



## SHIRYAEV-ZHOU INDEX – A NOBLE APPROACH TO BENCHMARKING AND ANALYSIS OF REAL ESTATE STOCKS

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**ABSTRACT.** Real estate markets and real estate stocks are interrelated and are important not only to the investors, but also to the academics. Real estate stocks are, in a sense, good measures of performance of the physical real estate market. The objective of this paper is to provide a preliminary study on gauging the performances of real estate stocks in Hong Kong using the Shiryayev-Zhou index. Evidence shows that the Shiryayev-Zhou index can gauge a real estate stock's performance, good or bad, according to the sign of the Shiryayev-Zhou index. Thus a trading strategy can be formulated as follows: buy a stock if its Shiryayev-Zhou index changes from negative to positive, then hold it until its Shiryayev-Zhou index turns negative, when it is time to sell the stock. We examine the Shiryayev-Zhou indices of the real estate stocks in Hong Kong, and from this we deduce the latest best selling dates of the stocks during the period of our study. The Shiryayev-Zhou index could be an indicator of whether the market is bullish or bearish and consequently tells an investor to hold a stock or not, and it naturally leads to an optimal selling strategy that maximize the average ratio of the selling price to the maximum stock price when the underlying coefficients are assumed to be constant over a definite period of time.

**KEYWORDS:** Real estate markets; Optimal selling times; Buy-and-hold; Sell-at-once; Shiryayev-Zhou index

### 1. INTRODUCTION

When investors buy and sell stocks, they always look for a strategy to maximize their profit. A well-known and common stock trading strategy is the “buy and hold” strategy, i.e. one should buy a stock and hold it for a long time. The “buy and hold” strategy is based on the efficient market hypothesis (EMH). However, the EMH may not be true in reality. Hence some investors look for a strategy to beat the “buy and hold” strategy.

With the above background, there have been an increasing number of researches on trading strategies. For example, Berkelaar et al. (2004) analyzed the optimal investment strategy for loss-averse investors, but did not suggest the optimal time to buy/sell stocks, which is important in maximizing the profit. Bao and Yang (2008) proposed a new intelligent financial trading system -- a trading strategy by probabilistic model from high-level representation of time series with turning

points and technical indicators. Moreover, Alizadeh and Nomikos (2007) used a sample of price and charter rates over the period from January 1976 to September 2004. They proved the existence of a long-run cointegrating relationship between price and earnings, and used this relationship as an indicator of investment or divestment timing decisions in the dry bulk shipping sector.

Recent studies provided further extension of optimal strategy for secretary problems. For instance, Bearden et al. (2005) presented a generalization of a class of sequential search problems with ordinal ranks, referred to as “secretary” problems, in which applicants are characterized by multiple attributes. Their experiments found that, relative to the optimal search policy, subjects stop the search too early. The results suggest that this bias is largely driven by a propensity to stop prematurely on applicants of intermediate (relative) quality. In addition, Bearden (2006) made an extension of the secretary problem in which the decision maker (DM) sequentially observes up to  $n$  applicants whose values are random variables  $X_1, X_2, \dots, X_n$  are independently drawn from the uniform distribution on  $[0, 1]$ .

Consider a problem as follows: an investor buys a stock at one time and must sell it in a certain period of time, say, one year. We want to find the best time within the period to sell the stock. It would be ideal if the investor can sell the stock exactly at the maximum price over the period. Unfortunately, this is impossible as the time the stock price reaches the maximum can only be known at the end of the period. A more sensible problem is to find the time to sell the stock at the price closest to the maximum price, i.e., more precisely, to minimize the expected relative error between the selling price and the maximum price.

The problems along this direction was first analyzed by Graversen et al. (2001), who solved the problem to stop a Brownian motion so as to minimize the square error deviation from

the maximum. After that, there were more attempts to tackle similar problems. For example, Shiryaev (2002), after whom the Shiryaev-Zhou index was named, investigated the quickest detection problem for a change of market parameters, while Li and Zhou (2006) revealed the high chance of a Markowitz mean–variance strategy hitting the expected return target. Shiryaev et al. (2008) developed the Shiryaev-Zhou index to determine the optimal time to buy and sell a stock in order to minimize the average relative error of selling price to maximum price; a probabilistic proof of the result could also be found in the work of Du Toit and Peskir (2008). Inspired by the ideas in solving for the secretary problem, Yam et al. (2009, 2010) resolved the problem in the binomial tree setting and hence generalize the Shiryaev-Zhou index over the corresponding framework. The name of the index is attributed to its two founders, A. Shiryaev and X. Y. Zhou. The Shiryaev-Zhou index is constructed from publicly available stock prices, which can be updated easily. As an alternative to traditional real estate price indices, the Shiryaev-Zhou index provides an invaluable tool for investing in both real estate and real estate stock market. Investors are recommended to trade a stock according to its Shiryaev-Zhou index as follows: buy a stock when its Shiryaev-Zhou index, and then hold it until its Shiryaev-Zhou index turns negative, when it is time to sell the stock.

In this paper, we investigate the Shiryaev-Zhou indices of some real estate stocks in Hong Kong. Real estate is an important constituent in portfolios of many fund managers. Many traditional real estate indices measure changes by observing changes in prices. The traditional real estate indices are constructed from real transaction prices recorded in the market (Chau et al., 2005). Samples are taken from all registered transactions available. Since it takes time for transactions to be completed and the index would be weeks lagging behind, on-going transactions are sometimes

used. As an example, the Centa-City Leading Index comprises transactions mainly from on-going transactions of the Hong Kong property agent Centaline. Real estate price indices are also constructed from surveying of selected samples by professionals. Such indices are updated monthly or quarterly (Chau et al., 2005). Meanwhile, Hui and Wong (2004) developed the BRE Index which provides an objective tool and a statistical pointer that forecasts future housing price trends. Later, they used the index to further investigate the housing price in Hong Kong (Wong and Hui, 2005).

However, the observations of the traditional real estate indices cannot be done continuously. There is always time lag between price change and observation. Yet, sometimes a change in long-term value of real estate properties cannot be seen on their short-term prices. Stock prices, on the other hand, can be regarded as a continuous evaluation of co-operations by market practitioners. There have been studies about the relationship between real estate price and real estate stock price or the stock market in general (Ong, 1994; Newell and Chau, 1996). Some argue that the stock price is affected by short-term factors like market liquidity (i.e., money supply) and cannot reflect the change in real estate price. However, history tells that the property price is also affected by liquidity. Sing (2001) has shown that the demand for condominiums was negatively related to one-quarter lagged stock price change. Recently, researchers used econometric methods to study relationship between real estate and stock markets. For example, Okunev and Wilson (1997) tested whether or not there existed a relationship of co-integration between the REIT and the S&P 500 indices. The results indicated that the real estate and stock markets were fractionally integrated. Okunev et al. (2000) conducted both linear and nonlinear causality tests on the US real estate and the S&P 500 Index and concluded that there

existed unidirectional relationship from real estate to stock market when using the linear test, but there was a strong unidirectional relationship from the stock market to the real estate market when using the nonlinear test. Some researchers found a long-term positive correlation between real estate and stock prices. Using data from 17 different countries over 14 years, Quan and Titman (1999) found a significant positive relation between stock returns and changes in commercial real estate values. Tse (2001) studied the Impact of Property Prices on Stock Prices in Hong Kong from 1974 to 1998, and found that the property and stock prices are cointegrated. Liow (2006) also found long-term positive correlations between real estate and stock prices in general. Similar results were found by Hui et al. (2011), who examine the relationship between real estate and stock markets in the United Kingdom and in Hong Kong by the method of data mining. They found not only a positive correlation, but also a co-movement, between the two markets. These suggest that the stock price can actually be a leading indicator of the real estate price. Therefore, as an alternative, the indices we used are constructed from stock prices of real estate companies. The frequency of these indices is daily, instead of monthly for most housing price indices. Thus they can reflect the continuous change of value, rather than chasing after the prices.

In this paper, we gauge the performances of real estate stocks in Hong Kong using the Shiryayev-Zhou index. The rest of this paper is organized as follows: Section 2 highlights necessary preliminaries for stock pricing mathematics. With such basic knowledge, in Section 3, an optimal strategy is illustrated for a special case which motivates the concept behind the Shiryayev-Zhou index. Sections 4 and 5 present our data set and statistical method used respectively. The results are given in Section 6. Finally, we draw up a conclusion in Section 7.

## 2. STOCK PRICE MOVEMENTS AND TRADING STRATEGIES

This section reviews the mathematics behind the stock price modeling. Stock prices are considered to move randomly and continuously. We said that the stock price follows a *stochastic process*  $W_t$ , which is a collection of random variables that depend on time  $t$  and event  $\omega$ ,  $\{W(t, \omega), t \in T, \omega \in \Omega\}$ . For instance, Hang Seng Index (HSI) is a random variable and change continuously from 10am to 11am. Therefore, HSI can be interpreted as a realization of stochastic process  $W_t$ ,  $10 \leq t \leq 11$  (Mikosch, 1998).

*Brownian motion* is a stochastic process  $B_t$  having continuous sample paths, as well as its independent increment  $B_t - B_s$ ,  $s < t$ , following a normal distribution  $N(0, t - s)$ . It is very irregular and not predictable from history. This assumption is agreed with Weak-form Efficient Market Hypothesis, said *Markov Property*.

Since the sample paths of the Brownian motion are nowhere differentiable, we cannot use the ordinary integral to calculate it. Hence, we must use stochastic calculus and new chain rule, namely Ito's formula, to solve for the stochastic differential equations which will be satisfied by the dynamic of a stock price. Assume that  $f$  is a twice differentiable function of  $B_t$ , by Taylor expansion, we have:

$$f(B_t + dB_t) - f(B_t) = f'(B_t)dB_t + \frac{1}{2}f''(B_t)(dB_t)^2 + \dots \quad (2.1)$$

Since the higher order terms  $(dB_t)^n$  for  $n = 3, 4, \dots$  are very small, we can neglect them all; however, we cannot omit the term  $(dB_t)^2$ . Recall  $B_t \sim N(0, t)$ , we obtain:

$$\text{Var}(B_t) = \mathbb{E}(B_t^2) - \mathbb{E}^2(B_t) = \mathbb{E}(B_t^2) = t \quad (2.2)$$

We have  $(dB_t)^2 \approx dt$ . Omitting all the higher order differential terms and note that  $f(B_t + dB_t) - f(B_t) = df(B_t)$ , the Taylor expansion becomes Ito's formula:

$$df(B_t) = f'(B_t)dB_t + \frac{1}{2}f''(B_t)(dB_t)^2 \quad (2.3)$$

Or, by integrating both sides of (2.3), we obtain the integral form of Ito's formula:

$$f(B_t) - f(B_s) = \int_s^t f'(B_x)dB_x + \frac{1}{2} \int_s^t f''(B_x)(dB_x)^2 \quad (2.4)$$

More formal proofs of Ito's formula were given by Hull (2006), Klebaner (2005), Mikosch (1998) and Øksendal (2005).

### Geometric Brownian motion as Stock Price Model

The stock price  $W_t$  reflects the internal value of a company, whose proportional change consists of two parts. The first part is relatively predictable: the annual growth of business in normal circumstances. This part is modeled as  $\alpha dt$  where the constant term  $\alpha$  is a measure of the annual growth rate of the stock price, also known as the drift. The second part is unpredictable: all kinds of uncontrollable incidents like war, diseases, new government regulations and etc. This part is modeled as  $\sigma dB_t$  where  $B_t$  is a Brownian motion and  $\sigma$  is the constant annual volatility of the stock.

Hence we have:

$$\frac{dW_t}{W_t} = \alpha dt + \sigma dB_t \quad (2.5)$$

$W_t$  is said to follow the Geometric Brownian motion.

Suppose  $f(W_t) = \ln(W_t)$ , by Ito's formula:

$$\begin{aligned}
d\ln(W_t) &= \frac{dW_t}{W_t} + \frac{1}{2} \left( -\frac{1}{W_t^2} \right) (dW_t)^2 = \\
(\alpha dt + \sigma dB_t) &+ \frac{1}{2} \left( -\frac{1}{W_t^2} \right) (\alpha W_t dt + \sigma W_t dB_t)^2 = \\
(\alpha dt + \sigma dB_t) &- \frac{1}{2} \sigma^2 dB_t^2 = \\
\left( \alpha - \frac{1}{2} \sigma^2 \right) dt &+ \sigma dB_t
\end{aligned}$$

Integrate both sides, then

$$\ln W_t = \ln W_0 + \left( \alpha - \frac{1}{2} \sigma^2 \right) t + \sigma B_t \quad (2.6)$$

$$W_t = W_0 \cdot \exp \left( \left( \alpha - \frac{1}{2} \sigma^2 \right) t + \sigma B_t \right) \quad (2.7)$$

Since  $B_t$  follows a normal distribution  $N(0, t)$ ,  $W_t$  follows a lognormal distribution and

$$\ln W_t \sim N \left[ \ln W_0 + \left( \alpha - \frac{1}{2} \sigma^2 \right) t, \sigma^2 t \right] \quad (2.8)$$

where:  $W_t$  is the stock price at a future time  $t$ ;  $W_0$  is the stock price at time 0.

Similar proofs were given by Hull (2006) and Wilmott et al. (1995).

### 3. OPTIMAL TRADING STRATEGY AND SHIRYAEV-ZHOU INDEX

In this section, we introduce an index to indicate whether time  $t$  is the best time to buy or sell a stock. Since nobody can anticipate the future, it is virtually impossible to sell the stock at maximum price. Therefore, we try to minimize the gap between the selling price and the maximum price of the stock.

Consider the problem:

$$V^* = \max_{0 \leq \tau \leq T} \mathbb{E} \left[ \frac{W_\tau}{M_T} \right] \quad (3.1)$$

where:  $M_t = \max_{0 \leq s \leq t} W_s$  is the maximum of the stock price from time 0 to  $t$ .

We look for the best time to sell the stock as close as possible to its highest price over  $[0, T]$ . In order to pay respect to the original poser, Shiryaev and Zhou, of this optimal stopping time problem, we name the index as Shiryaev-Zhou index (see the index in (3.9) as shown below). We conclude that we should sell the stock at time 0 if the Shiryaev-Zhou index is negative and at time  $T$  if the Shiryaev-Zhou index is positive. For the proof of the statement and its generalizations, one can refer the work by Shiryaev et al. (2008), Du Toit and Peskir (2008), and Yam et al. (2008, 2009, 2010).

Optimal strategy appearing in the Secretary Problem could inspire us to develop a method to solve the mentioned optimal selling problem, for more details, one can consult the work by Yam et al. (2008, 2009, 2010). We here illustrate the key ideas of the proof which can motivate for further development of the concept behind Shiryaev-Zhou index. Secretary Problem is a well-known optimal stopping problem about how to choose the best secretary in  $N$  candidates. The main idea to solve this problem is to skip the first  $\tau - 1$  applicants and then to accept the next candidate that is better than all those previously interviewed (Bearden et al., 2005).

In financial terms, for any selling time  $\tau$ , we postpone the time of selling until the first time after (or at)  $\tau$  that the stock price hits its maximum over  $[0, \tau]$  or sell at the terminal time if the stock price never collides with the maximum process after  $\tau$ . We define this new selling time  $\rho_\tau$  with respect to  $\tau$  as follows:

$$\rho_\tau = \min \{ \tau \leq t : W_t = M_\tau \} \wedge T \quad (3.2)$$

Hence if after  $\tau$ , the stock price hits its running maximum again, i.e.  $W_{\rho_\tau} = M_\tau$ , we shall benefit from selling at this time. But there is a possibility that the stock price never hits its running maximum again and in that case  $W_{\rho_\tau} = W_T$  which may be smaller than  $M_\tau$ .

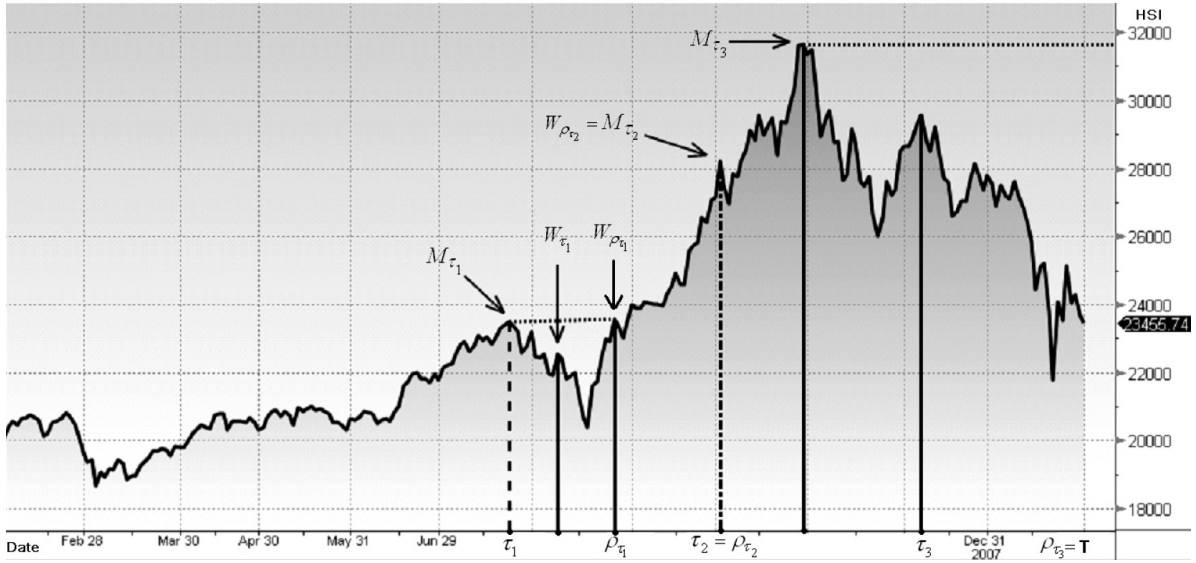


Figure 1. Relationship between  $\tau$  and  $\rho_\tau$  in HSI

Figure 1 shows the different relationships between  $\tau$  and  $\rho_\tau$  depending on where the maximum stock price lies, in  $[0, \tau]$  or  $[\tau, T]$ .

In the case  $\alpha \geq \frac{1}{2} \sigma^2$ ,  $\rho_\tau$  always dominates  $\tau$ , i.e.

$$\mathbb{E} \left[ \frac{W_\tau}{M_T} \right] \leq \mathbb{E} \left[ \frac{W_{\rho_\tau}}{M_T} \right] \tag{3.3}$$

with the equality in (3.3) holds if and only if

$$\tau = \rho_\tau \tag{3.4}$$

At the critical case  $\alpha = \frac{1}{2} \sigma^2$ , the optimal selling time is either at terminal time  $T$  or at the moment when the stock price hit its running maximum as (3.4) holds. Finally, the complete solution of the problem (3.1) is given by

$$\tau^* = \begin{cases} T & \alpha > \frac{1}{2} \sigma^2 \\ \text{any } \tau = \rho_\tau \text{ or } T & \alpha = \frac{1}{2} \sigma^2 \\ 0 & \alpha < \frac{1}{2} \sigma^2 \end{cases} \tag{3.5}$$

where:  $\tau^*$  is the optimal time to sell the stock in  $[0, T]$  and the solution can literally be interpreted as either buying-and-holding or selling-at-once depending on the quality of the underlying stock indicated by the Shiryayev-Zhou index as defined below in (3.9).

To introduce the index, let us first transform the problem (3.1) to a standard form. In view of (2.5), problem (3.1) is equivalent to

$$V^* = \max_{0 \leq \tau \leq T} \mathbb{E} \left[ \exp \left( \left( \alpha - \frac{1}{2} \sigma^2 \right) \tau + \sigma B_\tau - \max_{0 \leq s \leq T} \left( \left( \alpha - \frac{1}{2} \sigma^2 \right) s + \sigma B_s \right) \right) \right], \tag{3.6}$$

which by the scaling property of Brownian motion, is equivalent to

$$V^* = \max_{0 \leq \tau' \leq T'} \mathbb{E} \left[ \exp \left( \frac{\left( \alpha - \frac{1}{2} \sigma^2 \right)}{\sigma^2} \tau' + B_{\tau'} - \max_{0 \leq s \leq T'} \left( \frac{\left( \alpha - \frac{1}{2} \sigma^2 \right)}{\sigma^2} s + B_s \right) \right) \right]$$

$$\max_{0 \leq s \leq T} \left( \frac{\left( \alpha - \frac{1}{2} \sigma^2 \right)}{\sigma^2} s' + B_{s'} \right), \tag{3.7}$$

where:  $\tau' = \sigma^2 \tau$ ,  $T' = \sigma^2 T$  and  $s' = \sigma^2 s$ .

Therefore, the problem (3.1) become

$$V^* = \max_{0 \leq \tau \leq T} \mathbb{E} \left[ \exp \left( B_{\tau}^{\mu} - S_T^{\mu} \right) \right] \tag{3.8}$$

with the new index  $\mu = \frac{\left( \alpha - \frac{1}{2} \sigma^2 \right)}{\sigma^2}$ ,  $B_t^{\mu} = \mu t + B_t$ , and  $S_t^{\mu} = \max_{0 \leq s \leq t} B_s^{\mu}$ .

$$\tau^* = \begin{cases} T & \mu > 0 \\ \text{any } \tau = \rho_{\tau} \text{ or } T & \mu = 0 \\ 0 & \mu < 0 \end{cases} \tag{3.9}$$

Here,  $\mu$  is the Shiryaev-Zhou index. Again, the proof of the statement and its generalizations, one can refer the work by Shiryaev et al. (2008), Du Toit and Peskir (2008), and Yam et al. (2008, 2009, 2010). Note that optimal selling problem has only been solved for the case with constant parameters, under the cases with general stochastic or time-dependent parameters, the same problem still remains unsolved. In the rest of this paper, we shall provide some insight that, in general, the Shiryaev-Zhou index of any stock is still instructive in the sense that its sign essentially aligns with the market condition; that is to say, when the market is lull (resp. in growth), it will take negative values (positive values).

#### 4. SOURCES OF DATA

In this section, we select 26 real estate stocks in Hong Kong, which fall into three classes, for analysis. The three classes are: (a) Constituent stocks of HSI, which, by definition,

are all large and stable companies; (b) Normal real estate stocks; (c) Red Chips real estate corporations, which are held by or controlled by entities in the Mainland China. These stocks are chosen since they are all large and stable companies and hence we can assume that they have a relatively constant drift  $\alpha$  and volatility  $\sigma$  over a moving window for any given time point.

All the data we select are collected from Bloomberg terminal. There are a total of 18 years of daily closing prices for the 26 Hong Kong real estate stocks. The time line is from Mar 1990 to Sep 2008, which contains nearly 5,000 trading days. Another 5 of the 26 stocks have less data due to later IPOs, which are not later than 1993. In this paper, we assume that  $T$  is half year long, or precisely, 130 trading days. The indices for these stocks are recalculated each day with data from previous 130 days including the day under consideration.

Tables 1 to 3 show the details of the stocks we select.

**Table 1.** Constituent stocks

Securities code	Securities name	Weight*
0001.HK	Cheung Kong Holding	3.07%
0004.HK	WHARF Holding	0.75%
0012.HK	Henderson Land Development	0.94%
0016.HK	Sun Hung Kai Properties	3.10%
0017.HK	New World Development	0.54%
0019.HK	Swire Pacific Ltd	1.48%
0083.HK	Sino Land Co.	0.55%
0101.HK	Hang Lung Properties	0.92%

\*Remark: the weight is related to the stock price and keeps changing; these weights are read on 26<sup>th</sup> Sep, 2008

**Table 2.** Normal real estate stocks

Securities code	Securities name	Securities code	Securities name
0049.HK	Wheelock Properties Ltd	0173.HK	K. Wai International Holding
0056.HK	Allied Properties HK Ltd	0201.HK	Magnificent Estate
0075.HK	YT Realty Group Ltd	0225.HK	Pokfulam Development
0089.HK	Tai Sang Land Development	0247.HK	Tsim Sha Tsui Properties
0127.HK	Chinese Estate Holding Ltd	0278.HK	Wah Ha Realty
0158.HK	Melbourne Enterprise	0480.HK	HKR International

**Table 3.** Red chips

Securities code	Securities name	Securities code	Securities name
0028.HK	Tian An China Investment	0535.HK	Frasers Property China
0119.HK	Poly Hong Kong Investment	0688.HK	China Overseas Land & Investment
0123.HK	Guangzhou Investment	0755.HK	Shanghai Zengda Properties Ltd

## 5. THE STATISTICAL METHOD

In order to justify the use of the Geometric Brownian motion stock price model with varying coefficients, the random parts of the stock price data are tested if they are white noises. Then we will estimate the values of the drift  $\alpha$  and the volatility  $\sigma$  and hence the Shiryayev-Zhou index. The stock price data are transformed into relative ratio to facilitate the statistical calculations.

Recall (2.2), the logarithm  $\log(W_t)$  of stock price  $W_t$ , follows a Normal distribution  $N[\ln W_0 + (\alpha - 0.5\sigma^2)t, \sigma^2t]$ . We define a new notation  $u_i$

$$u_i \triangleq \log\left(\frac{W_i}{W_{i-1}}\right) \quad (5.1)$$

where:  $u_i$  means the relative ratio, or the daily return rate. Hence the mean of the relative ratio is:

$$\bar{u} = \frac{1}{n} \sum_{i=1}^n u_i \quad (5.2)$$

We set  $n = 130$  in this paper.

Therefore, the estimator of  $\alpha$  is simply  $\bar{u}$  multiplied by the number of trading days in one year, which we assume to be 250:

$$\hat{\alpha} = 250\bar{u} \quad (5.3)$$

Aside from the drift term, we would also estimate the volatility term, and thus the daily standard deviation by:

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (u_i - \bar{u})^2} \quad (5.4)$$

Consequently, the estimator of the volatility  $\sigma^2$  is:

$$\hat{\sigma}^2 = 250s^2 \quad (5.5)$$

And the estimator of the Shiryayev-Zhou index  $\mu$  is:

$$\hat{\mu} = \frac{(\hat{\alpha} - 0.5\hat{\sigma}^2)}{\hat{\sigma}^2} \quad (5.6)$$

Note that at any time point, the parameters estimated at the time would be based on those 130 data before that time point; besides, a moving window approach will be adopted.



## 6. RESULTS

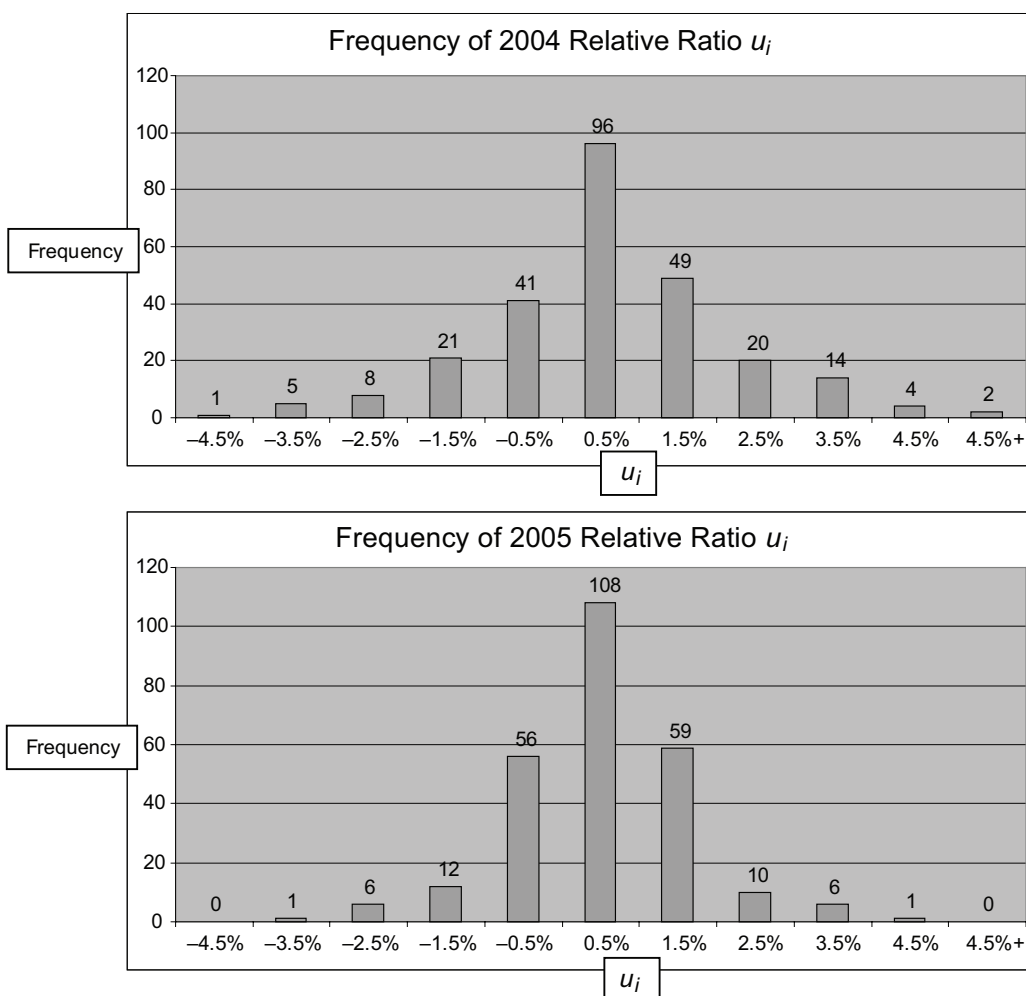
### 6.1. White noise test

Before we obtain the indices, it is essential for us to test whether the relative ratio is white noise, which is the basic assumption of Geometric Brownian Motion model with varying coefficients. The distribution of the relative ratios can be judged by observing their density plots. However, because of limited amount of data, we cannot plot continuous graphs. Therefore, we use frequency charts to approximate the distribution. The following graphs are the

frequency of Cheung Kong stock relative ratio  $u_i$  in the recent four years.

The bell-shapes in Figure 2 suggest that the relative ratio may follow a Normal distribution.

To further test for randomness, we regress the daily returns against time in each calendar year. If the R-square is small and the P-value is large, the relative ratio is white noise. Figure 3 illustrates Cheung Kong's relative ratio in recent four years, and Table 4 shows the R-squares and P-values of intercept and slope in the regression of Cheung Kong stock relative ratio against time  $t$ .



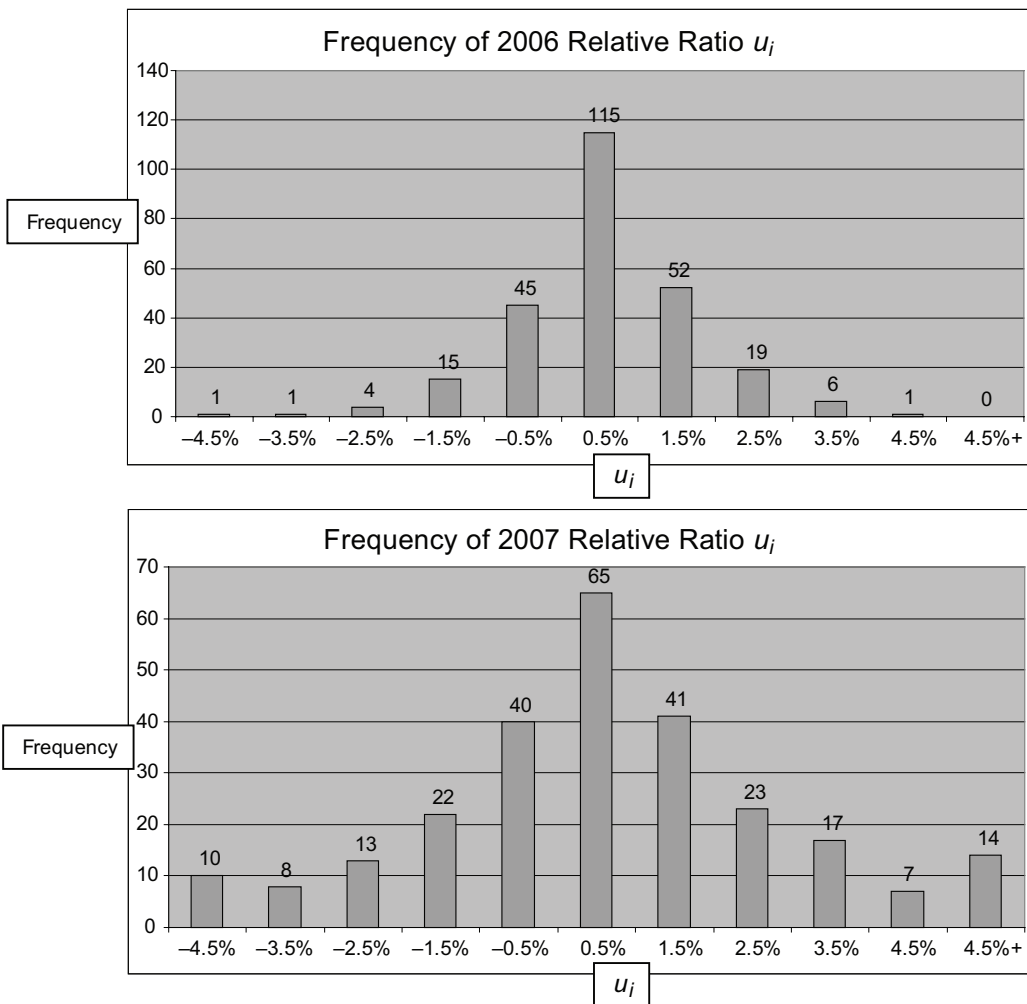


Figure 2. Relative ratios of Cheung Kong in year 2004-7

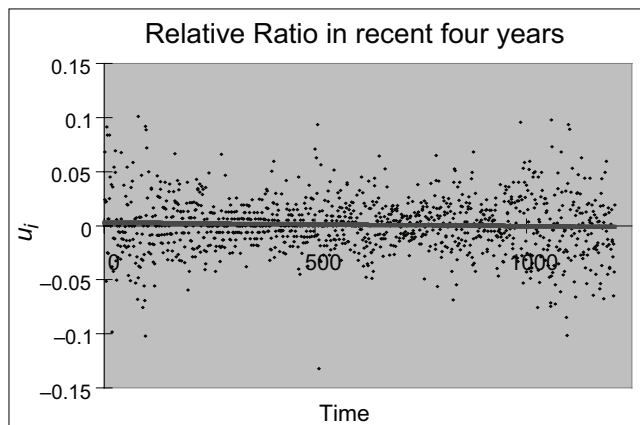


Figure 3. Relative ratios of Cheung Kong in Year 2004-7

**Table 4.** P-values of intercept and slope; R-square

Year	P-Intercept	P-Slope	R-square	Year	P-Intercept	P-Slope	R-square
1988	0.991784	0.607299	$1.67 \times 10^{-7}$	1999	0.962075	0.343128	$1.74 \times 10^{-8}$
1989	0.714656	0.857516	$1.68 \times 10^{-6}$	2000	0.610370	0.583500	$1.99 \times 10^{-6}$
1990	0.328826	0.602185	$4.13 \times 10^{-6}$	2001	0.240074	0.322867	$8.58 \times 10^{-6}$
1991	0.011738	0.055634	$1.91 \times 10^{-5}$	2002	0.349036	0.883162	$4.37 \times 10^{-6}$
1992	0.169108	0.105795	$6.65 \times 10^{-6}$	2003	0.766163	0.473868	$2.50 \times 10^{-7}$
1993	0.526344	0.328704	$2.90 \times 10^{-6}$	2004	0.851986	0.488391	$5.76 \times 10^{-8}$
1994	0.416477	0.791766	$5.11 \times 10^{-6}$	2005	0.756476	0.635841	$1.37 \times 10^{-7}$
1995	0.723229	0.681138	$7.14 \times 10^{-7}$	2006	0.959199	0.634572	$2.07 \times 10^{-8}$
1996	0.426591	0.985060	$2.19 \times 10^{-6}$	2007	0.832747	0.746133	$5.63 \times 10^{-7}$
1997	0.354935	0.138367	$8.24 \times 10^{-6}$	2008	0.317216	0.589850	$1.14 \times 10^{-5}$
1998	0.385462	0.271747	$1.37 \times 10^{-5}$				

The above table shows the p-values and R-squares of Cheung Kong's stock price data. All slope and intercept cannot pass T-test at the 95% confidence level – that means we cannot find any trend in daily return; henceforth the return is very likely to be white noise. After regressing all stocks' relative ratios ( $u_i$ ) to time  $t$ , we find that all the p-values of intercepts and slopes are very large. Also, all R-squares are quite small. Hence the Geometric Brownian Motion stock price model is justified (with varying coefficients which could be approximated by using moving window approach; here we used 130 data points before each time point to regress for the drift and volatility as if they were constant over the moving window).

## 6.2. Calculation of Shiryayev-Zhou index

By formulae (5.1)–(5.6) established in Section 5, we calculate the indices  $\mu$  of 26 stocks on 26<sup>th</sup> Sep, 2008, which is the ending day of our data. Table 5 summarizes the results.

As seen from Table 5, all stocks except three of them, of which the Shiryayev-Zhou indices are in bold face, had negative indices. It reveals that although 2008 is a year of bad investment

performance due to the Sub-prime mortgage crisis, there are still three “normal real estate stocks” that stay positive in terms of the Shiryayev-Zhou indices. The three stocks are, namely, YT Realty Group Ltd (0075), Melbourne Enterprise (0158) and Pokfulam Development (0225). Compared with the other 23 companies, these three companies are rather small in terms of capitalization and have adopted very conservative business strategies, so they were less adversely affected by the financial crisis. On the other hand, the stocks of the groups “Constituent stocks” and “Red chips” have greater market values, so their stock prices follow the economic cycle closer and hence they all showed negative Shiryayev-Zhou indices on 26<sup>th</sup> Sep, 2008. Another feature we observe is that on 26<sup>th</sup> Sep, 2008, the range of Shiryayev-Zhou indices of constituent stocks and red chips were from  $-13.92$  to  $-3.56$  and from  $-18.18$  to  $-2.36$  respectively. However, the Shiryayev-Zhou indices of the normal real estate stocks varied from  $-24.77$  to  $17.28$ , which is a much larger range. This is because the normal real estate stocks have generally smaller market values than the constituent stocks and red chips, so their stock prices have

a larger fluctuation, and hence their Shiryaev-Zhou indices varied larger.

By calculating the Shiryaev-Zhou indices for every day except the first 129 days within the period under study, we obtain the right time to sell the stock, when the Shiryaev-Zhou index  $\mu$  just turns from positive to negative. The following Table 6 shows the latest time

(as on the date of the end of our preliminary study) when we should sell the stocks.

As an example, it can be deduced from Figure 4 that for the index of 0001.HK, the latest best buying time is 23/08/2007, i.e. the latest day that the Shiryaev-Zhou index turns from negative to positive. Also, the latest best selling time is 10/03/2008.

**Table 5.** The Shiryaev-Zhou index  $\mu$  of Hong Kong real estate stocks

Securities code	Index $\mu$	Securities code	Index $\mu$
(a) Constituent stocks			
0001.HK	-3.56	0017.HK	-9.23
0004.HK	-11.14	0019.HK	-10.96
0012.HK	-13.92	0083.HK	-7.00
0016.HK	-8.54	0101.HK	-4.65
(b) Normal real estate stocks			
0049.HK	-12.73	0173.HK	-10.06
0056.HK	-7.48	0201.HK	-8.58
0075.HK	0.10	0225.HK	1.18
0089.HK	-10.03	0247.HK	-24.77
0127.HK	-3.72	0278.HK	-3.75
0158.HK	17.28	0480.HK	-17.16
(c) Red chips			
0028.HK	-9.77	0535.HK	-5.96
0119.HK	-18.18	0688.HK	-2.36
0123.HK	-11.78	0755.HK	-6.17

**Table 6.** The latest best selling time of Hong Kong real estate stocks

Securities code	Best date for selling	Securities code	Best date for selling
(a) Constituent stocks			
0001.HK	10/3/2008	0017.HK	12/3/2008
0004.HK	28/3/2008	0019.HK	17/3/2008
0012.HK	14/3/2008	0083.HK	14/3/2008
0016.HK	25/3/2008	0101.HK	22/1/2008
(b) Normal real estate stocks			
0049.HK	22/1/2008	0173.HK	11/3/2008
0056.HK	12/3/2008	0201.HK	25/9/2007
0075.HK	Keep holding	0225.HK	Keep holding
0089.HK	4/1/2008	0247.HK	17/3/2008
0127.HK	10/1/2008	0278.HK	30/11/2007
0158.HK	Keep holding	0480.HK	11/3/2008

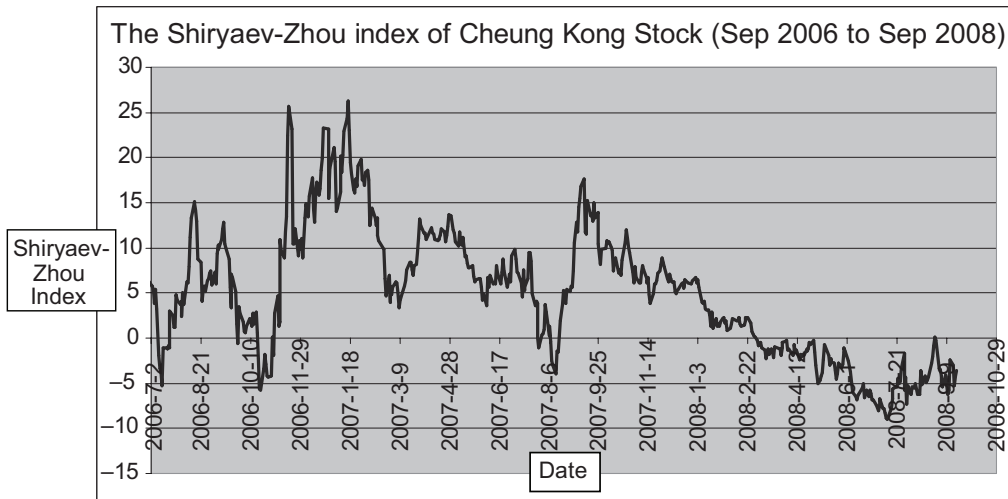


Figure 4. The Shiryayev-Zhou index of Cheung Kong Stock

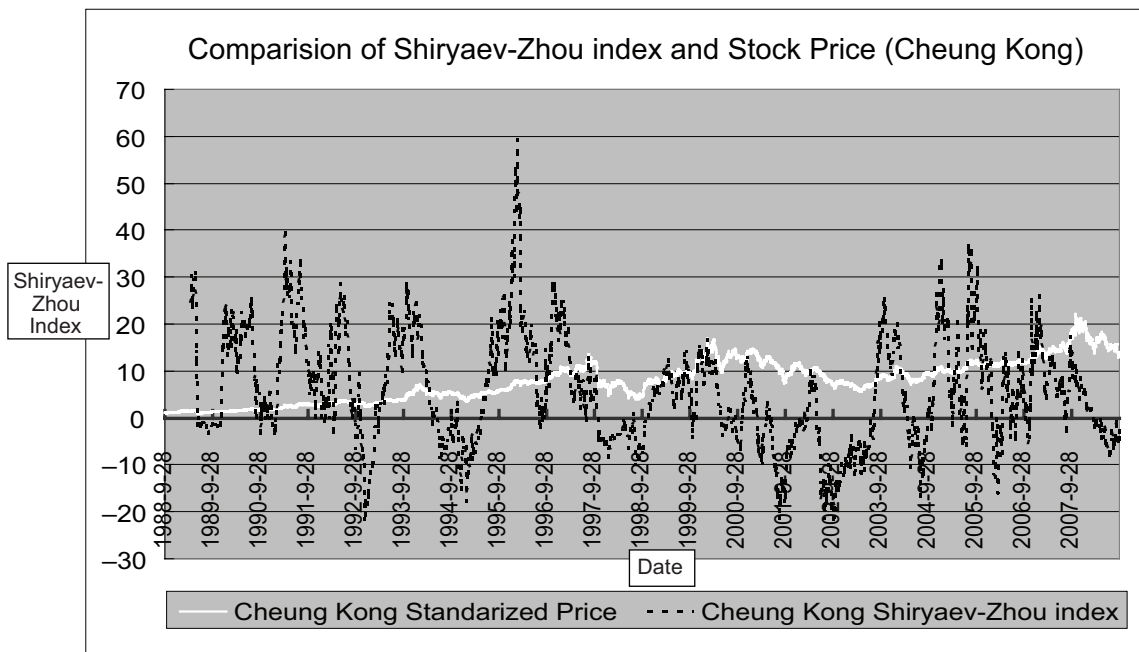


Figure 5. Comparison of Shiryayev-Zhou index and Stock Price of Cheung Kong

Figure 5 shows the relationship between stock price movement and Shiryayev-Zhou index for Cheung Kong stock. We can see that the Shiryayev-Zhou index usually peaks before the stock price reaching maximum.

### 7. CONCLUSION

From the above results, we find that the sign of the Shiryayev-Zhou index is an indicator of whether the market is bullish or bearish

and consequently tells an investor to hold a stock or not. For example, from the graph of the Shiryaev-Zhou index of Cheung Kong Stock in Figure 4, the Shiryaev-Zhou index of 0001.HK soared to a peak of about 26 in early 2007, and remained positive most of the time in that year, indicating that we should hold the stock at that time. However, as the subprime crisis in the U.S. broke out, the Shiryaev-Zhou index fell below zero in early 2008, and remained negative in that year. This indicated that we should sell the stock in early 2008, and should not buy that stock again in that year.

The Shiryaev-Zhou index is based on the optimization of time to sell a stock near its highest price. It naturally leads to a trading strategy that maximizes the selling prices. Under this strategy, the corresponding stock will be sold immediately or be held until the end of period under study, depending on the sign of the Shiryaev-Zhou index. We assume the stock prices follow the Geometric Brownian Motion model with varying coefficients, and statistics shows that stock price data fit well with the model. However, in reality, this strategy may not be as ideal as one thought. One reason is the presence of market frictions, mostly in the form of entry and/or transaction costs.

In our future research, we shall use more statistical methods to find out the optimal moving window, which is assumed to be 130 days here, and try to estimate the index with more sophisticated time series models (e.g. through different graduation methods) to make our prediction better. A too small interval will generate too many signals, whilst a too large interval will make the index inert to changes. A balance should be made based on the statistical characteristics of individual stocks. In the future research, one may try to use some time series model to fit every single stock, which is collected from all over the world. The findings of this paper can be applicable to other types of stocks. In particular, the Shiryaev-Zhou index proves to be a useful tool in gauging performances and

formulating trading strategies of real estate stocks in the global markets, such as Mainland China, Singapore, Japan, the US and UK.

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