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Simplified Log-Likelihood Ratio Calculation for Binary LDPC Codes

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Abstract

Combining binary LDPC codes and high order constellations is a simple but effective way to improve the bandwidth efficiency. Since the binary LDPC codes require soft decisions as the input information, the constellation has to provide soft information calculated by Log-Likelihood Ratio (LLR) to it. In this paper, a simplified algorithm to calculate LLR for binary turbo-codes is applied for binary LDPC codes. Simulation results show that the simplified algorithm have a very small performance loss over Gaussian and Rayleigh channels.

Keywords: Low-density parity-check (LDPC) codes, non-binary, Log-likelihood Ratio (LLR), Quadrature Amplitude Modulation (QAM), Gray mapping.

1. Introduction

Combining high errors correcting codes and high order constellations, such as Quadrature Amplitude Modulation (QAM), is an effective way to improve the bandwidth efficiency with a high transmission quality. Turbo-codes and LDPC codes are powerful error correcting codes can approach the Shannon limit [1].

Turbo-codes [2] are obtained by the concatenation of two or more low complexity codes to obtain a powerful code with reasonable complexity. Their decoding is done according to the principle of iterative decoding or turbo to improve code performance.

After the power of iterative decoding, which was shown by the invention of turbo-codes. Binary LDPC codes, which have been neglected because of their complexity, for many years since they were introduced by Gallager in 1962 [3, 4], have been rediscovered by Mackay [5] in 1995 and Spieleman and others [6] in 1996. LDPC codes are linear block codes, based on sparse parity check matrices, i.e. the number of non-zero elements in the matrix are less than the number of zeros, and decoding according to the iterative decoding principle.

LDPC decoders and turbo decoders must operate in soft decisions which can be calculated using the LLR at QAM. However, the number of operations performed to calculate the soft decisions used by the decoder increases with the order of the constellation. Thus, this calculation varies with the type of the transmission channel.

Several algorithms have been introduced in order to simplify the exact calculation of the LLR. The pragmatic algorithm, introduced in [7, 8], attempts to simplify the calculation assuming that the likelihood values are Gaussian variables. The max-log-MAP (Maximum A Posteriori) algorithm is the most popular simplifying the exact algorithm [9].

In this work, we apply the pragmatic algorithm for binary LDPC codes. It is programmed to adapt as perfectly as possible the transmission system to the type of channel concerned. This simplification leads to simplify the implementation of the system. We restrict our description of combining binary LDPC code with square Gary-QAM constellations MAQ-16, over Gaussian and Rayleigh Channels.

The rest of the paper is organized as follows. Section 2 introduces the exact calculation of LLR for Gray-QAM with square constellation over Gaussian and Rayleigh channels. In Section 3, the simplified calculation of LLR is investigated, respectively. Finally, the simulation results and concluding remarks are given in Section 4 and 5, respectively.

2. Exact LLR calculation for Gray-QAM with square constellation over Gaussian and Rayleigh channels

2^m-QAM transmit at each instant nT m bits $\{u_{n,i}\}, i \in \{1, \dots, m\}$, that is represented by $a_n + jb_n$ where $a_n \; and \; b_n \in \{\pm 1, \pm 3, \pm 5, \dots, m \pm 1\}$. After passing through the transmission channel, the observation relating to the couple (a_n, b_n) is represented by a couple (a'_n, b'_n).

In the case of Rayleigh channel a'_n and b'_n are given by:

$$a'_n = \alpha_n a_n + z_n \tag{1}$$

$$b'_n = \alpha_n b_n + z_n \tag{2}$$

Where z_n is a Gaussian noise, centered, with variance σ^2 and α_n is a variable characterizes the attenuation of the transmitted signal. In the case of Gaussian channel $\alpha_n = 1$.

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At the reception, we treat the couples (a'_n, b'_n) to extract m samples $\{\hat{u}_{n,i}\}, i \in \{1, \dots, m\}$ each representative of a bit $u_{n,i}$ associated. The sample $\hat{u}_{n,i}$ is obtained using the relationship $LLR(u_{n,i})$.

$LLR(u_{n,i}), i \in \{1, \dots, m\}$, is calculated as follows [10]:

$$LLR(u_{n,i}) = log\left[\frac{Pr\{(a'_n,b'_n)\ /u_{n,i}=0\}}{Pr\{(a'_n,b'_n)\ /u_{n,i}=1\}}\right] \qquad (3)$$

Where $Pr\{(a'_n, b'_n)\ /u_{n,i} = w\}$ is the probability that the available couple is (a'_n, b'_n); knowing the binary symbol $u_{n,i}$ is equal to w.

For a square constellation $m = 2p$, 2^{2p}-QAM has the particularity to be reduced to two amplitude modulations with 2^P states independently acting on two carriers in phase and quadrature [11]. According to the this property (the case of a square constellation):

- The p expressions in phase, obtained from the equation (3) are consequently the following:

$$LLR(u_{n,i}) = log\left[\frac{Pr\{a'_n\ /u_{n,i}=0\}}{Pr\{a'_n\ /u_{n,i}=1\}}\right] \qquad i \in \{1, \dots, p\} \qquad (4)$$

Where:

$$Pr\{a'_n\ /u_{n,i} = w\} = \frac{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(a'_n - \alpha_n a^w_{i,j}\right)^2\right\}}{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(a'_n - \alpha_n a^0_{i,j}\right)^2\right\} + \sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(a'_n - \alpha_n a^1_{i,j}\right)^2\right\}},$$
$$\{1, \dots, p\} \qquad (5)$$

With $a^k_{i,j}$ are possible values of the symbol a_n when the symbol $u_{n,i}$ to be transmitted has the value k (k = 0 or 1); $w = 0\ or\ 1$; For a Gaussian channel $\alpha_n = 1$.

Therefore, the equation (4) yields to:

$$LLR(u_{n,i}) = log\left[\frac{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(a'_n - \alpha_n a^0_{i,j}\right)^2\right\}}{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(a'_n - \alpha_n a^1_{i,j}\right)^2\right\}}\right] \qquad (6)$$

- The p relations in quadrature eventually lead to the following expressions:

$$LLR(u_{n,i}) = log\left[\frac{Pr\{b'_n\ /u_{n,i}=0\}}{Pr\{b'_n\ /u_{n,i}=1\}}\right] \quad i \in \{p+1, \dots, 2p\} \qquad (7)$$

With the same demonstration as precedent, the equation (7) yields to:

$$LLR(u_{n,i}) = log\left[\frac{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(b'_n - \alpha_n b^0_{i,j}\right)^2\right\}}{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(b'_n - \alpha_n b^1_{i,j}\right)^2\right\}}\right]$$
$$i \in \{p+1, \dots, 2p\} \qquad (8)$$

With $b^k_{i,j}$ are possible values of the symbol b_n when the symbol $u_{n,i}$ to be transmitted has the value k (k = 0 or 1).

Equations (6) and (8) are the exact calculation of the LLR, it is the optimal calculation that represents the log-MAP algorithm [12-14]. However, it involves several operations. Several algorithms have been introduced in order to simplify the exact calculation of the LLR.

In this work, we use a simplified algorithm, a pragmatic algorithm, that used for binary turbo-code. we apply this simplified algorithm for binary LDPC codes. In [7], the authors show that, for turbo-code, the pragmatic algorithm got on a Gaussian channel can be reused efficiently on a Rayleigh channel (Figure 1), this provided insert an additional operation to accommodate, each time nT, the channel attenuation α_n.

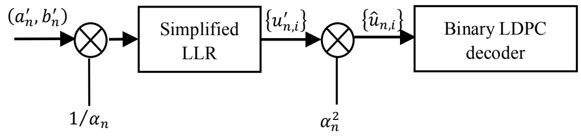

Fig. 1. Principle of simplified LLR calculation

3. Simplified LLR calculation

Gaussian channel

The pragmatic algorithm introduced in [7] shows that the p relations in the phase and p relations in the quadrature, multiplied by $(\sigma^2/2)$, are given respectively by the equation (9.a) and the equation (9.b):

$$\begin{aligned}
&LLR(u_{n,1}) = -a'_n \\
&LLR(u_{n,2}) = |LLR(u_{n,1})| - 2^{p-1} \\
&\vdots \\
&LLR(u_{n,i}) = |LLR(u_{n,i-1})| - 2^{p-i+1} \qquad (9.a) \\
&\vdots \\
&LLR(u_{n,p}) = |LLR(u_{n,p-1})| - 2
\end{aligned}$$

And

$$\begin{aligned}
&LLR(u_{n,p+1}) = -b'_n \\
&LLR(u_{n,p+2}) = |LLR(u_{n,p+1})| - 2^{p-1} \\
&\vdots \\
&LLR(u_{n,p+i}) = |LLR(u_{n,p+i-1})| - 2^{p-i+1} \qquad (9.b) \\
&\vdots \\
&LLR(u_{n,2p}) = |LLR(u_{n,2p-1})| - 2
\end{aligned}$$

For a good approximation of equations (6) and (8) without multiplication by $(\sigma^2/2)$, we multiply the equations (9) by $(2/\sigma^2)$, we get:

$$LLR(u_{n,i}) = (2/\sigma^2) \times LLR(u_{n,i}), i \in \{1, \dots, 2p\} \qquad (10)$$

Rayleigh channel

In [15], the authors assume that the attenuation of the Rayleigh channel α_n, at time nT, is known perfectly by the receiver. He shows that the pragmatic algorithm got on a Gaussian channel can be reused efficiently on a Rayleigh channel, as follow [8]:

First, as the variable α_n at time nT is known, it is possible to divide the two samples a'_n and b'_n, equations (1) and (2), available at the channel output by α_n [8]. Samples a''_n and b''_n thus obtained are expressed in the form:

$$a''_n = \frac{a'_n}{\alpha_n} = a_n + z'_n \qquad (10)$$

$$b''_n = \frac{b'_n}{\alpha_n} = b_n + z'_n \qquad (11)$$

Where z'_n is a Gaussian noise, centered, with variance σ_n^2 equals to σ^2/α_n^2.

Second, Since the samples a''_n and b''_n are modeled by Gaussian variables, it is possible to apply directly on the samples a''_n and b''_n available, simplified algorithms of LLR strictly identical to those used when the transmission channel is Gaussian, and this irrespective of the modulation used.

Finally, it is necessary to multiply by α_n^2 the samples $u'_{n,i} = LLR(u_{n,i})$.

4. Simulation results

In this section, we will show the effect of simplifying calculation of the LLR for 16-QAM, using Gray mapping, on the performance of binary LDPC codes, with different block lengths and iterations number, over Gaussian and Rayleigh channels. Simulation results, obtained by computer simulations using Matlab, are given in terms of Bit Error Rate (BER) versus E_b/N_0, where E_b is the energy per information and N_0 is the spectral density noise.

Figure 2 shows performance comparisons, on a Gaussian channel, between a binary LDPC code using the simplified LLR and a binary LDPC code using the exact LLR, with the same 512 input bits, code rate of 1/2 and frame equals 20. LDPC code is made by parity check matrix of size 512×1024; with two numbers of iterations 4 and 6.

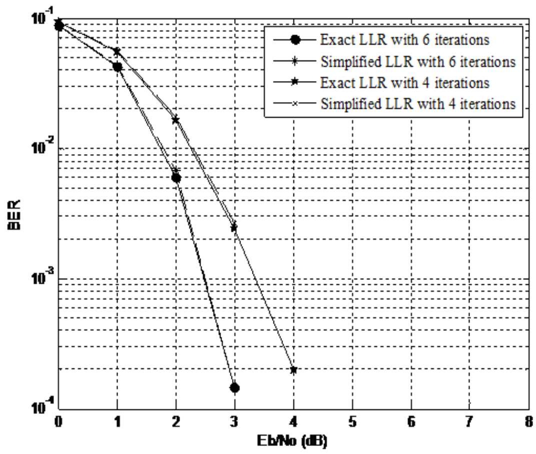

Fig. 2. Performance comparisons, under Gaussian channel, of (512, 1024) LDPC code using exact and simplified LLR algorithms, with 4 and 6 iterations (frame =20)

Under Gaussian channel and with different iteration number, as seen in figure 2, the simplification of LLR calculation has a very small performance loss. In order to study the influence of the simplified calculation on the performance of a binary LDPC code on a Rayleigh channel, in figure 3 a same performance comparison obtained on a Gaussian channel are performed on a Rayleigh channel. In this figure, we show performance comparisons, on a Gaussian and Rayleigh channels, between a binary LDPC code using the simplified LLR and a binary LDPC code using exact LLR, with the same 512 input bits, code rate of 1/2 and frame = 30. LDPC code is made by parity check matrix of size 512×1024; with number of iteration equals to 4. Also, we can see that the simplified LLR has a very small performance loss over Rayleigh channel.

The remarks obtained in figure 2 and 3 can be see when we increase the matrix size of LDPC code as shown in figure 4.

Figure 4 shows performance comparisons, on a Gaussian and Rayleigh channels, between a binary LDPC code using the simplified LLR and a binary LDPC code using exact LLR, with the same 1024 input bits, code rate of 1/2 and frame = 10. LDPC code is made by parity check matrix of size 1024×2048; with number of iteration equals 4.

As a result, the simplification of LLR calculation can achieve a good performance with a simple calculation.

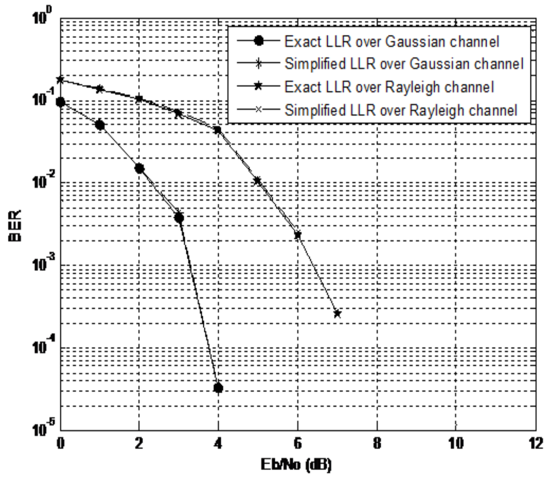

Fig. 3. Performance comparisons, under Gaussian and Rayleigh channels, of (512, 1024) LDPC code using exact APP and simplified LLR algorithms (frame = 30)

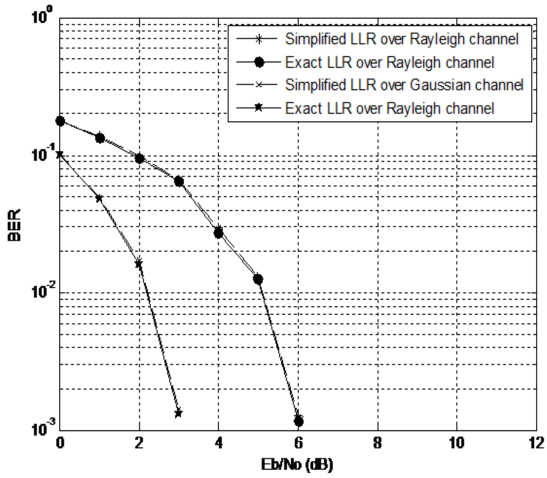

Fig. 4. Performance comparisons, under Gaussian and Rayleigh channels, of (1024, 2048) LDPC code using exact APP and simplified LLR algorithms (frame = 10)

5. Conclusion

In this work, we used the simplified calculation of the LLR for binary LDPC codes. This simplification is programmed to adapt as perfectly as possible the system to the type of channel in question. Also, it ensures an efficient decoding regardless of the channel type.

References

1. C. E. Shannon .A Mathematical Theory of Communication. Bell System Technical Journal, vol. 27, pp. 379-423 et 623-656, Juillet et Octobre 1948.
2. C. Berrou, A. Glavieux et P. Thitimajshima .Near Shannon Limit Error Correcting Coding and Decoding: Turbo-Codes. IEEE ICC'93, vol. 2/3, pp. 1064-1070, Geneva, May 1993.
3. R. Gallager .Low-Density Parity-Check Codes. IEEE Transaction Information Theory, vol. 8, no. 1, pp. 21–28, 1962.
4. R. G. Gallager .Low-density parity-check codes. Ph.D. dissertation, Department of Electrical Engineering, M.I.T., Cambridge, Mass., July 1963.
5. D. J. C. MacKay, R. M. Neal .Good codes based on very sparse matrices. in Cryptography and Coding, 5th IMA Conference, December 1995.
6. M. Sipser et D. A. Spieleman .Expender Codes. IRE Transaction on Information Theory, vol. 42, no. 06,pp. 1710-1722, November 1996.
7. S. Le Goff, C. Berrou et A. Glavieux .Turbo-Codes and High Spectral Efficiency Modulations. IEEE International Conference on Communications (ICC'94), vol. 2/3, pp.645-649, New Orleans, May 1994.
8. S. Y. Le Goff .Bit-Interleaved Turbo-Coded Modulations for Mobile Communications. European Signal Processing Conference (EUSIPCO), Finlande, 2000.
9. Xiang Liu and Martin Kosakowski .Max-Log-MAP Soft Demapper with Logarithmic Complexity for M-PAM Signals. IEEE Signal Processing Letters, pp. 50-53, vol. 22, no. 1, January 2015.
10. Md.Z. Alam, C. S. Islam et M. A. Sobhan .Using Log Likelihood Relation for BER Analysis of QAM in Space Diversity. journal of communications, vol. 4, no. 6, pp.371-379, Juillet 2009.
11. R. Ghaffar and R. Knopp .Low Complexity Metrics for BICM SISO and MIMO Systems. in 2010 IEEE 71st Vehicular Technology Conf. (VTC 2010-Spring), pp. 1–6, 6–19 May, 2010.
12. F. Tosato and P. Bisaglia .Simplified Soft-Output Demapper for Binary Interleaved COFDM with Application to HIPERLAN/2. in IEEE International Conference Communication, vol. 2, pp. 664–668 7 August, 2002.
13. K. Hyun and D. Yoon .Bit Metric Generation for Gray Coded QAM Signals. Proc. Inst. Elect. Eng., Commun., vol. 152, no. 6, pp. 1134–1138, Dec. 2005.
14. L. Wang, D. Xu, and X. Zhang, "Low Complexity Bit Metric Generation for PAM Signals Based on Non-Linear Function", Electron. Lett., vol. 17, pp. 966–967,18 August, 2011.
15. S. Le Goff .Les Turbo-Codes et leurs Applications aux Transmissions à Forte Efficacité Spectrale. PhD PhD Thesis, University of Western Brittany, Brest, November 1995.