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NEW DESIGN EQUATIONS FOR ELASTIC MODULUS OF CONCRETE USING MULTI EXPRESSION PROGRAMMING

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Abstract. An innovative multi expression programming (MEP) approach is used to derive new predictive equations for tangent elastic modulus of normal strength concrete (NSC) and high strength concrete (HSC). Similar to several building codes, the modulus of elasticity of NSC and HSC is formulated in terms of concrete compressive strength. Furthermore, a generic model is developed for the estimation of the elastic modulus of both NSC and HSC. Comprehensive databases are gathered from the literature to develop the models. For more verification, a parametric analysis is carried out and discussed. The proposed formulas are found to be accurate for the prediction of the elastic modulus of NSC and HSC. The predictions made by the MEP-based models are more accurate than those obtained by the existing models.

Keywords: tangent elastic modulus, normal and high strength concrete, multi expression programming, compressive strength, formulation.

Introduction

The elastic modulus of concrete is an important parameter in structural and material engineering. This parameter plays a key role in the determination of the immediate and time-dependant deformation, modular ratio and the stiffness of buildings and members. Moreover, the modulus of elasticity is widely used for the assessment of creep, shrinkage and crack control in reinforced and pre-stressed concrete (Mesbah *et al.* 2002; Khan 1995; Gandomi *et al.* 2010a). This parameter can be determined from the slope of a stress-strain curve depicting the results of tensile tests made on a sample of concrete. Figure 1 shows a typical stress-strain diagram (Gandomi *et al.* 2010a). As can be observed from this figure, the first part of the curve is almost a straight line. The initial slope of the stress-strain curve defines the initial or tangent modulus (E_c) used with the parabolic stress method. Despite its importance, the elastic modulus is not usually measured at the site. It is often estimated using empirical relationships proposed by various codes of practice. Such models formulate the elastic modulus in terms of the measured compressive strength. Consequently, there is no need to go through laborious and time-consuming direct measurements from load-deformation curve to determine the elastic modulus (ASTM 1994; Gandomi *et al.* 2010a).

Recently, pattern recognition systems have received much attention for tackling civil engineering tasks. These systems learn from experience and extract various discriminators. Artificial neural networks (ANNs), fuzzy logic (FL), Adaptive neuro fuzzy inference system (ANFIS), and support vector machine (SVM) are the well-known

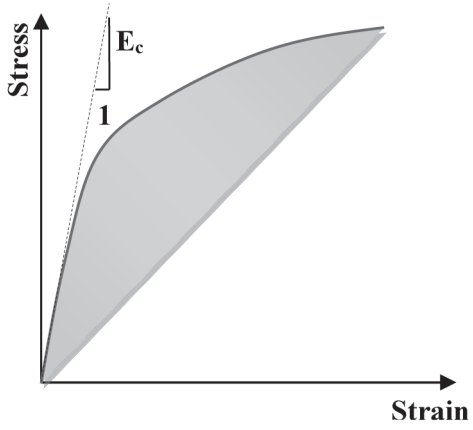

Fig. 1. Typical stress-strain diagram for concrete

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pattern recognition methods. These techniques have been utilized for the prediction of the elastic modulus of normal and high strength concrete (NSC and HSC) (Demir 2005, 2008; Yan, Shi 2010; Aydin 2006; Ahmadi-Nedushan 2012). Although ANNs, FL, ANFIS, and SVM are successful in prediction, their inability to produce prediction equations limits their use by researchers.

Genetic programming (GP) (Koza 1992) is a new alternative approach to overcome the limitations of ANNs, FL, ANFIS, and SVM. One of the main features of GP over other pattern recognition tools is its ability to generate simplified prediction equations without assuming prior form of the existing relationship (Alavi *et al.* 2011). GP is an extension to genetic algorithms in which computer programs are evolved to find solutions to problems (Banzhaf *et al.* 1998). For the last decade, GP and its variants has been pronounced as a powerful method for simulating the behavior of civil engineering problems (e.g. Gandomi *et al*. 2009, 2010b, 2012; Sadrossadat *et al.* 2012; Shahnazari *et al.* 2012). Multi expression programming (MEP) (Oltean, Dumitrescu 2002) is a recent variant of GP that uses a linear representation of chromosomes. MEP has a special ability to encode multiple computer programs of a problem in a single chromosome. The MEP approach is able to significantly outperform similar techniques (Oltean, Grosşan 2003). Some of the limited studies focused on applying MEP to the civil engineering tasks include predicting limestone compressive and tensile strength (Baykasoglu *et al.* 2008), formulation of soil classification (Alavi *et al.* 2010), ground-motion prediction (Alavi *et al.* 2011), prediction of uplift capacity of suction caissons (Gandomi *et al.* 2011), formulation of geotechnical engineering systems (Alavi, Gandomi 2011), soil liquefaction assessment (Alavi, Gandomi 2012), formulation of secant and reloading soil deformation moduli (Alavi *et al.* 2012), and modelling of concrete and steel structures (Gandomi, Alavi 2013).

MEP is capable of extracting the knowledge contained in the experimental data. Thus, it can be used to derive prediction models for the elastic modulus of concrete. The main purpose of this paper is to utilize the MEP technique to obtain mathematical relationships between the tangent elastic modulus and compressive strength of NSC and HSC. Reliable databases of previously published test results are utilized to develop the models. A comparative study is conducted between the results obtained by MEP and those obtained from the buildings codes (ACI-318-95 1996; NBS 2006; CEB-FIB 1993; BS-8110 1985; CSA-A23.3 1995; NS-3473 1992; TS-500 2000), compatibility aided (Wee *et al.* 1994; Gardner, Zhao 1993), regression (Demir 2005), FL (Demir 2005), ANN (Demir 2008), and SVM (Yan, Shi 2010) models.

1. Multi expression programming

GP creates computer programs to solve a problem by simulating the biological evolution of living organisms (Koza 1992). Generally, in GP, inputs and corresponding output data samples are known and the main goal is to find a program that connects them. Most of the genetic operators used in GA can be implemented in GP with minor changes. The main difference between GP and GA is the representation of the solution. GA creates a string of numbers that represent the solution. The classical GP solutions are computer programs represented as tree structures and expressed in a functional programming language (such as LISP) (Koza 1992; Alavi *et al*. 2011). In GP, a random population of individuals (programs) is created to achieve high diversity. A comprehensive description of GP can be found in Koza (1992) and Banzhaf *et al.* (1998). MEP is a linear variant of GP. The linear variants make a clear distinction between the genotype and the phenotype of an individual. Thus, the individuals are represented as linear strings that are decoded and expressed like nonlinear entities (trees) (Oltean, Grosşan 2003; Gandomi *et al.* 2008). MEP is another subarea of GP. It was first introduced by Oltean and Dumitrescu (2002). Linear chromosomes are used by MEP for solution encoding. This technique encodes multiple computer programs in a single chromosome. A program with the best fitness represents the chromosome. The MEP decoding process is not more complicated than other GP variants storing a single program in a chromosome (Oltean, Grosşan 2003; Alavi *et al.* 2010). The steady-state algorithm of MEP starts by the creation of a random population of computer programs. MEP uses the following steps to evolve the best program until a termination condition is reached (Oltean, Grosşan 2003; Alavi *et al*. 2010):

I. Selection of two parents using a binary tournament procedure and recombination of them with a fixed crossover probability.
II. Obtaining two offspring by the recombination of two parents.
III. Mutation of the offspring and replacement of the worst individual in the current population with the best of them (if the offspring is better than the worst individual in the current population).

The representation of the MEP solutions is similar to the procedure followed by *C* and *Pascal* to convert expressions into machine code. Functions and terminals are a part of a population member created by MEP. The terminal and function symbols are elements in the terminal and function sets, respectively. A function set can contain the basic arithmetic operations or any other mathematical functions. The terminal set can contain numerical constants, logical constants and variables (Alavi *et al.* 2010). Each gene encodes a terminal or a function symbol. The first symbol in a chromosome is a terminal symbol. An example of a MEP chromosome can be seen below. Using the set of functions $F = \{+, \times, /\}$ and the set of terminals $T = \{v_1, v_2, v_3, v_4\}$, the example is given as follows:

0: v_1
1: v_2
2: × 0, 1

3: v_3
4: + 2, 3
5: v_4
6: / 4, 5.

The translation of the MEP individuals into computer programs can be obtained by reading the chromosome top-down starting with the first position. In the present example, genes 0, 1, 3, and 5 encode simple expressions formed by a single terminal symbol. These expressions are: $E_0 = v_1$, $E_1 = v_2$, $E_3 = v_3$, $E_5 = v_4$.

Gene 2 indicates the operation × on the operands located at positions 0 and 1 of the chromosome. Therefore gene 2 encodes the expression: $E_2 = v_1 \times v_2$. Gene 4 indicates the operation + on the operands located at positions 2 and 3. Therefore, gene 4 encodes the expression: $E_4 = (v_1 \times v_2) + v_3$. Gene 6 indicates the operation / on the operands located at positions 4 and 5. Therefore gene 6 encodes the expression: $E_6 = ((v_1 \times v_2) + v_3)/ v_4$.

Each of MEP chromosomes encodes a number of expressions equal to the chromosome length (the number of genes). Due to its multi expression representation, each MEP chromosome may be viewed as a forest of trees rather than a single tree (Fig. 2). Each of these expressions can be considered as a possible solution to a problem. The fitness of each expression encoded in an MEP chromosome is defined as the fitness of the best expression encoded by that chromosome (Alavi *et al.* 2010). For solving symbolic regression problems, the fitness of an MEP chromosome may be computed by using the following formula (Oltean, Grosşan 2003; Alavi *et al.* 2010):

$$F = \min_{i=1,m}\left\{\sum_{j=1}^{n}\left|h_j - o_j^i\right|\right\}, \quad (1)$$

where: n is the number of fitness cases; h_j is the expected value for the fitness case j; o_j^i is the value returned for the j^{th} fitness case by the i^{th} expression encoded in the current chromosome; and m is the number of chromosome genes.

2. MEP-based modeling of elastic modulus of NSC and HSC

Deriving relationships between the elasticity modulus of concrete and its component characteristics has been of interest for many researchers (Larrard, Belloc 1997). The modulus of elasticity is frequently expressed as a function of the compressive strength of concrete. Most of the national and international codes use this way to express the modulus of elasticity of concrete (e.g. American Concrete Code (ACI-318-95 1996), British Concrete Code (BS-8110 1985), Canadian Concrete Code (CSA-A23.3 1995)). Thus, this study is aimed at developing explicit formulas for the tangent elastic modulus (E_c) of NSC and HSC in terms of compressive strength (f_c) as follows:

$$E_c = f\left(f_c\right). \quad (2)$$

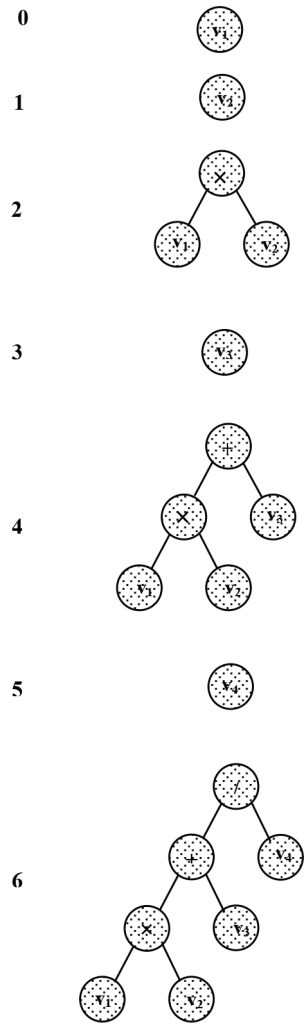

Fig. 2. Expressions encoded by an MEP chromosome represented as trees

Hence, one parameter is used for the MEP models as the input variable. The NSC and HSC databases are separately used to derive two different MEP-based formulas for the elastic modulus of each of NSC and HSC. In order to propose a generic model for both of NSC and HSC, another MEP model is developed based on the whole of available test results. Various parameters are involved in the MEP predictive algorithm. In this study, basic arithmetic operators and mathematical functions were utilized to get the optimum MEP models. The number of programs in the population that MEP will evolve is set by the population size. A run will take longer with a larger population size. The number of generation sets the number of levels the algorithm will use before the run terminates. The proper number of population and generation depends on the number of possible solutions and complexity of the problem. A large number of generations are tested to find models with minimum error. Mutation rate is the probability that an offspring will be subject to mutation. This parameter was set to 10%. Crossover rate is the probability that an offspring will be subject to crossover. At the low level

Table 1. Parameter settings for the MEP algorithm

Parameter	Setting
Function set	+, –, ×, /, exp, ln, sin, cos
Population size	500 – 2000
Chromosome length	20–50 genes
Number of generations	1000
Number of tournaments	4
Crossover probability (%)	50, 90
Crossover type	Uniform
Mutation probability (%)	10
Terminal set	Problem input

the crossover rate is 50% and at the high level it is 95%. The values of the other involved parameters are selected based on some previously suggested values (Baykasoglu *et al.* 2008; Alavi *et al.* 2010) and also after a trial and error approach. The parameter settings are shown in Table 1. For the analysis, source code of MEP (Oltean 2004) in C++ is modified by the authors to be utilizable for the available problem.

The best MEP models are chosen on the basis of a multi-objective strategy as below (Gandomi *et al.* 2010a):

1. The simplicity of the model, although this is not a predominant factor.
2. Providing the best fitness value on the training set of data.
3. Providing the best fitness value on a test set of unseen data.

The first objective can be controlled by the user through the parameter settings (e.g. chromosome length). For the other objectives, the following objective function (Obj) is considered as a measure of how well the model predicted output agrees with the experimentally measured output. The selections of the best MEP models are deduced by the minimization of the following function (Gandomi *et al.* 2010a):

$$Obj = \left(\frac{No._{Train} - No._{Test}}{No._{All}}\right)\frac{MAE_{Train}}{R^2_{Train}} + \frac{2No._{Test}}{No._{All}}\frac{MAE_{Test}}{R^2_{Test}}, \quad (3)$$

where: $No._{Train}$, $No._{Test}$ and $No._{All}$ are, respectively, the number of training, testing and whole of data; R and MAE are, respectively, correlation coefficient and mean absolute error given in the form of formulas as follows:

$$R = \frac{\Sigma_{i=1}^{n}\left(h_i - \overline{h}_i\right)\left(t_i - \overline{t}_i\right)}{\sqrt{\Sigma_{i=1}^{n}\left(h_i - \overline{h}_i\right)^2 \Sigma_{i=1}^{n}\left(t_i - \overline{t}_i\right)^2}}; \quad (4)$$

$$MAE = \frac{\sum_{i=1}^{n}\left|h_i - t_i\right|}{n}, \quad (5)$$

in which h_i and t_i are, respectively, actual and predicted outputs for the i^{th} output; $\overline{h}_i$ and $\overline{t}_i$ are the average of the actual and predicted outputs, respectively; n is the number of sample. The constructed objective function takes into account the changes of R and MAE together. Higher R values and lower MAE values result in lowering Obj and, consequently, indicate a more precise model. In addition, the above function considers the effects of different data divisions for the training and testing data.

2.1. Experimental database

An experimental database of the previously published test results (Wee *et al.* 1994; Shannag 2000; Turan, Iren 1997; Ozturan 1984; Gesoglu 2002) is utilized to develop the MEP based models. This database has been previously employed by Demir (2005, 2008) and Yan, Shi (2010) to develop the FL, ANN and SVM models, respectively. This database has been already been used by Gandomi *et al.* (2010a) to develop linear genetic programming models. The database contains 70 and 89 test results for the elastic modulus of NSC and HSC, respectively. The concrete specimens are tested at the age of 28 days. NSC and HSC are considered as types of concrete having compressive strength lower and higher than 50 MPa, respectively (Mehta, Monteiro 2005). It should be noted that one of the data sets in the HSC database has a compressive strength lower than 50 MPa, which is mistakenly considered in the development of the other existing models such as regression (Demir 2005), FL (Demir 2005), ANN (Demir 2008), and SVM (Yan, Shi 2010) . In the current study, this data set is also included in the HSC database in order to conduct a fair comparison between the predictions provided by MEP and other existing models. However, in the present study, a general model was further proposed for both of NSC and HSC using the entire data. Descriptive statistics of the variables included in the analysis are shown in Figure 3. To visualize the samples distribution, the data are presented by frequency histograms (Fig. 4).

For the analysis, the data sets are divided into the training and testing subsets. Out of the 89 data sets for HSC, approximately 78% of the data (69 values) are taken for the training of the MEP algorithm and the remaining 22% (20 values) are used to test the generalization capability of the models. For NSC, approximately 80% of the data (57 values) are taken to train and the remaining 20% (13 values) are used to test the models. Out of the total 159 data sets for NSC and HSC, almost 80% of the data (126 values) are taken for the training of the MEP algorithm and the remaining 20% (33 values) are used for the testing of the proposed NSC and HSC generic model.

2.2. Explicit formula for the elastic modulus of NSC

The optimal formulation of the E_c of NSC in terms of f_c is as given below:

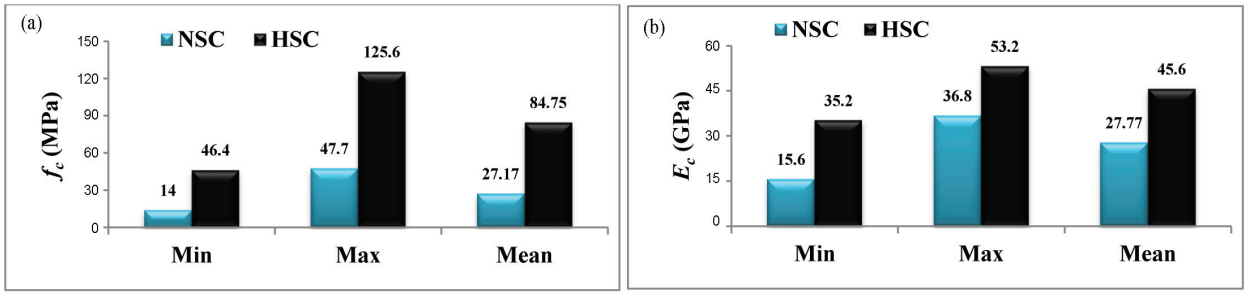

Fig. 3. Descriptive statistics of the variables

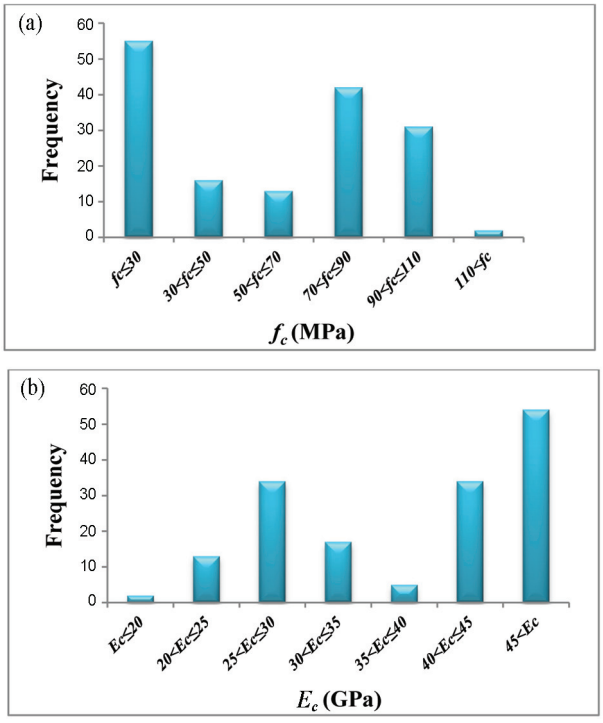

Fig. 4. The histograms of: a) compressive strength; and b) elastic modulus for all data

$$E_{c,NSC} = 19 + \frac{f_c}{3} - \sin\left(\left(f_c + 70\right)\left(f_c - 10\right)\right). \quad (6)$$

Figure 5 shows a comparison between the predicted and experimental E_c values for NSC. In Figure 5, residual is the difference between the experimental value and the value predicted by the MEP model. The proposed model for the E_c of NSC yields an Obj value equal to 6.542.

2.3. Explicit formula for the elastic modulus of HSC

The optimal formulation of the E_c of HSC in terms of f_c is as follows:

$$E_{c,HSC} = -9 + \frac{f_c}{9} + 6\left(8 - \frac{36}{f_c}\right) + \cos\left(\frac{f_c}{-3}\right). \quad (7)$$

Figure 6 presents a comparison between the predicted and experimental E_c values for HSC. As can be observed in this figure, the proposed model for E_c of HSC yields a low Obj value equal to 3.976.

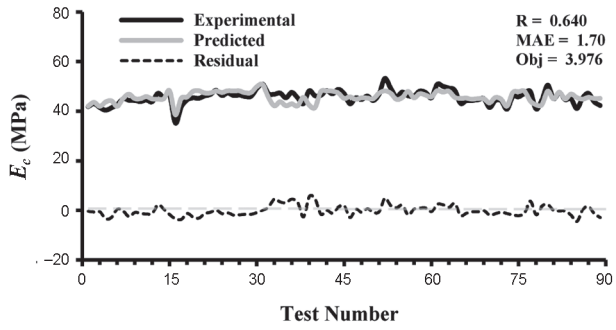

Fig. 5. Predicted versus experimental E_c of HSC using the MEP model

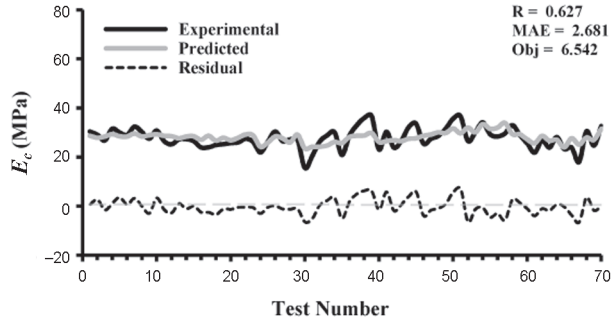

Fig. 6. Predicted versus experimental E_c of NSC using the MEP model

2.4. Explicit formula for the elastic modulus of NSC and HSC

The best prediction model for the E_c of NSC and HSC in terms of f_c is as given below:

$$E_{c,NSC-HSC} = \frac{-96}{f_c^2} + \frac{9\left(f_c + 9\right)}{f_c/8 + 8}. \quad (8)$$

A comparison of the MEP predicted values against experimental E_c of NSC and HSC is shown in Figure 7. The proposed generic model yields a low Obj value equal to 2.428.

3. Performance analysis

Figures 8 and 9 illustrate the prediction performance of the MEP models, American (ACI-318-95 1996), Iranian (NBS 2006), European (CEB-FIB 1993), British (BS-8110 1985), Canadian (CSA-A23.3 1995), Norwegian

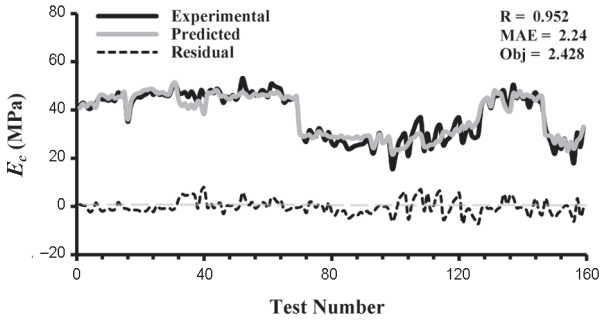

Fig. 7. Predicted versus experimental E_c of HSC and NSC using the MEP model

(NS-3473 1992), and Turkish (TS-500 2000) codes, two compatibility aided models (Wee *et al.* 1994; Gardner, Zhao 1993), regression (Demir 2005), FL (Demir 2005), ANN (Demir 2008), and SVM (Yan, Shi 2010) models for the E_c of NSC and HSC, respectively. Moreover, the predictions made by different models for the NSC and HSC databases are presented in Tables 2 and 3, respectively. It can be seen from Figures 8 and 9 and Tables 2 and 3 that the proposed MEP models provide a significantly better performance than the available codes and models. The only exception is the SVM model for NSC

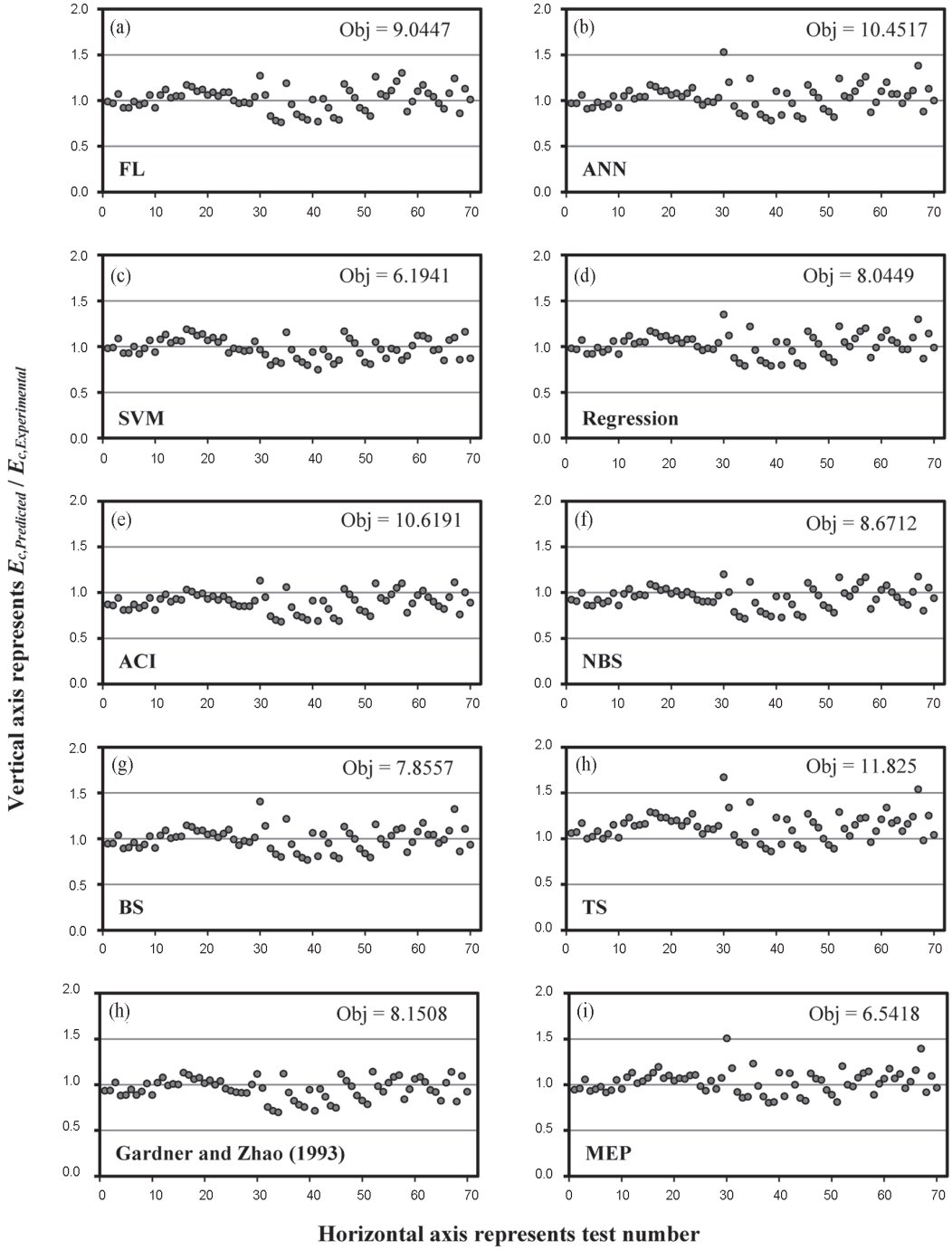

Fig. 8. A comparison of the ratio between the predicted and experimental E_c of NSC using different models

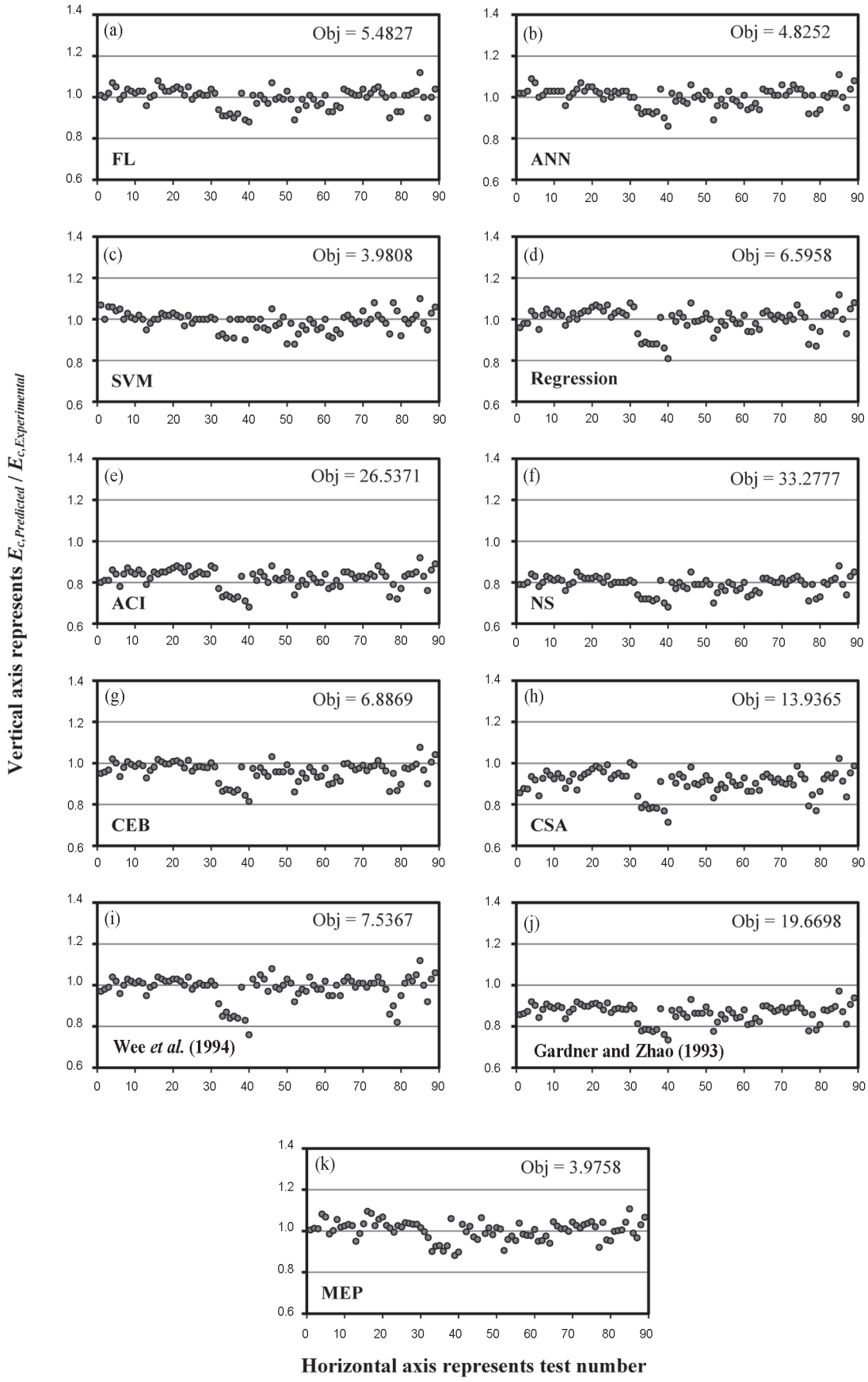

Fig. 9. A comparison of the ratio between the predicted and experimental E_c of HSC using different models

Table 2. Predictions made by different models for the E_c of NSC

Test No.	f_c (MPa)	E_c (GPa)	$E_{c,FL}/E_c$	$E_{c,ANN}/E_c$	$E_{c,SVM}/E_c$	$E_{c,Regression}/E_c$	$E_{c,ACI}/E_c$	$E_{c,NBS}/E_c$	$E_{c,BS}/E_c$	$E_{c,TS}/E_c$	$E_{c,Gardner\ and\ Zhao}/E_c$	$E_{c,MEP}/E_c$
1	31.4	30.4	0.99	0.97	0.98	0.98	0.87	0.92	0.94	1.06	0.93	0.94
2	27.8	29.1	0.97	0.97	0.99	0.97	0.86	0.91	0.95	1.07	0.94	0.96
3	28.5	26.8	1.07	1.06	1.09	1.07	0.94	1.00	1.04	1.17	1.03	1.06
4	29.4	31.5	0.92	0.91	0.93	0.92	0.81	0.86	0.89	1	0.88	0.93
5	26.4	30	0.92	0.92	0.93	0.92	0.81	0.86	0.90	1.02	0.89	0.95
6	28.5	29	0.99	0.98	1.00	0.99	0.87	0.92	0.96	1.08	0.95	0.98
7	32.6	32.4	0.95	0.93	0.92	0.94	0.83	0.88	0.90	1	0.89	0.91
8	29.9	30.2	0.97	0.96	0.98	0.97	0.86	0.91	0.94	1.05	0.92	0.94
9	29.8	27.5	1.06	1.05	1.07	1.06	0.94	0.99	1.03	1.15	1.01	1.05
10	28	30.8	0.92	0.92	0.94	0.92	0.81	0.86	0.90	1.01	0.89	0.95
11	27.3	26.5	1.06	1.05	1.08	1.06	0.93	0.99	1.03	1.17	1.02	1.08
12	27.5	25.2	1.12	1.11	1.13	1.12	0.98	1.04	1.09	1.23	1.08	1.13
13	27	27.2	1.03	1.02	1.04	1.03	0.9	0.96	1.00	1.14	0.99	1.02
14	28.5	27.3	1.05	1.04	1.07	1.05	0.93	0.98	1.02	1.15	1.01	1.04
15	26.4	26.5	1.05	1.04	1.06	1.05	0.92	0.97	1.02	1.16	1.00	1.07
16	27.1	23.9	1.17	1.17	1.19	1.17	1.03	1.09	1.14	1.29	1.13	1.13
17	26.3	24	1.15	1.15	1.17	1.15	1.01	1.07	1.13	1.28	1.11	1.19
18	26.1	24.9	1.10	1.10	1.12	1.11	0.97	1.03	1.08	1.23	1.06	1.07
19	27.8	25.3	1.12	1.11	1.14	1.12	0.99	1.04	1.09	1.23	1.08	1.10
20	25.7	25.7	1.06	1.06	1.07	1.07	0.93	0.99	1.04	1.19	1.02	1.04
21	27.8	26	1.09	1.08	1.10	1.09	0.96	1.01	1.06	1.2	1.05	1.07
22	28.6	27.5	1.05	1.04	1.05	1.04	0.92	0.97	1.01	1.14	1.00	1.06
23	27.9	26.2	1.09	1.08	1.10	1.08	0.96	1.01	1.05	1.19	1.04	1.10
24	18.4	21.9	1.09	1.14	0.93	1.08	0.92	0.98	1.10	1.27	0.95	1.11
25	23.4	26.3	1.00	1.01	0.98	1.00	0.87	0.92	0.99	1.13	0.93	0.98
26	29.9	30.4	0.97	0.95	0.97	0.96	0.85	0.90	0.93	1.05	0.92	0.93
27	22.9	26.5	0.98	0.99	0.95	0.98	0.85	0.90	0.98	1.11	0.91	1.04
28	23.7	27.2	0.97	0.98	0.96	0.97	0.85	0.89	0.96	1.1	0.91	0.95
29	27.4	27.1	1.04	1.03	1.06	1.04	0.91	0.97	1.01	1.14	1.00	1.07
30	14	15.6	1.27	1.53	0.97	1.35	1.13	1.20	1.41	1.67	1.12	1.51
31	16.9	20.5	1.06	1.20	0.91	1.12	0.95	1.00	1.14	1.34	0.96	1.18
32	17.1	26.3	0.83	0.94	0.80	0.88	0.74	0.79	0.89	1.04	0.76	0.92
33	18	28.8	0.78	0.86	0.84	0.82	0.7	0.74	0.83	0.96	0.72	0.86
34	18.5	30.1	0.76	0.83	0.82	0.79	0.68	0.71	0.80	0.93	0.70	0.87
35	21.8	20.9	1.19	1.24	1.16	1.22	1.06	1.12	1.22	1.4	1.12	1.23
36	25.8	28.6	0.96	0.96	0.97	0.96	0.84	0.89	0.94	1.07	0.92	0.98
37	27.3	32.9	0.85	0.85	0.87	0.85	0.75	0.79	0.83	0.94	0.82	0.87
38	30.3	35.9	0.82	0.81	0.83	0.82	0.73	0.77	0.79	0.89	0.78	0.80
39	29.6	36.8	0.79	0.78	0.80	0.79	0.7	0.74	0.76	0.86	0.76	0.81
40	19.6	23.1	1.01	1.10	0.94	1.05	0.91	0.96	1.06	1.23	0.94	1.13
41	19.4	30.3	0.77	0.84	0.75	0.80	0.69	0.73	0.81	0.94	0.71	0.87
42	20.9	23.9	1.02	1.08	0.97	1.05	0.91	0.96	1.05	1.21	0.95	1.13
43	21.2	26.5	0.92	0.97	0.89	0.95	0.82	0.87	0.95	1.09	0.87	1.00

Continued Table 2

Test No.	f_c (MPa)	E_c (GPa)	$E_{c,FL}/E_c$	$E_{c,ANN}/E_c$	$E_{c,SVM}/E_c$	$E_{c,Regression}/E_c$	$E_{c,ACI}/E_c$	$E_{c,NBS}/E_c$	$E_{c,BS}/E_c$	$E_{c,TS}/E_c$	$E_{c,Gardner\ and\ Zhao}/E_c$	$E_{c,MEP}/E_c$
44	23.6	32.1	0.81	0.83	0.81	0.82	0.72	0.76	0.81	0.93	0.77	0.85
45	24.2	33.6	0.79	0.80	0.85	0.79	0.69	0.73	0.78	0.89	0.75	0.82
46	31.8	25.5	1.18	1.17	1.17	1.17	1.04	1.11	1.13	1.27	1.12	1.12
47	32.2	27.4	1.11	1.09	1.09	1.10	0.98	1.04	1.06	1.18	1.04	1.06
48	30.6	28.6	1.03	1.03	1.04	1.03	0.92	0.97	1.00	1.12	0.98	1.05
49	29.6	31.6	0.92	0.91	0.93	0.92	0.81	0.86	0.89	1	0.88	0.94
50	35	35.6	0.89	0.88	0.83	0.88	0.79	0.83	0.84	0.93	0.83	0.89
51	32.8	36.7	0.83	0.82	0.81	0.83	0.74	0.78	0.79	0.89	0.78	0.81
52	38.4	26.6	1.26	1.24	1.05	1.22	1.1	1.16	1.15	1.29	1.14	1.20
53	35.7	30.1	1.07	1.05	0.97	1.05	0.94	0.99	1.00	1.11	0.98	1.00
54	42.7	34.1	1.05	1.03	0.87	1.00	0.91	0.96	0.93	1.03	0.92	0.98
55	36.8	29.3	1.11	1.10	0.98	1.09	0.98	1.04	1.03	1.15	1.02	1.08
56	40.1	28.4	1.21	1.19	0.96	1.17	1.05	1.11	1.10	1.22	1.08	1.13
57	47.7	29.6	1.30	1.26	0.85	1.20	1.1	1.17	1.11	1.23	1.10	1.15
58	29.4	33	0.88	0.87	0.90	0.88	0.78	0.82	0.85	0.96	0.84	0.89
59	28.8	29	0.99	0.98	1.01	0.99	0.88	0.93	0.96	1.08	0.95	1.01
60	27.7	25.6	1.10	1.10	1.12	1.10	0.97	1.03	1.08	1.21	1.06	1.06
61	22.1	21.8	1.17	1.20	1.12	1.18	1.02	1.08	1.17	1.34	1.08	1.17
62	28.9	26.8	1.08	1.07	1.09	1.07	0.95	1.00	1.04	1.17	1.03	1.07
63	20.6	23.9	1.04	1.07	0.96	1.04	0.9	0.95	1.04	1.2	0.94	1.12
64	25.3	28.1	0.97	0.97	0.97	0.97	0.85	0.90	0.95	1.08	0.92	0.96
65	16.2	23.3	0.91	1.05	0.85	0.97	0.82	0.86	0.99	1.16	0.82	1.03
66	23.2	23.9	1.08	1.11	1.07	1.10	0.95	1.01	1.09	1.24	1.02	1.16
67	17.9	18	1.24	1.38	1.10	1.30	1.11	1.18	1.32	1.54	1.14	1.39
68	23.9	30.5	0.86	0.88	0.86	0.87	0.76	0.80	0.86	0.98	0.82	0.92
69	27.1	24.7	1.13	1.13	1.16	1.14	1	1.05	1.11	1.25	1.09	1.09
70	37.5	32.6	1.01	1.00	0.87	0.99	0.89	0.94	0.93	1.04	0.92	0.97

which provides slightly better results than the MEP model. As shown in Figure 7, the proposed MEP model for both of NSC and HSC yields the best (lowest) Obj value on the entire database. The superior performance of the generic model implies the reasonability of developing comprehensive models for the E_c of both NSC and HSC rather than developing separate models for each of them.

Besides, Figure 10 shows the ratios of the experimental E_c values of NSC and HSC to the values predicted by the MEP solutions with respect to f_c. As the scattering increases in these figures, the model accuracy will consequently decrease. It can be observed from these figures that the predictions obtained by the proposed models have a very good accuracy with no significant trend with respect to the design parameters. In cases of the model for NSC and generic model for both NSC and HSC (Fig. 10(a) and (c)), the scattering slightly decreases with increasing f_c.

Although the ANN, FL and SVM models have a good performance, they do not give a certain function to calculate the outcome. ANN has only final synaptic weights to obtain outcome in parallel manner. Determination of the fuzzy rules in FL is also a difficult task (Yan, Shi 2010). The major limitation of SVM, which is not entirely solved, lies in the choice of the kernel for a given problem. Other serious problems with SVMs are the high algorithmic complexity and low speed of algorithm training process (Burges 1998; Platt 1999). More, the ANN, FL and SVM approaches are appropriate to be used as a part of a computer program and are not suitable for practical calculations. On the other hand, empirical modelling based on statistical techniques has significant limitations. The regression analyses can have large uncertainties. In regression analyses, the nature of corresponding problem is modelled by a limited number of pre-defined equations, either linear or nonlinear. Contrary to the existing modelling techniques, MEP introduces

Table 3. Predictions made by different models for the E_c of HSC

Test No.	f_c (MPa)	E_c (GPa)	$E_{c,FL}/E_c$	$E_{c,ANN}/E_c$	$E_{c,SVM}/E_c$	$E_{c,Regression}/E_c$	$E_{c,ACI}/E_c$	$E_{c,CEB}/E_c$	$E_{c,NS}/E_c$	$E_{c,CSA}/E_c$	$E_{c,Wee\ et\ al.}/E_c$	$E_{c,Gardner\ and\ Zhao}/E_c$	$E_{c,MEP}/E_c$
1	63.2	41.8	1.01	1.02	1.07	0.96	0.80	0.95	0.79	0.86	0.97	0.86	1.00
2	70.2	43	1.00	1.02	1.00	0.98	0.81	0.96	0.79	0.88	0.98	0.86	1.01
3	65.1	41.5	1.02	1.03	1.06	0.98	0.81	0.97	0.80	0.87	0.99	0.87	1.01
4	70.5	40.4	1.07	1.09	1.06	1.04	0.86	1.02	0.84	0.94	1.04	0.92	1.08
5	71.5	41.4	1.05	1.07	1.04	1.02	0.84	1.00	0.83	0.92	1.02	0.90	1.07
6	63.6	42.6	0.99	1.00	1.05	0.95	0.78	0.94	0.78	0.84	0.96	0.84	0.99
7	85.9	45	1.01	1.01	1.00	1.02	0.84	0.98	0.80	0.93	1.00	0.88	1.00
8	90.2	44.4	1.04	1.03	1.03	1.05	0.87	1.01	0.83	0.96	1.03	0.91	1.06
9	85.9	44.3	1.03	1.03	1.01	1.03	0.85	0.99	0.82	0.94	1.02	0.90	1.02
10	81.2	43.9	1.02	1.03	1.00	1.02	0.84	0.98	0.81	0.92	1.01	0.89	1.03
11	88.1	44.5	1.03	1.03	1.02	1.04	0.86	1.00	0.82	0.95	1.02	0.90	1.03
12	81.6	43.8	1.03	1.03	1.00	1.02	0.84	0.99	0.81	0.93	1.01	0.89	1.03
13	84.8	47.2	0.96	0.96	0.95	0.97	0.79	0.93	0.76	0.88	0.95	0.84	0.95
14	85.6	45.6	1.00	1.00	0.98	1.00	0.82	0.96	0.79	0.91	0.99	0.87	0.99
15	96.2	46.6	1.01	1.02	1.00	1.03	0.85	0.98	0.80	0.95	1.00	0.88	1.04
16	46.4	35.2	1.08	1.04	1.00	1.00	0.84	1.02	0.85	0.87	1.04	0.92	1.09
17	73.9	41.6	1.05	1.07	1.03	1.03	0.85	1.01	0.83	0.93	1.03	0.91	1.09
18	87.6	44.5	1.03	1.03	1.02	1.04	0.85	1.00	0.82	0.95	1.02	0.90	1.03
19	93.1	45.4	1.03	1.05	1.02	1.04	0.86	1.00	0.82	0.96	1.02	0.90	1.06
20	95.3	45.2	1.04	1.05	1.03	1.06	0.87	1.01	0.82	0.97	1.03	0.91	1.07
21	102.1	46.1	1.05	1.03	1.02	1.07	0.88	1.01	0.83	0.99	1.03	0.91	1.03
22	102.8	46.7	1.04	1.02	1.01	1.06	0.87	1.00	0.82	0.98	1.02	0.90	1.01
23	106.3	48.4	1.01	0.99	0.97	1.04	0.85	0.98	0.80	0.96	1.00	0.88	0.99
24	104.2	46.3	1.05	1.03	1.02	1.07	0.88	1.01	0.83	0.99	1.04	0.91	1.03
25	94.6	47.3	0.99	1.00	0.98	1.01	0.83	0.96	0.79	0.93	0.98	0.87	1.02
26	94	46.3	1.01	1.03	1.00	1.03	0.84	0.98	0.80	0.94	1.00	0.88	1.04
27	96.6	46.5	1.02	1.02	1.00	1.04	0.85	0.98	0.80	0.95	1.01	0.89	1.04
28	91.5	45.9	1.01	1.03	1.00	1.03	0.84	0.98	0.80	0.94	1.00	0.88	1.03
29	91.7	46	1.01	1.03	1.00	1.02	0.84	0.98	0.80	0.94	1.00	0.88	1.03
30	119.9	49.1	1.04	1.00	1.01	1.08	0.88	1.00	0.81	1.00	1.02	0.90	1.02
31	125.6	50.9	1.02	1.00	1.00	1.06	0.87	0.98	0.80	0.99	1.00	0.89	1.00
32	77.2	47.1	0.94	0.95	0.92	0.93	0.77	0.90	0.74	0.84	0.91	0.81	0.97
33	66.5	46.8	0.91	0.92	0.93	0.88	0.73	0.86	0.72	0.78	0.85	0.78	0.90
34	70.7	47.3	0.91	0.93	0.91	0.89	0.74	0.87	0.72	0.80	0.87	0.79	0.93
35	61.8	45.4	0.92	0.93	1.00	0.88	0.73	0.87	0.72	0.78	0.84	0.78	0.93
36	68.9	47.6	0.90	0.92	0.91	0.88	0.72	0.86	0.71	0.78	0.85	0.78	0.90
37	62.2	45.4	0.92	0.93	1.00	0.88	0.73	0.87	0.72	0.78	0.84	0.79	0.93
38	75.8	43	1.02	1.04	1.00	1.01	0.83	0.98	0.81	0.91	0.99	0.89	1.06
39	67.7	48.2	0.89	0.90	0.90	0.86	0.71	0.84	0.70	0.77	0.83	0.76	0.88
40	53.6	46.2	0.88	0.86	1.00	0.81	0.68	0.81	0.68	0.71	0.76	0.73	0.90
41	92.9	46.4	1.01	1.02	1.00	1.02	0.84	0.97	0.80	0.93	1.03	0.88	1.03
42	94	48.3	0.97	0.98	0.96	0.99	0.81	0.94	0.77	0.90	1.00	0.85	1.00
43	97.7	47	1.01	1.01	1.00	1.03	0.85	0.98	0.80	0.95	1.05	0.88	1.02
44	102	48.8	0.99	0.98	0.96	1.01	0.83	0.96	0.78	0.93	1.03	0.86	0.97

Continued Table 3

Test No.	f_c (MPa)	E_c (GPa)	$E_{c,FL}/E_c$	$E_{c,ANN}/E_c$	$E_{c,SVM}/E_c$	$E_{c,Regression}/E_c$	$E_{c,ACI}/E_c$	$E_{c,CEB}/E_c$	$E_{c,NS}/E_c$	$E_{c,CSA}/E_c$	$E_{c,Wee\ et\ al.}/E_c$	$E_{c,Gardner\ and\ Zhao}/E_c$	$E_{c,MEP}/E_c$
45	86.2	47.1	0.97	0.97	0.95	0.97	0.80	0.94	0.77	0.89	0.97	0.84	0.96
46	87.9	43	1.07	1.06	1.05	1.08	0.88	1.03	0.85	0.98	1.08	0.93	1.06
47	82.7	45.4	0.99	1.00	0.97	0.99	0.82	0.96	0.79	0.90	0.99	0.86	0.99
48	79.1	44.7	1.00	1.01	0.98	0.99	0.81	0.96	0.79	0.90	0.98	0.86	1.02
49	86.6	46.1	0.99	0.99	1.01	1.00	0.82	0.96	0.79	0.91	1.00	0.86	0.98
50	85.5	44.3	1.03	1.03	0.88	1.03	0.85	0.99	0.81	0.94	1.03	0.90	1.02
51	91.1	46.8	0.99	1.01	0.98	1.00	0.82	0.96	0.79	0.92	1.01	0.87	1.01
52	96.7	53.2	0.89	0.89	0.88	0.91	0.74	0.86	0.70	0.83	0.92	0.78	0.91
53	91.2	49.3	0.94	0.96	0.93	0.95	0.78	0.91	0.75	0.87	0.96	0.82	0.96
54	83.8	45.9	0.99	0.99	0.97	0.99	0.81	0.95	0.78	0.90	0.98	0.86	0.98
55	87.1	47.7	0.96	0.96	0.95	0.97	0.79	0.93	0.76	0.88	0.97	0.84	0.95
56	93.2	46.2	1.01	1.03	1.00	1.03	0.84	0.98	0.80	0.94	1.04	0.88	1.04
57	86.9	46.1	0.99	0.99	0.98	1.00	0.82	0.96	0.79	0.91	1.00	0.86	0.98
58	90.7	48.1	0.96	0.98	0.95	0.98	0.80	0.93	0.76	0.89	0.98	0.84	0.98
59	89.5	47.6	0.97	0.96	0.96	0.98	0.80	0.94	0.77	0.89	0.98	0.85	0.98
60	87.8	45.4	1.01	1.01	1.00	1.02	0.84	0.98	0.80	0.93	1.02	0.88	1.01
61	95.2	50.8	0.93	0.94	0.92	0.94	0.77	0.90	0.73	0.86	0.95	0.81	0.95
62	92.2	50	0.93	0.95	0.91	0.94	0.78	0.90	0.74	0.86	0.95	0.81	0.95
63	97.6	49.3	0.96	0.97	0.95	0.98	0.81	0.93	0.76	0.90	1.00	0.84	0.98
64	87.5	48.5	0.95	0.94	0.93	0.95	0.78	0.91	0.75	0.87	0.95	0.82	0.94
65	80.4	43.2	1.04	1.04	1.01	1.03	0.85	1.00	0.82	0.93	1.02	0.90	1.05
66	86.5	44.2	1.03	1.03	1.02	1.04	0.85	1.00	0.82	0.95	1.04	0.90	1.02
67	83.9	44.3	1.02	1.03	1.00	1.02	0.84	0.99	0.81	0.93	1.02	0.89	1.01
68	80.9	44.6	1.01	1.01	0.98	1.00	0.82	0.97	0.80	0.91	0.99	0.87	1.01
69	85.7	45.1	1.01	1.01	0.99	1.02	0.83	0.98	0.80	0.92	1.01	0.88	1.00
70	69.7	41.5	1.04	1.06	1.04	1.01	0.83	0.99	0.82	0.91	1.01	0.89	1.04
71	78.3	44.3	1.00	1.01	0.98	0.99	0.82	0.96	0.79	0.90	0.99	0.87	1.03
72	82.6	44.2	1.02	1.03	1.00	1.02	0.84	0.98	0.81	0.93	1.01	0.89	1.01
73	65.8	40.8	1.04	1.06	1.08	1.00	0.83	0.99	0.82	0.89	1.01	0.89	1.03
74	100.6	45.8	1.05	1.04	1.02	1.07	0.88	1.01	0.83	0.99	1.04	0.91	1.04
75	92.8	45.8	1.02	1.04	1.00	1.03	0.85	0.99	0.81	0.95	1.01	0.89	1.05
76	93.6	47.1	1.00	1.01	0.98	1.01	0.83	0.96	0.79	0.92	0.98	0.87	1.02
77	71.5	48	0.90	0.92	0.93	0.88	0.73	0.86	0.71	0.79	0.86	0.78	0.92
78	59.1	40.9	1.01	1.01	1.08	0.96	0.79	0.95	0.79	0.85	0.90	0.86	1.04
79	57.9	44.5	0.93	0.92	1.04	0.87	0.72	0.87	0.72	0.77	0.82	0.78	0.96
80	93.7	50.5	0.93	0.94	0.92	0.94	0.77	0.90	0.73	0.86	0.95	0.81	0.95
81	85.3	45	1.01	1.01	1.00	1.02	0.83	0.98	0.80	0.92	1.01	0.88	1.00
82	99.7	47.6	1.01	1.00	0.98	1.03	0.84	0.97	0.79	0.94	1.04	0.88	1.00
83	85.1	44.7	1.02	1.02	1.00	1.02	0.84	0.98	0.81	0.93	1.02	0.89	1.00
84	90.3	45	1.03	1.02	1.02	1.04	0.85	0.99	0.82	0.95	1.05	0.90	1.04
85	87.2	41.1	1.12	1.11	1.10	1.12	0.92	1.08	0.88	1.02	1.12	0.97	1.11
86	84.5	45.3	1.00	1.00	0.98	1.00	0.83	0.97	0.79	0.91	1.00	0.87	0.99
87	77	47.2	0.90	0.95	0.95	0.93	0.76	0.90	0.74	0.84	0.92	0.81	0.97
88	86	43.8	1.00	1.04	1.03	1.05	0.86	1.01	0.83	0.95	1.03	0.91	1.03
89	86	42.3	1.04	1.08	1.06	1.08	0.89	1.04	0.85	0.99	1.06	0.94	1.07

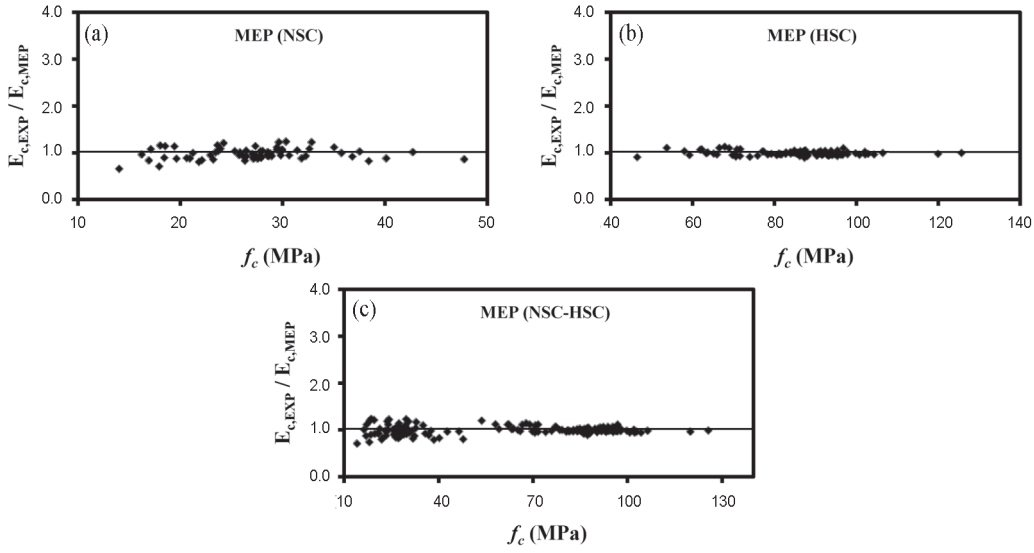

Fig. 10. The ratio between the predicted and experimental E_c values of NSC and HSC with respect to f_c

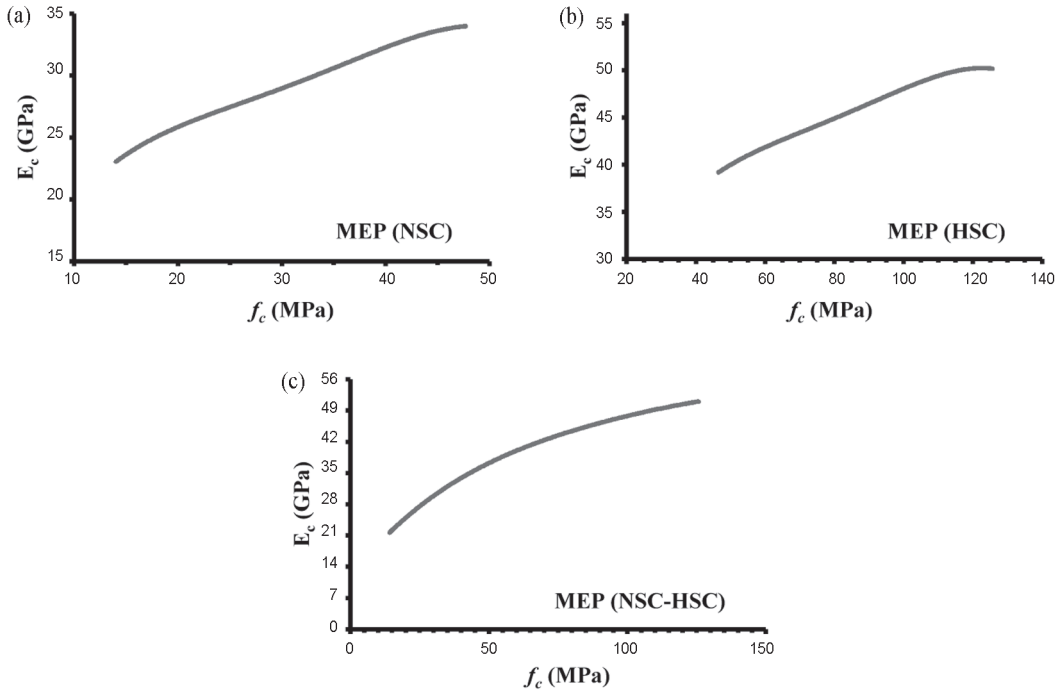

Fig. 11. Parametric analysis of the E_c of NSC and HSC

completely new characteristics. One of the major advantages of the MEP approach over other pattern recognition methods and traditional statistical analysis is its ability to derive explicit relationships for the E_c of NSC and HSC without assuming prior forms of the existing relationships. The best equations evolved by this technique are determined after controlling numerous preliminary models, even millions of linear and nonlinear models.

4. Parametric analysis

For further verification of the MEP models, a parametric analysis is performed in this study. The parametric analysis investigates the response of the predicted E_c by the MEP models to a set of hypothetical input data. The robustness of the design equations is determined by examining how well the predicted E_c values agree with the underlying physical behaviour of NSC and HSC (Kuo *et al.* 2009).

Figure 11 presents the tendency of the predictions to the f_c variations. The results indicate that the E_c of NSC and HSC continuously increases due to increasing f_c. The parametric analysis results are expected cases from a structural engineering viewpoint. The results confirm that the proposed design equations are robust and can confidently be used.

Conclusions

In this research, a new variant of GP, namely MEP is utilized to formulate the tangent E_c of NSC and HSC. Three design formulas are obtained for the prediction of E_c. The proposed models are developed upon several test results obtained from the literature. The MEP models provide reliable estimations of the E_c of NSC and HSC and outperform the existing models nearly in all of the cases. The generic MEP model provides significantly accurate determinations of the E_c of both NSC and HSC. In addition to the acceptable accuracy, the MEP-based prediction equations are very simple. The robustness of the proposed MEP models is confirmed with the results of the parametric study. With the use of the MEP approach, E_c can be estimated without carrying out sophisticated and time-consuming laboratory tests. The models can be easily retrained and improved to make more accurate predictions for a wider range by including the data for other test conditions.

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