order to systematically study the main influential factors in bridge deflection measurements a probabilistic analysis can be performed. Due to the associated computational costs such investigations are limited. The predictions based on the highly scattered input parameters are associated with uncertainties. There is interest in alternative prediction models decoupled from complex analytical and computationally expensive numerical models, using measured structural responses. A gamma process is an example of such an alternative method. This process is suitable for capturing evolving structural response quantities and deterioration mechanisms like crack propagation, corrosion, creep, and shrinkage, as reported in Ohadi and Micic (2011). The objective of this paper is to illustrate the use of gamma process approaches for the prediction of the creep and shrinkage performance of prestressed concrete bridges. The presented approaches incorporate uncertainties and make predictions more reliable with the help of structural health monitoring (SHM) data. The creep-shrinkage response of a prestressed box girder bridge serves for the calibration and evaluation of the considered gamma process approaches.

Keywords: realistic creep model, gamma process, statically indeterminate structures, probability, box girder bridges.

Introduction

In many countries, well-defined specifications for the design and prediction of the structural behavior of infrastructure systems such as highway bridges are in use. These specifications need to ensure an adequate degree of reliability. The wide range of processes which affect any infrastructure system in the course of its lifetime leads to the requirement of periodic assessments based on inspection and sometime monitoring. When necessary, the performance of these infrastructure systems and their components is updated through carefully optimized maintenance measures. Notwithstanding such control mechanisms and adjustment procedures, it is of utmost importance for the national economy to be able to reliably predict both short term and long term behavior (Strauss *et al.* 2013; Wendner *et al.* 2010) in order to (a) efficiently assign both short term and long term

adjustment measures, (b) plan the relevant expenses, and (c) provide an acceptable degree of reliability throughout the structural lifetime as defined by the relevant standards and within the limits accepted by society. In recent years, new technologies were developed for the accurate description and thus identification of physical processes such as corrosion, creep, and shrinkage. In addition, the availability of data for calibration and validation, which can be gained through monitoring (Guo *et al.* 2011), is increasing rapidly. This also creates a solid foundation for reviewing the effectiveness of long established inspection procedures for infrastructure systems, which are paramount for the expected performance and safety of structural systems. The systematic collection of a wide range of site-specific data, if made available to the engineering community, is an essential element for the verification

and improvement of current models and design concepts. Furthermore, optimized inspection and check routines facilitate (among other things) cost effective maintenance management and scheduling of repair measures. In this paper, the focus will be on the problems caused by the creep and shrinkage processes in prestressed reinforced concrete structures. In particular, we will investigate the possibilities for their performance prediction based on stochastic processes as an efficient alternative to computationally expensive mechanical models.

Stochastic processes and their representation have to date been applied only to a very limited degree for modelling the degradation of the mechanical properties of structural components. van Noortwijk (2009) has, however, identified them as a suitable approach for modelling the life cycle of structures. In the context of life cycle based structural engineering, it seems appropriate to consider time dependent and very uncertain properties, such as the average degradation rate per unit time, as random variables. The most suitable methods for this approach are stochastic processes in general, and Markov processes in particular. However, a distinction is made between discrete Markov processes (i.e. Markov chains) and continuous Markov processes such as the Poisson, Levy and Gamma processes (van Noortwijk 2009). According to Pandey *et al.* (2009), gamma process representations are preferable to discrete Markov models for continuous processes of growth or degradation.

Gamma processes are continuous-time stochastic processes with independent, non-negative increments that follow gamma distributions with typically identical scale parameters and a time dependent shape parameter. As such they are well suited for modelling damage that gradually accumulates over time in a sequence of small increments (van Noortwijk 2009). Both, the scale parameter β and the time dependent shape parameter α can be estimated by the method of moments.

In this paper creep and shrinkage behavior of a prestressed concrete box girder bridge is analyzed. Initially the creep and shrinkage of the bridge based on the 2014 RILEM recommendation B4 for modeling creep and shrinkage (Bažant *et al.* 2015; Hubler *et al.* 2015; Wendner *et al.* 2015a, 2015b), the successor of the 1995 RILEM recommendation B3 (Bažant, Baweja 1995), which both are based on the micro-prestress solidification theory (Bažant, Prasanna 1989a, 1989b). These numerical simulations, see also Wendner *et al.* (2015c), then serve as a reference frame for assessing the gamma process prediction models. The primary aim of this investigation is to evaluate which observation period is required for which gamma process formulation to still obtain a reasonable prediction of the ongoing creep and shrinkage processes. For the determination of the gamma parameters the method of moments on the one hand and Polynomial 2nd and 3rd order approaches on the other hand had been applied based on maximum likelihood estimators.

It is highly important to stress that no unbiased multi-decade monitoring data on bridge deformations exists which might have served for the experimental end of this paper. The simulation of measurement results (for training and validation) utilizing the most advanced and repeatedly validated (Bažant *et al.* 2012a, 2012b; Yu *et al.* 2012) state of the art rate-type modeling techniques in combination with the B4 prediction model which is endorsed by RILEM turned out to be the only feasible way for investigating the capabilities of multi-decade prediction and updating concepts.

1. Rate-type formulation for creep structural analysis

Considering the size scale and time span required in creep structural analysis for the long-term performance assessment of creep sensitive structures as well as the complex interactions with other phenomena such as steel relaxation or damage evolution it is advantageous to abandon the traditional approach to creep analysis – the integral-type formulation based directly on the principle of superposition (Wendner *et al.* 2015c; Yu *et al.* 2012). Instead, the more efficient and versatile rate-type formulation is used. It is well known that any realistic creep compliance function following ageing linear viscoelasticity, which is the case for concrete (at least under service load), can be approximated at any desired accuracy by a rheological model represented by Kelvin or Maxwell units. In addition to accuracy, the rate-type formulation brings great benefits in computational efficiency and compatibility, which allows the coupling with other memory-independent phenomena (e.g., cracking, fatigue, and steel relaxation).

An improved rate-type formulation using Kelvin units (Yu *et al.* 2012) is employed to simulate the long-term creep shrinkage behavior even of the most complex structural systems. By taking advantage of the Laplace transformation inversion and Widder's approximation (Widder 1971), the ageing spectrum of each Kelvin unit at each time step can be uniquely and efficiently identified based on the given compliance function of the creep model used in the analysis (Bažant *et al.* 2012a, 2012b; Yu *et al.* 2012). With the aid of the exponential algorithm (Jirásek, Bažant 2002), the rate-type formulation can be further extended to a three dimensional quasi-elastic stress-strain incremental relation, which can be programmed in a user subroutine and then imported in a general FEM software like ABAQUS for structural modeling. The detailed derivation and implementation of this improved rate-type formulation can be found in a recent investigation (Yu *et al.* 2012).

The accuracy, efficiency and compatibility of the improved rate-type formulation have been documented in recent studies on simple structures and large-span bridges (Bažant *et al.* 2012a, 2012b; Yu *et al.* 2012). The improved rate-type formulation not only reproduces the theoretical prediction for the deformation of a column

under step-wise loading, but also realistically estimates the deflection history of large-span box girders constructed in a segmental manner. Furthermore, the 3D quasi-elastic stress-strain incremental relation allows taking into account the creep under shear stress, and thus capturing the shear lag, which is critical for thin-wall structures but generally ignored in simplified formulations. Therefore, it can be assumed that all simulations were carried out with the best available algorithms in order to ensure a realistic estimation of the time-dependent deformation of prestressed bridges.

2. Long-term prediction of creep effects

Degradation and aging processes of a structure can be described by nonnegative continuous functions. These functions can be characterized by nonnegative increments with independent path and variable uncertainty. Furthermore, the period of time until the occurrence of an observed undesirable event is typically associated with considerable uncertainty and dependent on structural behavior. Currently, the most established creep and shrinkage prediction models are deterministic and provide no quantification of uncertainty. Yet, the creep and shrinkage response clearly follows a non-negative and continuous function with independent path and variable uncertainty content. Also, the time to failure (exceedance of acceptable limits) is highly uncertain and definitely structure specific. Creep and shrinkage processes are subject to numerous uncertainties. This is in fact the reason why the stochastic processes are used as an approximate approach for capturing the macroscopic creep shrinkage behavior of structural components and systems. Already in the 1970s Bažant and his coworkers explored a stochastic process for the extrapolation of uncertain creep processes (Cinlar *et al.* 1977). Within the framework of life cycle modeling, stochastic process predictions and probabilistic modeling have become key elements as acknowledged by Frangopol *et al.* (2004) and van Noortwijk (2009). The lack of failure data for buildings and bridges, although fortunate for society, constitutes a major problem for the application of reliability methods as no calibration or validation is possible. Consequently, decisions based on lifetime distribution and/or very low structural failure rates are inconsistent and not particularly rigorous. Time dependent uncertain structural properties, as for example the deterioration rates of structures or structural components are frequently computed using random variables. To describe the stochastic processes of these random variables, Markov process representations which require predefined status categories are used frequently. Markov processes belong to the group of stochastic processes without auto-correlation. Regarding stochastic processes, van Noortwijk (2009) distinguishes between discrete Markov processes, such as Markov chains, continuous Markov processes, such as Brownian movement, and Levy and gamma processes. The physical processes

behind structural deterioration clearly are not independent of previous time-steps and thus cannot be realistically described by non-correlated incremental stochastic processes. According to Pandey *et al.* (2009), gamma processes are suitable for capturing such kinds of deterioration processes. In particular, van Noortwijk (2009) suggests using temporally continuous stochastic gamma processes with independent non-negative increments for the description of a gradually developing deterioration processes. These are, for example, corrosion of reinforcement bars, creep and shrinkage of concrete, fatigue of concrete and reinforcement, and crack propagation.

3. Long-term prediction of creep effects

The main advantage of using gamma function approaches for modeling deterioration processes is their relatively straightforward mathematical calculations (van Noortwijk 2009). For the mathematical determination of gamma process modeling, a random variable X is considered. A random variable X that follows a gamma distribution can be described using the shape parameter $\alpha > 0$, the scale parameter $\beta > 0$ and the following probability density function (PDF):

$$\text{Ga}(x|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} \exp(-\beta \cdot x), \quad (1)$$

where

$$\Gamma(\alpha) = \int_{z=0}^{\infty} z^{\alpha-1} e^{-z} dz \quad (2)$$

describes the gamma function for $\alpha > 0$.

The graph of a gamma process can be used to illustrate that successive gamma PDF or CDF functions are independent of each other. Consequently, the conditional distribution of variable X can only be formulated on the basis of the current observation (Ohadi, Micic 2011). This property is very well suited for capturing typical deterioration processes in structural engineering. In practice, the projection of the deterioration into the future is best based on current observations of the condition and on those events that have led to the present condition. Historical deterioration profiles may be helpful, in addition to the projections, as a reference value for the comparison with the predictions. The inclusion of such profiles into the projections is recommended, but there is no need to record them for future use. It can be assumed that for deterioration processes, the shape parameter $\alpha(t)$ increases with time t , where $t > 0$ and $\alpha(0) = 0$. The gamma process with the shape parameter $\alpha(t) > 0$ and the scale parameter $\beta > 0$ is a continuous stochastic time process $\{X(t), t \geq 0\}$. The time dependent probability density function of $X(t)$ is given by:

$$f_{X(t)}(x) = \text{Ga}(x|\alpha(t), \beta) \quad (3)$$

with an expected value of:

$$E(X(t)) = \frac{\alpha(t)}{\beta} \quad (4)$$

and a variance of:

$$\text{Var}(X(t)) = \frac{\alpha(t)}{\beta^2}, \quad (5)$$

where the coefficient of variation is defined as:

$$\text{COV}(X(t)) = \frac{1}{\sqrt{\alpha(t)}}. \quad (6)$$

Both, expected value and coefficient of variation, are solely functions of the gamma process parameters and only indirectly dependent on time (Ohadi, Micic 2011).

3.1. Gamma processes for deterioration modelling

Historically, it was assumed that gamma processes can be used to model any form of deterioration with a constant scale parameter β and a time-dependent power law based on shape parameter $\alpha(t)$ of the form:

$$\alpha(t) = ct^b. \quad (7)$$

It is generally assumed that sufficient engineering expertise on the expected form of the deterioration is available. Thus, van Noortwijk *et al.* (2007) recommended using a constant exponent b which can be adapted according to the considered deterioration process; for the corrosion of rebars, Ellingwood and Mori (1993) recommend $b = 1$ while for sulphate attacks, the suggested exponent is $b = 2$ and for diffusion controlled ageing, $b = 0.5$. Bažant and his coworkers investigated the application of gamma processes to predict concrete creep, for which $b = 0.125$ is recommended (Cinlar *et al.* 1977). The other two parameters c and β however are unknown quantities and need to be adapted by drawing on expert knowledge and/or statistical analyses such as the Maximum Likelihood Method, the Method of Moments or Bayesian Statistics. Ohadi and Micic (2011) have used the Method of Moments for determining both values. Regarding special cases of gamma process, the gamma process is called stationary if the deterioration is linear in time, i.e. when $b = 1$ in the power law Eqn (1), and non-stationary when $b \neq 1$. In mathematical terms a process has stationary increments if the probability distribution of the increments $X(t+h) - X(t)$ depends only on h for all $t, h \geq 0$. Due to the stationarity, both the mean value and the variance of the deterioration are linear in time. An additional property of the gamma process with stationary increments is that the gamma density transforms into an exponential density if $t = c^{-1}$. When the unit-time length is chosen to be c^{-1} , the increments of deterioration are exponentially distributed with mean c^{-1} and the probability of failure in unit time i reduces to a shifted Poisson distribution (Frangopol *et al.* 2004). In this paper, the pre-

viously established approaches and assumptions will be critically reviewed, ultimately arriving at modeling recommendations for creep and shrinkage processes.

4. Case study on the Colle Isarco Viaduct Bridge

The presented framework for gamma prediction processes will be applied to multi-decade creep and shrinkage predictions, calculated for an existing 167.5 m long fully post-tensioned box girder bridge. Each of the four identical main structural elements of the Colle Isarco Viaduct, located in Northern Italy, consists of a main span of 91.0 m, a long cantilever of 59.0 m, and a short cantilever of 17.5 m (Strauss *et al.* 2009; Wendner *et al.* 2015c). The cross-section height varies between 10.8 m at the main pier and 2.85 m at the tip of the cantilever. The box girder itself has a width of 6.0 m whereas the top slab is 10.6 m wide. The bridge is erected sequentially out of 44 segments, 497 pre-stressing tendons in total and 149 tendons above the main support (Fig. 1). The concrete quality is B45, mild steel BSt500 and pre-stressing strands of type 1570/1770 are used.

4.1. Deterministic analysis

Wendner *et al.* (2015c) recently investigated the most widely used creep and shrinkage models, namely ACI92 (ACI Committee 209 2008), B3 (Bažant, Baweja 1995), B4 (Bažant *et al.* 2015), CEB-FIP90 (CEB-FIP 1993), fib2010 (fib 2013) based on the Colle Isarco's design specifications. This full discussion of the different models' characteristics and the respective consequences for long term predictions is based on realistic deterministic and probabilistic structural analyses and provides important insights into the creep behavior that could not be derived based on laboratory data and theoretical considerations as presented in Wendner *et al.* (2015a) and Hubler *et al.* (2015). Exploiting symmetry, the bridge was modeled in ABAQUS using solid elements, for the concrete members and truss elements for the discretely modeled pre-stressing tendons, which were assigned to 26 groups according to the pre-stressing sequence in longitudinal direction. In total half the bridge model contains 26,558 hexahedral elements and 11,793 truss elements; as shown in Figure 1(c). For each group of tendons, the prestress is assumed to be applied 7 days after their anchoring segments are cast. Five prestress levels, namely, 600, 720, 840, 960 and 1080 MPa have been investigated in order to be able to separate the contributions of concrete creep and steel relaxation to the overall pre-stress loss.

4.2. Probabilistic study

In addition to the deterministic parameter study with the goal to compare the creep and shrinkage models that are currently endorsed by engineering communities, the influence of the prestressing level (systematic changes in the overall pre-stressing level as well as random variations in individual tendon groups) was also investigated

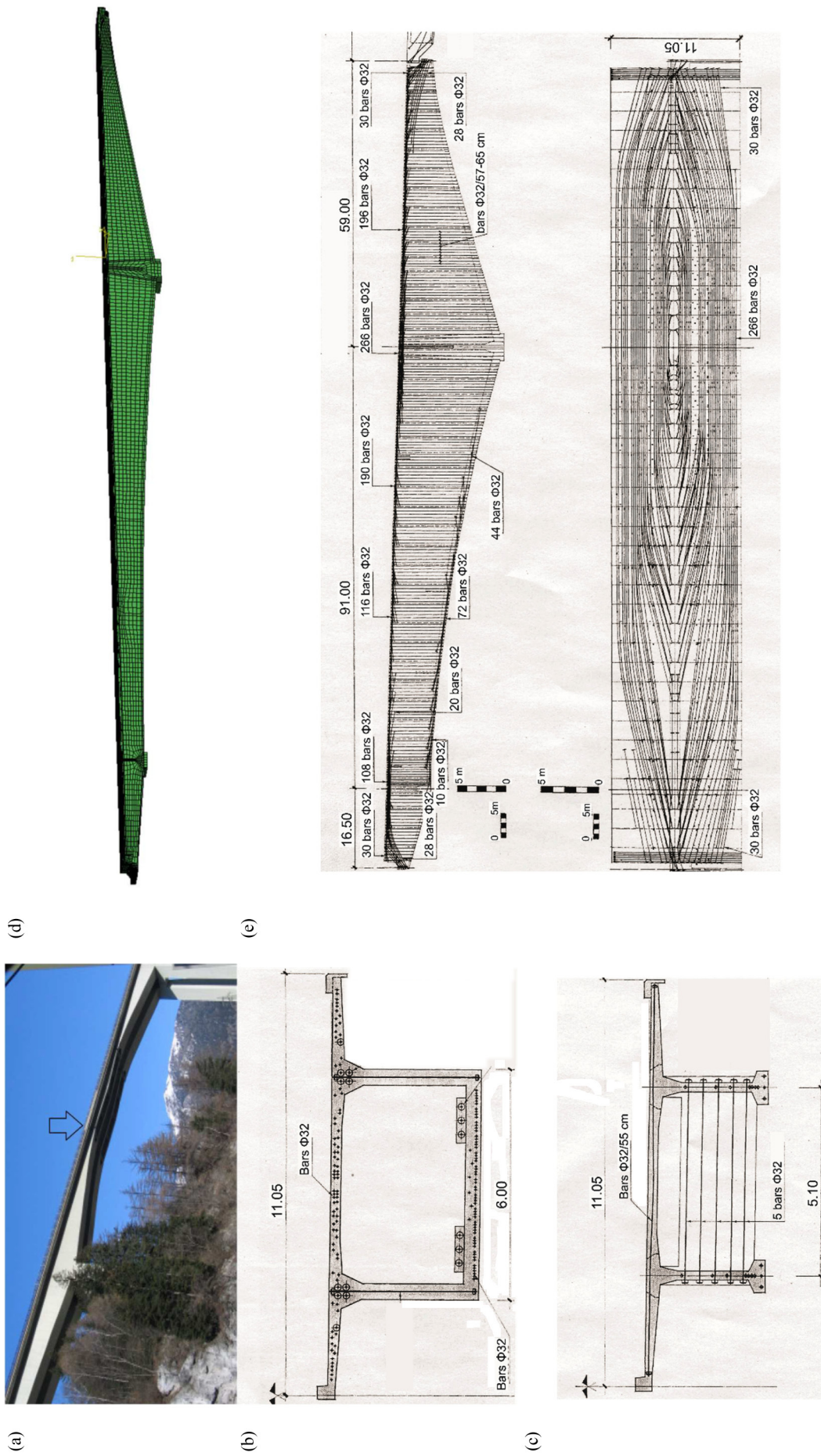


Fig. 1. Colle Isarco Viaduct: (a) Bottom view of the bridge structure, (b) and (c) cross section and its prestressing layout, and (d) ABAQUS Finite Element Model using solid elements, and (e) pre-stressing layout in vertical and longitudinal direction

Table 1. Mean and coefficient of variation of input parameters

Variable	Distribution type	Mean	COV
E_{28}	Lognormal	30,000 MPa	0.05
f_c	Lognormal	40.0 MPa	0.06
c	Normal	523.5 kg/m ³	0.20
w/c	Normal	0.35	0.20
a/c	Normal	3.50	0.20
h	Normal	0.70	0.05
T	Normal	20.0 °C	0.05
k	Normal	0.12	0.10
ρ_{1000}	Normal	0.035	0.10
$\alpha_{PS,i}$	Normal	1.00	0.10

(Wendner *et al.* 2015c). The main influence factors on bridge deformation measurements (vertical deflection and axial shortening) were determined by a probabilistic analysis that also yielded scatter bands for the predicted long-term response of the case study object. Due to the associated computational costs, this investigation was performed only once for model B3. The underlying stochastic models for all input variables are given in Table 1 in terms of mean values (corresponding to the inputs of the deterministic investigation) and coefficients of variation COV. In particular, the 28-day concrete modulus E_{28} and compressive strength f_c , the cement content c , the water to cement ratio w/c and the aggregate to cement ratio a/c were considered uncertain. The extrinsic parameters are the environmental humidity h and temperature T , both showing seasonal fluctuations in reality. k and ρ_{1000} are the primary parameters of the relaxation function, the values of which are taken according to *fib* 2010 Model Code recommendation (*fib* 2013). The uncertainties in the pre-stressing forces at the pre-stressing time are included in the model by a Normal distribution N (mean = 1.00;

Table 2. Correlation matrix of concrete properties (Strauss *et al.* 2013)

	E_{28}	f_c	c	w/c	a/c
E_{28}	1	0.19	0.06	-0.07	0.05
f_c	0.19	1	0.5	-0.52	0.36
c	0.06	0.5	1	-0.86	-0.86
w/c	-0.07	-0.52	-0.86	1	0.8
a/c	0.05	0.36	-0.86	0.8	1

COV = 0.10). With the exception of the concrete properties, all input variables are considered to be statistically independent. The correlation matrix for the concrete properties, described by pairwise linear correlation coefficients according to Pearson (1895) and Stigler (1989), is given in Table 2. A low yet statistically representative number of samples were generated utilizing the well-known Latin hypercube sampling scheme (Iman, Conover 1982) combined with simulated annealing (Kirkpatrick *et al.* 1983; Vořechovský, Novák 2009) with a population size of 45 samples.

The resulting scattering response not only allows the derivation of sensitivity factors between model inputs and structural response quantities (Bergmeister *et al.* 2007; Strauss *et al.* 2013) but also serves as input for the performance prediction using the concept of gamma processes as introduced in the subsequent section. Simulated deformations are available in 100 day increments starting with the end of construction up to an age of 32 years. In Figure 2 the scattering predictions (a) for the shortening of the main girder as well as (b) for the vertical deformation of the minor girder of the Colle Isarco Viaduct are plotted. The 45 grey lines represent single realizations of the simulated structural response with the mean response represented by the bold line and the dash-dotted 5% and 95% percentile lines.

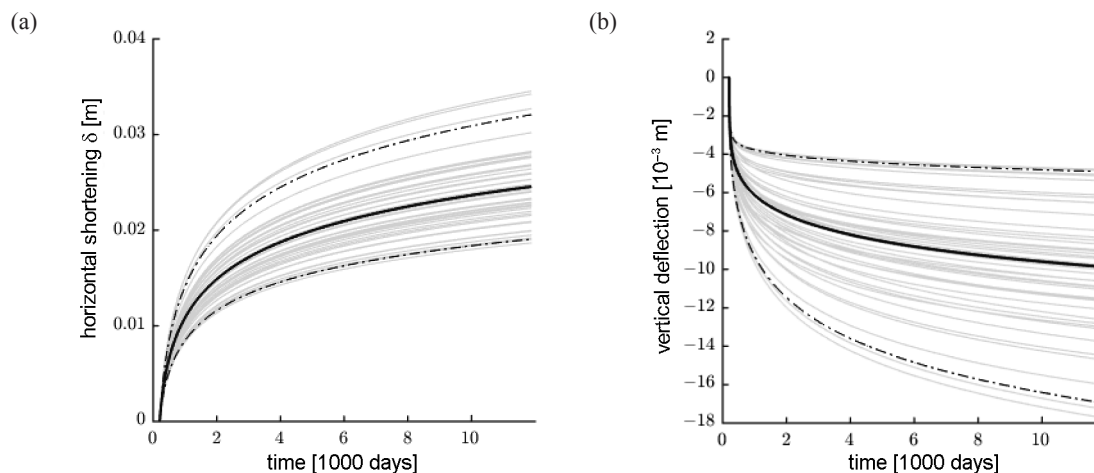


Fig. 2. Numerically computed creep shrinkage response of the Colle Isarco Viaduct: (a) shortening of the main girder w , and (b) vertical deflection of minor or cantilever girder (Wendner *et al.* 2015c)

4.3. Initial gamma process formulation and fitting procedure

In a first approach the gamma shape parameter $\alpha(t)$ and scale parameter β are formulated as proposed by van Noortwijk (2009) and again by Ohadi and Micic (2011). This approach is labeled fitting strategy I. The unknown parameters of the gamma process describing the creep and shrinkage response of the Colle Isarco Viaduct are estimated based on analytical equations. In those cases where the expected value and the variance of the accumulated deterioration at time t as well as the power exponent b are known, the non-stationary gamma process can easily be transformed into a stationary gamma process according to Ohadi and Micic (2011). This can be achieved by using a monotonic transformation from the real-time domain into the operative time domain:

$$z(t) = t^b; \quad t(z) = z^{\frac{1}{b}}. \quad (8)$$

This results in an expected value:

$$E(X(t(z))) = \frac{c \cdot z}{\beta} \quad (9)$$

and a variance of

$$\text{Var}(X(t)) = \frac{c \cdot z}{\beta^2}. \quad (10)$$

According to van Noortwijk *et al.* (2007), the observation periods can be transformed into:

$$z_i = t_i^b, \quad i = 1, \dots, n. \quad (11)$$

Accordingly, the monitoring periods on the transformed time axis are (van Noortwijk 2009) defined as:

$$w_i = t_i^b - t_{i-1}^b \quad (12)$$

and

$$\gamma_i = X_i - X_{i-1}. \quad (13)$$

The deterioration increment γ_i , according to van Noortwijk (2009), roughly corresponds to a gamma distribution with a shape factor $c \cdot w_i$ and a scale parameter β for $i = 1, 2, \dots, n$. Based on these assumptions, van Noortwijk (2009) recommends the following formulation for estimating the form parameters:

$$\frac{\hat{c}}{\hat{\beta}} = \frac{\sum_{i=1}^n \gamma_i}{\sum_{i=1}^n w_i} = \frac{x_n}{t_n^b} = \bar{\gamma} \quad (14)$$

and

$$\frac{x_n}{\hat{\beta}} \left(1 - \frac{\sum_{i=1}^n w_i^2}{\left[\sum_{i=1}^n w_i \right]^2} \right) = \sum_{i=1}^n (\gamma_i - \bar{\gamma} \cdot w_i)^2. \quad (15)$$

The Method of Moments can therefore be used for a simple parameter estimation of the gamma distributions.

Conversely, this simple method can be used to determine the shape parameters for optimizing the inspection intervals (Ohadi, Micic 2011). The Method of Moments mentioned above is a first approximation for determining the gamma process parameters that are then continuously adapted for subsequent time increments. As first approximation, the gamma parameters for the investigated creep and shrinkage processes have been determined for an exponent $b = 1$ describing the rate of deterioration in Eqns (7) and (8). In a second step we investigate the optimal exponent b for the investigated creep and shrinkage process.

4.4. Alternative formulation

In a second approach alternative to the fitting approach based on statistical moments as shown above are investigated. In particular, a 2nd and 3rd order polynomial fitting procedure for determining the shape parameter $\alpha(t)$ and scale parameter β of the gamma distribution are explored.

Two alternative fitting strategies are investigated: in strategy IIa a continuous adaptation of the shape parameter $\alpha(t)$ only, (e.g. by using the Maximum Likelihood Method) is performed while in strategy IIb the shape parameter $\alpha(t)$ is updated jointly with the scale parameter $\beta(t)$. In each case the shape and scale parameters are fitted to the available information t_k , where $k = 1, \dots, n$ denotes the evaluation or assessment point and n = the total number of observations.

Independently of the fitting strategy, two formulations for the prediction of the future evolution of the gamma process parameters are investigated – a polynomial 2nd and 3rd order. Therefore, the parameters $\alpha(t)$ and β of the gamma process can be predicted up to the assessment horizon k and extrapolated to the prediction horizon $i = k+1, \dots, n$.

The main goal is now the direct comparison of creep and shrinkage predictions between the van Noortwijk Moment Method on one side and the alternative strategy IIa and IIb on the other side. In particular, the basic features of the different fitting strategies and prediction approaches are discussed and compared with respect to their ability to predict the ‘true’ (accurately simulated) creep and shrinkage response. All investigations are based on the horizontal shortening data of the main girder and the vertical deflection data of the minor girder as presented in Figure 2. It is assumed that only $n = 10$ observations at times $t_{1, \dots, n} = 212, 512, 773, 1533, 3033, 4533, 6033, 7533, 9033, 10533$ days are available.

4.5. Discussion of the 2nd order polynomial fitting approach

The strategy IIb uses the 2nd order polynomial approach for the prediction of shape and scaling parameters of the gamma distribution based on a restricted set of calculated structural response characteristics (e.g. up to a predefined assessment point). Figures 3(a) and (b) portray the development of the shape and scaling parameters for

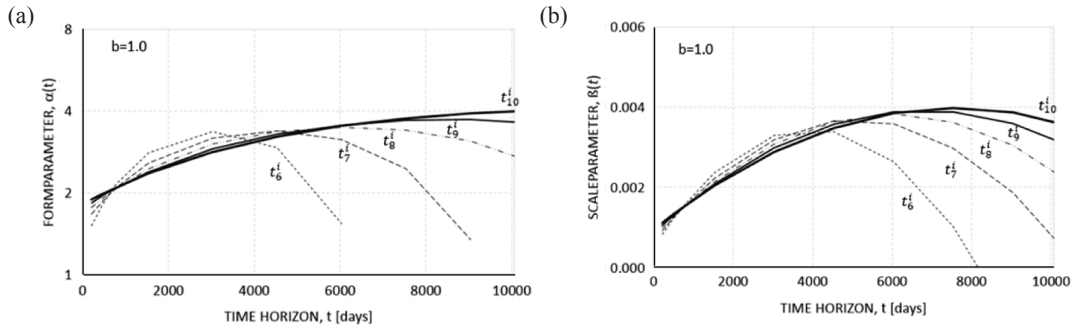


Fig. 3. Parameter evolution of the gamma process associated with the shortening of the main girder w following the fitting strategy IIb in combination with a 2nd order polynomial prediction approach: (a) evolution of shape parameter $\alpha(t)$, (b) evolution of scale parameter $\beta(t)$; based on the information from the time periods up to $k = 6, \dots, 10$, equivalent to $t_k = 4533, \dots, 10533$ days

prediction time horizons $i = 7, \dots, 10$ equivalent to $t_i = 6033, \dots, 10533$ days based on data up to the assessment points $k = 6, \dots, 10$ equivalent to $t_k = 4533, \dots, 10533$ days. As can be revealed from Figure 3, for a power law exponent $b = 1$ and a 2nd order polynomial prediction approach, $\alpha(t)$ increases from 1.5 to 4.0 and $\beta(t)$ from 0.001 to 0.004 with increasing k . It is interesting to note that the curvature of the parabola decreases with increasing data basis and the parameters of the gamma distribution approach their values for perfect information $k = n$ equivalent to data between day 0 and day 10533.

Both the shape and the scaling parameter should be positive and continuously increasing. These conditions are not automatically satisfied by a polynomial fitting function. Nonetheless, the 2nd order polynomial prediction represents a simple and flexible approximation of, arguably, sufficient accuracy, especially for modeling a well-known process. However, this approach should not be used for predictions far beyond the information basis.

The alternative to updating both the shape parameter and scale parameter (strategy IIb) is the assumption of a stationary scale parameter $\beta(t) = \beta$ (fitting strategy IIa). Figure 4 compares both fitting strategies utilizing the

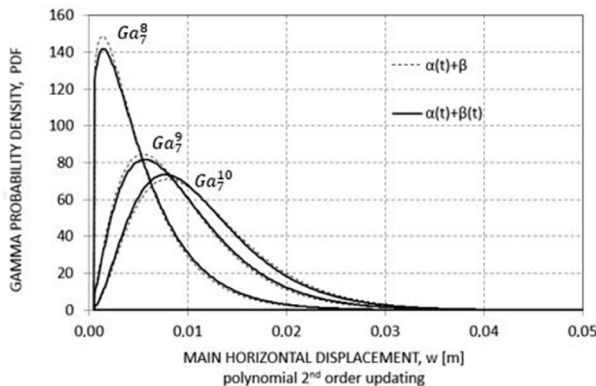


Fig. 4. Gamma process prediction for the shortening of the main girder w for time horizons $i = 8, \dots, 10$ equivalent to $t_i = 7533, \dots, 10533$ days based on the information up to time period $k = 7$ equivalent to $t_k = 6033$ days; comparison between a constant scale parameter β , and time-continuous adjustment of the scale parameter $\beta(t)$

shortening data of the main girder for an assessment time horizon $k = 7$ equivalent to $t_k = 6033$ days. The predictions for strategy IIa (stationary β) are plotted as dashed lines; those for strategy IIb as solid lines. The differences between both strategies are marginal for prediction times $i = 8, \dots, 10$, equivalent to $t_i = 7533, \dots, 10533$ days. Consequently, the simplification of assuming a stationary scale parameter seems justified.

Results of the 2nd order polynomial fitting approach are presented in Figure 5 in terms of the developing probability density functions of the gamma process for prediction time horizons $i = 3, \dots, 10$, equivalent to $t_i = 773, \dots, 10533$ days, based on the information up to time periods $k = 3, \dots, 10$, equivalent to $t_k = 773, \dots, 10533$ days. In Figure 5(a) the gamma process, strategy IIa, for the time dependent shape parameter $\alpha(t)$ and the stationary scale parameter β is presented. Figure 5(b) shows the alternative strategy IIb in which both parameters are non-stationary. Although the time adjustment of the scale parameter β has only a minor effect (compare Figs 5(a) and 5(b)), as already concluded, the continuous adjustment of both gamma distribution parameters is characterized by a slightly better convergence as compared to a continuous adjustment of only the shape parameter $\alpha(t)$.

4.6. Discussion of the 3rd order polynomial fitting approach

All previously discussed analyses are repeated on the basis of a third order polynomial fitting approach. The time-dependent behavior of the scale and shape parameters, $\alpha(t)$ and $\beta(t)$, is presented in the Figures 6(a) and 6(b), respectively. When compared to Figure 3, in which the second order polynomial fitting procedure of the parameters is portrayed, it becomes clear that the third order approach provides more consistent results.

As concluded for the 2nd order polynomial, the stationarity of the scale parameter is confirmed also for the 3rd order approach (see Fig. 7). It is shown again that only minor improvements can be expected from updating the scale parameter.

In Figure 8 the gamma process is visualized in terms of the evolving gamma distribution for both fitting strate-

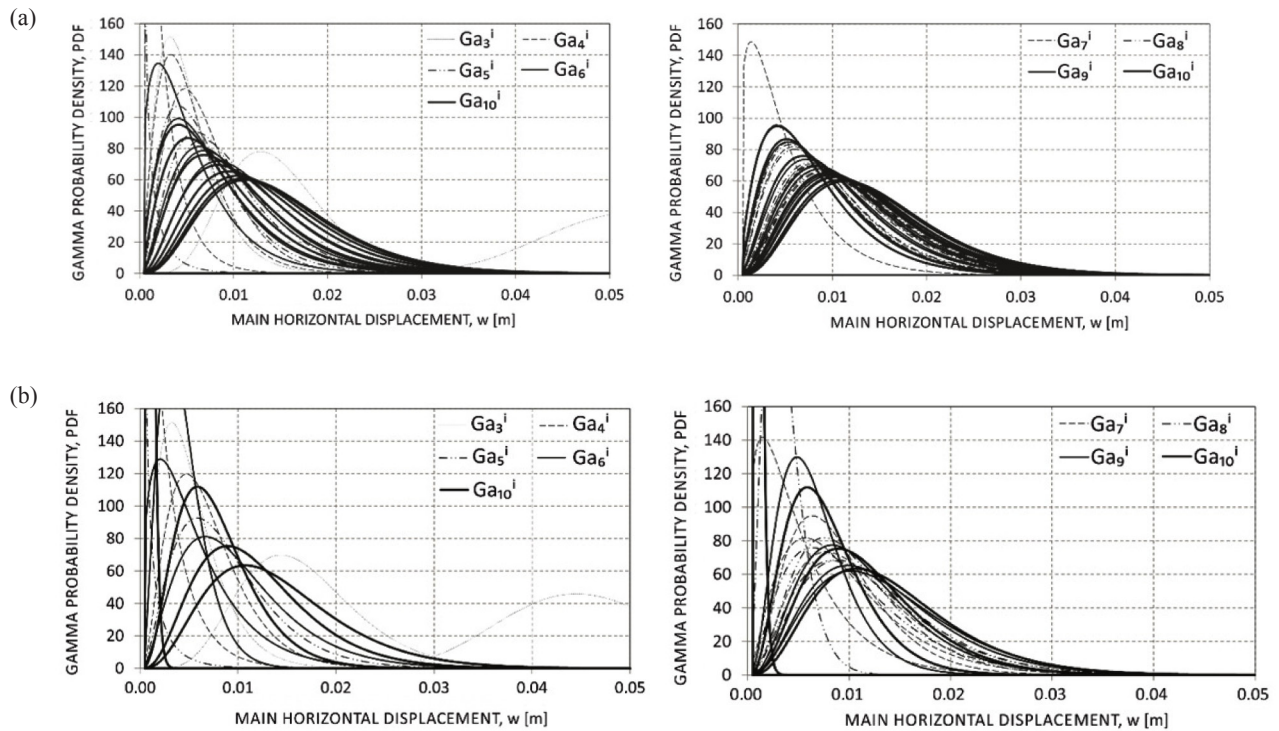


Fig. 5. Gamma process prediction of the *shortening of the main girder w* for time horizons $i = 3, \dots, 10$, equivalent to $t_i = 773, \dots, 10533$ days, based on the information from time periods $k = 3, \dots, 10$, using a polynomial 2nd order prediction of the gamma process associated shape parameter a and scale parameter β : (a) strategy IIa with a time-continuous adjustment of the gamma shape parameter $\alpha(t)$ and a constant scale parameter β , and (b) for strategy IIb with a time-continuous adjustment of the gamma shape parameter $\alpha(t)$ and the scale parameter $\beta(t)$

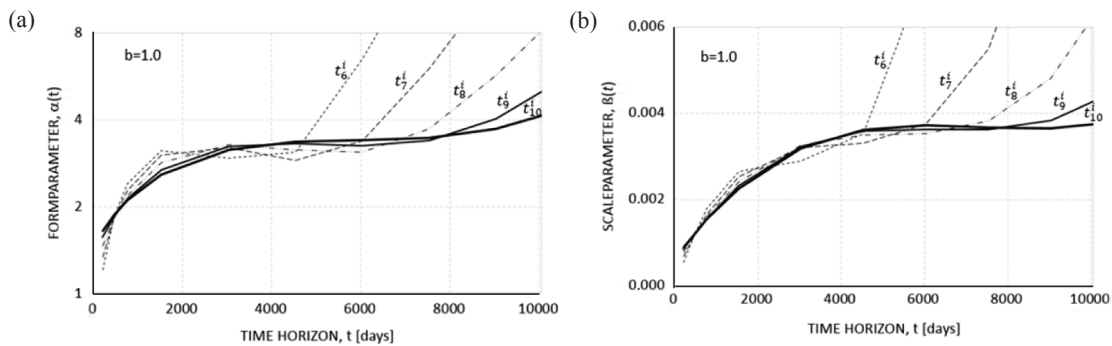


Fig. 6. Parameter evolution of the gamma process associated with the *shortening of the main girder w* following the fitting strategy IIb in combination with a 3rd order polynomial prediction approach: (a) development of shape parameter $\alpha(t)$, (b) development of scale parameter $\beta(t)$; based on the information from the time periods up to $k = 6, \dots, 10$, equivalent to $t_k = 4533, \dots, 10533$ days

gies IIa and IIb in combination with the 3rd order polynomial fitting approach for prediction time horizons $i = 3, \dots, 10$ based on information up to assessment time $k = 3, \dots, 10$, equivalent to $t_k = 773, \dots, 10533$ days.

4.7. Comparison to van Noortwijk’s approach

In further investigations the gamma prediction process was carried out according to the procedure recommended by van Noortwijk (2009) with a time dependent shape parameter following the power law $\alpha(t) = c \cdot t^b$, see Eqn (7) and a constant scale parameter β . Lacking a justification for another choice, a power law exponent $b = 1.0$ was

assumed (in obtaining Figs 9–11). Variations of parameter b will be discussed later.

The gamma prediction process of the main horizontal displacement based on the van Noortwijk’s approach is presented in Figure 9. Identical to its presentation for the 2nd order and 3rd polynomial prediction approaches in Figures 5 and 8 the gamma distributions are shown for prediction time horizons $i = 1, \dots, 10$ based on information up to time periods $k = 3, \dots, 10$.

It is important to note that the constant scale parameter β was determined based on information up to $k = 10$.

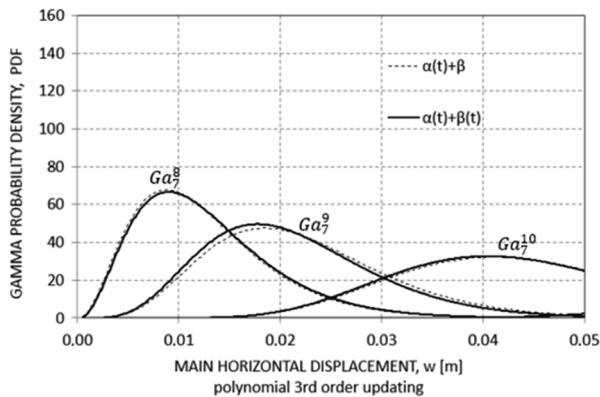


Fig. 7. Gamma process prediction for the shortening of the main girder w for time horizons $i = 8, \dots, 10$, equivalent to $t_i = 7533, \dots, 10533$ days, based on the information up to time period $k = 7$ equivalent to $t_k = 6033$ days; comparison between a constant scale parameter β , and time-continuous adjustment of scale parameter $\beta(t)$

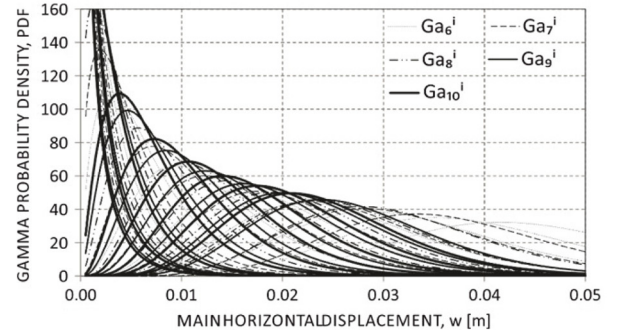
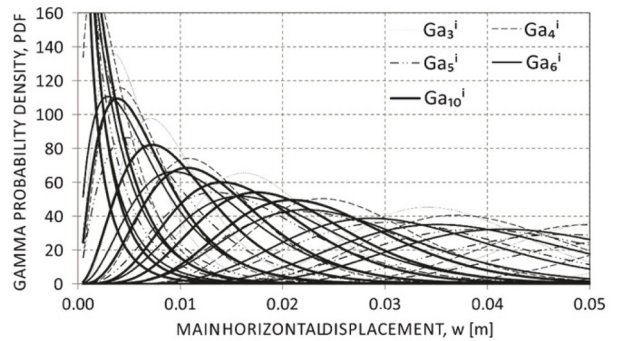


Fig. 9. Gamma process prediction associated with the shortening of the main girder w for prediction time horizons $t_i = 212, \dots, 10533$ days based on the information up to time periods $k = 3, \dots, 10$ according to the van Noortwijk's approach

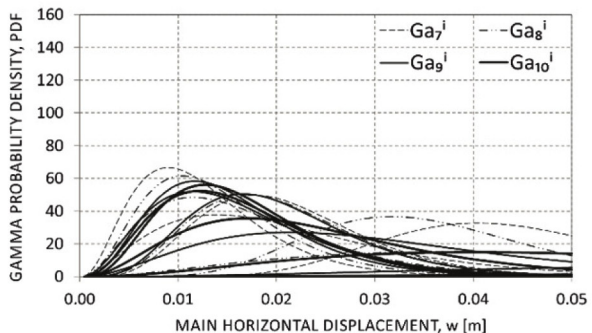
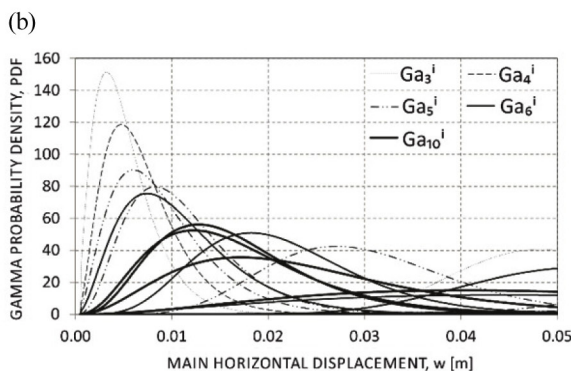
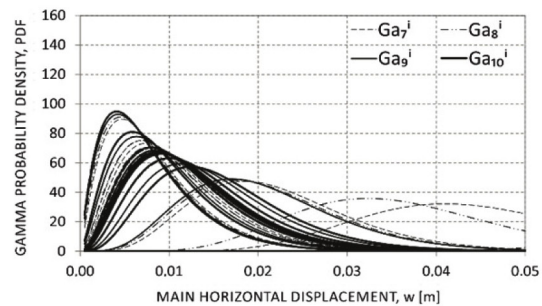
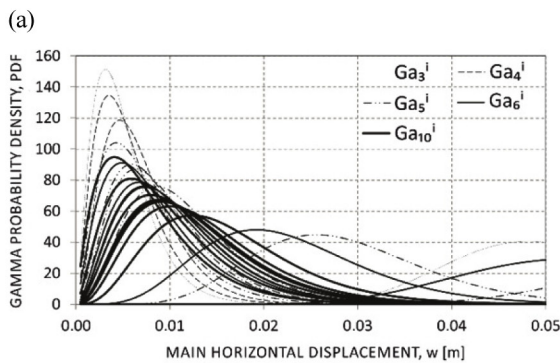


Fig. 8. Gamma process predictions associated with the shortening of the main girder w for time horizons $i = 3, \dots, 10$ based on the information from time periods $k = 3, \dots, 10$ using a polynomial 3rd order prediction of the shape parameter a and scale parameter β : (a) for strategy IIa with a time-continuous adjustment of the gamma shape parameter $\alpha(t)$ and a constant scale parameter β , and (b) for strategy IIb with a time-continuous adjustment of the gamma shape parameter $\alpha(t)$ and the scale parameter $\beta(t)$

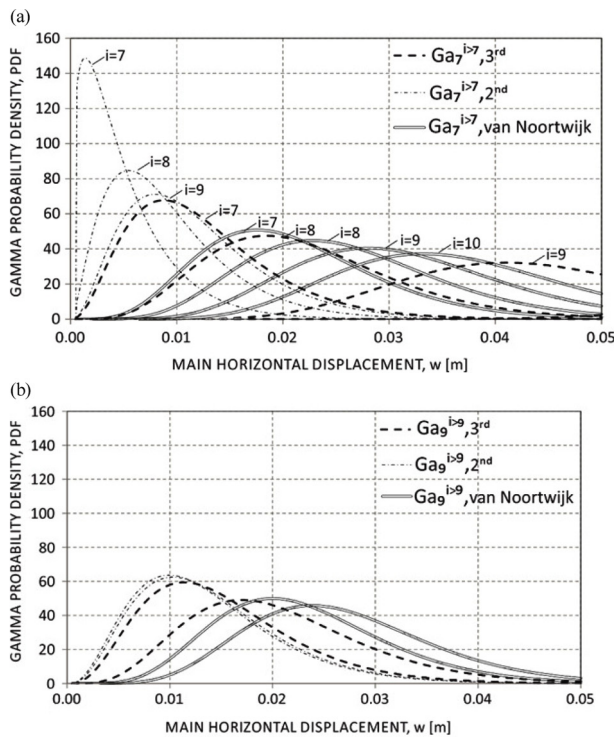


Fig. 10. Comparison between a 2nd and 3rd order polynomial prediction approach and the van Noortwijk's approach, based on information up to time periods: (a) $t_7 = 6033$ days, and (b) and $t_9 = 9033$ days; $Ga_7^{i>7}$ ($i = 7, 8, 9, 10$) and $Ga_9^{i>9}$ ($i = 9, 10$)

After analyzing the moment based prediction strategy I (van Noortwijk's approach) and the prediction strategy II (utilizing polynomial prediction functions) individually a comparison is performed. In particular, in Figure 10, (1) a second order polynomial, (2) a third order polynomial (both with constant scale parameter), and (3) the van Noortwijk's approach are compared for: (a) prediction time horizons $i = 7, \dots, 10$ based on information up to $t_7 = 6033$ days, and (b) prediction time horizons $i = 9, 10$ based on information up to $t_9 = 9033$ days, respectively.

The differences in PDFs obtained and presented again confirm greater adoptability of the third order polynomial function with respect to van Noortwijk's approach.

Furthermore, there is an interest in the assessment of the deviations between the investigated creep shrinkage prediction formulations, in particular the van Noortwijk's model, with respect to the numerically computed quasi-exact physical creep shrinkage behavior of the prestressed box girder bridge.

Figure 11(a) portrays the predicted expected values of the axial shortening of the main girder w based on the van Noortwijk's approach, and Figure 11(b) the associated relative deviations Δw_k^i with in percentage, where $k =$ assessment time and $i =$ prediction horizon. It is evident that the bias in the prediction decreases with increasing prior information (increasing k) and concurrent

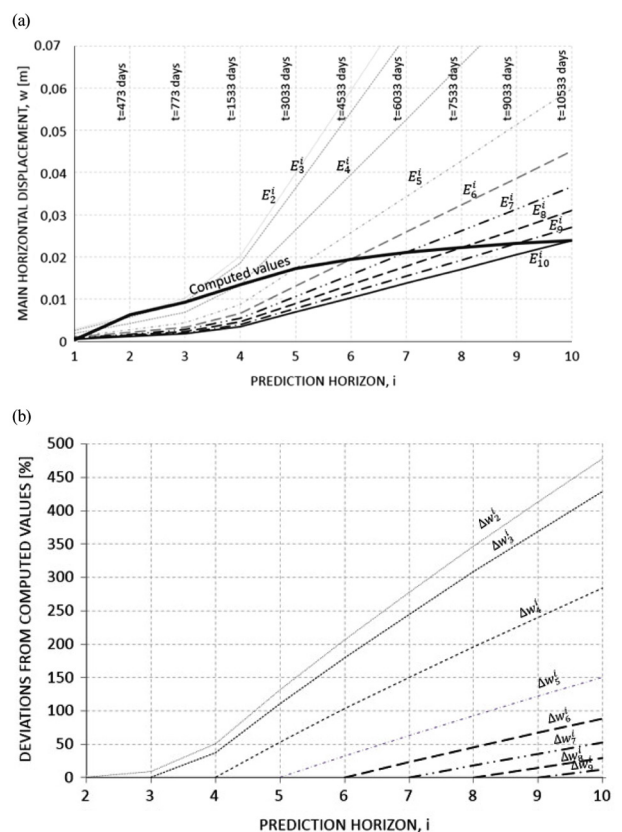


Fig. 11. Characteristics of the gamma process prediction associated with the shortening of the main girder w , according to van Noortwijk's approach: (a) expected values of time periods, and (b) deviations from real values of displacement

reduction in the distance to the prediction horizon i . The properties shown in Figure 11 for the van Noortwijk method, using the power law with $b = 1$, is also valid for the previously described alternative strategies IIa and IIb.

The information contained within Figure 11 (b) is essential for evaluating the required observation time for a given prediction quality or conversely for assessing the prediction quality for a given set of observations. For an accepted deviation between prediction and reality of $\Delta w_k^i < 50\%$ the van Noortwijk's model with an exponent $b = 1$ requires a distance between the prediction horizon i and the evaluation horizon k of $f = i - k = 3$, or in other words a Ga_{10}^7 prediction assures a deviation $\Delta w_k^i < 50\%$. For $\Delta w_k^i < 90\%$ we obtain $f = 6$ or e.g. a Ga_{10}^4 prediction for horizon $i = 10$. These considerations can be extended to arbitrary prediction horizons i .

Up to this point all analyses were based on a power law exponent of $b = 1.00$ for the transformation between real time and operational time. This corresponds to the generic recommendation by Ellingwood and Mori (1993) of $b = 1.00$ for stationary deterioration processes. Suggested values in literature go down to $b = 0.12$ for highly non-stationary processes. For creep the suggestion of $b = 0.125$ was formulated by Cinlar *et al.* (1977) and then again suggested by van Noortwijk *et al.* (2007).

In Figure 12 the prediction error Δw_k^i for non-stationary gamma processes is plotted as function of the power law exponent b . In particular, the recommendation $b = 0.125$ by Cinlar *et al.* (1977) for creep is presented in Figure 12(a). Figures 12(b)–(d) characterize the prediction quality for $b = 0.22$, $b = 0.30$, and $b = 0.39$. It can be seen that for values of b between $b = 0.125$, proposed by Cinlar *et al.* (1977), and $b = 0.22$, deviations Δw_k^i are uniformly distributed for increasing i and k . An increase to $b > 0.22$, see Figures 12(c)–12(d), is associated with an overall decrease in deviations Δw_k^i as well as to a decrease of errors with increasing i and k .

The graphs in Figures 12(c) and 12(d) also demonstrate that the optimization of the exponent b with respect to a physical phenomenon like creep and shrinkage effects is not only a characteristic of the phenomenon but also highly dependent on the assessment horizon k and the prediction horizon i ; in simple words a value dependent on the time periods.

For instance, Figure 12(b) presents the deviations Δw_k^i for a non-stationary gamma process with $b = 0.22$. For Ga_2^{10} the deviation of the expected displacement is $\Delta w_2^{10} = 48.6\%$ while the error decreases for Ga_4^{10}

to $\Delta w_4^{10} = 14.7\%$. Generally, as we increase the amount of available observation data the prediction quality increases. If a threshold of acceptable prediction quality is chosen as e.g. 30% it is possible to satisfy the quality requirements for a prediction at time $i = 10$ already at time $k = 3$, seven time steps before (Δw_3^{10} to $\Delta w_{10}^{10} \leq 30\%$). The deviation in $i = 10$ for predicting a displacement in $k = 3$ is 31.08%. If only data up to $k = 2$ is considered the same prediction quality can only be reached in $i = 3$.

Based on a heuristic optimization of b with respect to the considered creep and shrinkage phenomenon as presented in Figures 12(a) to 12(d) allow the following conclusions: the smallest deviations Δw_k^i , in general, over the considered time period k and $i = 1$ to 10 can be achieved with $b = 0.30$ and $b = 0.39$, while the values between $b = 0.22$ and $b = 0.30$ are preferred to be used for predictions if at least 5 observations are available, denoted Ga_5^i , as presented in Figure 18(a).

In addition, Figures 11 and 12 show: The deviations Δw_k^i of predictions for creep related deformations are higher when a stationary gamma process with $b = 1$ is assumed. In other words, a power law exponent larger than $b = 0.39$ can be considered conservative in the prediction

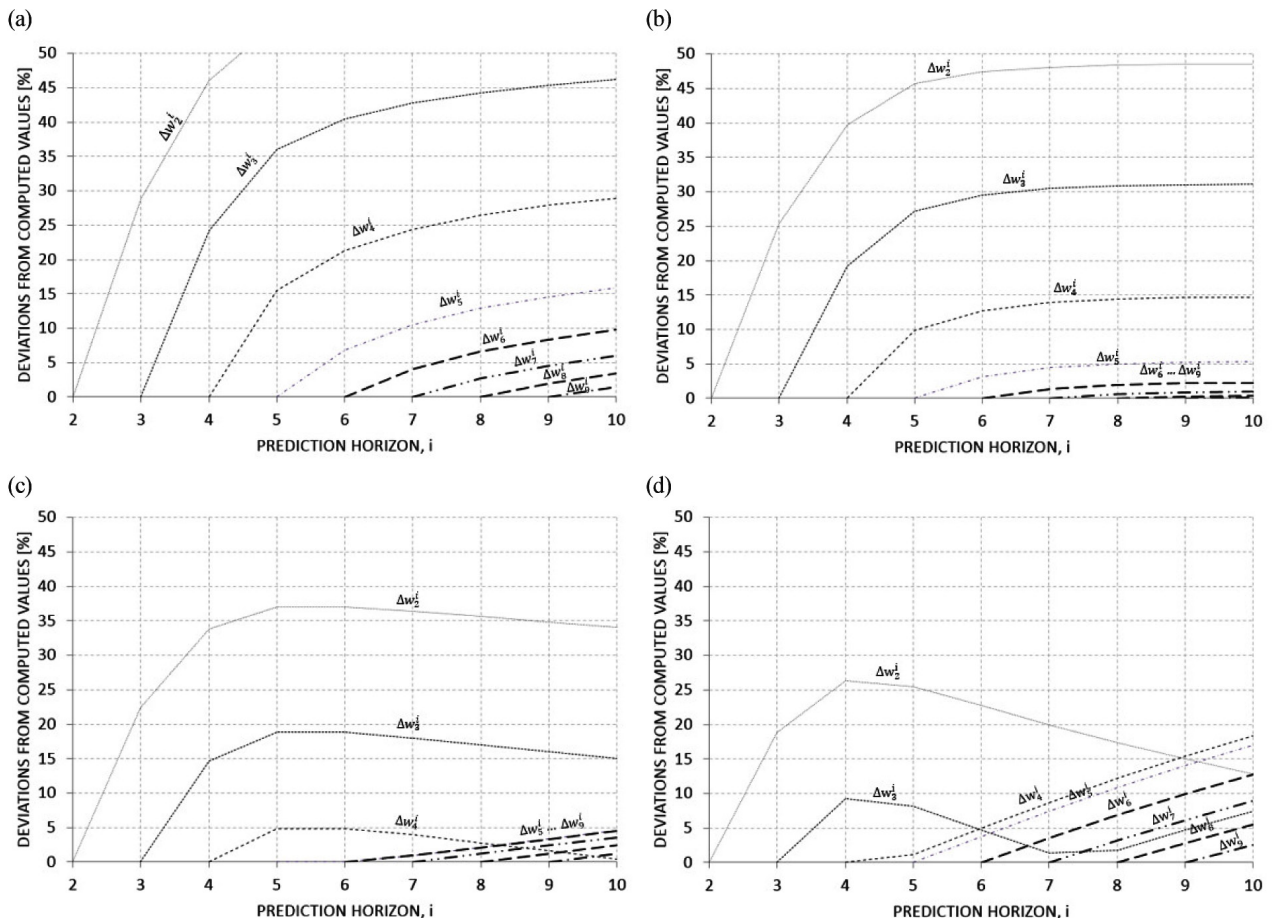


Fig. 12. Deviations of prediction of the shortening of the main girder w from computed values, according to van Noortwijk’s approach; in order to optimize a value of parameter b in the power law formulation $\alpha = c \cdot t^b$: a) $b = 0.125$; b) 0.22 ; c) 0.30 ; d) 0.39

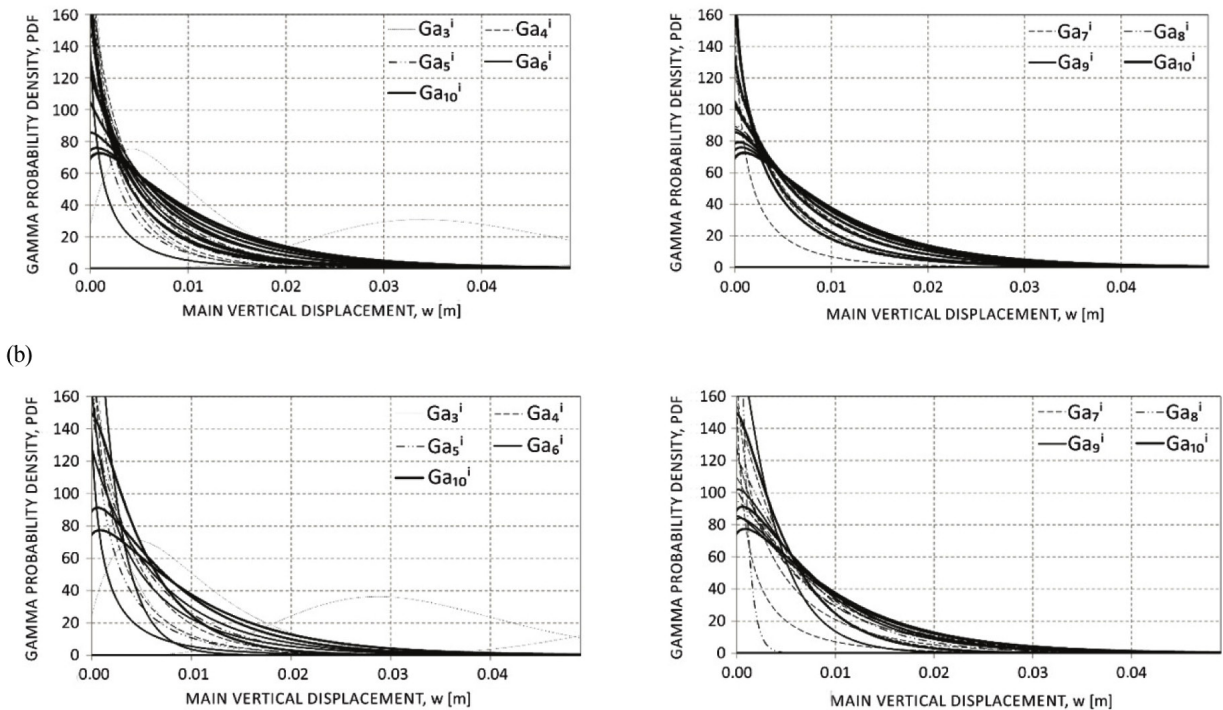


Fig. 13. Gamma process predictions associated with the vertical deflection of the main girder w for time horizons $i = 3, \dots, 10$ based on the information from time periods $k = 3, \dots, 10$ using a polynomial 2nd order prediction of the shape parameter α and scale parameter β : (a) for strategy IIa with a time-continuous adjustment of the gamma shape parameter $\alpha(t)$ and a constant scale parameter β , and (b) for strategy IIb with a time-continuous adjustment of the gamma shape parameter $\alpha(t)$ and the scale parameter $\beta(t)$

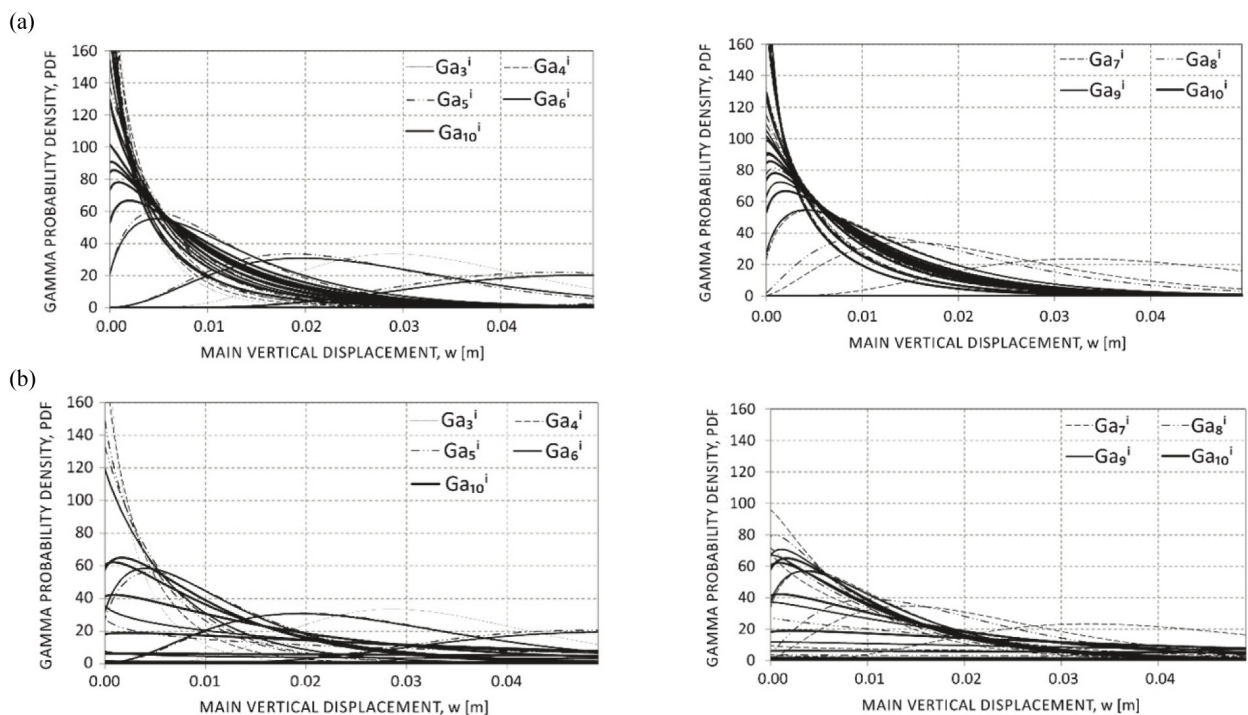


Fig. 14. Gamma process predictions associated with the vertical deflection of the main girder w for time horizons $i = 3, \dots, 10$ based on the information from time periods $k = 3, \dots, 10$ using a polynomial 3rd order prediction of the shape parameter α and scale parameter β : (a) for strategy IIa with a time-continuous adjustment of the gamma shape parameter $\alpha(t)$ and a constant scale parameter β , and (b) for strategy IIb with a time-continuous adjustment of the gamma shape parameter $\alpha(t)$ and the scale parameter $\beta(t)$

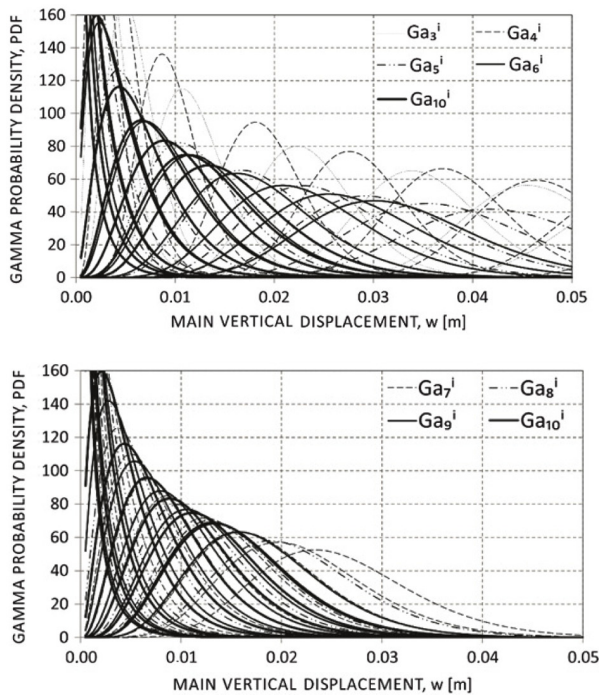


Fig. 15. Gamma process prediction associated with the vertical deflection of the main girder for time horizons $t_i = 212, \dots, 10533$ based on the information up to time periods $k = 3, \dots, 10$ according to the van Noortwijk's approach

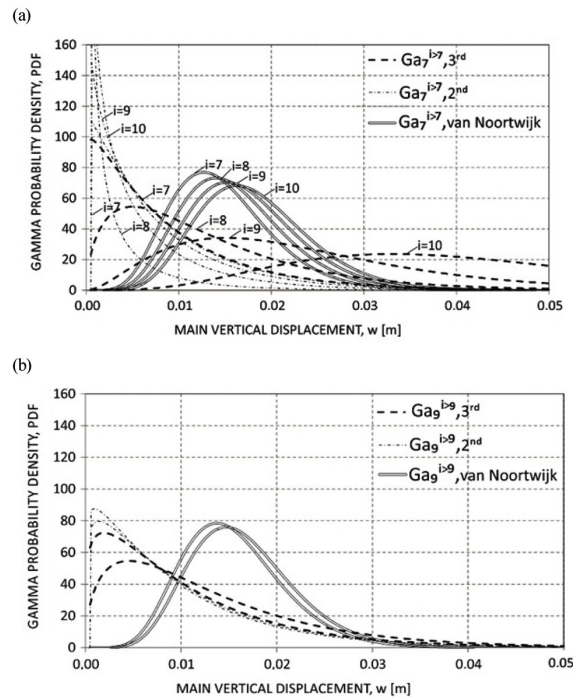


Fig. 16. Comparison between a polynomial 2nd and 3rd order updating procedure, and van Noortwijk's approach, based on the information up to (a) time period $t_7 = 6033$ days, and (b) $t_9 = 9033$ days; $Ga_7^{i>7}$ ($i = 7, 8, 9, 10$) and $Ga_9^{i>9}$ ($i = 9, 10$)

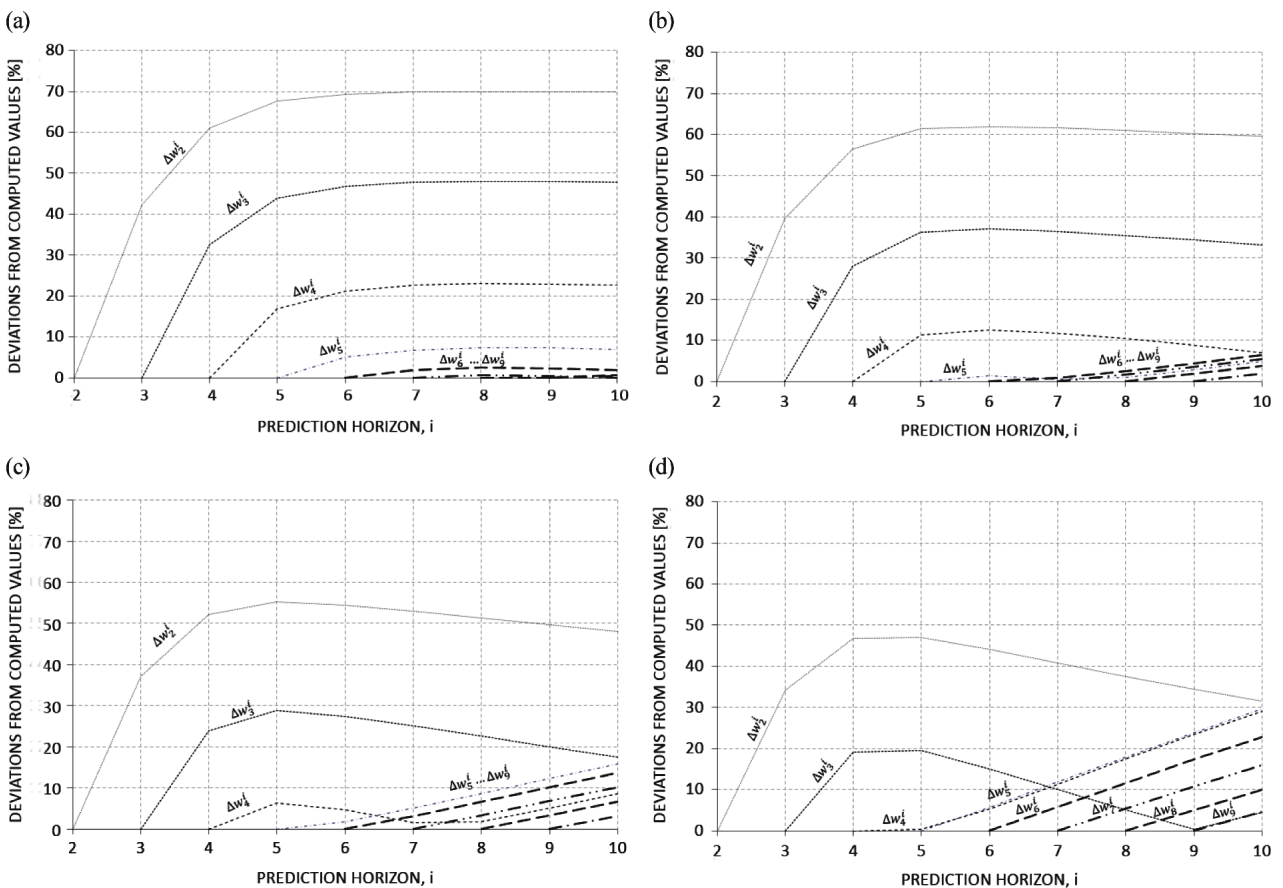


Fig. 17. Deviations of prediction of the vertical deflection of the main girder from computed values, according to van Noortwijk's approach; in order to optimize a value of parameter b in the power law formulation $\alpha = c \cdot t^b$: a) $b = 0.125$; b) 0.22 ; c) 0.30 ; d) 0.39

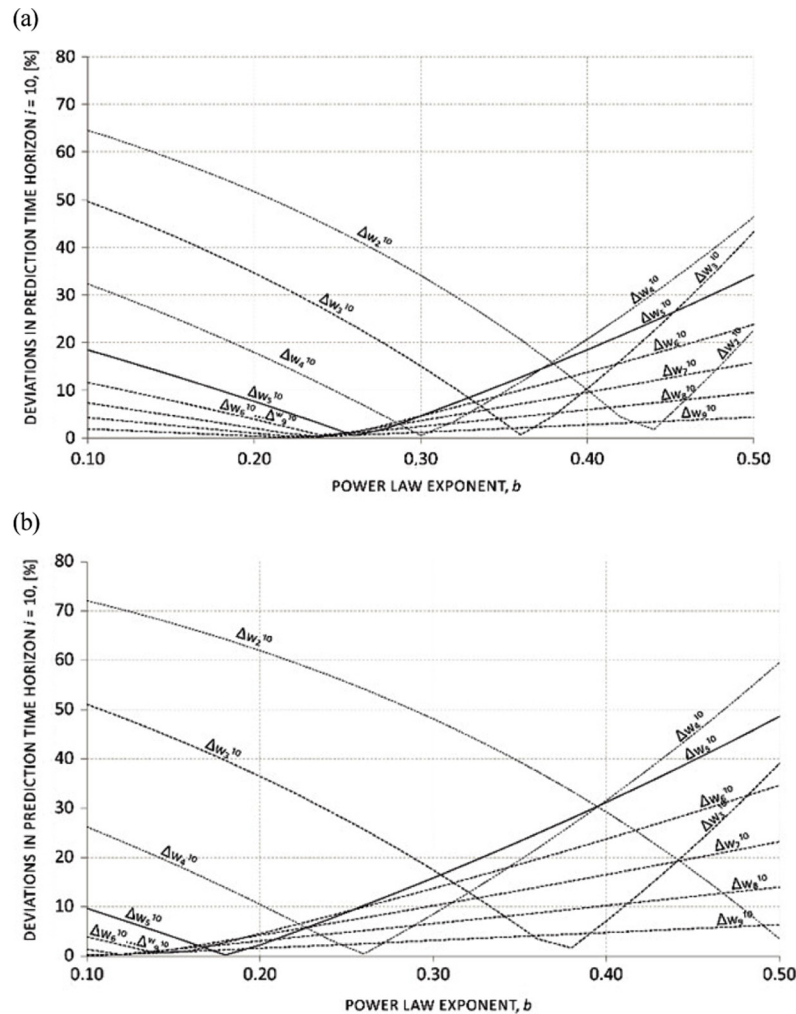


Fig. 18. Deviations of prediction from computed values Δw_k^{10} of the time periods $k = 2, \dots, 10$ in the time horizon $i = 10$, for different values of a parameter b of the power law formulation $\alpha(t) = ct^b$, for: a) shortening of the main girder, and b) vertical deflection of the main girder

of the structural creep shrinkage response and gets more conservative towards the stationary gamma power law with $b = 1$.

4.8. Validation based on vertical deflection data

The framework presented and applied to the creep and shrinkage determined long-term shortening of the main girder w can be applied analogously to the vertical deflection data. We find that the conclusions drawn with regard to e.g. the comparison between 2nd and 3rd order polynomial prediction or the stationarity of the scale parameter β also hold for this alternative creep and shrinkage process. Figures 13 and 14 present the gamma prediction processes associated with the vertical deflection of the main girder for fitting strategies IIa and IIb in combination with a second order and third order polynomial prediction approach, respectively. Figure 15 portrays the results for van Noortwijk’s approach. The comparison between all three approaches expressed in terms of gamma distribu-

tions for prediction time horizons $i = 7, \dots, 10$ based on information up to $t_7 = 6033$ days and for prediction time horizons $i = 9, 10$ based on information up to $t_9 = 9033$ days, respectively is given in Figures 16(a) and 16(b).

In order to discuss which would be the optimized value for exponent b , this time for the vertical deflection of the main girder, Figure 17 is presented. When compared with the Figure 12, it is seen that the deviations of the vertical deflection Δw_k^i are slightly larger than those of the shortening of the main girder. Nevertheless, similar conclusions are assigned, e.g. if a threshold of 30% is considered as acceptable, the deviation in $i = 10$ for predicting a displacement in $k = 3$ is 33.05%, and again, if only data up to $k = 2$ is considered the same prediction quality is reached already at $i = 3$. The main difference in these observations is that, for the axial shortening of the main girder, the values between $b = 0.12$ and $b = 0.22$ should rather be used for predictions if at least 5 observations are available, denoted Ga_5^i , see Figure 18(b).

Conclusions

The primary goal of the present study is the investigation of the suitability of the gamma process approach for the prediction of creep processes in prestressed concrete structures. In order to evaluate the applicability of the prediction models, the creep related long-term deformation of a prestressed concrete box girder bridge was simulated utilizing state of the art models and numerical techniques since no unbiased monitoring data was available.

Two fitting strategies – the van Noortwijk's approach based on statistical moments (strategy I) and individual fitting of observations (strategy II) in combination with polynomial prediction approaches of 2nd and 3rd order – are investigated. All formulations are applied to two sets of 10 observations each of the shortening of the main girder as well as the corresponding vertical deflection. Both structural response quantities are highly influenced by concrete creep and shrinkage. From these investigations, the following results become apparent:

- Time dependence of the scale parameter β has only a minor influence on the quality of the gamma process prediction and thus can be neglected.
- A 2nd order polynomial prediction approach provides a relatively good approximation of the gamma process but is unsuitable for predictions into the distant future.
- A third order polynomial prediction approach better approximates the evolution of the gamma parameters and provides good approximations.
- Alternative prediction equations with the correct asymptotic properties (ensuring e.g. a monotonous increase of the parameters α and β) might provide even better and more robust approximations.
- The 3rd order polynomial prediction shows higher flexibility than the van Noortwijk's approach, based on the comparison of predicted gamma distributions and 'true' response.
- The deviations of predictions for creep phenomenon are higher for a stationary gamma process with a power law exponent $b = 1.0$.
- The optimum parameter b of the power law exponent is not only a characteristic of deterioration phenomenon considered but it also depends on the time period.
- The best agreement between prediction and monitoring data, in general, can be achieved by a non-stationary gamma process with a power law exponent $b \approx 0.35$ for the shortening data and $b \approx 0.30$ for the deflection data.

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