A NOVEL SOLUTION FOR GROUND REACTION CURVE OF TUNNELS IN ELASTOPLASTIC STRAIN SOFTENING ROCK MASSES

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Abstract. Ground Reaction Curve (GRC) is one of the most important elements of convergence-confinement method generally used to design tunnels. Realistic presentation of GRC is usually assessed based on the advanced rock strength criteria, also, rock mass behavior (including plasticity and softening treatments). Since taking these parameters into account is not simply possible for practitioners and needs complicated coupled theoretical-numerical solutions, this paper presents a simple novel approach based on Evolutionary Polynomial Regression to determine GRC of rock masses obeying both Mohr-Coulomb and Hoek-Brown criteria and strain softening behaviors. The proposed models accurately present support pressures based on radial displacement, rock mass strength and softening parameter (determination coefficient of 97.98% and 94.2% respectively for Mohr-Coulomb and Hoek-Brown strain softening materials). The accuracy of the proposed equations are approved through comparing the EPR developed GRCs with the ground reaction curves available in the literature. Besides, the sensitivity analysis is carried out and in-situ stress, residual Hoek-Brown's *m* constant and residual dilation angle are introduced as parameters with the most influence on the support pressure in Hoek-Brown and peak and residual geological strength index are the most affective parameters on the support pressure of tunnels in the strain softening Mohr-Coulomb rock mass.

Keywords: ground reaction curve, circular tunnel, rock mass, Mohr-Coulomb, Hoek-Brown, strain softening, evolutionary polynomial regression, sensitivity analysis.

Introduction

Tunneling projects consist of different phases including pre-construction studies, risk assessment, cost evaluation, design phase, construction, monitoring (sometimes some of the mentioned phases will be merged when performance based design is the selected design method) (Sayadi *et al.* 2012; Hasanzadehshooiili *et al.* 2012a, 2012b; Ghorbani *et al.* 2013; Zavadskas *et al.* 2015; Zhang *et al.* 2016). One of the major issues in design of tunnels is the loads carried by tunnel linings (Ghorbani *et al.* 2013). Nevertheless, there are some main problems such as uncertainties in rock nature, variation of ground condition, ground deformation induced by redistribution of in-situ stresses and difficult computational procedure, which makes it difficult to accurately and easily obtain the pressure on and sustained by internal support of rock circular tunnels (Hasanzadehshooiili *et al.* 2012b). In this regard, it is common among practitioners to design tunnel structures more conservatively. Design of support system of tunnels, due to the problem complexities, has always been one of the most important and complicated problems in the tunnel engineering field (Hasanzadehshooiili *et al.* 2012a). Since rock mass properties are not the only parameters controlling rocks' stress-strain behavior, special attentions should be paid to the support type and the time of support installation, which makes it an interaction rock-support problem. Solutions proposed for the rock-support interaction problem can be categorized in the following ones: closed-form analytical methods, empirical methods, numerical methods (e.g. finite element, finite difference, distinct element and boundary element methods). Among the mentioned methods, convergence-confinement method is one of the most in demanding design methods widely used by tunnel and rock mechanics engineers. This method is founded based on three fundamental elements consisting longitudinal deformation profile (LDP), support characteristic curve (SCC) and ground reaction curve (GRC) (Brown *et al.* 1983).

As a simple explanation, the radial support pressure, P_i, required at a particular point on the boundary of excavation to limit the radial convergence to U_i, is represented by the ground reaction curve (Fig. 1), where U_i is radial tunnel displacement and P_i represents corresponding internal support pressure of tunnels.
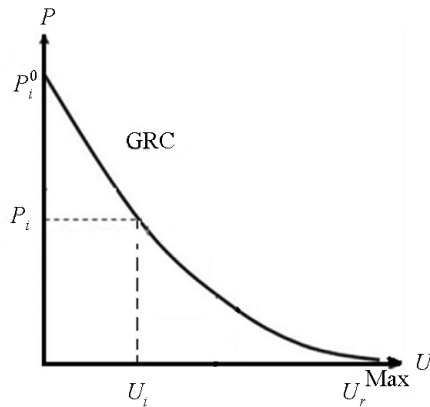
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Fig. 1. Schematic view of an idealized GRC for circular tunnel

Fig. 2. Different post peak rock mass behaviors (Hoek, Brown 1997)

Considering the parameters affecting on the problem, constituting elements of convergence-confinement method, especially ground reaction curve, are studied based on different theoretical, numerical or coupled theoretical-numerical methods. In development of ground reaction (response) curve, changes in the support pressures due to the tunneling is tracked near the tunnel face and the concerning radial tunnel displacement-support pressure curve is reported as the GRC, which is conjunctly used with SCC to gain the optimum time of installation of the support system.

1. Literature survey

Ground reaction curves are usually developed based on elastic-brittle, elastic perfect plastic and elastic brittle plastic rock mass behavior considering strain softening. Fully theoretical schemes only present solutions to the elastic brittle or elastic perfect plastic rock mass behaviors (Sharan 2003, 2005, 2008). Besides, in such cases, simple strength criteria, post peak behaviors or some main simplifications are often assumed to idealize the problem. Hence, since it is not possible to find the analytical closed form solutions to the more realistic rock mass behavior, complicated strain-softening material behavior, the GRC is often calculated using coupled theoretical-numerical approaches (Alonso *et al.* 2003; Guan *et al.* 2007; Lee, Pietruszczak 2008; Park *et al.* 2008; Alejano *et al.* 2009, 2010; Wang, Yin 2011; Zhang *et al.* 2012; Zareifard, Fahimifar 2012; González-Cao *et al.* 2013). Figure 2 presents different post peak behaviors of rock mass. In this figure, GSI represents geological strength index originally proposed by Hoek and Brown (1997).

As it can be seen in Figure 2, elastic brittle plastic and elastic perfectly plastic behaviors can be referred as two special cases for the strain softening model (Alejano *et al.* 2009). In strain softening models, material's behavior is commonly described using the peak and residual strength parameters for the materials. Indeed, it is assumed that the material follows a gradual transition from
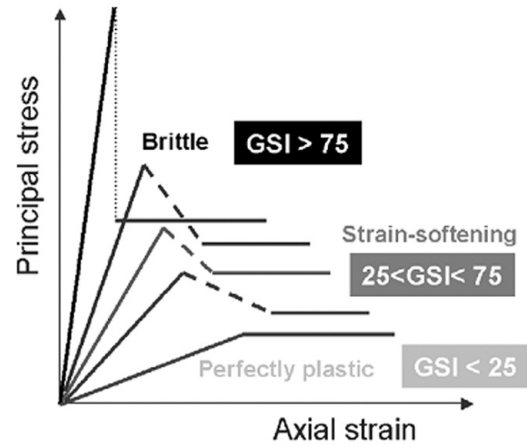
peak to the residual strength and the material's strength parameters are linearly decreased from their peak values to the corresponding residual ones. Slope of the transition line and the initiation of residual behavior are defined based on plasticity calculations (in a direct or an indirect way).

The proposed relations between materials peak and residual strength parameters for Hoek-Brown failure criterion are presented in Eqns (1)–(2) (Alejano *et al.* 2010):

$$
m(\eta) = \begin{cases} m^{peak} - \dfrac{m^{peak} - m^{res}}{\eta^*}\eta & 0 < \eta < \eta^* \\ m^{res} & \eta \ge \eta^* \end{cases}; \quad (1)
$$

$$
s(\eta) = \begin{cases} s^{peak} - \dfrac{s^{peak} - s^{res}}{\eta^*}\eta & 0 < \eta < \eta^* \\ s^{res} & \eta \ge \eta^* \end{cases}, \quad (2)
$$

where m^{peak} and s^{peak} present peak strength Hoek-Brown parameters and m^{res} and s^{res} are its residual parameters. Also, η^* is controlling softening parameter, which can be calculated in different ways. Hence, in order to derive GRC curve for a strain softening rock mass, it is essential to calculate strain softening parameter and to solve the governing differential equations simultaneously (equilibrium, compatibility and consistency equations along with plasticity rules and assumptions). Some of the GRC problems have been recently solved for different rock strength criteria showing different plasticity treatments and softening behaviors (Alonso *et al.* 2003; Mitaim, Detourney 2005; Xu, Yu 2006; Guan *et al.* 2007; Lee, Pietruszczak 2008; Fahimifar, Ranjbarnia 2009; Alejano *et al.* 2009, 2010; Osgoui 2009; Zhang *et al.* 2010; Zareifard, Fahimifar 2012; Zhang *et al.* 2012; González-Cao *et al.* 2013; Park 2014). Nevertheless, there is not still a general solution for some nonlinear failure criteria and softening behaviors for variable and stress dependent

dilatancies under different loading conditions. Besides, using such computations is not so easy for practitioners and tunnel engineers. This is the main reason that advanced GRC solutions (strain softening) are not commonly used by tunnel engineers using simple calculations. In this regard, in this paper, two different rock strength criteria (Mohr-Coulomb and Hoek-Brown failure criteria) are considered and some of the available authenticated strain softening solutions are gathered and back calculated. Following, a simple novel approach to determine GRC curve of circular rock mass tunnels has been described and relations for both Mohr-Coulomb and Hoek-Brown failure criteria are presented. It is believed that these relationships can be efficiently used by practitioners to gain the GRC of Mohr-Coulomb and Hoek-Brown strain softening rock masses in preliminary design step of tunnels with some simple calculations.

2. Problem statement

As described in previous section, two different failure criteria, Mohr-Coulomb and Hoek-Brown strength criteria are considered and the available authenticated coupled theoretical-numerical solutions to the GRC of strain softening materials have been gathered. Following, general assumptions and governing rules of conventional solutions to the problem of ground reaction curve of a circular opening in an elastic-plastic strain softening rock mass are presented.

2.1. Conventional GRC problem of circular tunnels in an elastoplastic strain softening rock mass

In theses methods, material's strength criteria are assumed as the yield criteria and are used for the solution of material's governing differential equations. Also, plastic potential function, flow rule and treatment with strain softening parameters are other three important elements defining GRC. Figure 3 shows different regions formed around circular tunnels, elastic and plastic regions.
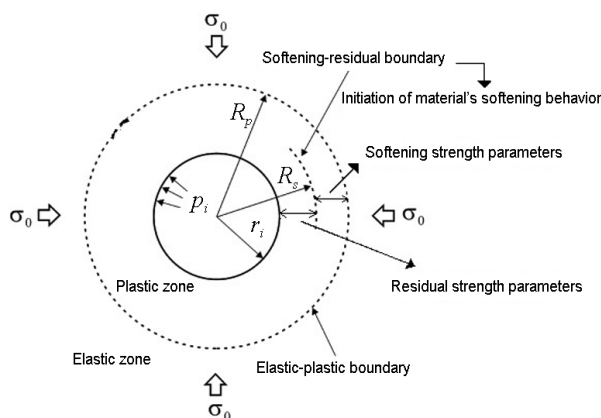

Fig. 3. General schematic regions around circular tunnel under hydrostatic stress field

Moreover, softening-residual boundary showing the boundary where material behavior is changed from softening to the residual is depicted. Indeed, beyond elastic-plastic boundary, material behaves perfectly elastic. Moving toward the tunnel boundary, plastic behavior initiates. In this zone, firstly, the peak strength parameters are mobilized and linearly softened to the residual values of strength parameters. There are different treatments to find the softening-residual boundary, also, the corresponding strength parameters (Lee, Pietruszczak 2008; Alejano *et al.* 2009, 2010). Some of them define a strain softening parameter (generally $\eta = \varepsilon_1^{plas} - \varepsilon_3^{plas}$) and an iterative procedure to cover the whole plastic region, calculating η successively and comparing it with the critical strain softening parameter, η^* (Lee, Pietruszczak 2008). The values of strain softening parameter beyond η^* declares that the material's strength parameters are changed to their residual ones, while lower values show that they are still in softening zone and can be determined using Eqns (1) and (2). On the other hand, some other methods use $\sigma_1 - \sigma_3$, $\sigma_1 - \varepsilon_1$, the slope of $\varepsilon_1^{plas} - \varepsilon_3^{plas}$ curve (defined by coefficient of dilation) and the slope of transition line in strain softening region to obtain the values of ε_1^{plas} and ε_3^{plas} to calculate the value of η and its critical value (Alejano *et al.* 2009, 2010). Such methods use an implicit scheme to solve the governing equilibrium and compatibility equations. The governing equilibrium equation in plane strain condition and cylindrical coordinate system for axial symmetry situation is as following (Park *et al.* 2008; Lee, Pietruszczak 2008):

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \quad (3)$$

where σ_r and σ_θ, respectively, present radial and hoop stresses. Also, r shows distance from the tunnel's centre. Moreover, relationship between radial and tangential plastic strains is defined using a suitable plastic potential function and applying associative/non-associative flow rule. The common general form of assumed plastic potential function, also, applied flow rule (commonly non-associative flow rule is considered) are respectively presented in Eqn (4) and Eqn (5) (Park *et al.* 2008; Lee, Pietruszczak 2008; Alejano *et al.* 2009, 2010; Zareifard, Fahimifar 2012):

$$G(\sigma_r, \sigma_\theta, \eta) = \sigma_\theta - k(\eta)\sigma_r; \quad (4)$$

$$d\varepsilon_r^{plas} = -k(\eta)d\varepsilon_\theta^{plas}, \quad (5)$$

where G is the plastic potential function and $d\varepsilon_r^{plas}$, $d\varepsilon_\theta^{plas}$ and $k(\eta)$ are radial increment of plastic strain, tangential increment of plastic strain and coefficient of dilation, respectively. Also, η represents strain softening parameter.

Considering the described methods, GRC of circular tunnels are calculated using one of the iterative or implicit solutions. In this paper, in order to develop some

new relationships for GRC, some of these solutions are gathered and further analyzed. In this regard, results of Park *et al.* (2008), Lee and Pietruszczak (2008), Alejano *et al.* (2009, 2010), Zareifard and Fahimifar (2012) presenting GRC curves for Mohr-Coulomb or Hoek-Brown criterion and satisfying strain softening situation are more focused and investigated.

3. Data division for model development

As explained earlier, GRC of the available sources along with their peak and residual strength parameters, also, critical strain softening parameters are gathered. Back calculating the obtained ground reaction curves, different P_i, internal support pressures of tunnels, and U_i, radial displacements of tunnel wall, values are digitized. Indeed, as an example and for a specific case, the entire peak, residual and strain softening parameters are assumed as input parameters. Besides, the points of GRC of the studied case are digitized. The values of displacements gained from the curves are also considered as the input parameter (each of the values of displacements makes a dataset, considering other strength and softening parameters) and the corresponding support pressures are used as the output parameters. Hence, it can be noted that each GRC constructs a few dozen of input-output datasets. Then, P_i, which is now considered as a function of tunnel wall radial displacement, rock mass strength and residual parameters and softening parameters, is then predicted using evolutionary polynomial regression modeling.

Regarding the fact that during data gathering phase, a wide range of material qualities and behaviors have been covered, from poor quality to hard rock, the resulting relations for P_i (with a high degree of correlations with available solutions) can then be used for new rock strength and softening parameters in the early design stage of tunneling projects. The resulting $P_i - U_i$ relation can then be used as a first order estimation of GRC of tunnels in strain softening rock mass.

3.1. GRC modeling based on the strain softening Mohr-Coulomb strength criterion

In order to cover a wide range of material strength properties, three different cases studied by Alejano *et al.* (2009) are investigated and the concerning parameters are back calculated.

Tables 1–3, respectively, present the materials' strength parameters and strain softening parameter for soft, medium and hard rock masses, where "peak" and "res" superscripts respectively represent peak and residual strength parameters and GSI, σ_{ci}, γ, E, ϑ, ψ and σ_0 are respectively geological strength index, uni-axial compressive strength, unit weight, Young's modulus, Poisson's ratio, dilation angle and in-situ stress of rock mass.

Also, c, φ and m_i are material constants. Besides, r_i is the radius of tunnel and η^* represents the strain softening parameter and is determined using

Table 1. Strength and softening parameters for soft rock mass used in Mohr-Coulomb model (Alejano *et al.* 2009)

Parameters	Unit	Value
GSI^{peak}	–	21.4
GSI^{res}	–	15.1
σ_{ci}	MPa	100
m_i	–	20
γ	kN/m^3	26
E	GPa	1.1
ϑ	–	0.3
c^{peak}	MPa	0.34
φ^{peak}	°	43
c^{res}	MPa	0.27
φ^{res}	°	40
σ_0	MPa	10.4
ψ	°	0
η^*	–	0.1394

Table 2. Strength and softening parameters for medium rock mass used in Mohr-Coulomb model (Alejano *et al.* 2009)

Parameters	Unit	Value
GSI^{peak}	–	55
GSI^{res}	–	33
σ_{ci}	MPa	23
m_i	–	10
γ	kN/m^3	26.7
E	GPa	3.837
ϑ	–	0.25
c^{peak}	MPa	0.744
φ^{peak}	°	24.81
c^{res}	MPa	0.397
φ^{res}	°	15.69
σ_0	MPa	12
ψ	°	3.1
η^*	–	0.0929

Table 3. Strength and softening parameters for hard rock mass used in Mohr-Coulomb model (Alejano *et al.* 2009)

Parameters	Unit	Value
GSI^{peak}	–	64.9
GSI^{res}	–	27.8
σ_{ci}	MPa	162
m_i	–	19
γ	kN/m³	26
E	GPa	24
ϑ	–	0.25
c^{peak}	MPa	3.7
φ^{peak}	°	57.8
c^{res}	MPa	0.96
φ^{res}	°	51
σ_0	MPa	26
ψ	°	14
η^*	–	0.0465

Eqns (6)–(8) for soft, medium and hard rock, respectively (Alejano *et al.* 2009):

$$\eta^* = 6\frac{\sigma_1^{peak}(\sigma_3) - \sigma_1^{res}(\sigma_3)}{E}\left(1 + \frac{k_\psi}{2}\right); \qquad (6)$$

$$\eta^* = 4\frac{\sigma_1^{peak}(\sigma_3) - \sigma_1^{res}(\sigma_3)}{E}\left(1 + \frac{k_\psi}{2}\right); \qquad (7)$$

$$\eta^* = 2\frac{\sigma_1^{peak}(\sigma_3) - \sigma_1^{res}(\sigma_3)}{E}\left(1 + \frac{k_\psi}{2}\right), \qquad (8)$$

where k_ψ is dilation parameter and is defined as the slope of $\sigma_1 - \sigma_3$ diagram in Alejano *et al.* (2009) for the strain softening model with constant dilatancy and constant drop modulus. It is noteworthy to imply that the effect of depth of tunnel is also taken into account considering the far field intact stress, σ_0.

Regarding described cases and considering concerning strength and softening parameters, $P_i - U_i$ curve has been digitized and back calculated. Considering U_i as an input parameter, different values of P_i will be gained as a function of strength parameters, softening parameter and U_i. Regarding the described procedure, 168 independent data sets $P_i(r_i, GSI^{peak}, GSI^{res}, \sigma_{ci}, \gamma, E, \vartheta, c^{peak}, \varphi^{peak}, c^{res}, \varphi^{res}, m_i, \psi, \sigma_0, \eta^*, U_i)$ have been constructed ready for evolutionary polynomial regression modeling.

3.2. GRC modeling based on the strain softening Hoek-Brown strength criterion

The source for strength and softening parameters used to model GRC of Hoek-Brown strain softening rock mass (along with their treatment with strain softening parameter) are presented in Table 4, where M represents drop modulus (slope of transition line in strain softening zone) and E is material's Young's modulus.

Ground reaction curves developed using the input parameters of these cases are gathered. Besides, all the (U_i, P_i) data pairs are back calculated from these obtained GRCs. Combining other input parameters, strength and softening parameters, with the back calculated (U_i, P_i) data pairs, a total 547 numbers of data series are achieved considering all the studied cases. Based on the mentioned 547 gathered data series, support pressure is sorted as the function of 17 input parameters $P_i\left(r_i, GSI^{peak}, GSI^{res}, \sigma_{ci}, m^{peak}, s^{peak}, a^{peak}, m^{res}, s^{res}, a^{res}, E, \vartheta, \psi^{peak}, \psi^{res}, \sigma_0, \eta^*, U_i\right)$, where m, s and a represent Hoek-Brown constants of materials.

Then, using evolutionary polynomial regression modeling, the prepared data series are used to predict the available support pressure based on the input parameters and their relationship with the available support pressure.

4. Sensitivity analysis

In order to investigate the strength of relationship between input parameters and internal support pressure (to compare the degree of importance of each input parameter), the Cosine Amplitude Method (CAM) is considered. It should be noted that since U_i was modeled as a fictional input parameter to model GRC, investigating its strength of relationship with internal support pressure as a variable does not mean and hence, here, its effect has not been investigated.

It should be noted that before carrying out the sensitivity analysis, all the input and output data are normalized and dimensionless in 0–1 scale using Eqn (9) (Ghorbani *et al.* 2014):

Scaled Value = (*unscaled value* – min. *value*) / (max. *value* – min. *value*).   (9)

Thus, using CAM, the express similarity relation between the target function and the input parameters is attained. To do this, all of data pairs are expressed in the common *X*-space. They would form a data array *X* defined as the Eqn (10) (Ghorbani *et al.* 2014):

$$X = \{x_1, x_2, x_3, x_4, ..., x_i, ... x_n\}, \qquad (10)$$

where each element, x_i, as shown in Eqn (11), is a vector of the length of *m* (Ghorbani *et al.* 2014):

$$x_i = \{x_{i1}, x_{i2}, x_{i3}, ..., x_{im}\}. \qquad (11)$$

Therefore, each of the datasets can be considered as a point in the *m*-dimensional space, where each point

Table 4. The applied references for gathering strength parameters and deriving softening parameter used to model GRC based on HB and strain softening

Hoek and Brown (1997)	In conjunction with Cai *et al.* (2007) and Alejano *et al.* (2010)
Cai *et al.* (2007)	In conjunction with Hoek and Brown (1997) and Alejano *et al.* (2010)
Lee and Pietruszczak (2008)	$\gamma_i^p = \gamma_{(i-1)}^p + \left(\Delta\varepsilon_{\theta(i)}^p - \Delta\varepsilon_{r(i)}^p \right)$
Park *et al.* (2008)	$\eta^* = (\alpha - 1)\varepsilon_{\theta e}$
Zareifard and Fahimifar (2012)	$\gamma^p = \varepsilon_\theta^p - \varepsilon_r^p$ $\dot{\varepsilon}_r^p = \dot{\lambda}\dfrac{\partial g}{\partial \sigma_r}$ $\dot{\varepsilon}_\theta^p = \dot{\lambda}\dfrac{\partial g}{\partial \sigma_\theta}$
Alejano *et al.* (2010)	$\varepsilon_1^{plas} = \left(\sigma_1^{peak}(\sigma_3) - \sigma_1^{res}(\sigma_3) \right)\left[\dfrac{1}{E} - \dfrac{1}{M} \right]$ $\varepsilon_3^{plas} = -\dfrac{1}{2}\dfrac{1+sin\psi}{1-sin\psi}\varepsilon_1^{plas}$ $\gamma^{p*} = \eta^* = \varepsilon_1^{plas} - \varepsilon_3^{plas}$

requires *m*-coordinates to be fully described. Eqn (12) presents the strength of the relationship between x_i and x_j (Hasanzadehshooiili *et al.* 2012b; Ghorbani *et al.* 2014):

$$r_{ij} = \frac{\sum\limits_{k=1}^{m} x_{ik}x_{jk}}{\sqrt{\sum\limits_{k=1}^{m} x_{ik}^2 \sum\limits_{k=1}^{m} x_{jk}^2}}. \qquad (12)$$

Figures 4 and 5, respectively, show the strength of relationship between input parameters and internal sup-

port pressure for Mohr-Coulomb and Hoek-Brown strain softening cases.

As it can be seen, in-situ far field stress σ_0, m^{res} and ψ^{res} are the most affective parameters on the internal support pressure of tunnels in Hoek-Brown strain softening rock mass and peak and residual geological strength index of materials will more affect the support pressure of a strain softening Mohr-Coulomb rock mass. Besides, as it can be seen, γ and ϑ are the least affective parameters on the GRC of strain softening Mohr-Coulomb material. Moreover, a^{res} and η^* have the least influence on the GRC of strain softening Hoek-Brown
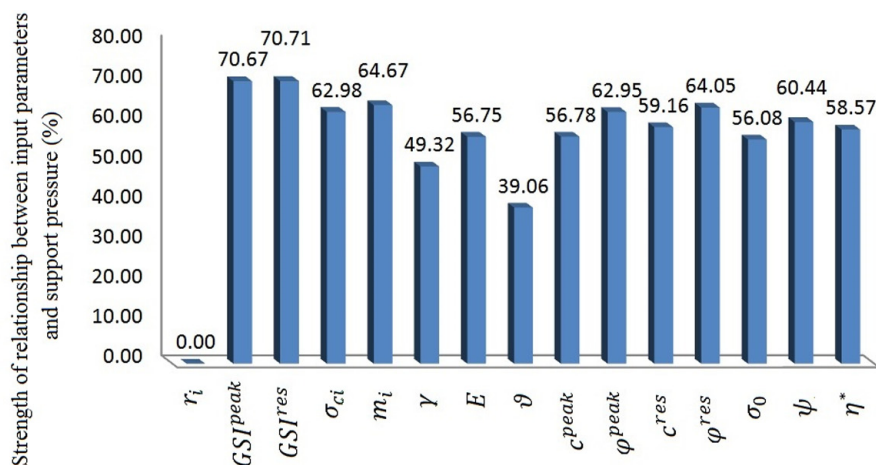

Fig. 4. Strength of relationship between internal support pressure and concerning parameters for the case of Mohr-Coulomb strain softening model
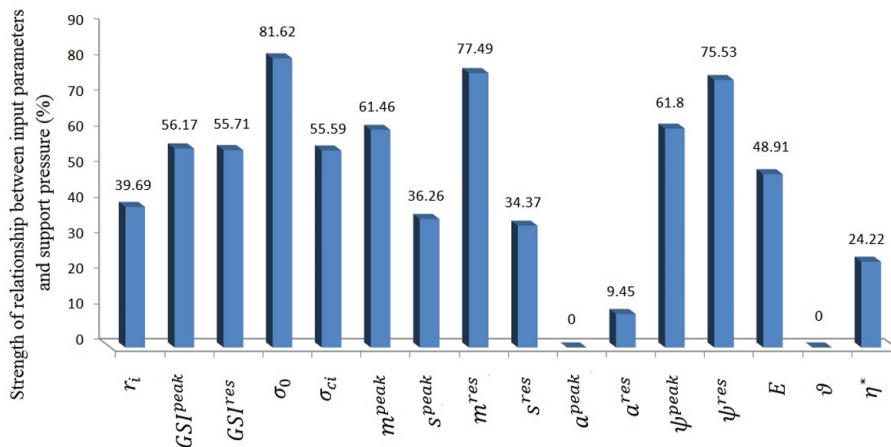
Fig. 5. Strength of relationship between internal support pressure and concerning parameters for the case of Hoek-Brown strain softening model

material. In addition, radius of tunnel in strain softening Mohr-Coulomb material, also, a^{peak} and ϑ in Hoek-Brown strain softening material are constant input parameters in the development of all the studied ground reaction curves. Hence, the strength of relationship between these parameters and internal support pressure is reported as zero. Nevertheless, their values will vary in the case of studying larger databases, in which different r_i, a^{peak} and ϑ values are available and used.

5. Evolutionary polynomial regression modeling

Different branches of artificial intelligence (AI) are applied to predict various complicated civil engineering functions (Baziar, Ghorbani 2005; Ghorbani *et al.* 2012; Sadrossadat *et al.* 2013; Sadowski, Nikoo 2014; Hasanzadehshooiili *et al.* 2014; Fiore *et al.* 2014; Gandomi *et al.* 2015; Sadowski, Hoła 2015; Yin *et al.* 2016; Turskis *et al.* 2016). Among the AI based modeling methods, EPR is classified as a symbolic grey box technique which can clearly make structured model expressions for a given dataset in a way that is shown in Figure 6. This figure

Fig. 6. EPR classification along with other modeling techniques (Rezania 2008)

represents EPR classification along with other modeling methods (Rezania 2008).

EPR, which is a new hybrid regression method, combines the best features of conventional numerical regression techniques with the genetic programming symbolic regression technique. The main idea of this method is to employ an evolutionary computing methodology to search for a model of the system/process being modeled and to employ parameter estimation to obtain constants using least squares (Giustolisi, Savic 2006).

In genetic programming, a general evolutionary search is conducted in the evolutionary procedure while the exponents of a polynomial function with a fixed maximum number of terms are searched in EPR. Hence, the problem of mathematical expressions will be kept away from growing rapidly in length with time in evolutionary polynomial regressions. Moreover, selection of the optimum term numbers is permitted during one execution. Indeed, to do this, it returns some expressions with increasing numbers of terms up to a user defined threshold. The general expression form used in EPR is as Eqn (13) (Giustolisi, Savic 2006):

$$y = \sum_{j=1}^{m} F\left(X, f\left(X\right), a_j\right) + a_0, \qquad (13)$$

where y represents the estimated vector of output, a_j is a constant, F is a function constructed by the process, X is the matrix of input variables, f is a user defined function and m is the maximum term numbers of the target. Transferring Eqn (13) to the following vector form (Eqn (14)) is the first model identification step (Giustolisi, Savic 2006):

$$Y_{N \times 1}\left(\theta, Z\right) = \begin{bmatrix} I_{N \times 1} & Z_{N \times m}^j \end{bmatrix} \times \begin{bmatrix} a_0 & a_1 & \dots & a_m \end{bmatrix}^T =$$
$$Z_{N \times d} \times \theta_{d \times 1}^T, \qquad (14)$$

where $Y_{N \times 1}\left(\theta, Z\right)$ represents the least squares estimation vector of the N target values, $\theta_{1 \times d}$ is the vector
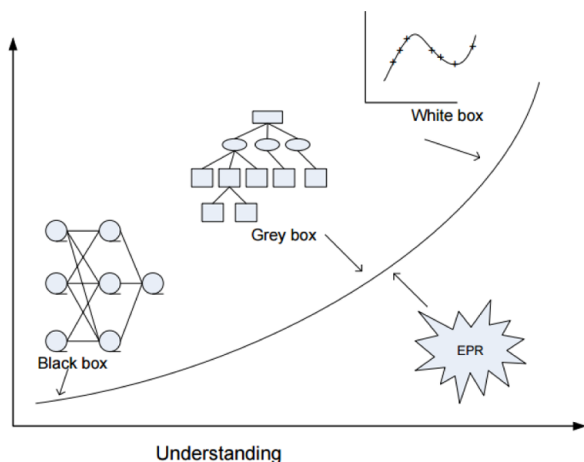
of $d = m+1$ parameters a_j and a_0 (superscript "T" represents the transposed vector), and $Z_{N \times d}$ is a matrix established by unitary vector for bias a_0, and m vectors of variables Z^j that for fixed j are a product of the independent forecasters vectors of inputs, $X = \langle X_1 \ X_2 \ ... \ X_K \rangle$. Generally, construction of EPR models is formed in two stages. Firstly, the best function structure is realized using standard genetic algorithm and secondly, for each input combinations, the adjustable parameters are found conducting a least squares regression. Hence, a global search algorithm is put into the action for both the best set of input combinations and related exponents simultaneously, based on the cost function defined by the user. Eqn (15) presents matrix of inputs (Giustolisi, Savic 2009):

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} & ... & x_{1k} \\ x_{21} & x_{22} & x_{23} & ... & x_{2k} \\ x_{31} & x_{32} & x_{33} & ... & x_{3k} \\ ... & ... & ... & ... & ... \\ x_{N1} & x_{N2} & x_{N3} & ... & x_{Nk} \end{bmatrix} = [X_1 \ X_2 \ X_3 \ ... X_K \,],$$

(15)

where jth term in Eqn (14) is represented by kth column of X in Eqn (15). Hence, jth term of Z in Eqn (14) will be re-written as Eqn (16) (Giustolisi, Savic 2009):

$$Z_{N \times 1}^j = \left[(X_1)^{ES(j,1)} . (X_2)^{ES(j,2)} . (X_3)^{ES(j,3)} \, ... \right.$$
$$\left. (X_k)^{ES(j,k)} \right] \ \forall j = 1...m.$$

(16)

In Eqn (16), Z^j represents jth column vector and product of candidate independent inputs makes its elements. Also, ES is exponents' matrix. The final goal will be finding $ES_{m \times k}$ matrix of exponents in a way that its elements hold the values between user-defined limits. Then, based on the calculated $ES_{m \times k}$ matrix and using least square method (LS) and trying to minimize sum of squared errors (SSE) as the cost function, a_j will be evaluated. The expression used in EPR as the typical flow diagram is shown in Figure 7 (Giustolisi, Savic 2009).

In the evolutionary process of building EPR models, a number of constraints can be implemented to control the output models in terms of the type of functions used,
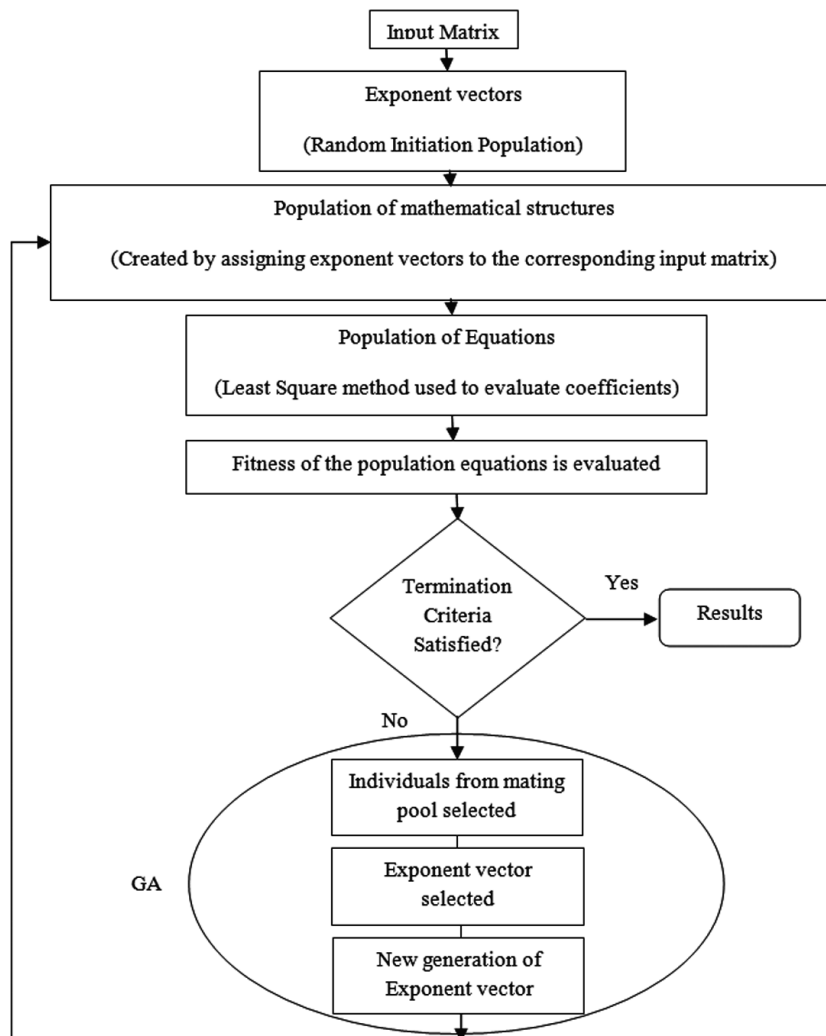

Fig. 7. Typical flow diagram for EPR procedure (Ahangar-Asr *et al.* 2011)

number of terms, range of exponents, number of genera-tions, etc. (Rezania *et al.* 2011). By increasing the num-ber of evolutions, it gradually picks up different partici-pating parameters in order to form equations describing the relationship between the parameters of the system. The level of accuracy of the output models is evaluated using the coefficient of determination. And, if the fitness of the model is not acceptable or the other termination criteria (in terms of maximum number of generations and maximum number of terms) are not satisfied, in order to gain a new model, the current model goes through an-other evolution (Ahangar-Asr *et al.* 2011).

6. Results and discussion

6.1. EPR models based on Mohr-Coulomb strength criterion and strain softening

As it was described in the previous sections, 16 input parameters (including U_i) is used to train and predict the values of P_is, P_i(r_i, GSI^{peak}, GSI^{res}, σ_{ci}, γ, E, ϑ, c^{peak}, φ^{peak}, c^{res}, φ^{res}, m_i, ψ, σ_0, η^*, U_i).

Eqn (17) presents the relation obtained for predict-ing P_i based on the input parameters:

$$P_i = \left(4.4062 \times 10^{-7} \times e^{2\psi}\right) + \left(5.9109 \times 10^{-11} \times e^{E} \times U_i\right) -$$
$$\left(0.0015057 \times (\eta^*)^2 e^{\sigma_0}\right) + \left(0.0032054 \times \vartheta \times (\varphi^{res})^{0.5} \times U_i\right) -$$
$$\left(0.00044979 \times (GSI^{peak})^{0.5} \times \sigma_{ci} \times e^{\left(\vartheta + 0.5c^{peak} + c^{res}\right)} \times U_i^{0.5}\right) +$$
$$5.7746. \tag{17}$$

Figure 8 compares the predicted P_i values (gained based on the new proposed equation) along with the cal-culated P_i values (available in the authenticated literature for different number of datasets). In Eqn (17), the values of U_i are based on mm. Also, P_i values are calculated in MPa.

Besides, Figure 9 shows the GRC points gained by EPR modeling versus their corresponding values avail-able in the literature. As it can be seen, high value of coefficient of determination, 97.98%, shows the high ca-pability of new proposed model in prediction of GRC curve of circular tunnels in strain softening rock masses obeying Mohr-Coulomb criterion.
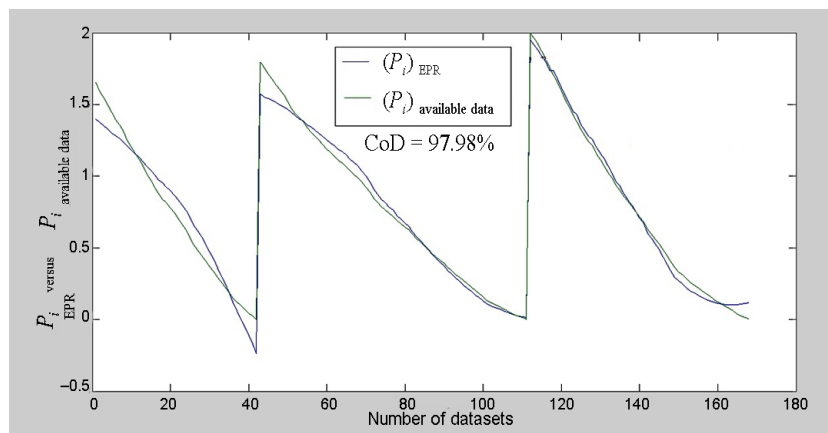

Fig. 8. Comparing the predicted P_i with those available in the literature (based on Mohr-Coulomb strength criterion and strain softening)
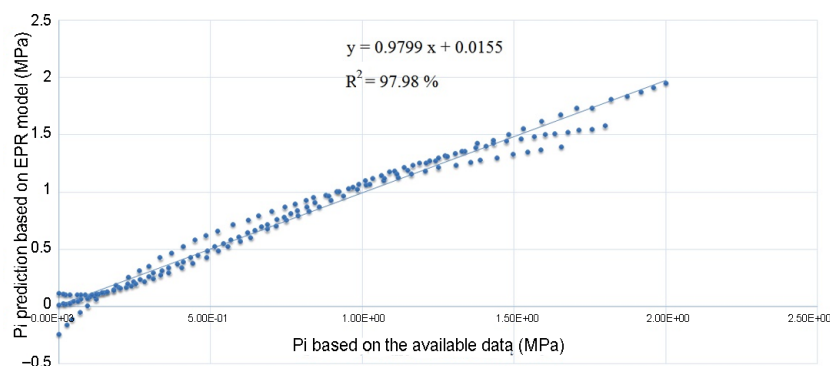

Fig. 9. Coefficient of determination for new proposed EPR-based GRC model for Mohr-Coulomb strength criterion and strain softening

6.1.1. Verification of developed EPR equation for Mohr-Coulomb criterion

In order to evaluate the accuracy of the proposed equation, ground reaction curves available in the literature and used to develop EPR relation are used and the results gained from the new EPR equations are compared with those available in the literature.

Figure 10 presents EPR-proposed GRC versus their corresponding curves in the literature for three cases of Alejano *et al.* (2009), soft, medium and hard quality rock masses. As it can be seen, the proposed equation can be efficiently used as a first order estimation in preliminary design step of tunnels and can accurately predict the GRC of such cases.

## 6.2. EPR models based on Hoek-Brown strength criterion and strain softening

Similar to the previous section, the values of $P_i\left(r_i, GSI^{peak}, GSI^{res}, \sigma_{ci}, m^{peak}, s^{peak}, a^{peak}, m^{res}, s^{res}, a^{res}, E, \vartheta, \psi^{peak}, \psi^{res}, \sigma_0, \eta^*, U_i\right)$ are predicted using EPR modeling based on 547 used data sets.

Equation (18) represents the gained relation for P_i (MPa) based on the concerning input parameters. In this equation, dimension of U_i is meter:

$$P_i = \left(1555.515 \times (a^{peak})^{0.5} \times e^{\left(s^{res} + 0.5U_i\right)}\right) +$$

$$\left(33.5852 \times r_i^{0.5} \times e^{0.5U_i} \times U_i^{0.5}\right) -$$

$$\left(103.3917 \times r_i^{0.5} \times e^{\left(2s^{peak} + 0.5m^{res} + 2U_i\right)} \times U_i^{0.5}\right) +$$

$$\left(7.3712 \times 10^{-5} \times r_i^{0.5} \times \sigma_0^{0.5} \times m^{res} \times (\eta^*)^{0.5} \times e^{\left(0.5\sigma_0 + U_i\right)}\right) -$$

$$\left(0.00016622 \times r_i \times GSI^{res} \times (s^{peak})^{0.5} \times E^{0.5} \times e^{0.5\psi^{res}}\right) -$$

$$1084.1708.$$

$$\text{(18)}$$

Figures 11 and 12 show the performance of the developed equation. As it can be clearly seen in the figures, high acceptable coefficient of correlation, 94.2%, proves the efficiency and the applicability of newly proposed equation.

It should be noted that introducing new strength and softening parameters, both developed models can be efficiently used in the preliminary design step of circular tunnels in strain softening rock masses.
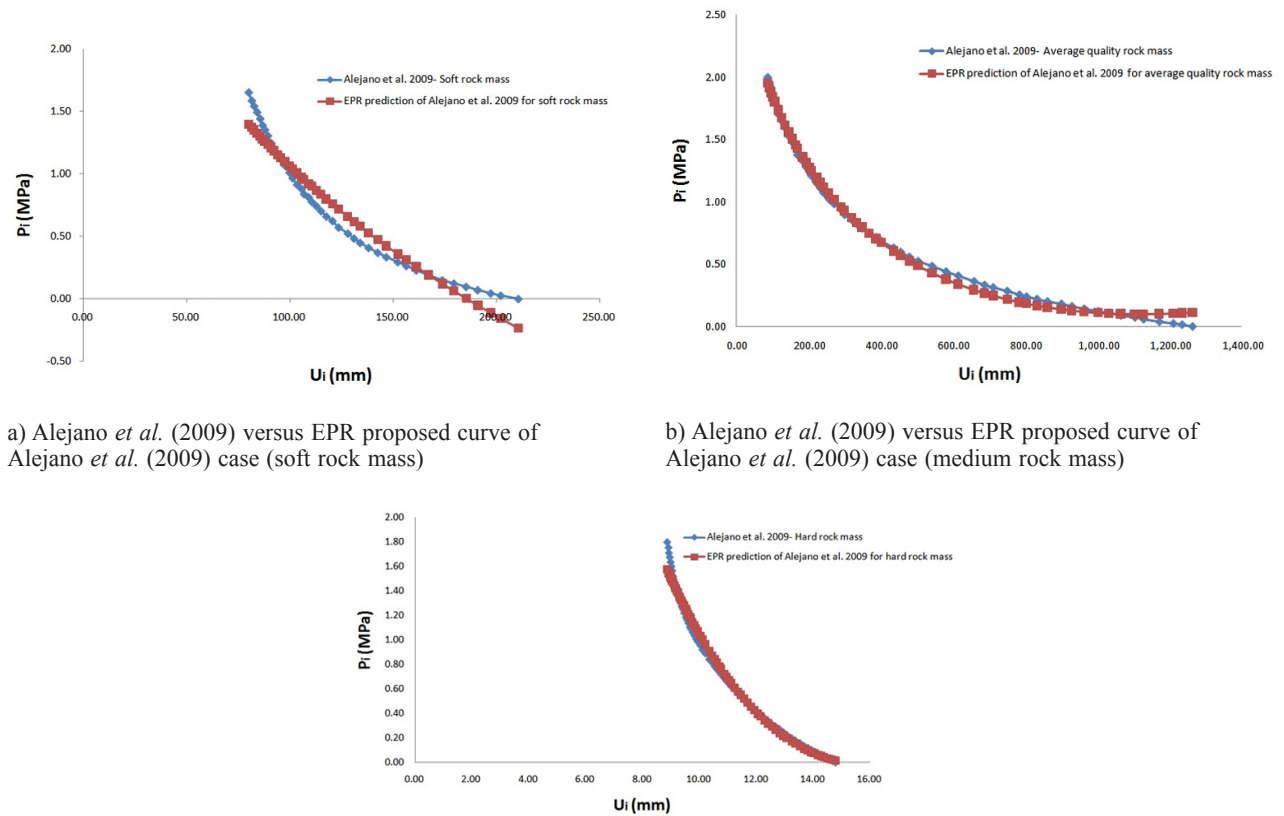
a) Alejano *et al.* (2009) versus EPR proposed curve of Alejano *et al.* (2009) case (soft rock mass)

b) Alejano *et al.* (2009) versus EPR proposed curve of Alejano *et al.* (2009) case (medium rock mass)

c) Alejano *et al.* (2009) versus EPR proposed curve of Alejano *et al.* (2009) case (hard rock mass)

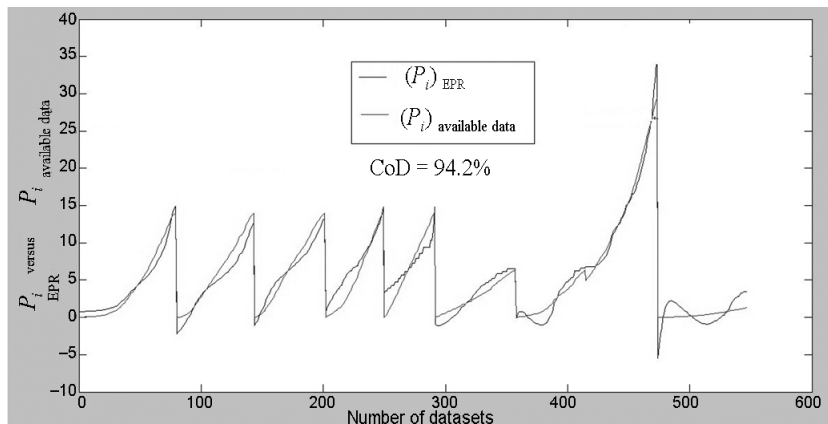Fig. 10. Comparison between available Mohr-Coulomb based GRC and their corresponding curves proposed by EPR

Fig. 11. Comparing the predicted with those available in the literature (based on Hoek-Brown strength criterion and strain softening)
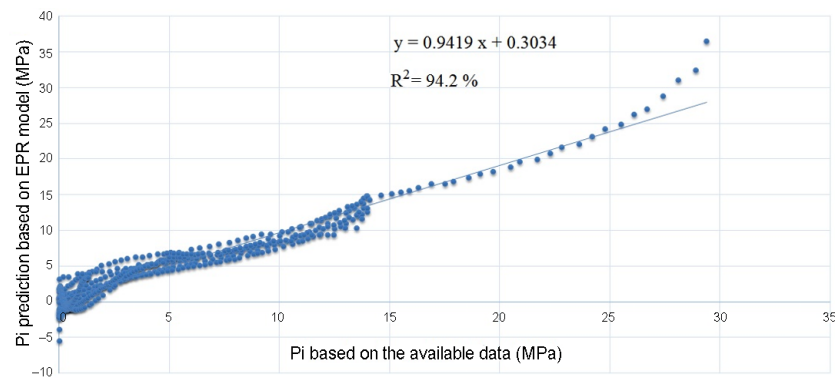

Fig. 12. Coefficient of determination for new proposed EPR-based GRC model for Hoek-Brown strength criterion and strain softening

6.2.1. Verification of developed EPR equation for Hoek-Brown criterion

In this section, accuracy of the proposed equation is examined through modeling five different cases available in the literature. In this regard, the corresponding GRC curves of all these five cases are predicted using the new proposed EPR equation and compared with those available in the literature. As it is shown in Figure 13, the acceptable prediction accuracy of new equation makes it useful for tunnel engineers and practitioners.

Conclusions

Because of difficulties in the development of realistic coupled theoretical-numerical modeling of GRC for strain softening rock masses, this papers aims to present an alternative simple novel model useful for practitioners and tunnel engineers to preliminary design the tunneling projects. In this regard, based on two well-known and widely used rock strength criteria, Mohr-Coulomb and Hoek-Brown failure criteria, two models for prediction of GRC of strain softening rock masses, believed to be more accurate, are presented based on the evolutionary poly-

nomial regression modeling. The developed models are trained using available authenticated solutions for different quality strain softening rock masses and cover a wide range of rock mass qualities. Also, in order to investigate the role of strain softening of rock materials in GRC calculations, considering different plastic strain treatments, the parameter controlling softening of rock materials is also taken into account. It has been shown that the proposed equations are highly capable to predict the values of tunnel's internal support pressure based on rock masses strength and strain softening parameters. It is proved that the new proposed approach can be an efficient alternative for common available methods in the preliminary design steps of tunneling projects. Besides, based on the sensitivity analysis, the most and the least effective parameters on the values of tunnel's internal support pressure are introduced. Based on the CAM, in-situ stress, m^{res} and ψ^{res} are the most affective parameters on the support pressure of rock tunnels in Hoek-Brown strain softening rock mass and peak and residual geological strength index will more affect the support pressure of a strain softening Mohr-Coulomb rock mass.
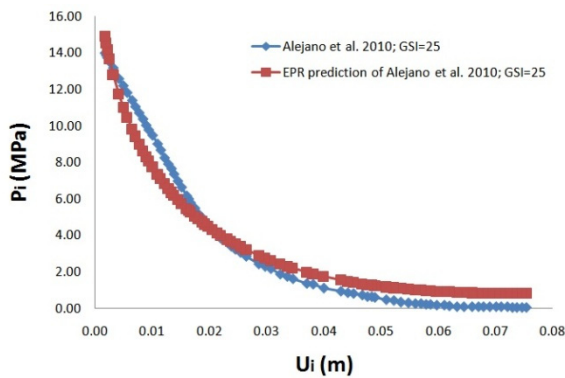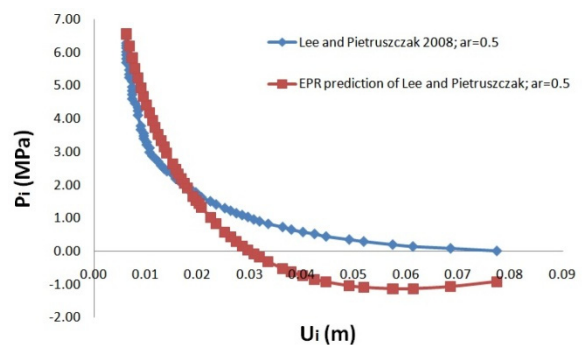
a) Alejano *et al.* (2010) versus EPR proposed curve of Alejano *et al.* (2010) case; GSI = 25
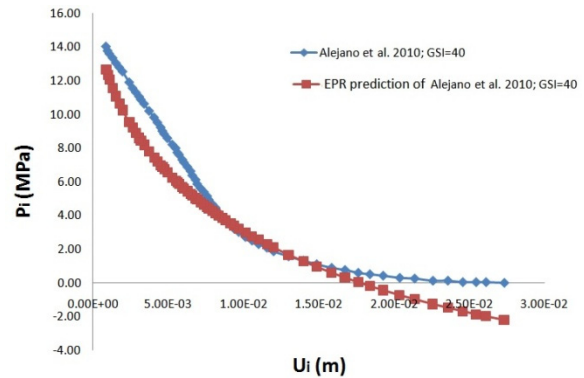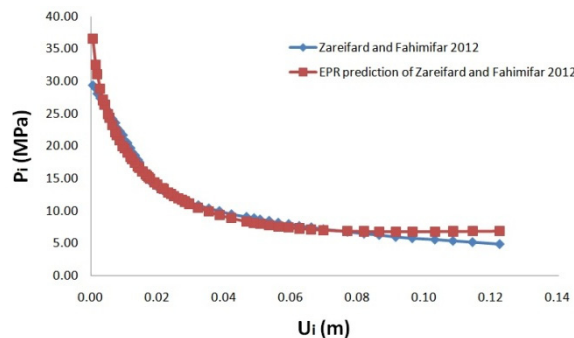
b) Alejano *et al.* (2010) versus EPR proposed curve of Alejano *et al.* (2010) case; GSI = 40
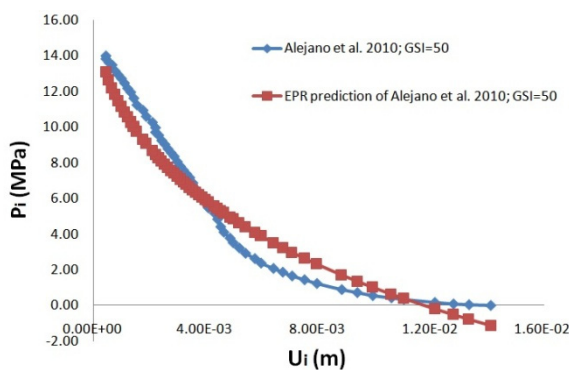
c) Alejano *et al.* (2010) versus EPR proposed curve of Alejano *et al.* (2010) case; GSI = 50

d) Lee and Pietruszczak (2008) versus EPR proposed curve of Lee and Pietruszczak (2008) case; a_r = 0.5

e) Zareifard and Fahimifar (2012) versus EPR proposed curve of Zareifard and Fahimifar (2012) case

Fig. 13. Comparison between some available Hoek-Brown based GRC and their corresponding curves proposed by EPR

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