



FORECASTING PROPERTY PRICE INDICES IN HONG KONG BASED ON GREY MODELS

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ABSTRACT. The real estate market in Hong Kong plays an important role in its economy. The property prices have been increasing a lot since 2009, which have become a major concern. However, few studies have been done to forecast the property price indices in Hong Kong. In this paper, two grey models, GM(1,1) and GM(0,N), are introduced for the forecasting. The results show that GM(1,1) has a better performance when forecasting with stable trend data, while GM(0,N) is more suitable for forecasting data in fluctuating trend. The sensitivity analysis for GM(0,N) shows that Population (POP) and Best Lending Rate (BLR) are significantly sensitive factors for data in stable trend. While for the fluctuating data, sensitivity of each factor presents uncertainties. This study also compares the forecasting performance of grey models with the ANN model and ARIMA model. The study demonstrates that grey models are more suitable for forecasting the Hong Kong property price indices than others.

KEYWORDS: Grey model; Forecast; Real estate market; Property price indices; Hong Kong

1. INTRODUCTION

The real estate market in Hong Kong plays a very important role in its economy, which is one of the most significant economic pillars of the territory. The Hong Kong real estate market is known as the most active one in the world and has experienced ups and downs over the past twenty years or so. There has been an increase in property bubbles in Hong Kong since 1997 (Teng *et al.* 2013). A newly-developed bubble detection method (Phillips *et al.* 2015) has been employed and the results reveal that positive bubbles emerged in the Hong Kong residential property market in the years 1997, 2004, and 2008, while negative bubbles have been identified in the years 2000 and 2001 (Yiu *et al.* 2013). Since 2009, the price indices rose by around 10% annually. The sharp increase of the Hong Kong property prices from 2009 led to the concerns of the emerging of property bubbles (Ahuja, Porter 2010). In the early 2011, bubbles in the Hong Kong residential property market have been identified and the primary cause is the demand pressure for the small-to-medium sized apartments (Yiu *et al.* 2013).

Currently, property prices in the territory are notoriously expensive by international standards. According to the Rating and Valuation Department Annual Report 2015, housing affordability has worsened to 58% in 2014, exceeding the long-term average of 47% over 1994–2013. Vulnerability is accumulating during the price inflation period. The real estate market is vulnerable to the future interest rate increases and global financial volatility. The Government has been vigilant and prudent, and continues to closely monitor the real estate market and evolving external environment. Property prices are always at issue in Hong Kong. How can an effective forecasting be made for future property prices? It attracts attention from various parties, including government, developers and researchers. This paper aims to find proper forecasting approaches for the Hong Kong property prices and the accurate forecasting results will be good references for all the stakeholders.

The property price forecasting has been studied by many researchers. Existing studies show that there are many types of classification of the forecasting methods, including multivariate models and univariate models (Hepsen, Vatansever

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2011), complex models and simple models (Jadevicius *et al.* 2013), traditional models and advanced models (Pagourtzi *et al.* 2003), statistical models and artificial intelligence models (Kayacan *et al.* 2010), etc. Several studies have been focused on the comparison among different methods. The same method may be grouped into different types of classification. For example, the Artificial Neural Network (ANN) has been grouped into the complex models, the advanced models and the artificial intelligence models (Jadevicius *et al.* 2013; Pagourtzi *et al.* 2003).

The autoregressive integrated moving average (ARIMA) models could provide accurate forecasting performance, especially in the short-term forecasting (Vishwakarma 2013; Crawford, Fratantoni 2003). The ARIMA models have been considered as the traditional model, the univariate model and the complex model. ARIMA models have frequently been used in forecasting the real estate market. For example, the models have been applied in forecasting the real estate market of Hong Kong and Dubai (Hepsen, Vatansever 2011; Tse 1997). There are different models in the ARIMA family, including ARIMAX and ARIMAX-GARCH. The comparison has been conducted among different ARIMA models. In addition, the comparison between ARIMA model and other models, such as GARCH and regime-switching has also been deployed. The ARIMA models could provide more accurate forecasting performance than other methods, especially in the short-term forecasting (Vishwakarma 2013; Crawford, Fratantoni 2003). However, there are some limitations with ARIMA models. Stevenson (2007) pointed out that ARIMA models have substantial different forecasting performance when choosing alternative specifications. Besides, some researchers pointed out that forecasting needs a sufficient number of observations (not less than 50) to ensure the ARIMA model is efficient (Jadevicius *et al.* 2012; Stevenson 2007; Tse 1997; McGough, Tsolacos 1995; Holden *et al.* 1990). Together with the relevant Auto Regressive (AR), Moving Average (MA), Auto Regressive Moving Average (ARMA) and Box-Jenkins models, all the above models can be mentioned as the statistical models (Kayacan *et al.* 2010).

Compared with the “traditional” multiple utilization of time series forecasting methods, some “advanced” models such as the ANN model, fuzzy logic model (FL), hedonic pricing model, etc. have been adopted by many researchers. The advanced models have some advantages when processing data with nonlinearity, fuzziness and discontinuity. ANN and FL are artificial intelligence models

and have been used to forecast the housing price. The results show that ANN and FL methods perform very well in forecasting real estate prices (Kuşan *et al.* 2010; Wilson *et al.* 2002; Aiken, Bsat 1999). Comparing with ARIMA models and hedonic regression, ANN models have lower errors (Selim 2009; Ho *et al.* 2002; Kohzadi *et al.* 1996). The forecasting performance of statistical models is less accurate and more complex to be used, compared to the artificial intelligence methods, in forecasting future values of time series (Kayacan *et al.* 2010). However, ANN models also have limitations (Zhang *et al.* 1998). The proper utilization of ANN is based on a large amount of training data and relatively long training period (Jo 2003).

The market situation not only varies with its economic development, but also with many demand and supply factors at work. The relationship between the price indices and different factors is difficult to define. Therefore, uncertainties exist in the system, which can be considered as a “grey” system. Grey system theory was originally introduced by Deng (Deng 1982). The theory can deal with the grey systems with incomplete or inadequate information (Deng 1989a). During the past three decades, the grey system theory has been developed rapidly and applied widely in many fields such as social, financial, economic, technological systems. Various Grey models have been developed. The basic and simple Grey model is the single-variable first-order grey model, abbreviated as GM(1,1). The GM(1,1) is suitable for forecasting the competitive environment where decision-makers have limited historical data. GM(1,1) has been widely used in real estate market (Cheng *et al.* 1999). The accuracy of the simulation results depend on the extent of the original data. The forecasting precision for data sequences with large random fluctuation is low (Qian *et al.* 2013; Jia *et al.* 2012). Tan (2000) also pointed that, GM(1,1) often makes a delay error when fitting a high growth index series. Some researchers explored innovative methods to enhance the forecasting performance of grey models for the fluctuating data. For example, Zhao *et al.* (2014) integrated a Markov chain model into GM(1,1) and the results can effectively reflect the fluctuation characteristics of the data. Besides, various hybrid models based on GM(1,1) were developed for the time series data forecasting (Tsaor 2010; Lin *et al.* 2009; Lin, Lee 2007; Mao, Chirwa 2006; Zhou *et al.* 2006; Yang, Xing 2006; Hsu, Chen 2003). GM(0,N) has been used in many areas when considering relevant factors (Li *et al.* 2011; Wei *et al.* 2011; You *et al.* 2009). He *et al.*

(2010) developed an improved model, I-GM(0,N), for cross-sectional data simulation and forecasting. With incomplete information, grey models perform very well in time-series forecasting. In the recent decade, the Hong Kong property price indices have shown an increasing trend with some fluctuation. For GM(0,N) considers the variation of relevant factors, it can be used to forecast a system with large fluctuations to improve the forecasting performance of GM(1,1).

In this paper, both the two mainly used grey models, GM(1,1) and GM(0,N), employ the rolling mechanism in the calculation processes. Monthly data is adopted in this study and every 12 months' original data is used to forecast the 13th month's value. The rolling mechanism could be further illustrated from two aspects. First, based on 12 months' data, several months' values in periods afterwards could be forecasted, such as the 13th, 14th, 15th, etc. Since property prices are unstable in some periods, it is difficult to obtain accurate forecasting performance using long-term data, and thus short-term forecasting is more effective (Li et al. 2012). To maximize the forecast accuracy, the study only adopts the one step simulation value (Liu, Deng 2000), the 13th month's value. Second, the rolling mechanism has been employed by many researchers in the grey models' forecasting. Kumar and Jain (2010) used grey model with a rolling mechanism to forecast the coal and electricity consumption. In the coal consumption forecasting case, recent 6 years' data has been employed to predict the 7th year's value. While in the electricity consumption forecasting case, recent 5 years' data has been employed for the 6th year's value forecasting. The results obtained from the rolling mechanism are comparable to the data in official planning. Akay and Atak (2007) implemented the rolling mechanism with the utilization of recent four years data (k-3, k-2, k-1, k) to forecast the (k+1)th year's value. It has been emphasized that the rolling mechanism is very efficient to increase the forecasting accuracy of grey prediction for the chaotic data (Wang 2007). The rolling mechanism was also employed in using the data from 1994 to 2004 to forecast the values from 2006 to 2015 by using the forecasted values as original data to forecast future values year by year. Thus, for the high accuracy that can be obtained from the rolling mechanism, it can be employed to achieve the long-term forecasting. In this study, GM(1,1) and GM(0,N) will be used to forecast the property price indices in Hong Kong. And the results will also be compared with ARIMA and ANN models' forecasting performance.

2. METHODOLOGY

Grey models have been widely and successfully applied in various fields by researchers around the world (Lee, Tong 2011; Zhou et al. 2006; Hsu, Chen 2003). And many universities offer courses in grey systems theory (Liu et al. 2010a,b). Two grey models, GM(1,1) and GM(0,N), are used in this study. The calculation of the two grey models is conducted with the Grey System Theory Modeling Software (7.0), which is developed by the Institute for Grey Systems Studies (IGSS) in the Nanjing University of Aeronautics and Astronautics (Zeng, Liu 2015). The software is developed in according to the book Grey System Theory and Application (Liu et al. 2010a, 2010b). The algorithm of the two models is shown as follows.

2.1. GM (1,1) model

According to Liu and Lin (2006) and Liu et al. (2010a, 2010b), the algorithm of GM(1,1) can be summarized as follows:

Step 1: Generate $X^{(1)}$ by 1-AGO.

For an original non-negative sequence $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}$, it can be transformed into a new sequence $X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\}$ by applying a first-order accumulated generation operator (1-AGO).

$$x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i), k = 1, 2, \dots, n. \quad (1)$$

In most cases, the original data do not have an obvious development tendency. After applying the accumulating operator one time, the new sequence will show a development tendency, an exponential growth pattern. Further, the characteristics hidden in the original data can be revealed and interpreted.

Step 2: Check the quasi-smoothness on $X^{(0)}$ by:

$$\rho(k) = \frac{x^{(0)}(k)}{x^{(1)}(k-1)}, k = 3, 4, \dots, n, \quad (2)$$

when: $k > 3$; $\rho(k) < 0.5$

Step 3: Check the quasi-exponentially of $X^{(1)}$ by:

$$\sigma^{(1)}(k) = \frac{x^{(1)}(k)}{x^{(1)}(k-1)}, k = 3, 4, \dots, n, \quad (3)$$

when: $k > 3$; $\sigma^{(1)}(k) \in [1, 1.5]$

Step 4: The sequence $X^{(1)}$ is then modelled by a differential equation (whitening equation) as follows:

$$\frac{dx^{(1)}(k)}{dt} + ax^{(1)}(k) = b, k = 1, 2, \dots, n, \quad (4)$$

where: $\frac{dx^{(1)}(k)}{dt}$ is the derivative of the unknown function; $x^{(1)}(k)$ is the background value, and a and b are the parameters. The grey derivative of $X^{(1)}$ is:

$$d(k) = x^{(0)}(k), k = 1, 2, \dots, n. \tag{5}$$

Then the equation (4) can be written as:

$$x^{(0)}(k) + ax^{(1)}(k) = b, k = 1, 2, \dots, n. \tag{6}$$

Because the grey derivative $x^{(0)}(k)$ and elements in the set of background values $\{x^{(1)}(k), x^{(1)}(k-1)\}$ do not satisfy the horizontal mapping relation, a generated sequence $Z^{(1)} = \{z^{(1)}(1), z^{(1)}(2), \dots, z^{(1)}(n)\}$ is introduced by $z^{(1)}(k) = \alpha x^{(1)}(k) + (1 - \alpha)x^{(1)}(k-1)$, $\alpha \in [0, 1]$, where α is the generation coefficient. When $\alpha > 0.5$, the generated sequence $z^{(1)}(k)$ gives more emphasis on new data, when $\alpha < 0.5$, the generated sequence $z^{(1)}(k)$ gives more emphasis on old data, normally, a mean generated sequence is used with $\alpha = 0.5$.

The equation (6) is translated into a grey differential equation as follows:

$$x^{(0)}(k) + az^{(1)}(k) = b, k = 1, 2, \dots, n. \tag{7}$$

Step 5: The least-squares error method is used to get the parameters a and b :

$\hat{a} = (a, b)^T = (B^T B)^{-1} B^T Y$, where

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \dots \\ x^{(0)}(n) \end{bmatrix}, B = \begin{bmatrix} -z^{(0)}(2) & 1 \\ -z^{(0)}(3) & 1 \\ \dots & \dots \\ -z^{(0)}(n) & 1 \end{bmatrix}. \tag{8}$$

Step 6: With the parameters a and b , the simulation value of $\hat{X}^{(1)}$ is calculated by:

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} + \frac{b}{a}, \tag{9}$$

where: $\hat{x}^{(1)}(k+1)$ is the forecast value of $x^{(1)}(k+1)$; \wedge denotes the forecast value. According to the equation (9), it can be seen that GM(1,1) is an exponential curve.

Step 7: Restore the $\hat{X}^{(1)}$ value to get the simulation value of $\hat{X}^{(0)}$ by:

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1). \tag{10}$$

Step 8: Check errors. The sum of squared errors and relative errors can be checked.

Error: $\varepsilon(k) = x^{(0)}(k) - \hat{x}^{(0)}(k)$.

Relative error: $\Delta_k = \frac{|\varepsilon(k)|}{x^{(0)}(k)}$.

Mean Absolute Percentage Error:

$$\Delta = \frac{1}{n-1} \sum_{k=2}^n \Delta_k.$$

Sum of squared errors:

$$s = \varepsilon^T \varepsilon = [\varepsilon(2), \varepsilon(3), \dots, \varepsilon(n)] \begin{bmatrix} \varepsilon(2) \\ \varepsilon(3) \\ \dots \\ \varepsilon(n) \end{bmatrix}.$$

Step 9: Forecast the future value.

2.2. GM (0,N) model

GM(0,N) is a static model which does not contain derivatives. The GM(0,N) verifies the uncertainty in quantitative relations between the main system's characteristics and a number of primary influencing behavioral factors. According to Liu and Lin (2006) and Liu *et al.* (2010a, 2010b), the algorithm of the GM(0,N) can be summarized as follows:

Step 1: Get the 1-AGO sequence $X_i^{(l)}$ of $X_i^{(0)}$.

Assume that $X_i^{(0)}$ is a data sequence with system's characteristics, $X_i^{(0)}$, $i = 2, 3, \dots, N$, are relevant factor sequences, and $X_i^{(l)}$ are the 1-AGO sequences of $X_i^{(0)}$, $i = 1, 2, 3, \dots, N$. Then the GM(0,N) can be presented as:

$$X_i^{(l)} = b_2 X_2^{(l)} + b_3 X_3^{(l)} + \dots + b_N X_N^{(l)} + a, \tag{11}$$

where: a is the development coefficient of the system; b_i is the driving coefficient.

Step 2: Make matrix B and Y .

$$B = \begin{bmatrix} X_2^{(1)}(2) & X_3^{(1)}(2) & \dots & X_N^{(1)}(2) & 1 \\ X_2^{(1)}(3) & X_3^{(1)}(3) & \dots & X_N^{(1)}(3) & 1 \\ \dots & \dots & \dots & \dots & \dots \\ X_2^{(1)}(n) & X_3^{(1)}(n) & \dots & X_N^{(1)}(n) & 1 \end{bmatrix}, \tag{12}$$

$$Y = \begin{bmatrix} X_1^{(1)}(2) \\ X_1^{(1)}(3) \\ \dots \\ X_1^{(1)}(n) \end{bmatrix}.$$

The least squares estimate of the parameter sequence $\hat{b} = [b_2 \ b_3 \ \dots \ b_N \ a]^T$ is:

$$\hat{b} = [B^T B]^{-1} B^T Y. \tag{13}$$

Step 3: Apply the parameter sequence \hat{b} to the equation (11), get the GM(0,N) estimation model.

$$X_1^{(1)}(k) = b_2 X_2^{(1)}(k) + b_3 X_3^{(1)}(k) + \dots + b_N X_N^{(1)}(k) + a. \tag{14}$$

Step 4: Restore the 1-AGO sequence through the equation.

$$\hat{X}_1^{(0)}(k) = \hat{X}_1^{(1)}(k) - \hat{X}_1^{(1)}(k-1). \tag{15}$$

Step 5: Check errors. The sum of squared errors and relative errors can be checked.

$$\text{Error: } \varepsilon(k) = x^{(0)}(k) - \hat{x}^{(0)}(k).$$

$$\text{Relative error: } \Delta_k = \frac{|\varepsilon(k)|}{x^0(k)}.$$

Mean Absolute Percentage Error:

$$\Delta = \frac{1}{n-1} \sum_{k=2}^n \Delta_k.$$

Sum of squared errors:

$$s = \varepsilon^T \varepsilon = [\varepsilon(2), \varepsilon(3), \dots, \varepsilon(n)] \begin{bmatrix} \varepsilon(2) \\ \varepsilon(3) \\ \dots \\ \varepsilon(n) \end{bmatrix}.$$

Step 6: Forecast the future value.

3. DATA COLLECTION

The real estate market in Hong Kong fluctuates with the variation of economic development. Data collection based on two major considerations: economic development characteristics; and the availability of data. The trend in the Hong Kong economy was steadily increasing since 2004, except the downturn in 2008, which is due to the U.S. sub-prime crisis. Property price indices kept increasing in the past decade, as shown in Figure 1. From about Jul 2008 to Jan 2009, property price indices of the four sub-sectors all went down by around 20% over 7 months. While from about Feb 2009 to Jul 2009, the indices increased around 20% from the pull-back. Thus this period is defined as a fluctuating period. By contrast, if the data kept a continuous steady increasing trend without sudden large increase or decrease within a few months, the data are defined as being in a stable trend. In this

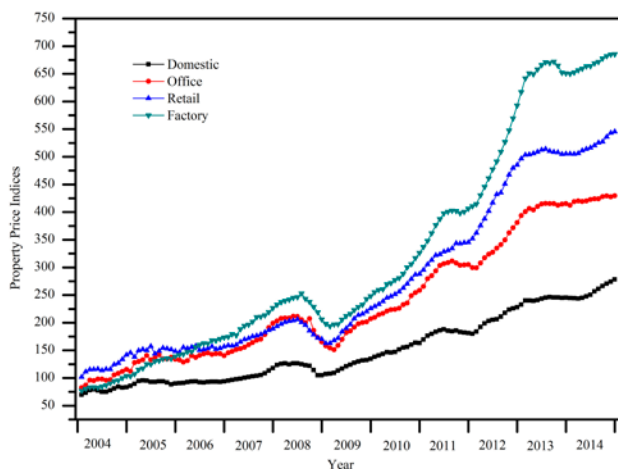


Fig. 1. Hong Kong property price index trend lines

Source: Property Market Statistics, Rating and Valuation Department of Hong Kong.

study, monthly data from Jan 2004 to Dec 2014 were used for analysis. Property price indices were obtained from the Hong Kong Rating and Valuation Department website, including the following four sectors: Private Domestic (domestic sector), Private Office (office sector), Private Retail (retail sector), Private Flatted Factories (factory sector).

There are many factors affecting the property price indices. Tse *et al.* (1999) focused on the role of independent variables, including population, transaction volume, best lending rate and inflation, in determining house prices. Hui *et al.* (2014) examined the inter-relationships between housing transaction volume and a variety of factors, including money supply, housing price, banks' loan-to-deposit ratio, stock market returns, and best lending rate.

Six factors have been selected and used in the forecasting of the property prices in this study. The first one is population (POP). With the population growth, more residential demand and investment demand are generated, thus leading to a large number of transactions in the real estate market. The increasing population causes the variation of the supply-demand relationship of the real estate market and affects the property prices. Lee (2009) demonstrated that the volatility of population is influential in affecting the fluctuation of housing prices. The past high volatility of population will cause an amplification effect for the current fluctuation of housing market. POP is adopted as a factor to forecast the property prices in this study.

The second factor that has been considered is the composite consumer price index (CCPI). According to the Hong Kong Annual Report on the Consumer Price Index (2015 Edition), the weight of housing in CCPI is the largest among the all sections, which is 31.66%. Diewert *et al.* (2009) proposed a new approach to account for owner occupied housing (OOH) in the CPI to consider the housing bubble in the real estate market. They all demonstrate the close relationship between CCPI and property prices. CCPI is adopted as the second factor in this study.

The third factor is the best lending rate (BLR). Lending rate is directly related with the mortgage amount of the property owner and affects property debt financing conditions. Agnello and Schuknecht (2011) examined the determinants for the booms and busts in a housing market and concluded that the annual growth rate of housing prices is negatively related to the short-term interest rates. Gerlach and Peng (2005) studied the correlation between residential property prices and BLR in

Hong Kong. The results show that the property prices influence the bank credit rather than conversely. So the close relationship between property prices and BLR is presented.

The fourth and the fifth factors are money supply (M3) and Hang Seng Index (HSI). Money supply is a factor that affects property demand, especially the investment option. A major source of new money supply in Hong Kong is through hot money from Mainland China or other foreign countries. Three flows of the incoming capital are included, namely the stock market, property markets, and banking system (Hui *et al.* 2014). The first two flows will generate shocks to the stock market and real estate market (Guo, Sophie 2010). The positive correlation between the real estate market and stock market has also been examined by Hui *et al.* (2011). The results also show the co-movement between the two markets. The last flow, from banking, is helpful to improve the liquidity condition of financial institutions and availability of loans for further investment. This can also be demonstrated by the last factor that has been considered in this study, the banks' loan-to-deposit ratio (LTD), that is used to assess a bank's liquidity and which demonstrates the availability of the loans for the real estate market. Thus, the main six factors adopted in this study include population (POP), composite consumer price index (CCPI), best lending rate (BLR), money supply (M3), Hang Seng Index (HSI), banks' loan-to-deposit ratio (LTD). The data of these six factors can be obtained from the Hong Kong Monetary Authority and the Census and Statistics Department websites.

4. DATA ANALYSIS

Grey models can be used with at least 4 historical data points for each variable to forecast the behavior of a system (Lee, Tong 2011; Deng 1989b). Mao and Chirwa (2006) deployed repeated computation and comparisons for the optimum sample number n for the application of grey models. The application of GM(1,1) for the forecasting of vehicle rollover fatal accidents indicated that the optimum of the sample number n is 11. Hsu and Chen (2003) proposed an improved GM(1,1), using a technique that combines residual modification with artificial neural network estimation. The number of the original observations is 14 for fitting the forecasting. In this study, the previous 12 months' data are used to forecast the 13th month's value. To maximize the forecast accuracy, the study only adopts the one step simulation value (Liu, Deng 2000). For example, the data of domestic sector from Jan 2004 to Dec 2004 are used to forecast the value of Jan 2005, the data from Feb 2004 to Jan 2005 are used to forecast Feb 2005. Therefore, the property indices from Jan 2005 to Dec 2014 can be simulated by GM(1,1) and GM(0,N).

In this data analysis section, the correlation analysis between six factors and property prices is conducted in the first part. Then taking the domestic sector data from Jan 2004 to Dec 2004 as an example, the simulation procedures of the GM(1,1) and GM(0,N) models have been demonstrated in the following section 4.2 and section 4.3. According to the presented calculation procedure, the property indices from Jan 2005 to Dec 2014 are

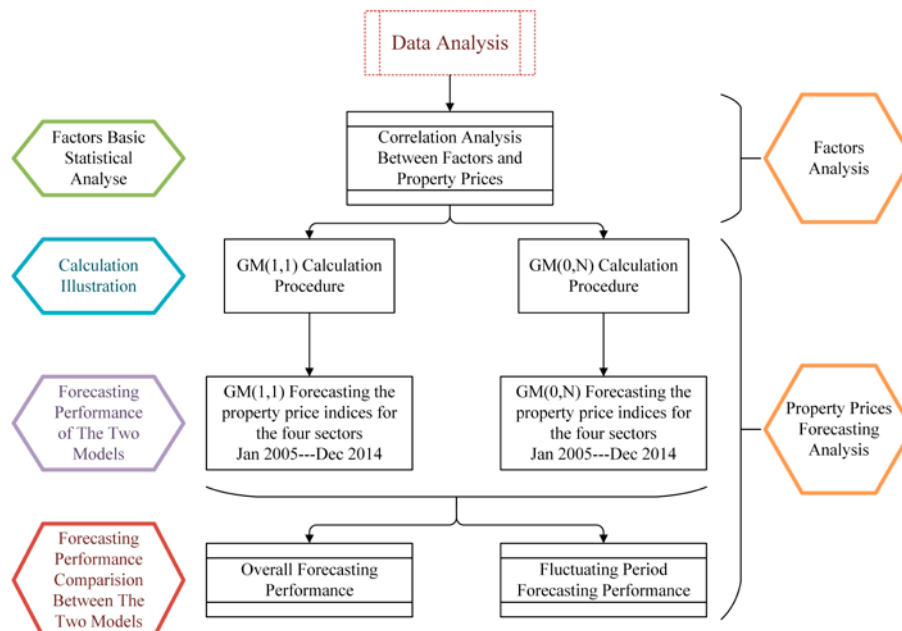


Fig. 2. Main steps of data analysis

Step 3: Check quasi-exponentially on $X^{(1)}$.

$$\begin{aligned} \sigma^{(1)}(3) &= \frac{x^{(1)}(3)}{x^{(1)}(2)} = \frac{220.8}{142.7} \approx 1.547; \\ \sigma^{(1)}(4) &= \frac{x^{(1)}(4)}{x^{(1)}(3)} = \frac{300.2}{220.8} \approx 1.360 \in [1, 1.5]; \\ \sigma^{(1)}(5) &= \frac{x^{(1)}(5)}{x^{(1)}(4)} = \frac{377.7}{300.2} \approx 1.258 \in [1, 1.5]; \\ \sigma^{(1)}(6) &= \frac{x^{(1)}(6)}{x^{(1)}(5)} = \frac{452.4}{377.7} \approx 1.198 \in [1, 1.5]; \\ \sigma^{(1)}(7) &= \frac{x^{(1)}(7)}{x^{(1)}(6)} = \frac{527.3}{452.4} \approx 1.166 \in [1, 1.5]; \\ \sigma^{(1)}(8) &= \frac{x^{(1)}(8)}{x^{(1)}(7)} = \frac{604.9}{527.3} \approx 1.147 \in [1, 1.5]; \\ \sigma^{(1)}(9) &= \frac{x^{(1)}(9)}{x^{(1)}(8)} = \frac{685.8}{604.9} \approx 1.134 \in [1, 1.5]; \\ \sigma^{(1)}(10) &= \frac{x^{(1)}(10)}{x^{(1)}(9)} = \frac{769.9}{685.8} \approx 1.123 \in [1, 1.5]; \\ \sigma^{(1)}(11) &= \frac{x^{(1)}(11)}{x^{(1)}(10)} = \frac{852.6}{769.9} \approx 1.107 \in [1, 1.5]; \\ \sigma^{(1)}(12) &= \frac{x^{(1)}(12)}{x^{(1)}(11)} = \frac{935.9}{852.6} \approx 1.098 \in [1, 1.5], \end{aligned}$$

when: $k > 3; \sigma^{(1)}(k) \in [1, 1.5]$. The quasi-exponentially of $X^{(1)}$ of is satisfied. Therefore, the GM(1,1) can be applied on $X^{(1)}$.

Step 4 ~ Step 7: Establish the GM(1,1).

The mean generated sequence of consecutive neighbours of $X^{(1)}$ is given by $Z^{(1)}$,

$$Z^{(1)} = \{106.10, 181.75, 260.50, 338.95, 415.05, 489.85, 566.10, 645.35, 727.85, 811.25, 894.25\}.$$

The development coefficient “a” and grey action “b” are calculated as $a = -0.01$, and $b = 73.43$;

A GM(1,1) is obtained as follows:

$$x^{(0)}(k) - 0.01z^{(1)}(k) = 73.43.$$

Step 8: Check the errors.

The results are shown in Table 2.

Step 9: Forecast the future value.

The one step forecast value is 83.98. The Jan 2005 true value of domestic sector is 85.7. It can be calculated that the absolute value of the forecasting error is 2.01%.

4.3. GM(0,N) calculation procedure

Step 1: Get the 1-AGO sequence $X_i^{(1)}$ of $X_i^{(0)}$.

The sequence of system’s characteristics is:

$$X_1^{(0)} = \{69.5, 73.2, 78.1, 79.4, 77.5, 74.7, 74.9, 77.6, 80.9, 84.1, 82.7, 83.3\};$$

$$X_1^{(1)} = \{69.50, 142.70, 220.80, 300.20, 377.70, 452.40, 527.30, 604.90, 685.80, 769.90, 852.60, 935.90\}.$$

The relevant factor sequences of $X_i^{(0)}$, $i = 2, 3 \dots 7$, are as follows,

$$X_2^{(0)} = \{6767.42, 6770.63, 6773.85, 6777.07, 6780.28, 6783.50, 6785.87, 6788.23, 6790.60, 6792.97, 6795.33, 6797.70\};$$

...

$$X_7^{(0)} = \{82.283, 82.734, 83.253, 84.606, 84.656, 85.282, 85.967, 86.047, 85.371, 83.761, 83.992, 82.597\}.$$

Then the 1-AGO sequences of the relevant factor sequences are:

$$X_2^{(1)} = \{6767.42, 13538.05, 20311.90, 27088.97, 33869.25, 40652.75, 47438.62, 54226.85, 61017.45, 67810.42, 74605.75, 81403.45\};$$

...

$$X_7^{(1)} = \{82.28, 165.02, 248.27, 332.88, 417.53, 502.81, 588.78, 674.83, 760.20, 843.96, 927.95, 1010.55\}.$$

Step 2: Make matrix B and Y,

$$B = \begin{bmatrix} 13538.1 & 178.7 & \dots & 165.0 & 1 \\ 20311.9 & 267.8 & \dots & 248.3 & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 81403.15 & 1071.9 & \dots & 1010.5 & 1 \end{bmatrix},$$

$$Y = \begin{bmatrix} 142.7 \\ 220.8 \\ \dots \\ 935.9 \end{bmatrix}.$$

The least squares estimate of the parameter sequence $\hat{b} = [b_2 \ b_3 \ \dots \ b_N \ a]^T$ is:

$$\begin{aligned} \hat{b} &= [B^T B]^{-1} B^T Y = [a, b]^T; \\ a &= -30.1; \\ b &= [-0.16; 10.29; 48.27; 0.00; 0.01; 0.36]^T. \end{aligned}$$

Step 3: The GM(0, N) estimation model as follows:

$$X_1^{(1)}(k) = -0.16X_2^{(1)}(k) + 10.29X_3^{(1)}(k) + 48.27X_4^{(1)}(k) + 0.01X_6^{(1)}(k) + 0.36X_7^{(1)}(k) - 30.1.$$

Step 4~ Step 5: Restore the 1-AGO sequence and calculate the mean absolute percentage error and sum of squared errors. The results are shown in Table 2.

Table 2. The GM(1,1) and GM(0,N) forecasting errors

Year	Month	Actual	Simulation		Absolute errors $\varepsilon(k) = x^{(0)}(k) - \hat{x}^{(0)}(k)$		Relative errors $\Delta_k = \left \frac{\varepsilon(k)}{x^{(0)}(k)} \right $	
			GM(1,1)	GM(0,N)	GM(1,1)	GM(0,N)	GM(1,1)	GM(0,N)
2004	Jan	69.5	–	–	–	–	–	–
	Feb	73.2	74.58	84.24	-1.38	-11.04	1.89%	15.08%
	Mar	78.1	75.39	79.59	2.71	-1.49	3.47%	1.91%
	Apr	79.4	76.21	78.79	3.19	0.61	4.02%	0.77%
	May	77.5	77.03	76.04	0.47	1.46	0.60%	1.88%
	Jun	74.7	77.87	75.77	-3.17	-1.07	4.25%	1.43%
	July	74.9	78.72	75.64	-3.82	-0.74	5.09%	0.99%
	Aug	77.6	79.57	77.68	-1.97	-0.08	2.54%	0.10%
	Sep	80.9	80.43	79.41	0.47	1.49	0.58%	1.84%
	Oct	84.1	81.31	84.3	2.79	-0.2	3.32%	0.24%
	Nov	82.7	82.19	84.07	0.51	-1.37	0.62%	1.66%
	Dec	83.3	83.08	82.2	0.22	1.1	0.26%	1.32%

MAPE(mean absolute percentage error): GM(1,1) $\Delta = \frac{1}{11} \sum_{k=2}^{12} \Delta_k = 2.42\%$; GM(0,N) $\Delta = \frac{1}{11} \sum_{k=2}^{12} \Delta_k = 2.47\%$.

Sum of squared errors: GM(0,N) $s = \varepsilon^T \varepsilon = 133.63$

Step 6: Forecast the future value.

The simulation result is 75.47 and the true value is 85.7. It can be calculated that the absolute value of the forecasting error is 11.94%.

4.4. Forecasting performance of GM(1,1) and GM(0,N)

According to the calculation processes of the Grey System Theory Modeling Software 7.0 (Zeng, Liu 2015), the monthly property price indices of the four sectors from Jan 2005 to Dec 2014 could be simulated by GM(1,1) and GM(0,N) through the employment of the software. The percentages in the forecasting error (absolute value) scope of the two models are shown in Table 3. The simulation values and the true values for the four sectors

are shown in Figures 3 to 6. It can be seen that the forecasting performance of both GM(1,1) and GM(0,N) are acceptable. Around 80% forecasting errors are less than 5%. The GM(1,1)'s forecasting lines fit much better with the true value lines than GM(0,N) for the whole forecasting period. Some sudden changes exist in the GM(0,N)'s forecasting lines. However, during the fluctuating period (Jul 2008–Jul 2009), GM(0,N) performances are much better than GM(1,1), as shown in Figure 7 and Table 4. Further, the simulation results of GM(1,1) present obvious forecasting lags compared with the true values for the fluctuating period. The forecasting performance of GM(0,N) is much better than GM(1,1). Therefore, GM(0,N) should be considered for use when a fluctuating period occurs.

Table 3. Statistics of forecasting performance of two grey models (Jan 2005–Dec 2014)

Percentage	Error scope	Forecasting error (absolute value) scope		
		0–5%	5%–10%	>10%
Domestic	GM(1,1)	79.2%	16.7%	4.2%
	GM(0,N)	77.5%	14.2%	8.3%
Office	GM(1,1)	71.7%	20.8%	7.5%
	GM(0,N)	75.8%	13.3%	10.8%
Retail	GM(1,1)	85.0%	8.3%	6.7%
	GM(0,N)	78.3%	16.7%	5.0%
Factory	GM(1,1)	78.3%	15.0%	6.7%
	GM(0,N)	73.3%	19.2%	7.5%

Table 4. Statistics of forecasting errors of four sectors for fluctuating data (Jul 2008–Jul 2009)

Error scope	Domestic (Percentage)		Office (Percentage)		Retail (Percentage)		Factory (Percentage)	
	GM(1,1)	GM(0,N)	GM(1,1)	GM(0,N)	GM(1,1)	GM(0,N)	GM(1,1)	GM(0,N)
0~5%	15.38%	84.62%	23.08%	76.92%	15.38%	69.23%	7.69%	84.62%
5%~10%	46.15%	0%	23.08%	23.08%	23.08%	15.38%	46.15%	15.38%
>10%	38.46%	15.38%	53.85%	0.00%	61.54%	15.38%	46.15%	0.00%
MAPE	8.60%	4.35%	11.53%	4.33%	9.29%	3.10%	8.51%	2.38%

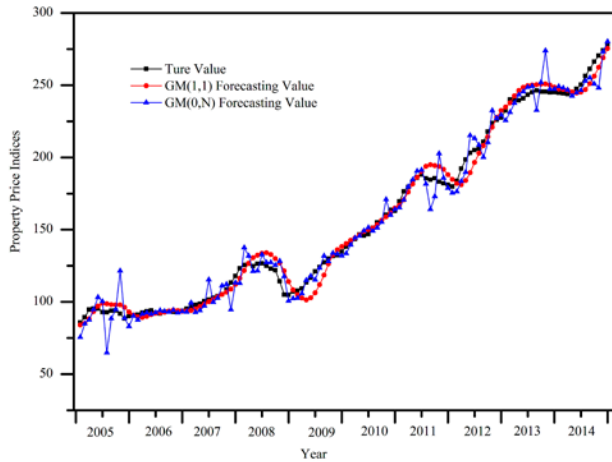


Fig. 3. Fitting lines of forecast values and true values (domestic sector)

Source: Authors' calculations & Property Market Statistics, Rating and Valuation Department of Hong Kong.

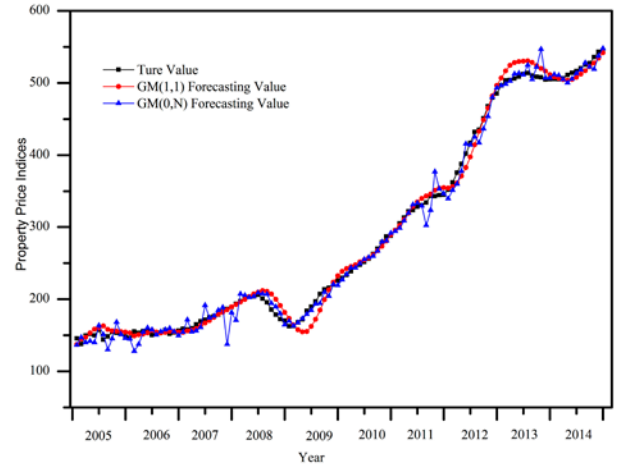


Fig. 5. Fitting lines of forecast values and true values (retail sector)

Source: Authors' calculations & Property Market Statistics, Rating and Valuation Department of Hong Kong.

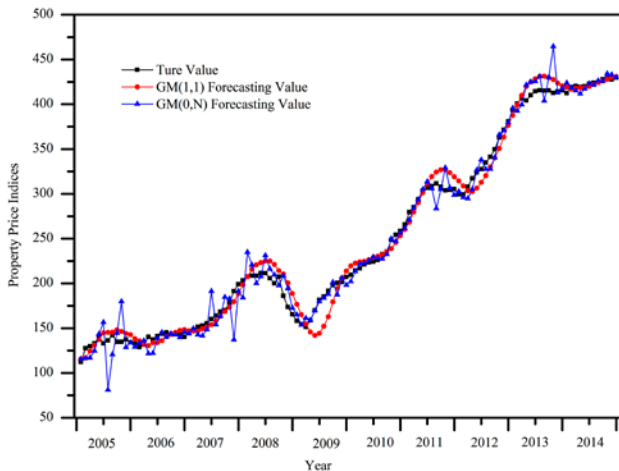


Fig. 4. Fitting lines of forecast values and true values (office sector)

Source: Authors' calculations & Property Market Statistics, Rating and Valuation Department of Hong Kong.

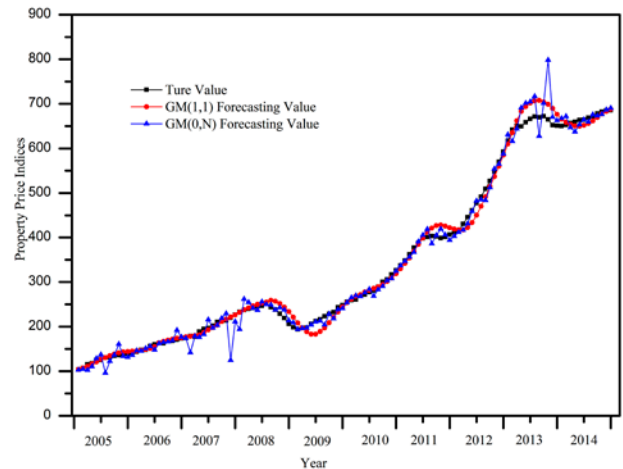


Fig. 6. Fitting lines of forecast values and true values (factory sector)

Source: Authors' calculations & Property Market Statistics, Rating and Valuation Department of Hong Kong.

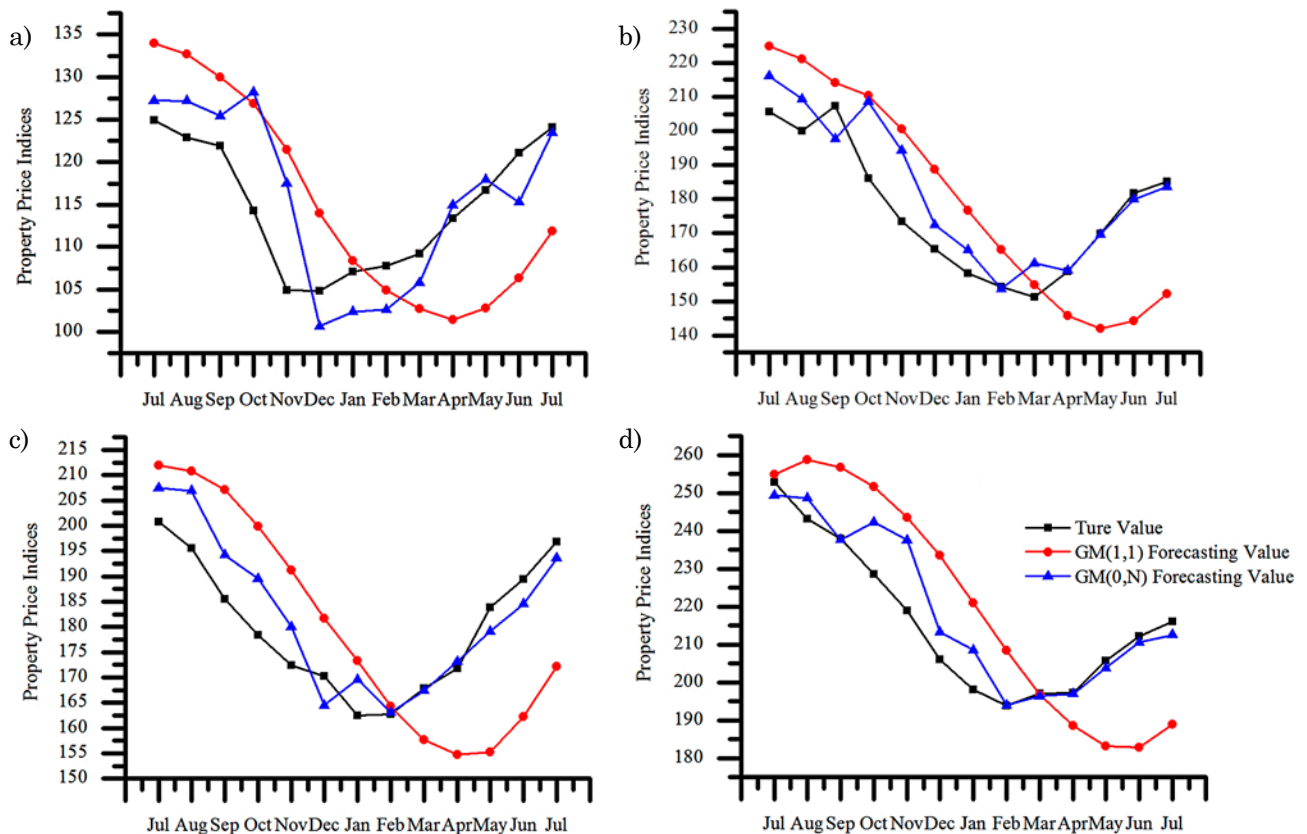


Fig. 7. Fitting lines of four sectors for fluctuating data (Jul 2008–Jul 2009)
(a) domestic (b) office (c) retail (d) factory

Source: Authors' calculations & Property Market Statistics, Rating and Valuation Department of Hong Kong.

5. SENSITIVITY ANALYSIS

From the graphs presented in Section 4 above, the model GM(0,N) shows its outperformance during the fluctuating period with considering relevant factors. In practice, the impact of factors varies. In this section, the impact of factors on property prices was examined by sensitivity analysis for two periods, the stable period from Jan 2010 to Dec 2010, and the fluctuating period from Jul 2008 to Jul 2009. During the selected stable period from Jan 2010 to Dec 2010, the Hong Kong's economy showed an increase and the four sectors' property prices are in stable increasing trends without sudden change, which could represent stable period. For the period from Jul 2008 to Jul 2009, the Hong Kong's economy development and four sectors' property prices present obvious fluctuation according to the previous analysis in Section 3, which could represent fluctuate periods. The sensitivity coefficients were calculated when one factor's value increases or decreases 10% respectively. The results are shown in Figures 8 and 9.

For the period from Jan 2010 to Dec 2010, the sensitivity coefficients of POP and BLR are much higher than for the other four factors, which indicate that POP and BLR have high influence on the property prices during stable periods. Therefore, for a stable trend, the POP and BLR are relatively sensitive factors for forecasting property indices. The sensitivity of these two sensitive factors to the property price has also been emphasized by some existing studies (Jeanty *et al.* 2010; Gerlach, Peng, 2005; Iacoviello, Minetti 2003).

For the period from Jul 2008 to Jul 2009, the sensitivity coefficients of the six factors present much more uncertainty. Specifically, when the original data went through a decrease and then increased 20% in about half year, the sensitivity coefficients of some factors present are shown to be very low or very high. Especially from Feb 2009, when the economy began to rebound, some sensitivity coefficients of the six factors become very high. The sensitivity coefficients of POP vary in the largest degree. There are no definite high or low sensitive factors during the fluctuating period.

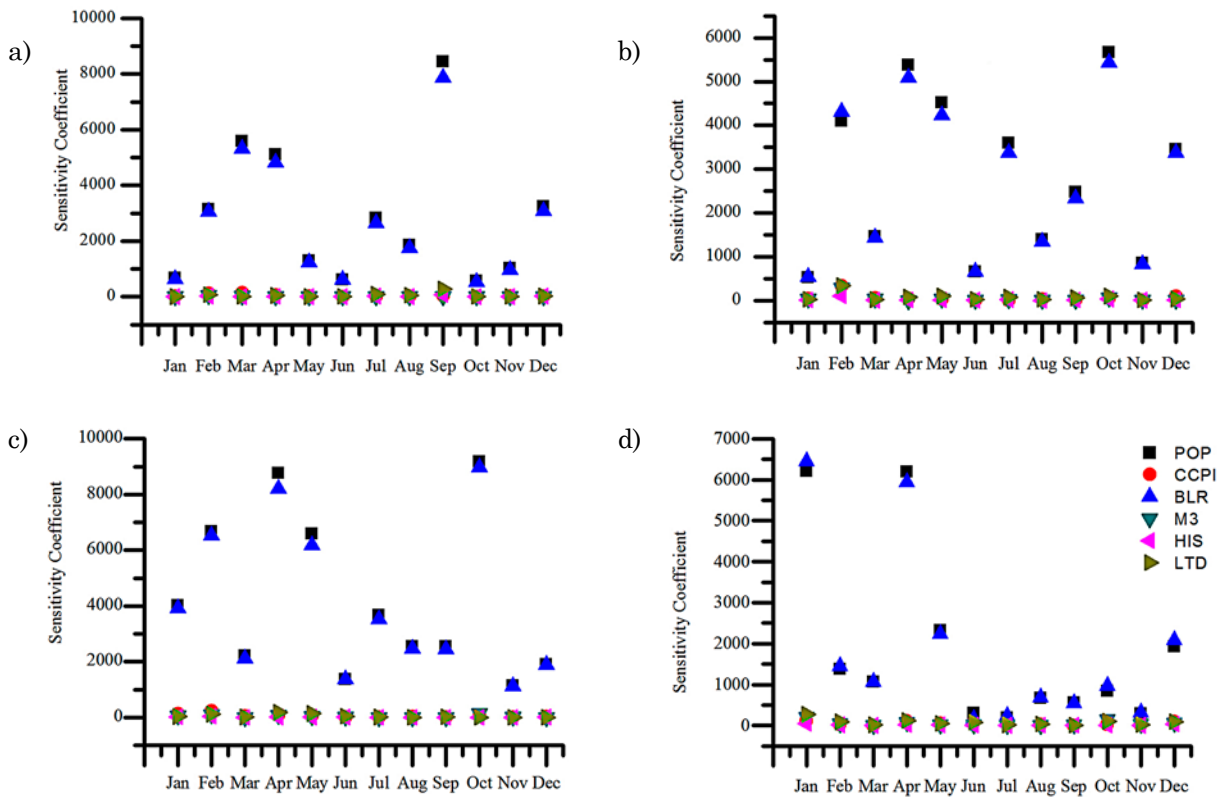


Fig. 8. Scatter plots of sensitivity coefficients for stable trend data (Jan 2010–Dec 2010)
 (a) domestic (b) office (c) retail (d) factory

Source: Authors' calculations.

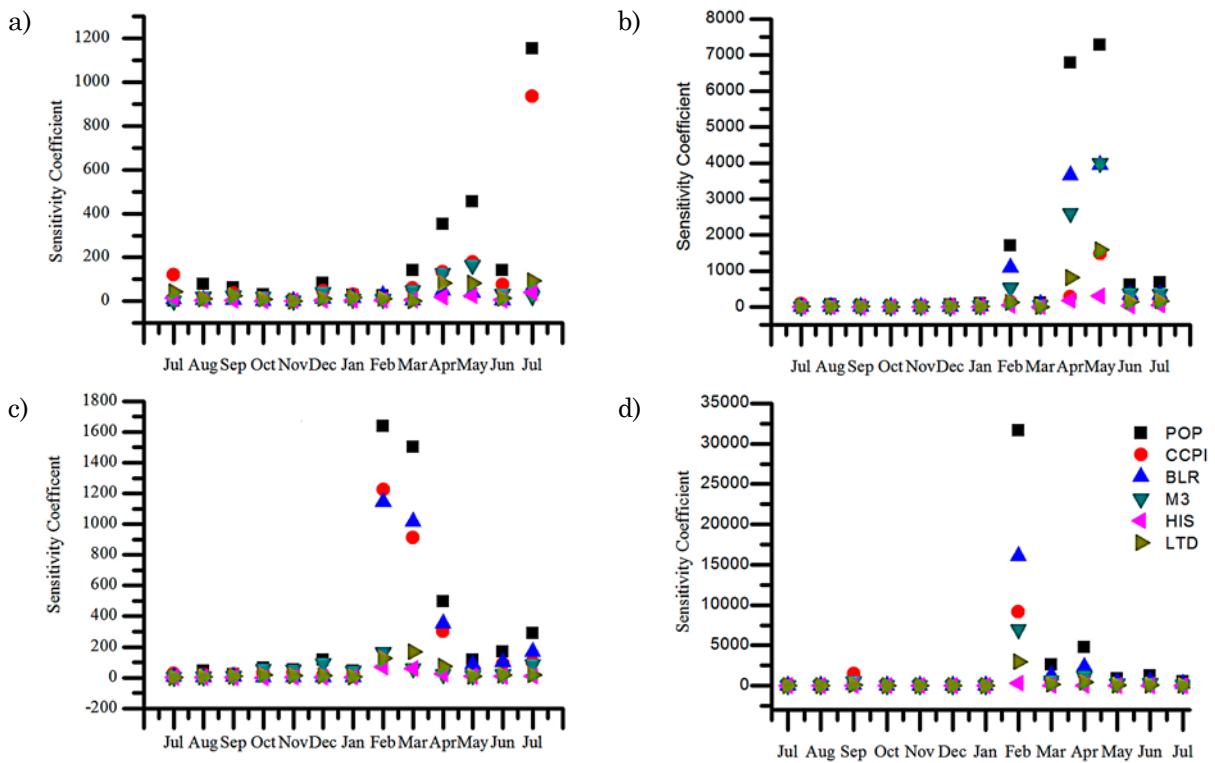


Fig. 9. Scatter plots of sensitivity coefficients for fluctuating trend data (Jul 2008–Jul 2009)
 (a) domestic (b) office (c) retail (d) factory

Source: Authors' calculations.

6. FORECASTING PERFORMANCE OF FOUR MODELS

Both the ANN model and ARIMA model are also widely used in property price forecasting. In accordance with the above grey models, the monthly price indices from Jan 2005 to Dec 2014 are forecast by both the ANN model and ARIMA model for the comparison with the two grey models.

First, the ANN model has been employed in the forecasting. The type of ANN model that has been used in this study is the Back Propagation Neural Network (BPNN) model, which was proposed in 1986. BPNN model opened the way for multi-layer ANN using and overcame shortcomings of a single layer model. In ANN models, BPNN model is the most popular model used and so it has also been developed further by many researchers (Graupe 2007). The BPNN model used in this study is based on the software Matlab. In the calculation programming, using the function of `newff` () in Matlab to establish the network structure and two layers are set. The number of hidden neurons in the hidden layer is decided through a validation process (Huang *et al.* 2006). The neuron numbers of BPNN model in the validation process are in the range of [3, 5, 10, 15, 20, 25, 30, 35, 40]. The results show that with the increase of neuron numbers, the training performance and forecasting performance become better. However, when the neuron numbers reach to 30 or more, the training speed is noticeably slower than in the previous runs. Thus, the number of hidden neurons is set as 25 in the hidden layer. In the calculation processes, every year's 12 month price indices' forecast is based on the former 90 months' data, which are used as training data. Six factors, POP, CCPI, BLR, M3, HSI, LTD are inputs, and prices indices are outputs. The price indices for four sectors were calculated and then the Mean Absolute Percent Errors (MAPEs) and the percentages of the forecasting error in the scope of 0~5% could be obtained, as shown in Table 5.

Second, the ARIMA model has been employed in the forecasting. For the ARIMA model, as mentioned in the introduction section, it needs not less than 50 observations. Thus 5 years' monthly data, or 60 observations, are used for the forecasting of the following 5 months' values in this study. The calculation of the ARIMA model is based on the software SPSS. Specifically, the "forecasting" function of SPSS is used for the establishment of the ARIMA model and it then conducts a calculation process. The first time period input in SPSS

is from Jan 2000 to Dec 2004, which was used to forecast Jan 2005 to May 2005. The procedure was repeated for the four sectors. The MAPEs and the percentages of forecasting error in the scope 0~5% of ARIMA model are shown in Table 5.

Comparing the forecasting performance between grey models and ANN model, the MAPEs of grey models are lower than from ANN models. Through the t test, the MAPE differences between ANN model and grey models for the four sectors are statistically significant. So MAPE is a useful evaluation for the forecasting performance of the four models. Also, the percentages of forecasting error in the scope 0~5% of grey models are much higher than ANN model. Thus grey models perform much better than ANN models in the forecasting of Hong Kong property price indices.

For the ARIMA model, the four sectors' MAPEs are similar with grey models' MAPEs. The t test shows that the differences between ARIMA model and grey models are not statistically significant. So MAPE is not a useful judgement for the forecasting performance of the four models. For the percentages of forecasting error less than 5%, grey models present higher than ARIMA model for the domestic, office and retail sectors and lower for the factory sector. However, it should be noted that the ARIMA model has its limitations: 1) ARIMA model requires more observations (not less than 50) than grey models; 2) Uncertainties exist in the forecasting errors of ARIMA model. For example, The forecasting errors above 10% in ARIMA model have appeared randomly, while GM(1,1) performs very well with the stable trend data and GM(0,N) for the fluctuating data. Thus, grey models are more suitable for forecasting the Hong Kong property price indices. With GM (1,1) and GM(0,N), the property price indices of four sectors in 2015 are forecast, as shown in Figure 10 and Table 6. The forecasting results of two grey models are very close.

The final observations to make about the four forecasting models are that price indices will still increase with expectations of a stable economy development. The government has implemented many measures to regulate the market, but not effectively. According to the sensitivity analysis, property prices are more sensitive to changes in population and best lending rate. The shortage of housing supply has been a problem for long time due to limited land supply. This problem is likely to continue for two reasons. First, the Hong Kong population is projected to increase at an average

Table 5. Forecasting performance of four models (Jan 2005–Dec 2014)

Model	Domestic		Office		Retail		Factory	
	MAPE	Percentage (0–5%)	MAPE	Percentage (0–5%)	MAPE	Percentage (0–5%)	MAPE	Percentage (0–5%)
GM(1,1)	3.20%	79.17%	4.21%	71.67%	3.08%	85.00%	3.22%	78.33%
GM(0,N)	3.94%	77.50%	4.42%	75.83%	3.20%	78.33%	4.07%	73.33%
ANN	8.19%	35.83%	8.48%	33.33%	7.59%	40.00%	8.49%	35.83%
ARIMA	3.61%	74.17%	5.81%	65.00%	3.62%	76.67%	2.87%	87.50%

Table 6. Forecast of property price indices of 2015

Year	Month	Domestic		Office		Retail		Factory	
		GM(1,1)	GM(0,N)	GM(1,1)	GM(0,N)	GM(1,1)	GM(0,N)	GM(1,1)	GM(0,N)
2015	Jan	281.51	282.44	430.99	430.47	548.21	548.78	690.93	690.57
	Feb	286.87	287.43	432.36	432.93	553.32	552.81	694.53	694.71
	Mar	292.01	292.14	433.90	434.04	558.24	558.67	698.33	698.44
	Apr	296.96	292.04	435.08	430.03	563.47	570.90	702.11	694.77
	May	301.72	297.59	436.12	432.45	568.75	574.11	706.11	700.19
	Jun	306.20	303.75	437.37	435.97	573.80	575.83	709.58	706.54
	July	310.94	310.66	438.59	439.67	578.76	577.44	713.18	715.35
	Aug	315.82	313.38	439.52	439.39	583.77	583.85	716.52	715.01
	Sep	320.97	319.00	440.90	442.29	588.18	585.92	720.30	720.38
	Oct	326.28	325.14	442.51	445.44	593.12	588.72	724.34	727.12
	Nov	331.63	331.20	443.64	448.60	598.75	591.11	728.46	733.69
	Dec	337.04	337.18	444.84	451.56	604.17	593.85	732.03	739.72

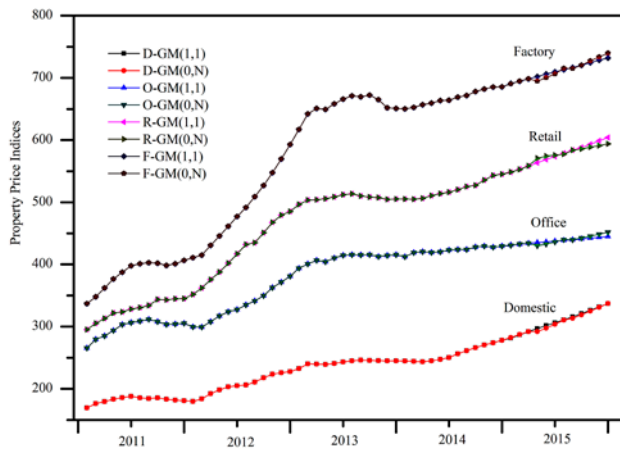


Fig. 10. Forecast of property price indices of 2015

annual rate of 0.6% and to 8.47 million by mid-2041 (Census and Statistics Department 2012). If the economy is stable, the property prices will continually increase due to the shortage of supply. Secondly, low interest rates are likely to continue in Hong Kong. The government may try to introduce policy changes to reduce this problem. Therefore, there is a need to analyse the factors affecting the property prices. When using GM(0,N) model, different scenarios can be identified based on factor analysis which will provide real forecasting of future market.

7. CONCLUSIONS

Hong Kong’s property prices have doubled since 2009. Its future development is still unknown. People are facing high pressures with the increasing housing prices. Therefore, there is a need to analyse the factors affecting the real estate market and forecast the price indices in future. This paper introduces two grey models, GM(1,1) and GM(0,N), to forecast Hong Kong property price indices. Six factors are considered in the GM(0,N). And it is found that Population and Best Lending Rate are more sensitive than other factors. Both the two grey models can forecast the property price indices very well with low average errors. When the property price indices have a stable increasing trend, the use of the GM(1,1) model is preferred. When the property price indices fluctuate, GM(0,N) has better forecasting performance than GM(1,1), when considering relevant affecting factors. Also, owing to the good forecasting performance obtained from the rolling mechanism, the two grey models can also be employed to achieve the long-term forecasting by using the forecast values as ‘original’ data to forecast values even further into the future. Furthermore, the comparison among two grey models, ANN model and ARIMA model has been done in this paper. The results show that grey models are

simple and appropriate for forecasting Hong Kong property price indices.

This study is new to the forecasting of property price indices. The two grey models can provide precise forecasting accuracy of future property prices with limited data. The results provide an indication of the likely future market. The results can help investors make better decisions based on their own analyses. Developers can use these models to test the market changes in future. Factors in the GM(0,N) can be changed with different purposes. The results may be different with different scenarios when using GM(0,N). Thus the models can also help test the effects of some policies before implementation. Therefore, the findings in this paper provide useful references for various stakeholders in the industry.

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