TRANSPORT
ISSN 1648-4142 / eISSN 1648-3480
Special Issue on Travel Demand Management / 2014 Volume 29(3): 248–259
doi:10.3846/16484142.2014.914970

COMPARISON AND OPTIMIZATION OF CORDON AND AREA PRICINGS FOR MANAGING TRAVEL DEMAND

Lihui Zhang[1], Huiyuan Liu[2], Daniel (Jian) Sun[3]

[1]*Institute of Transportation Engineering, Zhejiang University, China*
[2]*School of Transportation and Logistics, Dalian University of Technology, China*
[3]*Dept of Transportation Engineering, Shanghai Jiaotong University, China*

Submitted 3 November 2013; accepted 31 March 2014; first published online 28 May 2014

Abstract. This paper analyses both the cordon and area pricings from the perspective of travel demand management. Sensitivity analysis of various performance measures with respect to the toll rate and demand elastic parameter is performed on a virtual grid network. The analysis shows that cordon pricing mainly affects those trips with origins outside of the Central Business District and destinations inside, while area pricing imposes additional cost on the trips with either origins or destinations in the Central Business District. Though both pricing strategies are able to alleviate traffic congestion in the charging area, area pricing seems more effective, however, area pricing owns the risk to detour too much traffic and thus cause severe congestion to the network outside of the Central Business District. Following the sensitivity analysis, a unified framework is proposed to optimize the designs of the both pricing strategies, which is flexible to account for various practical concerns. The optimization models are formulated as mixed-integer nonlinear programs with complementarity constraints, and the solution procedure is composed of solving a series of nonlinear programs and mixed-integer linear programs. Results from the numerical examples are in line with the findings in the sensitivity analysis. Under the specific network settings, cordon pricing achieves the best system performance when the toll rate reaches the maximum allowed, while area pricing finds the optimal design scheme when the toll rate equals half of the maximum allowed.

Keywords: travel demand management; cordon pricing; area pricing; sensitivity analysis; toll location; toll rate; demand elastic parameter; mixed-integer; complementarity constraints; dual-based heuristic.

Reference to this paper should be made as follows: Zhang, L.; Liu, H.; Sun, D. (J.). 2014. Comparison and optimization of cordon and area pricings for managing travel demand, *Transport* 29(3): 248–259.
http://dx.doi.org/doi:10.3846/16484142.2014.914970

Introduction

The spreading traffic congestion in most metropolitan areas mainly results from the unbalanced travel demand and supply. Realizing that the improvement can be made in the supply side is limited, scholars and researchers pay more and more attention to travel demand management. Travel demand management aims to reduce automobile demand or to redistribute the demand in space and in time, by applying various control instruments and strategies, among which congestion pricing has long been recognized as an important one (Hensher, Puckett 2007).

Congestion pricing refers to charging a substantial fee for operating a motor vehicle at times and places where there is insufficient road capacity to easily accommodate demand. The intention is to alter people's travel behaviour enough to reduce congestion (Small, Gomez-Ibañez 1999). Extensive research has been carried out in the literature to investigate the effectiveness and efficiency of congestion pricing. For a recent survey on various pricing methodologies and technologies, readers may refer to De Palma and Lindsey (2011). Due to practical restrictions on the settings of road pricing, the first-best pricing, including the marginal-cost pricing (see e.g. Walters 1961; Vickrey 1963), usually cannot be realized. Instead, the second-best pricing is widely adopted in field applications (Zhang, Ge 2004; Zhang, Yang 2004). The current implementations of second-best pricing across the world are mainly in three forms, namely the area pricing (e.g. in Singapore), cordon pricing (e.g. in Stockholm) and high-occupancy/toll lane (e.g. SR91 in California). This study focuses on the former two strate-

Corresponding author: Lihui Zhang
E-mails: *lihui.zhang@dlut.edu.cn; zhanglh02@gmail.com*

gies. It is assumed that cordon pricing charges vehicles that are entering a specific area enclosed by a charging cordon. Each time vehicles pass the tolling points on the cordon, they will be charged the prescribed toll rates. Area pricing charges vehicles driving in the charging area, no matter they are entering, leaving or travelling within the area.

In the literature, endeavors have been made to compare the performance of various second-best pricing strategies. May and Milne (2000) compared cordon pricing with other three network-wide charging strategies: time-based, congestion and distance pricings, to find that cordon pricing is relatively the least effective among the four pricing strategies. Mitchell *et al.* (2005) developed a comprehensive procedure to compare the effect of cordon and distance based pricing on air quality control. Ieromonachou *et al.* (2007) employed a Strategic Niche Management theory to analyse different cordon and area pricing practices, but did not provide quantitative comparison. Maruyama and Sumalee (2007) compared social welfare and equity impact of the cordon and area pricings using the network of Utsunomiya city. The results showed that in general the area-based schemes performed better than the cordon-based schemes in terms of social welfare and level of spatial equity impact. Safirova *et al.* (2008) compared six types of second-best pricings including single cordon, double cordon, freeway toll, vehicle miles travelled tax, and distance-based comprehensive toll on their overall performance like social welfare, air pollution and congestion effect. Zuo *et al.* (2010) compared cordon pricing with other two pricing schemes, which have toll locations same as in the first-best pricing. Numerical tests performed on the network of Nagoya Metropolitan Area showed that cordon pricing was again the least effective. Fujishima (2011) proposed a multi-regional general equilibrium model to compare the cordon and area pricing. Tests on the network of Osaka city found that if long-distance commuting was prevalent in the city, then cordon pricing was better than area pricing, and if the city had large central urban area, then area pricing with some discount rate would be a better choice. Other researchers focused on case studies and comparisons, for example, Ison and Rye (2005) compared the Central London pricing scheme with other two pricing attempts in Hong Kong, China and Cambridge, UK, and provided several key points that may help to achieve a successful pricing practice; Santos (2005) compared London and Singapore pricings, and emphasized that alternative travel mode should be provided to travellers.

It can be observed that direct comparison between area and cordon pricing is limited in the literature, especially from the perspective of travel demand management. This paper performs quantitative analyses of the two pricing instruments on a virtual network, trying to reveal information that may assist in planning and designing of the two pricing instruments. Following the comparison, we propose a unified framework to optimize the designs of the two pricing strategies with objective of maximizing the performance of the whole network. Most of the studies on the optimal design of second-best pricing focus on cordon pricing. Mainly three solution approaches can be found in the literature, namely, the judgmental approach by May *et al.* (2002), the cutset based approach by Zhang and Yang (2004) and the branch tree based approach by Sumalee (2004). Studies on the design of area pricing is limited, and the majority investigate the toll rate structure with given charging area design (see e.g. Lawphongpanich, Yin 2012). The optimization framework proposed here follows the structure developed in Zhang and Sun (2013), with modifications made to account for different practical considerations.

The remainder of the paper is organized as follows: Section 1 performs sensitivity analysis of various performance measures with respect to the toll rate and the demand elastic parameter, with given charging cordon and charging area. Section 2 illustrates the unified framework for optimizing the designs of cordon and area pricings, followed by Section 3 to demonstrate the optimization framework, with two numerical examples. Concluding remarks are provided in the last section.

The symbols used in this paper are summarized as follows:

a – link designation;
(i, j) – link designation;
v_a – the flow on link a;
$\mathbf{v}$ – the aggregated link flow vector;
$\mathbf{x}^w$ – the link flow vector for OD (abbreviation in the main text) pair w;
t_a – the travel time of link a;
t_a^0 – the free flow travel time of link a;
CAP_a – the capacity of link a;
$\boldsymbol{\tau}$ – the link toll vector;
τ_a – the toll imposed on link a;
τ^0 – the unit increment in toll rate;
τ_U – upper bound on toll rate;
τ_L – lower bound on toll rate;
w – OD pair designation;
W – the set of all OD pairs;
d_w – the elastic travel demand of OD pair w;
$\mathbf{d}$ – the realized demand vector;
D_w^0 – the potential demand level of OD pair w;
D_w^{-1} – the inverse demand function of OD pair w;
u_w – the demand elastic parameter of OD pair w;
C_w – the realized general travel cost between OD pair w;
c_a – the general link cost, $c_a = t_a + \tau_a$;
C_w^0 – the travel cost between OD pair w under the potential demand level;
V^F – the set of all feasible flow-demand vector;
ρ^w – the KKT (abbreviation in the main text) multiplier associated with the flow conservation constraints for OD pair w in V^F;
$\mathbf{A}$ – the link-node incidence matrix;

$\mathbf{E}^w$ – a vector in R^n, with only two alternative values: 1 indicating the corresponding origin node of OD pair w, and -1 indicating the destination node;

β_{ij} – a binary decision variable, which equals 1 if link (i, j) is covered by the sub-network, and equals 0 if not;

χ_i – a binary decision variable, which equals 1 if node i is covered by the sub-network, and equals 0 if not;

o_i – a binary decision variable, which equals 1 if node i is selected as the starting node of the sub-network, and equals 0 if not;

z_{ij} – the auxiliary flow on link (i, j);

γ_{ij} – a binary decision variable, which equals 1 if – link (i, j) is tolled, and equals 0 if not;

BM – a big number;

$DI(i)$ – the in-degree of node i;

b^k – a binary decision variable;

$\bar{K}$ – an integer parameter;

L^U – the upper bound on the number of links to be enclosed by charging cordon;

G – link set always enclosed by the charging cordon;

Lth_{ij} – the length of link (i, j);

λ_a – the Lagrange multiplier associated with β_a in problem R–APP (abbreviation in the main text);

η^k – the Lagrange multiplier associated with b^k in problem R–APP;

θ – the lower bound on the objective function of problem UPDATE;

κ – iteration designation;

Z_A – the objective value of APP.

1. Sensitivity Analyses Under Various Cordon and Area Pricing Designs

Our analyses were carried out on a virtual grid network as shown in Fig. 1. The network has 25 nodes, 80 links and 272 origin-destination (OD) pairs. All the links are assumed to have the same capacity 20000 veh/hour and free flow travel time 3 min. Table 1 presents the potential demand matrix. The area covering nodes 12, 13, 17 and 18 is assumed to be the Central Business District (CBD). Table 1 reveals that nodes in this area attract relatively larger demand. The pure user equilibrium assignment reveals that the majority of links in and around the CBD area have volume/capacity ratio over 0.9, and some even over 1.0.

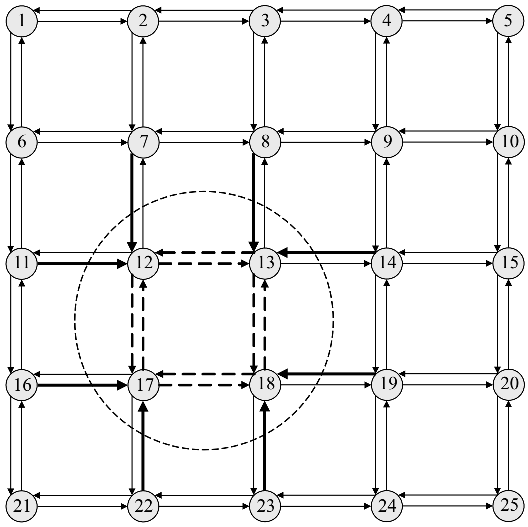

Fig. 1. Study grid network

Table 1. Potential OD table

	1	2	4	5	8	11	12	16	17	18	23	24	25	26	29	30	33
1		1*	1	0.5	1	2	1	2	2	1	2	2	1	1	0.5	1	1
2	1		1	0.5	1	2	1	2	2	1	2	2	1	1	0.5	1	1
4	1	1		1	1	2	1	2	2	1	2	2	1	1	1	1	1
5	1	1	1		1	2	1	2	2	1	2	2	1	1	1	1	1
8	1	1	1	1		2	1	2	2	1	2	2	1	1	1	1	1
11	1	1	1	1	1		1	2	2	1	2	2	1	1	1	1	1
12	1	1	1	1	1	2		3	3	1	2	2	1	1	1	1	1
16	1	1	1	0.5	1	2	1		5	1	5	4	1	1	0.5	1	1
17	1	1	1	0.5	1	2	1	5		1	4	5	1	1	0.5	1	1
18	1	1	1	1	1	2	1	2	2		2	2	1	1	1	1	1
23	1	1	1	0.5	1	2	1	5	4	1		5	1	1	0.5	1	2
24	1	1	1	0.5	1	2	1	4	5	1	5		1	1	0.5	1	2
25	1	1	1	1	1	2	1	2	2	1	2	2		1	1	1	1
26	0.5	0.5	0.5	0.5	1	1	1	2	2	1	2	2	1		1	1	1
29	0.5	0.5	0.5	0.5	0.5	0.5	0.5	3	3	1	3	3	1	1		1	0.5
30	1	1	0.5	0.5	1	1	1	3	3	1	3	3	1	1	1		1
33	1	1	0.5	0.5	1	1	1	2	2	1	2	2	1	2	1	1	

Note: * – $\times 10^3$ veh.

To compare the performance of cordon and area pricings, we virtually created a charging area that is composed of eight links ((12, 13), (13, 12), (12, 17), (17, 12), (13, 18), (18, 13), (17, 18), (18, 17)) and a charging cordon also composed of eight links ((7, 12), (11, 12), (8, 13), (14, 13), (16, 17), (22, 17), (19, 18), (23, 18)). The thicker links in Fig. 1 form the charging cordon, and the dashed links form the charging area. Vehicles using any of the links will be charged a toll (represented in time, varying from 1 to 10 min). For cordon pricing, it is assumed a vehicle is charged some fixed toll rate each time it passes a charging point on the cordon. For area pricing, it is assumed a distance-based toll structure is imposed, which means the toll rate charged on a vehicle is proportional to its travel distance in the charging area.

The Bureau of Public Road (BPR) function (Traffic Assignment Manual 1964) is used as the link travel time function:

$$t_a(v_a) = t_a^0\left(1+0.15\left(\frac{v_a}{CAP_a}\right)^4\right).$$

To investigate the response of traffic demand to the imposed toll, we solve the tolled user equilibrium problems with elastic-demand. The following demand function is borrowed from Zhang and Yang (2004):

$$d_w = D_w^0 \exp\left(u_w\left(1.0-\frac{C_w}{C_w^0}\right)\right),\ w \in W$$

and the inverse demand function can be derived as:

$$C_w = C_w^0 - \frac{C_w^0}{u_w}\ln\left(\frac{d_w}{D_w^0}\right),\ w \in W.$$

Tables 2 and 3 provide the computational results from the sensitivity analysis with the demand elastic parameter u_w set as 0.3 for all the OD pairs. The results are generated from GAMS implementation (Brooke *et al.* 1992) on a *Lenovo* computer with 3.40GHz *Intel Core* i7 CPU and 4GB of Ram. The nonlinear user equilibrium problems are solved by CONOPT (Drud 1994).

1.1. Analysis of Demand

For cordon pricing, the total travel demand (TTD) across the network generally decreases as the toll rate increases. The total demand is further split into four categories: demand with both origins and destinations in the CBD (DII), demand from origins in the CBD to the destinations outside (DIO), demand from origins outside of the CBD to the destinations inside (DOI), and demand with both origins and destinations outside of the CBD (DOO), which are depicted in Figs 2 and 3.

The decrease in DOI is the main contributor to the change in TTD. From rate 1 to 10 min, the toll reduces DOI by 27012 veh/hour, while it reduces TTD by 26105 veh/hour. The reduction in DOI alleviates the network congestion, and thus the elastic demand DII, DIO and

Table 2. Sensitivity analysis of cordon pricing

Rate	DII	DIO	DOI	DOO	TTD	TTT	TTI	TTO	TTDS	VII	VIO
1	55.997	54.010	110.647	154.549	375.203	3751.520	528.912	3222.608	3451.068	97.909	57.542
2	56.053	54.024	107.192	154.586	371.854	3711.701	523.456	3188.246	3423.803	99.352	55.005
3	56.106	54.037	103.860	154.621	368.624	3673.513	518.182	3155.331	3397.277	98.238	55.050
4	56.157	54.050	100.648	154.653	365.508	3636.894	513.087	3123.807	3371.463	98.035	54.210
5	56.205	54.062	97.552	154.683	362.502	3601.753	508.185	3093.568	3346.345	98.539	52.691
6	56.251	54.074	94.565	154.711	359.601	3568.003	503.462	3064.541	3321.909	101.154	49.088
7	56.315	54.084	91.683	154.738	356.820	3547.579	496.811	3050.768	3308.493	97.913	50.907
8	56.355	54.095	88.904	154.801	354.154	3516.975	492.611	3024.364	3285.883	102.459	45.462
9	56.394	54.105	86.222	154.861	351.582	3487.482	488.557	2998.925	3263.864	104.645	42.401
10	56.430	54.115	83.635	154.918	349.098	3459.071	484.639	2974.432	3242.446	101.685	44.507

Table 3. Sensitivity analysis of area pricing

Rate	DII	DIO	DOI	DOO	TTD	TTT	TTI	TTO	TTDS	VII	VIO
1	52.342	53.441	112.780	154.283	372.846	3759.460	396.215	3363.245	3442.992	84.872	40.110
2	48.071	52.982	111.853	154.274	367.180	3719.210	369.825	3349.386	3412.583	78.915	39.024
3	44.117	52.535	110.946	154.269	361.868	3681.708	346.357	3335.351	3383.712	73.378	38.031
4	40.468	52.098	110.060	154.264	356.890	3646.772	325.150	3321.622	3356.279	68.242	37.064
5	37.106	51.672	109.195	154.258	352.231	3614.100	305.883	3308.218	3330.226	63.487	36.121
6	34.013	51.258	108.350	154.252	347.873	3583.440	288.295	3295.145	3305.490	59.089	35.200
7	31.257	50.671	107.094	153.435	342.457	3775.611	191.106	3584.505	3421.415	36.742	26.664
8	28.803	49.657	104.680	167.151	350.291	4260.491	69.317	4191.175	3607.989	10.324	12.724
9	27.680	49.094	103.408	175.460	355.642	4487.647	31.773	4455.874	3662.671	6.172	4.411
10	27.257	48.840	102.959	181.457	360.513	4578.956	28.332	4550.624	3672.421	5.738	3.702

DOO all increase as the toll rate increases. The curves of DII, DIO and DOO are rather flat for cordon pricing, showing that the increments are very marginal. It can be figured out that cordon pricing is relatively more effective in managing the demand entering the CBD area.

For area pricing, the DII, DIO and DOI all decreases as the toll rate increases, which complies with the common sense that the imposed toll in CBD increases the cost of all trips associated with DII, DIO and DOI, thus the elastic demands all decreases. Fig. 3 shows DII is the most sensitive to the toll rate among the three. The DOO shows a more interesting pattern. At the first seven rates, DOO also decreases as the toll rate increases, however at the last three points, DOO increases dramatically. The cost of trips associated with DOO is affected by two aspects: the imposed toll and the congestion alleviation due to the reduction in DII, DIO and DOO. The congestion alleviation can generally save some cost for the DOO trips, however the increased toll will add additional cost for those DOO trips that need to traverse the CBD area, thus some may divert and subsequently offset the congestion alleviation brought by the reduction in DII, DIO and DOO. For the first seven data points, the cost increase for DOO trips due to increased toll is larger than the cost reduction due to congestion alleviation, thus the DOO decreases as the toll increases. However, for the last three data points, the cost reduction due to congestion alleviation becomes larger than the cost increase due to increased toll, and the DOO value increases very fast.

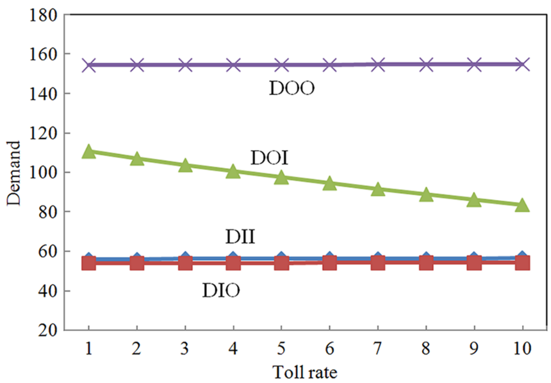

Fig. 2. Analysis of demand under cordon pricing

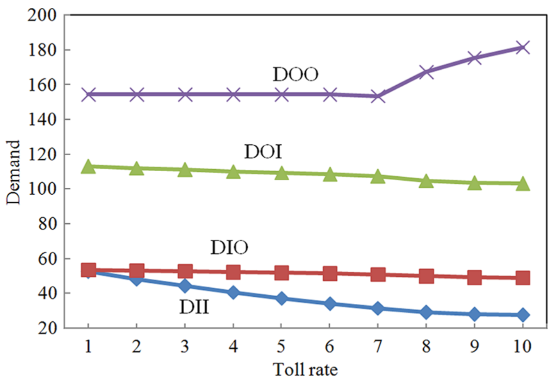

Fig. 3. Analysis of demand under area pricing

1.2. Analysis of Travel Time and Travel Distance

The total travel time (TTT) and total travel distance (TTDS) shown in Figs 4–6 exhibit similar patterns as the total travel demand for both pricing strategies. The total travel time is further split into total travel time in the CBD (TTI) and the total travel time outside of the CBD (TTO).

For cordon pricing, both the TTT and TTDS decrease as the toll rate increases, since the increase in toll generally results in reduction in TTD under cordon pricing. And the reduction in DOI causes both TTI and TTO to decrease as the toll rate increases.

For area pricing, TTT, TTO and TTDS all decrease as the toll rate increases at the first six data points, and then experience fast increase at the next four data points,

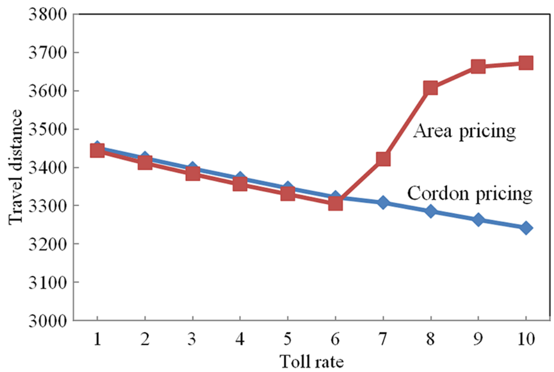

Fig. 4. Analysis of travel distance

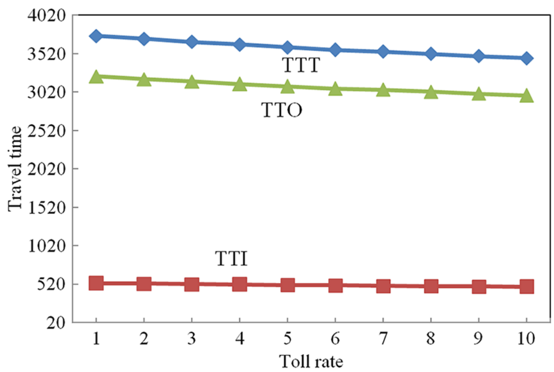

Fig. 5. Analysis of travel time under cordon pricing

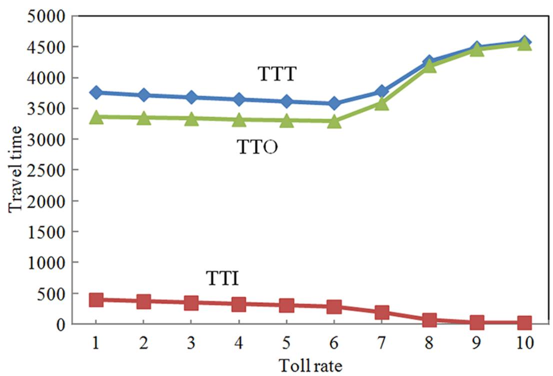

Fig. 6. Analysis of travel time under area pricing

which coincides with the trends for DOO and TTD, but the inflection point is one date point ahead. The reason may be relevant to TTI. TTI generally deceases as the toll rate increases, and the decrease, as in Fig. 6, is especially large when the rate changes to seven and eight, which should result from two aspects: the direct reduction in the elastic demand, and the detours of DII trips to the network outside of the CBD. The large amount of detours increase the congestion level of the network outside of the CBD, thus the TTT, TTO and TTDS start to increase at toll rate seven.

1.3. Analysis of Volume

We examined the total flow in the CBD area with origins in the CBD (VII) and origins outside of the CBD (VIO). The results from the two pricing strategies show very different patterns as shown in Figs 7 and 8.

For cordon pricing, since both DII and DIO are not very sensitive to the toll rate, the VII does not very much. There is no strictly increasing trend for VII, which may be due to the complex interactions among different paths for various OD pairs, sometimes a DIO trip may take a path that traverses the CBD, sometimes it may not. The VIO generally decreases mainly due to the reduction in DOI, but as can be observed, VIO increases at several data points, which may be because some DOO trips select the paths traversing the CBD area due to the congestion alleviation in the CBD.

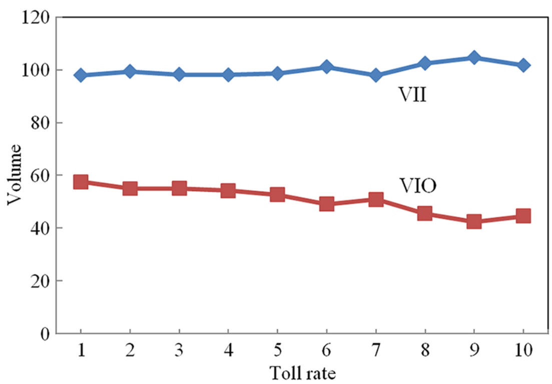

Fig. 7. Analysis of volume under cordon pricing

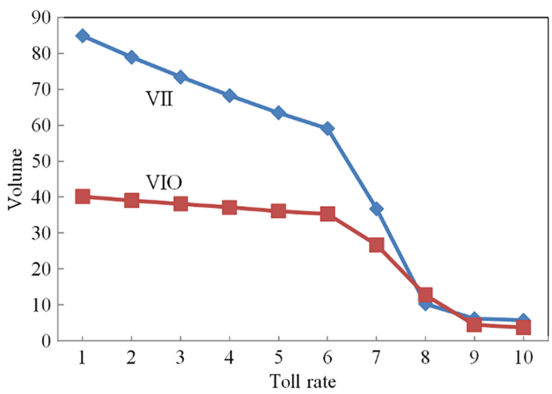

Fig. 8. Analysis of volume under area pricing

For area pricing, the toll in the CBD does not only reduce the elastic demands DII and DIO, but also detours trips to the network outside of the CBD, thus the VII flow gradually decreases to a very small amount of 5738 veh/hour, even much smaller then DII whose origins and destinations are both in the CBD. The VIO decreases slowly at the first six data points, then experience fast drop, and finally reaches a small amount of 3702 veh/hour, when few trips will choose routes traversing CBD to reach destinations in the CBD or behind the CBD.

Comparing Figs 7 and 8, the VII and VIO under area pricing are both smaller than those under cordon pricing. This is because area pricing imposes additional impedance on any trip that uses the roads in the CBD, while cordon pricing mainly affects those DOI trips.

1.4. Analysis of Demand Elastic Parameter

The demand elastic parameter affects the response of travel demand to the pricing strategies, thus will influence the congestion level of the network. The following analysis is performed with fixed pricing schemes and varying elastic parameter values (from 0.1 to 0.6). Without loss of generality, toll rate 5 min is used. Selected results are presented in Figs 9 and 10.

Fig. 9 shows the equilibrium demand patterns with different elastic parameters under both pricing schemes. For cordon pricing, the DOI is more sensitive to the

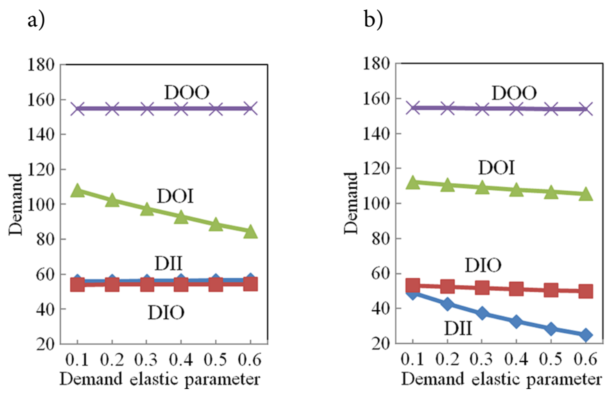

Fig. 9. Demand under different elastic parameters:
a – cordon pricing; b – area pricing

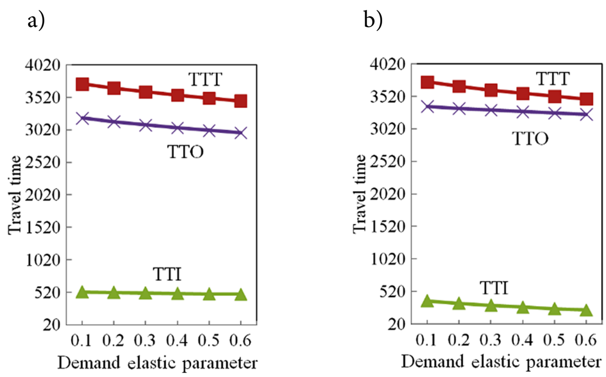

Fig. 10. Travel time under different elastic parameters:
a – cordon pricing; b – area pricing

elastic parameter since again the toll is mainly imposed on these DOI trips, and the curves for the other three demands are still flat. For area pricing, the DII is the most sensitive, DOI and DIO also decrease as the value of elastic parameter increases, but the changes are smaller. DOO is the least sensitive to the elastic parameter.

Fig. 10 shows the TTT, TTI and TTO under different elastic parameters. Generally, these measures of travel time decrease as the elastic parameter increases. Under the settings of the two specific schemes, TTT shows similar patterns under both pricing schemes, TTO is relatively more sensitive to the elastic parameter under cordon pricing and TTI is more sensitive under area pricing.

1.5. Short Summary

The sensitivity analysis of travel demand, travel time, travel distance and traffic volume are performed for two pricing strategies. With the given pricing settings, cordon pricing is relative more effective to manage the demand DOI, which is rather sensitive to both the toll rate and demand elastic parameter. Area pricing is able to reduce the demand related to nodes in the CBD, and DII is the most sensitive to the toll rate and demand elastic parameter. The change in demand leads to the observation that the VII and VIO are more sensitive under area pricing. Cordon pricing may be able to control the congestion level both inside and outside of the cordon in a much milder manor, while area pricing has the risk to cause severe congestion to the network outside of the CBD.

The computational results obtained may be case sensitive, but the above analysis at least reveals that cordon and area pricings may behave in quite different manors. Pricing schemes need to be carefully designed based on different control objectives. In the next section, we present a unified framework to optimize the both pricing strategies.

2. Optimizing Cordon and Area Pricing Designs

Firstly, the model for optimizing one cordon in Zhang and Sun (2013) is briefly illustrated. The formulation is a single-level mathematical program with complementarity constraints (MPCC), with the toll rate modelled as a continuous variable (CPDP):

$$\textbf{CPDP} \quad \min \sum_a t_a\left(v_a\left(\tau_a\right)\right)v_a\left(\tau_a\right) - \sum_w \int_0^{d_w(\tau)} D_w^{-1}(\omega)d\omega,$$

$$\text{s.t.} \quad \left(\mathbf{t}(\mathbf{v})+\boldsymbol{\tau}\right) \ge \mathbf{A}^T\rho^w,\ \forall w \in W; \tag{1}$$

$$D_w^{-1}\left(d_w\right) \le \mathbf{E}_w^T\rho^w,\ \forall w \in W; \tag{2}$$

$$\left(\mathbf{t}(\mathbf{v})+\boldsymbol{\tau}\right)^T\mathbf{v} \le \left(\mathbf{D}^{-1}(\mathbf{d})\right)^T\mathbf{d}; \tag{3}$$

$$(\mathbf{v},\mathbf{d}) \in V^F; \tag{4}$$

$$V^F = \left\{(\mathbf{v},\mathbf{d}) : \mathbf{v} = \sum_w \mathbf{x}^w, \mathbf{A}\mathbf{x}^w = \mathbf{E}_w d_w, \mathbf{x}^w \ge 0, \forall w\right\}; \tag{5}$$

$$\beta_{ij} = \beta_{ji},\ \forall(i,j); \tag{6}$$

$$\sum_{ij}\beta_{ij} \ge 2; \tag{7}$$

$$\chi_i \le \sum_j \beta_{ij} \le BM\cdot\chi_i,\ \forall i; \tag{8}$$

$$\chi_i + \chi_j - 1 \le BM\cdot\beta_{ij},\ \forall(i,j); \tag{9}$$

$$\sum_i o_i\chi_i = 1; \tag{10}$$

$$\chi_i\left(\sum_j \beta_{ij}z_{ij} - \sum_j \beta_{ji}z_{ji} - o_i\sum_j \chi_j + 1\right) = 0,\ \forall i; \tag{11}$$

$$\gamma_{ij}\beta_{ij} = 0,\ \forall(i,j); \tag{12}$$

$$\gamma_{ij} \le \chi_i + \chi_j,\ \forall(i,j); \tag{13}$$

$$\left(\sum_j \gamma_{ji} + \sum_j \beta_{ji} - DI(i)\right)\cdot\chi_i = 0,\ \forall i; \tag{14}$$

$$\tau_{ij} = \gamma_{ij}\tau,\ \forall(i,j); \tag{15}$$

$$\tau_L \le \tau \le \tau_U; \tag{16}$$

$$\gamma_{ij} \in \{0,1\},\ \forall(i,j); \tag{17}$$

$$\beta_{ij} \in \{0,1\},\ \forall(i,j); \tag{18}$$

$$\chi_i \in \{0,1\},\ \forall i; \tag{19}$$

$$o_i \in \{0,1\},\ \forall i; \tag{20}$$

$$z_{ij} \ge 0,\ \forall(i,j). \tag{21}$$

The objective function is to maximize the social welfare. Constraints (1)–(5) are the Karush–Kuhn–Tucker (KKT) conditions of the tolled user equilibrium problem with elastic demand. Constraints (6)–(11) are constraints to form a connected sub-network, like the dashed sub-network in Fig. 1, which is to be enclosed by some charging cordon. With the sub-network defined by constraints (6)–(11), constraints (12)–(14) further define the corresponding charging cordon. Links with end points in the sub-network and the start points outside are set as tolling links, which together form a charging cordon, like the thicker links in Fig. 1. Constraints (15) and (16) are to set the upper and lower bounds on the toll rate if a link is to be tolled. Constraints (17)–(21) define the types of the decision variables.

2.1. Cordon Pricing with Discrete Toll

The toll rate is assumed to be continuous in the above model, which is not practical, especially when there are no electric toll collection facilities available. Various field implementations show that it is more reasonable to select toll rate from some candidate set of discrete toll rates (see e.g. Olszewski, Xie 2005; Yin, Lou 2009; Rotaris *et al.* 2010).

Notice that the expression $\sum_{k=1}^{K} 2^{(k-1)}b^k,\ b^k \in \{0,1\}$ varies from 0 to $2^K - 1$. Any discrete toll between the upper bound τ_U and lower bound τ_L can be represented as follows:

$$\tau_{ij} = \tau_L + \tau^0 \cdot \sum_{k=1}^{\bar{K}} 2^{(k-1)}b^k,$$

where: $\bar{K}$ is some integer that satisfies

$$\tau^0 \cdot \sum_{k=1}^{\bar{K}} 2^{(k-1)}b^k \ge \tau_U - \tau_L.$$

The size of the charging area should be a big concern for the planners, and we here provide the flexibility to limit the size of charging area by changing the upper bound L^U on the total number of links enclosed by the cordon. On the other hand, in order to control the congestion level on some targeted links in the CBD, these links are always set to be enclosed by the cordon. According to different management objectives, various objective functions can be selected. We here try to minimize the total system travel time across the network. Thus the discrete version of the cordon pricing design problem (CPP) can be formulated as follow:

$$\textbf{CPP}\ \min Z_C=\sum_a t_a\left(v_a\left(\tau_a\right)\right)v_a\left(\tau_a\right),$$

$$\text{s.t.}\ \ \tau_{ij}=\gamma_{ij}\cdot\left(\tau_L+\tau^0\cdot\sum_{k=1}^{\bar{K}}2^{(k-1)}b^k\right),\ \forall(i,j);$$

$$\tau_{ij}\le\tau_U,\ \forall(i,j);$$

$$\sum_{ij}\beta_{ij}\le L^U;$$

$$\beta_{ij}=1,\ \forall(i,j)\in G;$$

$$b^k\in\{0,1\},\ \forall k$$

and constraints (1)–(14), (17)–(21).

2.2. Area Pricing with Discrete Toll

Area pricing charges travellers that use any road in the charging area. The charging area can be directly defined using constraints (6)–(11). Assuming a distanced-based toll strategy, the area pricing design problem (APP) can be formulates as follows:

$$\textbf{APP}\ \min Z_A=\sum_a t_a\left(v_a\left(\tau_a\right)\right)v_a\left(\tau_a\right),$$

$$\text{s.t.}\ \ \tau_{ij}=\beta_{ij}\cdot Lth_{ij}\cdot\left(\tau_L+\tau^0\cdot\sum_{k=1}^{\bar{K}}2^{(k-1)}b^k\right),\ \forall(i,j);$$

$$\tau_{ij}\le\tau_U,\ \forall(i,j);$$

$$\sum_{ij}\beta_{ij}\le L^U;$$

$$\beta_{ij}=1,\ \forall(i,j)\in G;$$

$$b^k\in\{0,1\},\ \forall k$$

and constraints (1)–(11), (18)–(21).

In this case, the optimized toll rate is the unit-distance toll rate. It can be seen that the APP problem contains fewer constraints than CPP problem, thus is smaller in scale, however the computation complexity is equivalent.

2.3. Solution Algorithm

Both the CPP and APP problems are MPCCs with many binary variables (Scheel, Scholtes 2000). MPCCs violate Mangasarian–Fromovitz Constraint Qualification, thus directly solving these programs with commercial nonlinear solvers will be numerically unsafe (Leyffer 2003). On the other hand, the programs contain large numbers of binary variables. The algorithms based on branch-and-bound will require too large memories to store the branching tree. To solve the two problems, we modify the dual-based heuristic proposed in Zhang and Sun (2013). Next we will take APP problem as an example to illustrate the algorithm.

We first construct two series of active sets $\Omega_\beta^0=\{a\mid\beta_a=0\}$, $\Omega_b^0=\{k\mid b^k=0\}$ and $\Omega_\beta^1=\{a\mid\beta_a=1\}$, $\Omega_b^1=\{k\mid b^k=1\}$ to restrict the values of β_a and b^k to be zero or one. And we further make $\Omega_\beta^0\cup\Omega_\beta^1=\{a\mid\forall a\}$ and $\Omega_b^0\cup\Omega_b^1=\{k\mid\forall k\}$ to ensure β_a and b^k always take binary values.

Given a connected sub-network and the discrete toll rate, in other words Ω_β^0, Ω_b^0, Ω_β^1 and Ω_b^1, the APP problem becomes the following relaxed problem (R–APP):

$$\textbf{R–APP}\ \min Z_A=\sum_a t_a(v_a(\tau_a))v_a(\tau_a),$$

$$\text{s.t.}\ \ \tau_{ij}=\beta_{ij}\cdot Lth_{ij}\cdot\left(\tau_L+\tau^0\cdot\sum_{k=1}^{\bar{K}}2^{(k-1)}b^k\right),\forall(i,j);$$

$$\beta_{ij}=0,\ \forall(i,j)\in\Omega_\beta^0;$$

$$\beta_{ij}=1,\ \forall(i,j)\in\Omega_\beta^1;$$

$$b^k=0,\ \forall k\in\Omega_b^0;$$

$$b^k=1,\ \forall k\in\Omega_b^1$$

and constraints (1)–(5).

The R-APP problem is a regular nonlinear program that can be solved directly via commercial solvers. However, the KKT conditions are only necessary conditions but are not sufficient to guarantee that the resulted flow-demand pattern is in user equilibrium with elastic demand. Our strategy is to first solve the following tolled user equilibrium with elastic demand (TUE) to guarantee equilibrium condition, and then feed the resulting flow-demand vector into problem R–APP to solve for the dual variables associated with β_{ij} and b^k:

$$\textbf{TUE}\ \min_{\mathbf{v},\mathbf{d}}\sum_a\int_0^{x_a}c_a\left(\omega,\tau_a\right)d\omega-\sum_w\int_0^{d_w}D_w^{-1}\left(\omega\right)d\omega,$$

$$\text{s.t.}\ \ \tau_{ij}=\beta_{ij}\cdot Lth_{ij}\cdot\left(\tau_L+\tau^0\cdot\sum_{k=1}^{\bar{K}}2^{(k-1)}b^k\right),\forall(i,j);$$

$$\beta_{ij}=0,\ \forall(i,j)\in\Omega_\beta^0;$$

$$\beta_{ij}=1,\ \forall(i,j)\in\Omega_\beta^1;$$

$$b^k=0,\ \forall k\in\Omega_b^0;$$

$$b^k=1,\ \forall k\in\Omega_b^1$$

and constraints (4), (5).

With the dual information obtained from solving the R–APP, the following UPDATE problem is built to search for new combinations of β_a and b^k values:

$$\textbf{UPDATE}\ \ \min\sum_a\lambda_a\beta_a+\sum_k\eta^k b^k,$$

$$\text{s.t.}\ \ b^k\in\{0,1\},\ \forall k;$$

$$\sum_a\beta_a\le L^U;$$

$$\beta_{ij}=1,\ \forall(i,j)\in G;$$

$$\sum_a\lambda_a\beta_a+\sum_k\eta^k b^k>\theta \tag{22}$$

and (6)–(11), (18)–(21).

Take η^k as an example, for some $b^k = 0$: if $\eta^k < 0$, then making b^k positive will possibly lead to a decrease in the total system travel time, and if $\eta^k \geq 0$, b^k should remain unchanged; for some $b^k = 1$: if $\eta^k > 0$, then it may be beneficial to decrease β_a^b, and if $\eta^k \leq 0$, b^k should remain 1. The objective function is built to maximize the potential improvement that can be made in the total system travel time, and at the same time guarantee the aforementioned updating logic. To ensure the resulting charging links form a connected sub-network, constraints (6)–(11) are also included in the problem.

Since λ_a and η^k are both binary, the area pricing scheme found by the immediate next discrete update through the UPDATE problem may not lead to an improvement in the objective function. In such cases, new charging schemes need to be searched, until no negative objective value of UPDATE can be achieved. To ensure that each next search finds a new charging scheme, constraint (22) is further added to the problem. The left hand side of (22) is the same as the objective function of UPDATE problem. The constraint ensures the objective value is greater than a parameter θ. Before any new search iteration, θ is set to be $\sum_a \lambda_a \tilde{\beta}_a + \sum_k \eta^k \tilde{b}^k$, where $\left(\tilde{\boldsymbol{\beta}}, \tilde{\mathbf{b}}\right)$ is the charging scheme found by the immediate previous discrete update.

Since the UPDATE problem contains the complementarity constraints (10) and (11), it is still difficult to directly solve UPDATE. An alternative approach that sequentially solves two sub-problems is adopted here.

Firstly, a knapsack problem is built with no complementarity constraints as follows to search for new combinations of β_a and b^k values:

KNAPSACK $\min \sum_a \lambda_a \beta_a + \sum_k \eta^k b^k$,

s.t. $b^k \in \{0,1\}, \ \forall k$;

$\sum_a \beta_a \leq L^U$;

$\beta_{ij} = 1, \ \forall (i,j) \in G$;

$\sum_a \lambda_a \beta_a + \sum_k \eta^k b^k > \theta$

and constraints (6)–(9), (18) and (19).

The feasibility of one portion of the solution $\left(\bar{\boldsymbol{\beta}}, \bar{\boldsymbol{\chi}}\right)$ from the knapsack problem is subsequently checked by the following auxiliary problem:

CHECK $\min \sum_a \bar{\beta}_a$,

s.t. constraints (10), (11), (20) and (21).

The CHECK problem is to check if the charging links can form a connected sub-network. Given $\left(\bar{\boldsymbol{\beta}}, \bar{\boldsymbol{\chi}}\right)$, the objective function is fixed, and constraints (10) and (11) both become linear. If the CHECK problem can be solved to optimality, then the updating scheme is a valid area pricing scheme, otherwise the KNAPSACK problem needs to be solved again with updated θ to find a next potential charging scheme.

The complete heuristic is listed below:

Step 1: Set $\kappa = 1$. Generate an initial area pricing scheme $\left(\boldsymbol{\beta}^\kappa, \mathbf{b}^\kappa\right)$, i.e. $\Omega_\beta^{0,\kappa}$, $\Omega_b^{0,\kappa}$, $\Omega_\beta^{1,\kappa}$ and $\Omega_b^{1,\kappa}$.

Step 2: Solve the TUE with $\left(\boldsymbol{\beta}^\kappa, \mathbf{b}^\kappa\right)$ to obtain $\left(\mathbf{v}^\kappa, \mathbf{d}^\kappa\right)$.

Step 3: Solve the R–APP with $\left(\boldsymbol{\beta}^\kappa, \mathbf{b}^\kappa, \mathbf{v}^\kappa, \mathbf{d}^\kappa\right)$ to obtain $\left(\boldsymbol{\lambda}^\kappa, \boldsymbol{\eta}^\kappa\right)$, the KKT multipliers of $\left(\boldsymbol{\beta}^\kappa, \mathbf{b}^\kappa\right)$, and the objective function value Z_A^κ.

Step 4:

a) Solve KNAPSACK with $\left(\boldsymbol{\lambda}^\kappa, \boldsymbol{\eta}^\kappa\right)$ for a potential charging scheme $\left(\bar{\boldsymbol{\beta}}^\kappa, \bar{\mathbf{b}}^\kappa\right)$, and the auxiliary variable $\bar{\boldsymbol{\chi}}^\kappa$. If the objective value is 0, then the optimal solution is found, terminate the algorithm. Otherwise, go to Step 4b.

b) Solve CHECK with $\left(\bar{\boldsymbol{\beta}}^\kappa, \bar{\boldsymbol{\chi}}^\kappa\right)$. If the problem is infeasible, update $\theta = \boldsymbol{\lambda}^\kappa \cdot \bar{\boldsymbol{\beta}}^\kappa + \boldsymbol{\eta}^\kappa \cdot \bar{\mathbf{b}}^\kappa$, and go to Step 4a; if optimal solution can be found, update:

$$\bar{\Omega}_\beta^0 = \left(\Omega_\beta^{0,\kappa} - \left\{a \in \Omega_\beta^{0,\kappa} : \bar{\beta}_a = 1\right\}\right) \cup \left\{a \in \Omega_\beta^{1,\kappa} : \bar{\beta}_a = 0\right\};$$

$$\bar{\Omega}_\beta^1 = \left(\Omega_\beta^{1,\kappa} - \left\{a \in \Omega_\beta^{1,\kappa} : \bar{\beta}_a = 0\right\}\right) \cup \left\{a \in \Omega_\beta^{0,\kappa} : \bar{\beta}_a = 1\right\};$$

$$\bar{\Omega}_b^0 = \left(\Omega_b^{0,\kappa} - \left\{k \in \Omega_b^{0,\kappa} : \bar{b}_k = 1\right\}\right) \cup \left\{a \in \Omega_b^{1,\kappa} : \bar{b}_k = 0\right\};$$

$$\bar{\Omega}_b^1 = \left(\Omega_b^{1,\kappa} - \left\{a \in \Omega_b^{1,\kappa} : \bar{b}_k = 0\right\}\right) \cup \left\{a \in \Omega_b^{0,\kappa} : \bar{b}_k = 1\right\}$$

and go to Step 4c.

c) Solve the TUE with $\left(\bar{\boldsymbol{\beta}}^\kappa, \bar{\mathbf{b}}^\kappa\right)$ to obtain $\left(\bar{\mathbf{v}}^\kappa, \bar{\mathbf{d}}^\kappa\right)$.

d) Solve the R–APP with $\left(\bar{\boldsymbol{\beta}}^\kappa, \bar{\mathbf{b}}^\kappa, \bar{\mathbf{v}}^\kappa, \bar{\mathbf{d}}^\kappa\right)$ to obtain $\left(\bar{\boldsymbol{\lambda}}^\kappa, \bar{\boldsymbol{\eta}}^\kappa\right)$ and $\bar{Z}_A^\kappa$. If $\bar{Z}_A^\kappa < Z_A^\kappa$, set $\kappa = \kappa + 1$, update $Z_A^\kappa = \bar{Z}_A^\kappa$, $\Omega_\beta^{0,\kappa} = \bar{\Omega}_\beta^0$, $\Omega_b^{0,\kappa} = \bar{\Omega}_b^0$, $\Omega_\beta^{1,\kappa} = \bar{\Omega}_\beta^1$, $\Omega_b^{1,\kappa} = \bar{\Omega}_b^1$, $\boldsymbol{\lambda}^\kappa = \bar{\boldsymbol{\lambda}}^\kappa$, $\boldsymbol{\eta}^\kappa = \bar{\boldsymbol{\eta}}^\kappa$ and $\theta = -\infty$, then go to Step 4a; if $\bar{Z}_A^\kappa \geq Z_A^\kappa$, update $\theta = \boldsymbol{\lambda}^\kappa \cdot \bar{\boldsymbol{\beta}}^\kappa + \boldsymbol{\eta}^\kappa \cdot \bar{\mathbf{b}}^\kappa$, and go to Step 4a.

The algorithm terminates at Step 4a, once no improvement can be achieved in the objective function. When a new combination of β_a and b^k does not form a connected sub-network, or does not lead to a decrease in the system travel time, the algorithm goes back to Step 4a with the current dual information to search for a next charging scheme, however when the new combination improves the system performance, the algorithm goes to Step 4a with updated dual information to find new pricing schemes. The convergence of the heuristic and the property of the final solution have been discussed in previous works like (Zhang *et al.* 2009; Wu *et al.* 2011; Zhang, Sun 2013), thus will not be presented here again.

The discrete cordon pricing design problem CPP can be solved with a similar solution procedure as discussed in this sub-section.

3. Numerical Examples

Numerical tests are performed on the network as in Fig. 1, with the same network settings for the sensitivity analysis. The demand elastic parameter is set to be 0.3 for all the OD pairs. Link pair (12, 13) and (13, 12) are forced to be in the charging area.

3.1. Generation of Cordon Pricing Scheme

The maximal number of links to be enclosed by the charging cordon is limited to be 12, in other words, L^U is set to be 12. The toll rate is allowed to have a maximum value of 10 min, with unit increment of 1 min.

Among all the smaller problems to be solved, the problem TUE contains 38358 continuous variables and 2372 constraints; R–CPP contains 39560 continuous variables and 5649 constraints; the KNAPSACK problem contains 113 discrete variables and 310 constraints; the CHECK problem contains 113 discrete variables, 81 continuous variables and 332 constraints. It takes 1525 seconds to achieve the optimal cordon pricing scheme, with a final total system travel time of 3211.776, over 15% reduction compared with 3783.433 resulted from the no toll equilibrium. The optimal cordon design is shown in Fig. 11.

It is interesting that the optimal charging cordon encloses a sub-network that is exactly a street corridor, connecting nodes 11, 12, 13, 14 and 15. The ten thicker links entering the corridor nodes are the links to be tolled. The optimal toll rate is the maximum 10 min, which coincides with the trend in the elasticity analysis.

Further tests are performed to see the effect of toll rate on the system performance. Fig. 12 shows the optimal objective values with different upper bounds on the toll rate.

As expected, as the feasible region of toll rate increases, the system performance becomes better, though the algorithm produces the same optimal cordon pricing scheme when the upper bound is 6 and 7.

3.2. Generation of Area Pricing Scheme

The maximal number of links to be tolled is also limited to be 12. Since the links in the test network share the same link attributes, the distance based toll is proportional to the number of tolled links a trip traversed. Thus a homogeneous toll rate can be applied to each tolled link. The toll rate is also allowed to vary from 1 to 10 min.

The smaller problems solved here share the same sizes as those for generating cordon pricing scheme, except that the CHECK problem contains 185 less constraints, since constraints (12)–(14) are not needed to determine toll locations for area pricing. The algorithm terminates after 656 seconds to obtain a total system

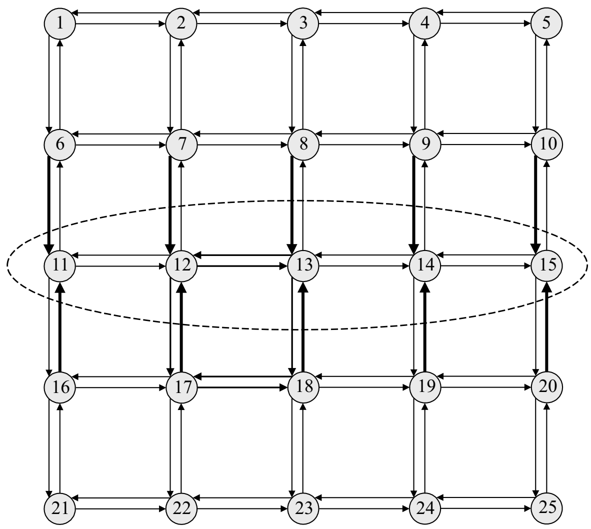

Fig. 11. Optimal charging cordon design

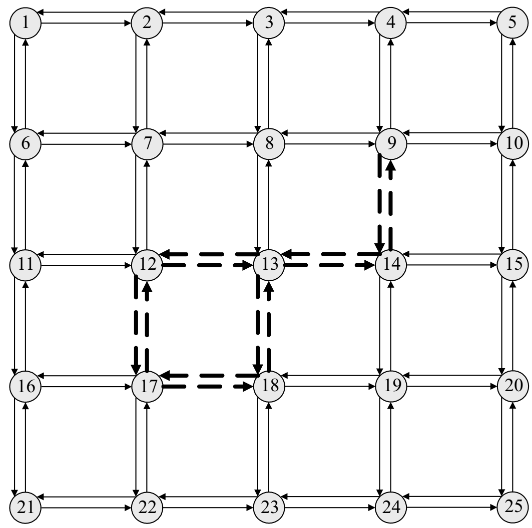

Fig. 13. Optimal charging area design

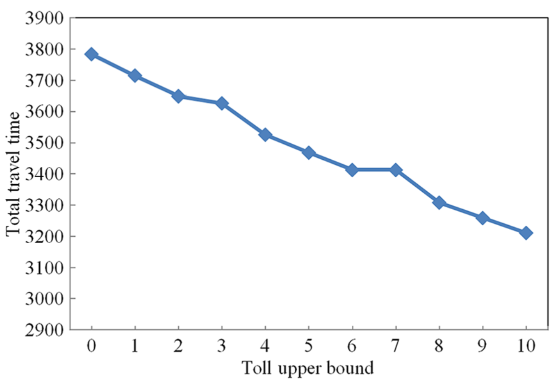

Fig. 12. Optimal objective values with different toll upper bounds under cordon pricing

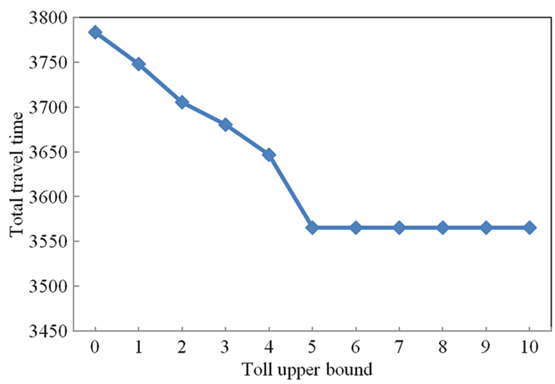

Fig. 14. Optimal objective values with different toll upper bounds under area pricing

travel time of 3565.473. This total system travel time is around 6% smaller than that from the no toll case, and around 11% larger than that from the cordon pricing scheme in the previous section. The pricing area includes the dashed links shown in Fig. 13. The optimal toll rate is 5 min per link, but not the allowed maximal value, which is 10 min.

Fig. 14 shows the best system travel time achieved with different feasible regions of the toll rate. The increased upper bound improves the system performance at the first five data points, however, leads to no further improvement at the next five. Upper bounds 6, 7, 8, 9 and 10 produce the same charging area schemes as the upper bound 5.

Concluding Remarks

This paper starts with the sensitivity analysis of various network performance measures with respect to the toll rate and demand elastic parameter, under both cordon and area pricings. The analysis, to some extent, reveals the effectiveness of the two pricing strategies. With the given network and pricing settings, the cordon pricing mainly affects the DOI trips with the origins outside of the CBD and destinations inside, while area pricing reduces the number of trips with either origins or destinations in the CBD. Area pricing is relatively better in managing the traffic condition in the charging area, but has the potential to cause unexpected congestion to the network outside. The sensitivity analysis also shows it is not valid that the larger the charging area is, or the higher the toll rate is, the better the system will perform.

The paper then proposes a unified framework to optimize the designs of the both pricing strategies. The optimization models are formulated as MPCC problems with multiple binary variables, which incorporated practical concerns on the charging locations and the toll rate settings. The solution procedure is composed of solving a series of sub-problems created, among which is a tolled user equilibrium problem with elastic demand to guarantee the flow-demand patterns during search iterations are in equilibrium. Numerical examples proved the potential of the optimization framework in dealing with design problems in actual-sized transportation network. The results in the numerical tests are in line with the findings in the sensitivity analysis.

Acknowledgements

The authors would like to thank three anonymous reviewers for their helpful comments. This research was supported in part by the Humanities and Social Science Research Project (11YJCZH242) and Scientific Research Foundation for Returned Scholars, Ministry of Education, China.

References

Brooke, A.; Kendirck, D.; Meeraus, A. 1992. *GAMS: a User's Guide*. Scientific Press. 289 p.

De Palma, A.; Lindsey, R. 2011. Traffic congestion pricing methodologies and technologies, *Transportation Research Part C: Emerging Technologies* 19(6): 1377–1399. http://dx.doi.org/10.1016/j.trc.2011.02.010

Drud, A. S. 1994. CONOPT – a large-scale GRG code, *ORSA Journal on Computing* 6(2): 207–216. http://dx.doi.org/10.1287/ijoc.6.2.207

Fujishima, S. 2011. The welfare effects of cordon pricing and area pricing simulation with a multi-regional general equilibrium model, *Journal of Transport Economics and Policy* 45(3): 481–504.

Hensher, D. A.; Puckett, S. M. 2007. Congestion and variable user charging as an effective travel demand management instrument, *Transportation Research Part A: Policy and Practice* 41(7): 615–626. http://dx.doi.org/10.1016/j.tra.2006.07.002

Ieromonachou, P.; Potter, S.; Warren, J. 2007. Analysing road pricing implementation processes in the UK and Norway, *European Journal of Transport and Infrastructure Research* 7(1): 15–38.

Ison, S.; Rye, T. 2005. Implementing road user charging: the lessons learnt from Hong Kong, Cambridge and Central London, *Transport Reviews* 25(4): 451–465. http://dx.doi.org/10.1080/0144164042000335788

Lawphongpanich, S.; Yin, Y. 2012. Nonlinear pricing on transportation networks, *Transportation Research Part C: Emerging Technologies* 20(1): 218–235. http://dx.doi.org/10.1016/j.trc.2011.05.010

Leyffer, S. 2003. Mathematical programs with complementarity constraints, *SIAG/OPT Views-and-News* 14(1):15–18.

Maruyama, T.; Sumalee, A. 2007. Efficiency and equity comparison of cordon- and area-based road pricing schemes using a trip-chain equilibrium model, *Transportation Research Part A: Policy and Practice* 41(7): 655–671. http://dx.doi.org/10.1016/j.tra.2006.06.002

May, A. D.; Liu, R.; Shepherd, S. P.; Sumalee, A. 2002. The impact of cordon design on the performance of road pricing schemes, *Transport Policy* 9(3): 209–220. http://dx.doi.org/10.1016/S0967-070X(02)00031-8

May, A. D.; Milne, D. S. 2000. Effects of alternative road pricing systems on network performance, *Transportation Research Part A: Policy and Practice* 34(6): 407–436. http://dx.doi.org/10.1016/S0965-8564(99)00015-4

Mitchell, G.; Namdeo, A.; Milne, D. 2005. The air quality impact of cordon and distance based road user charging: an empirical study of Leeds, UK, *Atmospheric Environment* 39(33): 6231–6242. http://dx.doi.org/10.1016/j.atmosenv.2005.07.005

Olszewski, P.; Xie, L. 2005. Modelling the effects of road pricing on traffic in Singapore, *Transportation Research Part A: Policy and Practice* 39(7–9): 755–772. http://dx.doi.org/10.1016/j.tra.2005.02.015

Rotaris, L.; Danielis, R.; Marcucci, E.; Massiani, J. 2010. The urban road pricing scheme to curb pollution in Milan, Italy: Description, impacts and preliminary cost–benefit analysis assessment, *Transportation Research Part A: Policy and Practice* 44(5): 359–375. http://dx.doi.org/10.1016/j.tra.2010.03.008

Safirova, E.; Houde, S.; Harrington, W. 2008. *Marginal Social Cost Pricing on a Transportation Network: a Comparison of Second-Best Policies*. Discussion Paper RFF DP 07-52. 23 p. Available from Internet: http://www.rff.org/RFF/Documents/RFF-DP-07-52.pdf

Santos, G. 2005. Urban congestion charging: a comparison between London and Singapore, *Transport Reviews* 25(5): 511–534. http://dx.doi.org/10.1080/01441640500064439

Scheel, H.; Scholtes, S. 2000. Mathematical programs with complementarity constraints: stationarity, optimality, and sensitivity, *Mathematics of Operations Research* 25(1): 1–22. http://dx.doi.org/10.1287/moor.25.1.1.15213

Small, K. A.; Gomez-Ibañez, J. A. 1999. Road pricing for congestion management: the transition from theory to policy, in Button, K. J.; Verhoef, E. T. (Eds.). *Road Pricing, Traffic Congestion and the Environment: Issues of Efficiency and Social Feasibility*, 213–246.

Sumalee, A. 2004. Optimal road user charging cordon design: a heuristic optimization approach, *Computer-Aided Civil and Infrastructure Engineering* 19(5): 377–392. http://dx.doi.org/10.1111/j.1467-8667.2004.00364.x

Traffic Assignment Manual. 1964. U.S. Dept. of Commerce, Bureau of Public Roads, Office of Planning, Urban Planning Division, Washington D.C. 300 p.

Vickrey, W. S. 1963. Pricing in urban and suburban transport, *The American Economic Review* 53(2): 452–465.

Walters, A. A. 1961. The theory and measurement of private and social cost of highway congestion, *Econometrica* 29(4): 676–699. http://dx.doi.org/10.2307/1911814

Wu, D.; Yin, Y.; Lawphongpanich, S. 2011. Pareto-improving congestion pricing on multimodal transportation networks, *European Journal of Operational Research* 210(3): 660–669. http://dx.doi.org/10.1016/j.ejor.2010.10.016

Yin, Y.; Lou, Y. 2009. Dynamic tolling strategies for managed lanes, *Journal of Transportation Engineering* 135(2): 45–52. http://dx.doi.org/10.1061/(ASCE)0733-947X(2009)135:2(45)

Zhang, H. M.; Ge, Y. E. 2004. Modeling variable demand equilibrium under second-best road pricing, *Transportation Research Part B: Methodological* 38(8): 733–749. http://dx.doi.org/10.1016/j.trb.2003.12.001

Zhang, L.; Lawphongpanich, S.; Yin, Y. 2009. Reformulating and solving discrete network design problem via an active set technique, in *Proceedings of 18th International Symposium 'Transportation and Traffic Theory'*, 16–18 July 2009, Hong Kong, China, 283–300.

Zhang, L.; Sun, J. 2013. Dual-based heuristic for optimal cordon pricing design, *Journal of Transportation Engineering* 139(11): 1105–1116. http://dx.doi.org/10.1061/(ASCE)TE.1943-5436.0000591

Zhang, X.; Yang, H. 2004. The optimal cordon-based network congestion pricing problem, *Transportation Research Part B: Methodological* 38(6): 517–537. http://dx.doi.org/10.1016/j.trb.2003.08.001

Zuo, Z.; Kanamori, R.; Miwa, T.; Morikawa, T. 2010. Comparison of cordon and optimal toll points road pricing using genetic algorithm, in *Proceedings of the Seventh International Conference on Traffic and Transportation Studies 2010*, 3–5 August 2010, Kunming, China, 535–544. http://dx.doi.org/10.1061/41123(383)51