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BEARING CAPACITY OF STRIP FOOTING ON REINFORCED LAYERED GRANULAR SOILS

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Abstract. In this study a limit equilibrium method is proposed to determine the bearing capacity of strip foundations on geosynthetic reinforced sand soils. A two-layered granular soil was foreseen to represent the loose in situ soil and the compacted fill above the reinforcement. First the modified bearing capacity factors N_q and N_γ were derived for the two layered granular reinforced soil. The bearing capacities were also calculated for different reinforcement geometries and soil properties using Finite Element analyses. The bearing capacities obtained from Finite Element and Limit Equilibrium analyses were compared, it was seen a good agreement. Therefore, it was concluded that the new limit equilibrium method proposed in this paper for reinforced two-layered soils can be successfully used in calculating the bearing capacities of geosynthetic reinforced soils.

Keywords: bearing capacity, reinforced soil, tensile force, layered soils, strip footing, finite elements.

Introduction

Geosynthetic reinforcement is used to increase the bearing capacity of foundation soils and to reduce differential settlements under surface foundations. As it is known, bearing capacity of reinforced foundation soil is affected by the position, size, stiffness and tensile strength of reinforcement and there is no common agreement upon a simple design method available to calculate the bearing capacity of a geosynthetic reinforced foundation. In this study, a simple method for determining the bearing capacity of a geosynthetic reinforced soil was developed. In order to determine the effect of the reinforcement in the foundation soil, a simple analysis technique developed by Ghazavi and Eghbali (2008) to calculate the bearing capacity of a two layered foundation soil was adapted. To validate the newly proposed limit equilibrium method, several cases were analyzed using a Finite Element Model and the results were compared.

1. Summary of literature

1.1. Bearing capacity of reinforced foundations

Many researchers have investigated the bearing capacity of geosynthetic reinforced foundation soils using experimental, analytical and numerical methods. Binquet and Lee (1975a) carried out one of the first experimental studies to analyze the bearing capacity of reinforced soils. After them a lot of researches implemented the experimental approach to understand the behavior of reinforced granular soils (Huang, Tatsuoka 1990; Yetimoglu *et al.* 1994; Adams, Collin 1997; Patra *et al.* 2006; Chen 2007; El Sawwaf, Nazir 2010; Tafreshi, Dawson 2010; Lavasan, Ghazavi 2012). Several researches used numerical method to determine the bearing capacity of reinforced foundation soils (Yamamoto, Otani 2002; Nogueira *et al.* 2008; Madhavi, Somwanshi 2009; Gu 2011; Abu-Farsakh *et al.* 2012). In the experimental and numerical studies the optimum vertical spacing of reinforcement, the maximum total depth of reinforced soil, the effective length of reinforcements and the optimum depth of the first reinforcement layer were investigated.

Analytical models for estimating the bearing capacities of foundations on reinforced soil have also been developed (Binquet, Lee 1975b; Wayne *et al.* 1998; Kumar, Saran 2003; Huang, Meng 1997; Michalowski 2004; Sharma *et al.* 2009; Dey 2010). Binquet and Lee (1975b) reported the failure mechanism of strip footings on reinforced soil. They determined three failure mechanisms depending on the configuration and tensile force of reinforcement, namely: a) shear above reinforcements; b) tie pullout; and c) tie break. They calculated the tensile load on the reinforcement, as a function of the difference between the bearing capacities of reinforced and unreinforced soils. Huang and Tatsuoka (1990) conducted a series of plane strain model tests with a strip footing and measured the tensile forces in the reinforcement.

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They conducted tests with different reinforcement length, rigidity, arrangement and tensile strength. They used these measured reinforcement stresses to calculate the additional bearing capacity due to the reinforcement. Huang and Meng (1997) based their studies on two failure mechanisms, namely deep-footing and wide-slab mechanisms. They considered that the effect of reinforcement is to distribute the foundation load to a greater area. This phenomenon was called 'the wide slab effect' by the researchers. Regression analyses were performed to find a relationship between the factors that control the scheme of reinforcement and the effect of reinforcing on the load-spreading angle. Michalowski (2004) suggested determining the reinforcement force based on the geometry and the tensile stress of the reinforcement. However, they directly used tensile strength value as the tensile stress in the reinforcement. Sharma *et al.* (2009) also developed analytical solutions for bearing capacity of geogrid reinforced soils. They performed an experimental study with sand and silty clay soils. In their study, the tensile force was estimated with a formula based on the elastic settlement suggested by Schmertmann *et al.* (1978). For a given footing the settlement distribution in reinforced soil was assumed to be the same as that in unreinforced soil and the strain in the reinforcement were calculated based on the shape of the settlement bowl (Sharma *et al.* 2009). Dey (2010) made a critical evaluation of two different bearing capacity theories of reinforced foundation beds. An enhanced data-set has been used to carry out a refined linear and nonlinear multivariable regression analysis. It was found that the relative depth, number of layers of reinforcement and the angle of internal friction of soil significantly influence the sensitivity of the system.

1.2. Bearing capacity based on Coulomb theory

In this paper the Coulomb failure mechanism technique was adapted to determine the bearing capacity of geosynthetic reinforced soil. The Coulomb failure mechanism was proposed by Lambe and Whitman (1969) to model the failure of soil under surface foundations. This theory is based on the fact that the soil underneath the foundation tends to expand and hence is in an active state. The soil expanding below the foundation tries to compress the soil outside of the foundation and therefore the soil outside the footing is considered to be in a passive state. Richards *et al.* (1993) adapted this technique and added to the formulation the effect of shear stresses that develop on the vertical plane that separates the active and passive zones. They adapted this modified Coulomb method to calculate the seismic bearing capacities of footings. Ghazavi and Eghbali (2008) also took the simple Coulomb method proposed by Lambe and Whitman (1969) and adapted it to a two layered soil. They checked the results obtained by using the Coulomb approach with the results obtained from Finite Element analyses. Ghazavi and Eghbali (2008) concluded that the results obtained from the Finite Element analysis matched very well with the results obtained from the Coulomb formulation.

1.3. Direction of geosynthetic reinforcement in failure zone

Michalowski (1998) stated that as failure occurs, the direction of a flexible reinforcement will change and try to align itself with the failure plane. So it is assumed that the direction of the tensile force will have the same direction as the failure plane. Michalowski (1998) stated further that failure normally occurs along a band and therefore the position of the reinforcement just before failure depends not only on the direction of the failure plane but also on the width of the shear zone.

2. Model for geosynthetic reinforced foundations

When a foundation soil is reinforced using geosynthetic reinforcement, the reinforcement is always covered with a soil layer. Therefore, in general there are two soil layers below the foundation, namely the natural soil and fill soil. Therefore, the limit equilibrium approach which allows the calculation of bearing capacity of a two layered soil proposed by Ghazavi and Eghbali (2008) was seen as a good tool to analyze the reinforced foundation problem. Since the soil above the reinforcement will be compacted properly, we considered the natural soil to be less dense than the fill soil. The literature generally reported that the effect of reinforcement changes with increasing distance of the reinforcement from the base of the footing. Many researchers investigated the optimum location of the top reinforcement layer. This value was changed between 0.25 and 0.75. Therefore, in our model it is assumed that the reinforcement is laid directly on the loose soil, namely as far away from the footing as possible.

Although the literature suggest that it is beneficial to extend the reinforcement outside the footprint of the foundation, construction restraints may not allow this and this will lead to the condition that the reinforcement can be placed only as wide as the foundation itself. Therefore, two extreme models were considered in this study: a) the length of the geosynthetic reinforcement is just equal to the width of the foundation; and b) a very long reinforcement.

The failure surfaces were assumed as proposed by Ghazavi and Eghbali (2008) and given in Figure 1 for a reinforced two-layered soil. In Figure 1, the line KLM represents the failure line of the active zone and line MPR line represents the failure line in the passive zone according to the Coulomb failure mechanism. Here, q is the surcharge load applied to the soil outside of the foundation. h_1 is the thickness of the fill layer and at the same time, the distance between the base of the foundation and the reinforcement. For the system to be in equilibrium for an unreinforced soil, the active and passive forces must be equal to each other. However, due to the addition of the reinforcement, the tensile load on the reinforcement also contributes to the resisting forces and must be considered in the equilibrium equation.

The free body diagrams of the active and passive zones for both soil layers and the tensile force within the

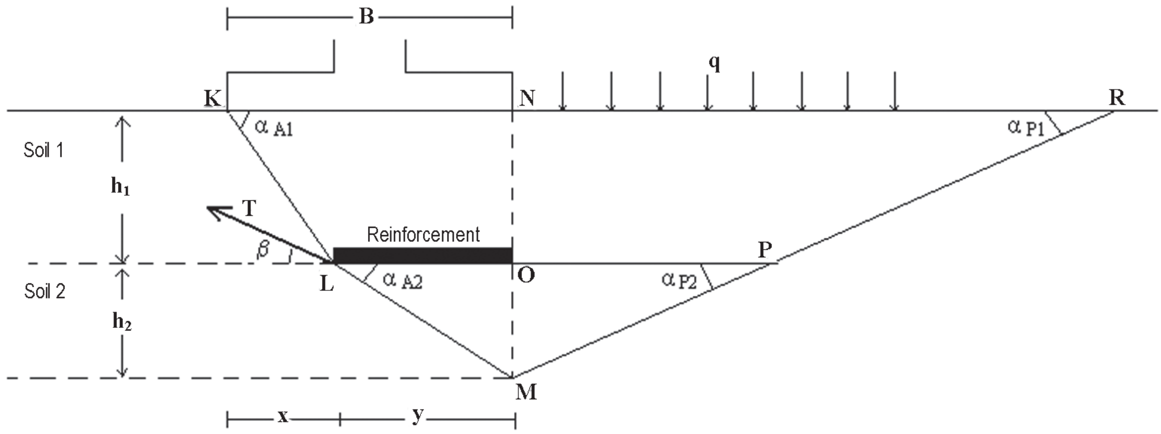


Fig. 1. Simplified slip area with Coulomb wedges and geosynthetic reinforcement

reinforcement are shown in Figure 2. The variables used in Figure 2 are: α_{Ai} – angle of slip surface in the active zone; ϕ_i – internal friction angle; γ_i – unit weight of the soil layers; δ_i –friction angle along surface between active and passive zones; K_a, K_p – coefficients of active and passive pressures; h_1 – thickness of the fill layer; h_2 – the height of the failing soil in the second layer; T – tensile force in the reinforcement; β – angle between the tensile force and horizontal plane.

The value of α_{Ai} is given as (Ghazavi, Eghbali 2008):

$$\alpha_{Ai} = \phi_i + tan^{-1}$$
$$\left[\frac{\left[tan\phi_i(tan\phi_i + cot\varphi_i)(1+tan\delta_i cot\phi_i)\right]^{1/2} - tan\phi_i}{1+tan\delta_i(tan\phi_i + cot\phi_i)}\right]. \quad (1)$$

From the geometry, the height of the failing soil (h_2) can be calculated as (Fig. 2):

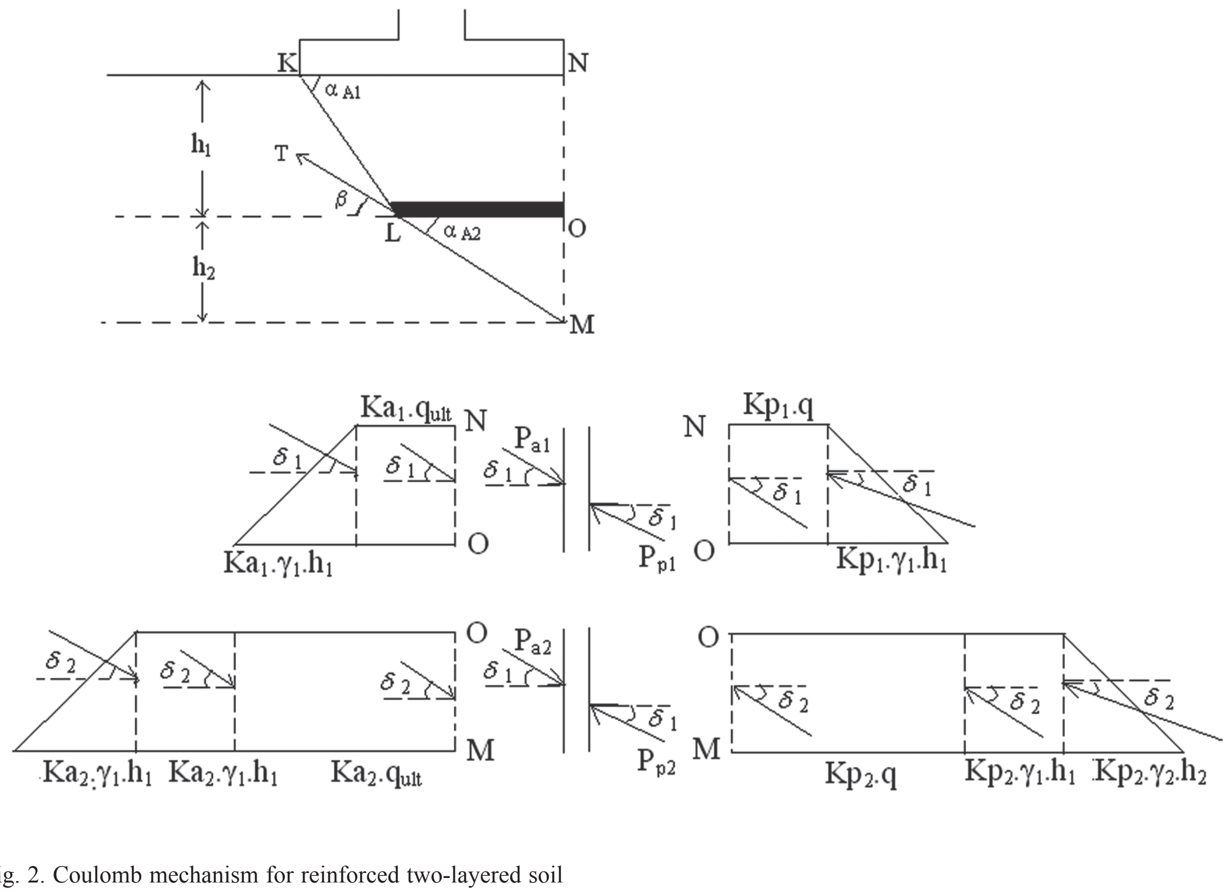


Fig. 2. Coulomb mechanism for reinforced two-layered soil

$$h_2 = (B - h_1 \cot \alpha_{A1}) \tan \alpha_{A2}. \quad (2)$$

The active and passive earth pressure coefficient for each soil layer is calculated as follows (Ghazavi, Eghbali 2008):

$$K_{ai} = \frac{\cos^2\phi}{\left(\cos\delta_i\left[1+\sqrt{\sin(\varphi_i+\delta_i)\sin\varphi_i/\cos\delta_i}\right]^2\right)}; \quad (3)$$

$$K_{pi} = \frac{\cos^2\phi}{\left(\cos\delta_i\left[1-\sqrt{\sin(\varphi_i+\delta_i)\sin\varphi_i/\cos\delta_i}\right]^2\right)}. \quad (4)$$

The theoretical base of Eqns (1), (3) and (4) is Coulomb earth pressure theory. Resultants of active and passive earth pressures for each layer are defined as follows:

$$P_{a1} = K_{a1}\,(q_{ult}\,h_1 + 0.5\gamma_1 h_1^2); \quad (5)$$

$$P_{a2} = K_{a2}\,(q_{ult}\,h_2 + \gamma_1 h_1 h_2 + 0.5\gamma_2 h_2^2); \quad (6)$$

$$P_{p1} = K_{p1}\,(q h_1 + 0.5\gamma_1 h_1^2); \quad (7)$$

$$P_{p2} = K_{p2}\,(0.5\gamma_2 h_2^2 + \gamma_1 h_1 h_2 + q h_2). \quad (8)$$

The horizontal component of the tensile force T_f depends on the angle of the shear plane and the width of the shear band as explained above and the rigidity of the reinforcement used. The extreme values for flexible and rigid reinforcement respectively can be given as follows:

$$(T_f)_{min} = T^*\cos\alpha_{A2}; \qquad (T_f)_{\max} = T. \quad (9)$$

The total driving force is calculated as follows:

$$P_{ah} = P_{a1}\cos\delta_1 + P_{a2}\cos\delta_2. \quad (10)$$

The total resisting force therefore can be determined as follows:

$$P_{ph} = P_{p1}\cos\delta_1 + P_{p2}\cos\delta_2 + T\cos\beta. \quad (11)$$

The bearing capacity of a strip footing on unreinforced cohesionless soil, q_{ult}, is computed by Terzaghi (1943) using the following equation:

$$q_{ult} = qN_q + 0.5B\gamma N_\gamma. \quad (12)$$

Ghazavi and Eghbali (2008) had expressed the bearing capacity of a two layered soil with the help of modified bearing capacity factors. This formulation was further developed in this paper for a geosynthetic reinforced soil. The modified N_q and N_γ values can be determined as derived below for two different assumptions, namely the case of "Weightless Soil" and the case of "No Surcharge". These are determined by applying the super position principle in subsequent section.

2.1. State I: Weightless soil ($q \neq 0$, $\gamma_1 = \gamma_2 = 0$)

In this case the contribution of the soil weight on the bearing capacity is neglected. With this assumption the value of N_q will be calculated. For this purpose Eqns (5) and (6) are combined for weightless soil condition and Eqn (13) is obtained:

$$P_{ah\,(\gamma=0)} = q_{ult}\,(K_{a1}h_1\cos\delta_1 + K_{a2}h_2\cos\delta_2). \quad (13)$$

Eqn (11) is modified to reflect the assumption of the weightless soil and the total passive earth pressure can be expressed as given in Eqn (14):

$$P_{ph\,(\gamma=0)} = q\,(K_{p1}h_1\cos\delta_1 + K_{p2}h_2\cos\delta_2) + T\cos\beta. \quad (14)$$

Eqn (15) is obtained from the equilibrium of horizontal forces. Eqn (16) gives the Terzaghi bearing capacity equation for a weightless soil. When Eqns (15) and (16) are combined, the value of N_q can be obtained as given by Eqn (17):

$$q_{ult(\gamma=0)} = \frac{\left(q(K_{p1}h_1\cos\delta_1 + K_{p2}h_2\cos\delta_2) + T\cos\beta\right)}{\left(K_{a1}h_1\cos\delta_1 + K_{a2}h_2\cos\delta_2\right)}; \quad (15)$$

$$q_{ult} = q.N_q\,; \quad (16)$$

$$N_q = \left[\frac{K_{p1}h_1\cos\delta_1 + K_{p2}h_2\cos\delta_2}{K_{a1}h_1\cos\delta_1 + K_{a2}h_2\cos\delta_2}\right] + \left[\frac{T\cos\beta}{q(K_{a1}h_1\cos\delta_1 + K_{a2}h_2\cos\delta_2)}\right]. \quad (17)$$

2.2. State II: No surcharge ($q = 0$, $\gamma_1, \gamma_2 \neq 0$)

In this case the formulation is derived by including the effect of the weight of the soil. However, this time the surcharge load in the surrounding soil is not considered in the formulation. The earth pressures in this case are given by Eqns (18) and (19):

$$P_{ah(q=0)} = q_{ult}(K_a h_1\cos\delta_1 + K_{a2}h_2\cos\delta_2) + 0.5K_{a1}\gamma_1 h_1^2\cos\delta_1 + K_{a2}\cos\delta_2(\gamma_1 h_1 h_2 + 0.5\gamma_2 h_2^2); \quad (18)$$

$$P_{ph(q=0)} = 0.5K_{p1}\gamma_1 h_1^2\cos\delta_1 + K_{p2}\cos\delta_2\,(0.5\gamma_2 h_2^2 + \gamma_1 h_1 h_2) + T.\cos\beta. \quad (19)$$

For the purposes of simplicity, a dimensionless parameter X is defined as proposed by Ghazavi and Eghbali (2008):

$$X = h_2 / h_1 = (B.\tan\alpha_{A2} / h_1) - \left(\tan\alpha_{A2} / \tan\alpha_{A1}\right). \quad (20)$$

As seen in Figure 2, we have the horizontal active and passive pressures on the right and left sides of the line NOM and the tensile force in the reinforcement. Considering the equilibrium of horizontal forces, Eqn (21) is obtained:

$$q_{ult(q=0)} = \frac{[0,5\gamma_1 cos\delta_1(K_{p1}-K_{a1}) + cos\delta_2(0,5\gamma_2 X^2 + \gamma_1 X)(K_{p2}-K_{a2})]h_1 + \frac{Tcos\beta}{h_1}}{(K_{a1}cos\delta_1 + K_{a2}Xcos\delta_2)}. \quad (21)$$

The Terzaghi bearing capacity for a granular soil with zero cohesion and no surcharge load outside the footing is given as:

$$q_{ult} = 0.5B\gamma\, N_\gamma. \quad (22)$$

The value of $N\gamma$ can be determined by combining Eqns (21) and (22). To do that, a parameter is defined as the equivalent unit weight ($\overline{\gamma}$) showing the proportion of each layer in the rupture zone as originally proposed by Ghazavi and Eghbali (2008):

$$\overline{\gamma} = \left(A_1\gamma_1 + A_2\gamma_2\right)/(A_1 + A_2), \quad (23)$$

where A_1 and A_2 are the effective area of each layer in rupture zone and therefore are a function of h_1 and h_2. So $\overline{\gamma}$ can be determined as:

$$\overline{\gamma} = \left(\frac{X^2\gamma_2 + (X(B/h_1)\tan\alpha_{A2})\gamma_1}{X^2 + (X(B/h_1)\tan\alpha_{A2})}\right). \quad (24)$$

Thus N_γ can be expressed as in Eqn (25):

$$N_\gamma = \frac{\begin{bmatrix} 0.5\gamma_1 cos\delta_1(K_{p1}-K_{a1}) + cos\delta_2 \\ (0.5\gamma_2 X^2 + \gamma_1 X)(K_{p2}-K_{a2}) \end{bmatrix} h_1 + \frac{Tcos\beta}{h_1}}{0.5B\overline{\gamma}(K_{a1}cos\delta_1 + K_{a2}Xcos\delta_2)}. \quad (25)$$

3. Numerical analysis

To verify the above derived formulation, Finite Element analyses were conducted. The software Plaxis V8 was chosen for these analyses. A plane strain model using triangular elements with 15 nodes was utilized and the Mohr-Coulomb Model was used to model the soil behavior. The parameters were chosen to be the same as given in the solutions of Ghazavi and Eghbali (2008), in order to be able to compare results of this study with their results. The width of the strip foundation (B) was chosen as 1, 2 and 3 m and the surcharge load q outside the footing was taken as 10 kN/m^2, 17.5 kN/m^2, and 25 kN/m^2, respectively. The footing thickness was 0.143 m and placed directly on the surface without any embedment. The footing's axial stiffness per unit width was chosen as 5×10^6 kN/m and it was modeled as an elastic material. In the Finite Element analyses the load applied on the footing is increased in equal steps until failure occurred. The reinforcement used was a geosynthetic reinforcement and its axial stiffness per unit width was selected as J = 2,000 kN/m to represent an average geogrid (El Sawwaf 2007). The reinforcement width (L) was selected as B and 20B. It was modeled as a linear elastic material. In the analyses, no specific interaction model between soil and reinforcement was used.

The thickness of the first soil layer which is at the same time the depth of the reinforcement was normalized by dividing it to the width of the footing. Three different h_1/B values (0.25, 0.50 and 0.75) were selected. In limit equilibrium calculations, the value of δ usually ranges between 0.35 ϕ and 0.45 ϕ (Ghazavi, Eghbali 2008). Table 1 shows the soil properties used in the analyses. These properties were taken same as values given by Ghazavi and Eghbali (2008) to be able to compare and verify the unreinforced results. Since the system is symmetric, only half of the model was analyzed. The mesh size was chosen as fine and the first layer was chosen to be even finer. The horizontal and vertical distances from the center of the footing to the respective model boundaries were chosen as 10B in each analysis. In this paper, two different soil combinations were investigated. The friction angle of the fill (top) sand layer is ϕ_1 and the friction angle of the natural (bottom) sand is ϕ_2. In the first case the internal friction angle values were chosen as ϕ_1 = 34° and ϕ_2 = 31°. In the second case they were taken as ϕ_1 = 39° and ϕ_2 = 36°.

To calculate the bearing capacity with the limit equilibrium analysis proposed in this study, the activated tensile force in the reinforcement must be known. Therefore, the tensile forces in the reinforcement were determined using the Finite Element analysis and were put into the new limit equilibrium equation and the bearing capacities of the reinforced two-layered soil were calculated. In this calculation the new developed limit equilibrium coefficients N_γ and N_q were used. Table 2 shows the bearing capacity and maximum reinforcement tensile forces obtained from the Finite Element analyses. The results obtained from limit analysis are shown in Table 3. The symbols used in these Tables are as follows: q_u is the ultimate bearing capacity of unreinforced soil, q_1 is the bearing capacity of reinforced soil for L = B, q_2 is the bear-

Table 1. The properties of soils in the analyses (Ghazavi, Eghbali 2008)

Friction angle	Dilatation angle	Unit weight	Young modules	Poisson's ratio	Friction angle along surface between active and passive zones*
ϕ_i (°)	ψ (°)	γ (kN/m^2)	E (kN/m^2)	υ	δ_i(°)*
31	4.2	19.3	20	0.327	12
34	6.4	20.1	27.5	0.306	14
36	8	20.5	35	0.291	15
39	10	20.9	50	0.27	17

*It has been stated by Ghazavi and Eghbali (2008) that for 30 < ϕ < 45, the value of δ ranges between 0.35 ϕ < δ < 0.45 ϕ linearly.

ing capacity of reinforced soil for L = 20B, T_1 is the max tensile load in reinforcement for L = B and T_2 is the max tensile load in reinforcement for L = 20B. In Table 3 results of limit equilibrium analyses are displayed, where maximum tensile loads of reinforcement are taken as $T_f = T$ and $T_f = T\cos\alpha_{A2}$.

From the obtained results it was seen that the tensile forces calculated by the finite element analyses are individually between the values given in Table 3 (where $T_f = T$ and $T_f = T\cos\alpha_{A2}$). This is in good agreement with the assumption of Michalowski (1998) which states that failure occurs along a shear band and therefore the inclination of the reinforcement along the failure plane is not completely in alignment with the shear surface. The same calculations were done for the second model where the loose soil has an internal friction angle of $\phi = 36^o$ and the fill soil is assumed to have an internal friction angle of $\phi = 39°$. The results for the second model are shown in Tables 4 and 5.

Table 2. Bearing capacity and maximum tensile forces of reinforcements obtained from FEM ($\phi = 34°$ on $\phi = 31°$)*

B (m)	h_1/B	h_1 (m)	q_{u*} (kPa)	q_1 (kPa)	q_2 (kPa)	T_1 (kN/m)	T_2 (kN/m)
1	0.25	0.25	491	541	617	21.1	57.0
1	0.5	0.5	526	635	687	63.6	74.6
1	0.75	0.75	578	732	768	73.4	81.4
2	0.25	0.5	879	989	1047	70.8	90.8
2	0.5	1	910	1055	1115	102	125
2	0.75	1.5	1001	1232	1283	141	161
3	0.25	0.75	1086	1160	1216	104	136
3	0.5	1.5	1133	1286	1305	172	197
3	0.75	2.25	1203	1405	1486	219	231

*q_u – unreinforced bearing capacity, q_1– bearing capacity of reinforced soil where L = B, q_2 – bearing capacity of reinforced soil where L = 20B, T_1 = Max. tensile load in reinforcement where L = B, T_2 = Max. tensile load in reinforcement where L = 20B.

Table 3. Bearing capacity obtained from the new Limit equilibrium method ($\phi = 34°$ on $\phi = 31°$)

				$T_f = T$		$T_f = T\cos\alpha_{A2}$	
B (m)	h_1/B	h_1 (m)	q_u (kPa)	q_1 (kPa)	q_2 (kPa)	q_1 (kPa)	q_2 (kPa)
1	0.25	0.25	411	503	661	460	543
1	0.5	0.5	435	717	766	585	610
1	0.75	0.75	463	792	828	637	656
2	0.25	0.5	780	935	979	862	885
2	0.5	1	826	1052	1103	946	973
2	0.75	1.5	880	1194	1240	1046	1070
3	0.25	0.75	1150	1301	1348	1229	1254
3	0.5	1.5	1217	1471	1508	1352	1371
3	0.75	2.25	1296	1623	1641	1469	1478

Figure 3 shows the reinforcement tensile load distributions for the models with B/2 = 0.5 m; $h_1 = 0.75$ m; $\phi_1 = 39°$ on $\phi_2 = 36°$. It is seen that no tensile force in the reinforcement developed beyond a distance of 3B outside the foundation. Similar observations were made for other configurations as well.

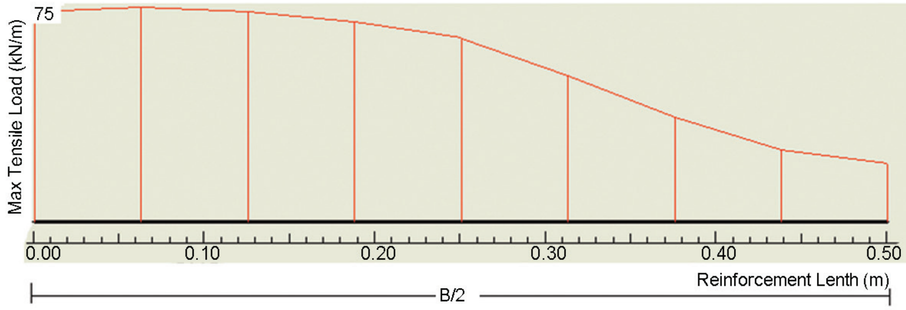
These calculations show that if the reinforcement force can be known, the bearing capacity of a reinforced foundation can be directly calculated by the limit equilibrium formulation proposed. Therefore, an attempt was made to express the tensile force in the reinforcement as a function of the geometry and soil properties. The tensile forces obtained from the Finite Element analysis results, were evaluated with a multiple linear regression analysis. The comparison of bearing capacity values between Finite Element Method (FEM) and new Limit Equilibrium Method (LEM) can be seen as in Figures 4 and 5. Figure 4 shows the bearing capacity values of first case (B = 2 m, L = B and $\phi_1 = 34°$ on $\phi_2 = 31°$). Figure 5 is for second case (B = 2, L = B and $\phi_1 = 39°$ on $\phi_2 = 36°$). In these Figures, reinforcement tensile stress was taken as T.

Table 4. Bearing capacity and maximum tensile forces of reinforcements obtained from FEM ($\phi = 39°$ on $\phi = 36°$)

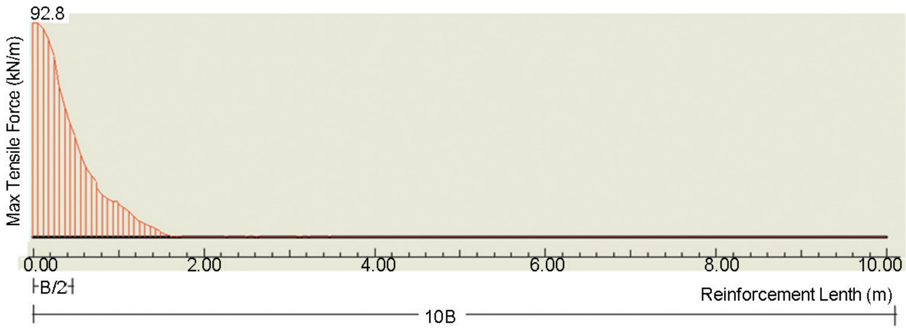
B (m)	h_1/B	h_1 (m)	q_u (kPa)	q_1 (kPa)	q_2 (kPa)	T_1 (kN/m)	T_2 (kN/m)
1	0.25	0.25	953	981	1153	30.0	62.1
1	0.5	0.5	998	1192	1269	68.4	78.7
1	0.75	0.75	1109	1244	1327	75.0	92.8
2	0.25	0.5	1808	1958	2067	97.4	128
2	0.5	1	1887	1994	2059	124	148
2	0.75	1.5	2043	2252	2326	137	154
3	0.25	0.75	2543	2723	2849	184	215
3	0.5	1.5	2674	2946	3035	218	259
3	0.75	2.25	2824	2999	3110	223	274

Table 5. Bearing capacity obtained from the new Limit Equilibrium Method ($\phi = 39°$ on $\phi = 36°$)

				$T_f = T$		$T_f = T\cos\alpha_{A2}$	
B (m)	h_1/B	h_1 (m)	q_u (kPa)	q_1 (kPa)	q_2 (kPa)	q_1 (kPa)	q_2 (kPa)
1	0.25	0.25	846	994	1152	919	996
1	0.5	0.5	893	1232	1284	1059	1084
1	0.75	0.75	947	1323	1413	1131	1175
2	0.25	0.5	1615	1855	1931	1732	1769
2	0.5	1	1703	2013	2070	1855	1883
2	0.75	1.5	1806	2152	2193	1975	1996
3	0.25	0.75	2383	2686	2736	2532	2556
3	0.5	1.5	2514	2875	2943	2690	2724
3	0.75	2.25	2666	3040	3125	2849	2891



a)



b)

Fig. 3. Reinforcement axial forces for B/2 = 0.5 m, h_1 = 0.75 m, ϕ_1 = 39° on ϕ_2 = 36°: a) L/2 = 0.5 m, b) L/2 = 10 m

The depth of the reinforcement to width of foundation ratio (*h*/B), the foundation width (B), the internal friction angle of the denser upper layer (ϕ) and length of the reinforcement to width of foundation ratio (L/B) were chosen as the independent parameters. The following relation was obtained from the multiple regression analysis:

$$T = 110h/\text{B} + 68.8\text{B} + 5.07\phi + 8.05\text{L/B}267. \quad (26)$$

In this equation T is the tensile stress in the reinforcement, h is the distance of the reinforcement from the base of footing, B is the footing width, f is the internal friction angle of the upper strata, L is the reinforcement length. In the regression analyses the L/B ratio was chosen as 1 for the reinforcement that has the same length as the footing. For the case of the long reinforcement the L/B ratio was taken as 2.

The regression coefficient of equation 26 was found as R = 0.97. This shows that there is a good correlation. The significance of every coefficient was investigated as well and as can be seen from the t and P distribution values given in Table 6. It can be stated that the relation between each individual parameter and the tensile stress (T) is significant.

The bearing capacity values of reinforced soil obtained from Finite Element analyses and the Limit1 Equilibrium method proposed in this paper is compared in Figure 6. In this Figure, $q_{1\max}$ and $q_{1\min}$ are the bearing capacity values of FEM and LEM for L = B with tensile load of reinforcement taken as T and $T\cos\alpha A_2$,

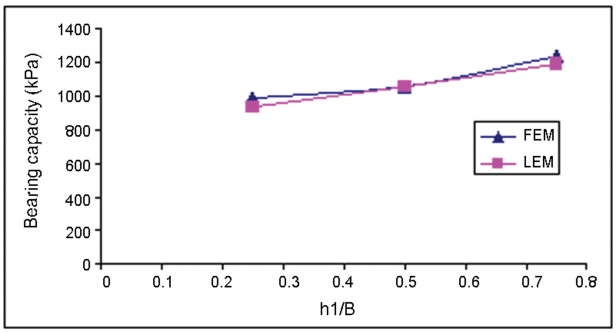


Fig. 4. Comparison of bearing capacity values with models (B = 2 m, L = B and ϕ_1 = 34° on ϕ_2 = 31°)

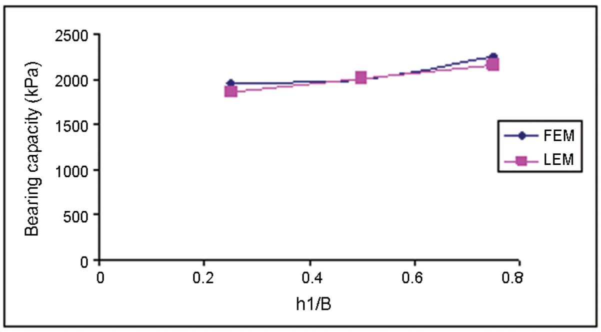


Fig. 5. Comparison of bearing capacity values with models (B = 2 m, L = B and ϕ_1= 39° on ϕ_2= 36°

Table 6. Results of multiple regression analysis

	Coefficients	Standard error	t Stat	P-value
Variable 1 (*h*/B)	109.9	14.77	7.44	2.22E–08
Variable 2 (B)	68.83	3.69	18.6	2.13E–18
Variable 3 (ϕ)	5.075	1.206	4.21	2.05E–4
Variable 4 (L)	8.055	2.01	4.01	3.58E–4
Intercept	–267.6	45.6	–5.86	1.80E–06

respectively. q_{2max} and q_{2min} are the bearing capacity values of FEM and LEM for L = 20B with tensile load of reinforcement taken as T and $T\cos\alpha A_2$, respectively. A solid line was drawn in these graphs indicating equal values of LE and FEM results. As can be seen from this figure the new limit equilibrium approach gives values very close to the bearing capacities obtained from Finite Element solutions. The differences are very small and within acceptable limits in civil engineering practice.

4. Worked example

A strip foundation of width B = 3 m rests on a reinforced two layered soil. The top sand layer has a thickness of h = 1.5 m, underlain by a deep bed of second sand soil. The friction angle of the fill (top) sand layer is $\phi_1 = 39°$ and the unit weigh is $\gamma_1 = 20.9$ kN/m^2. The friction angle of the natural (bottom) sand is $\phi_2 = 36°$ and the unit weigh is $\gamma_2 = 20.5$ kN/m^2. Friction angles along surface between active and passive zones are taken as $\delta_1 = 17°$ and $\delta_2 = 15°$. The surcharge on the soil is taken as $q = 25$ kN/m^2. The reinforcement is placed between the two sand layers. The bearing capacity is calculated for a reinforcement length equal to the foundation width (L = B = 3 m). The calculation of the bearing capacity of reinforced soil with the new proposed formulation is given below.

Firstly, the maximum tensile force in the reinforcement is calculated by making use of Eqn (26):

$$T_1 = 110\left(\frac{1.5}{3}\right) + 68.8*3 + 5.07*39 + 8.05*(1) - 267.$$

$$T_1 = 200 \text{ kN/m}.$$

Bearing capacity factors are calculated considering $(T_f)_{max} = T$, ($\beta = 0$). N_q and N_γ values are founded from Eqns (17) and (25) as 39.5 and 60, respectively. The equivalent unit weight of soil is found as 20.7 kN/m^3. Ultimate bearing capacity of reinforced soil is found by making use of Eqn (12) and the ultimate bearing capacity is found as 2845 kPa.

For $(T_f)_{min} = T*\cos\alpha_{A2}$ ($\beta = \alpha_{A2}$) the bearing capacity factors are calculated as $N_q = 36$ and $N_\gamma = 57$. Ultimate bearing capacity for $(T_f)_{min}$ is equal to 2676 kPa. The bearing capacity computed by using Finite Element Method as 2946 kPa. Therefore, it can be concluded that the values of bearing capacity of the reinforced foundation using the method proposed in this paper match the results of the Finite Element analyses well. For the same example without any reinforcement the bearing capac-

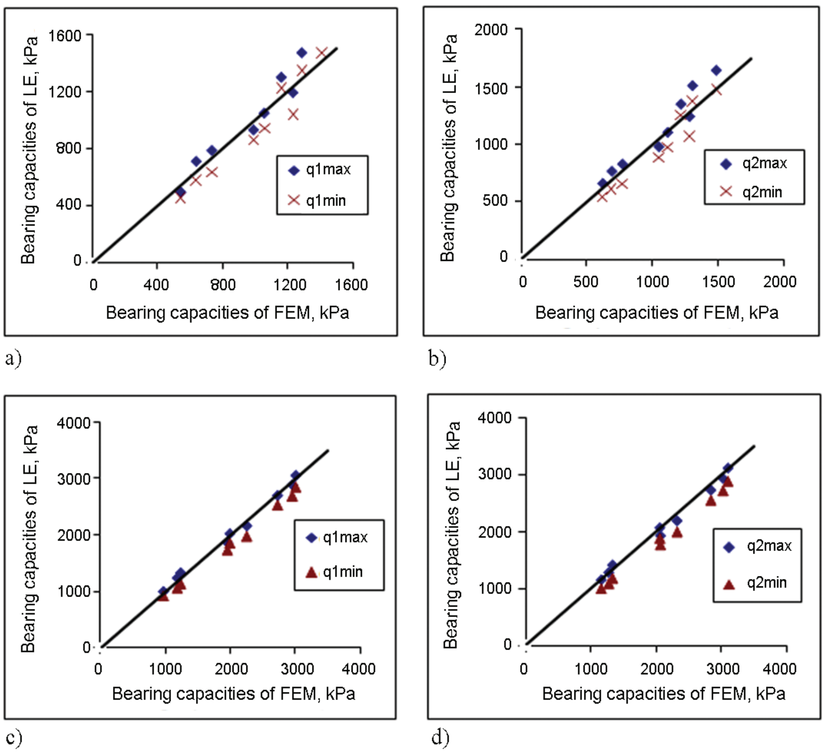


Fig. 6. Comparisons of bearing capacity obtained from FEM and the New Limit Equilibrium Approach: a) L= B and b) L/B = 2 for $\phi_1 = 34°$ on $\phi_2 = 31°$, c) L = B and d) L/B = 2 for $\phi_1 = 39°$ on $\phi_2 = 36°$

ity from Finite Element analysis was found as 2514 kPa. This indicates that with the reinforcement approximately 17% improvement in bearing capacity was obtained.

Conclusions

In this paper a limit equilibrium method is developed for calculating the bearing capacity factors of strip foundations on a two-layered reinforced granular soil. New formulations for the ultimate bearing capacity factors N_q and N_γ were derived for two layered soils which were reinforced with a single reinforcement layer. The results obtained were compared with the results obtained from Finite Element analysis. The ultimate bearing capacities of reinforced soils for two extreme geosynthetic lengths were considered. One extreme is the case where the reinforcement length is equal to the footing width (L = B). The second extreme is that a very long reinforcement is used. When results are evaluated it was determined that for L ≥ 4B, the case of long reinforcement is valid. The results obtained from the new proposed formulation were compared to the Finite Element analysis results. As a result it can be stated that closed form solution and the finite element analysis results are in agreement with each other. Therefore, it can be stated that the bearing capacities of footings on a two layered soil reinforced with one layered reinforcement can be estimated with the new limit equilibrium approach successfully.

In the limit equilibrium analysis proposed in this paper, the tensile force mobilized in the geosynthetic reinforcement is determined by the correlation formula given in Eqn (26).

When the bearing capacities of footings with short and long reinforcements are compared, it can be seen that the bearing capacities determined for the long reinforcement is 1.23 times higher than the bearing capacity obtained for the short reinforcement length (L = B). However, an improvement is also obtained for a reinforcement that is only as wide as the footing itself.

Considering the many results reported in the literature, it can be stated that no difference occurs for a reinforcement length above L = 4B. Also the tensile force distribution obtained in this study verifies this statement. Therefore, it can be stated that the bearing capacities obtained for long reinforcement is valid for L ≥ 4B and for shorter reinforcement lengths an interpolation can be made between the two extreme conditions.

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