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The impact of daily number talks on the development of mental math abilities of second graders within a reform-based classroom

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THE IMPACT OF DAILY NUMBER TALKS ON THE DEVELOPMENT OF MENTAL MATH ABILITIES OF SECOND GRADERS WITHIN A REFORM-BASED CLASSROOM

By

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Abstract

In this mixed methods case study I examined the impact of daily number talks (or strings) on the development of mental math abilities of second graders within a reform-based classroom. I also looked at whether or not the implementation of number talks would increase students' ability to calculate with accuracy, efficiency, and flexibility. Finally, I looked at whether or not the implementation of number talks would increase students' understanding of place value and number relationships. The sample included one class of 19 second-graders to determine the overall impact of number talks, with a focus on six embedded case studies which amplified how this change occurred. A preassessment interview, two midassessments, twenty-four number talks, a postnumber talk questionnaire, and a postassessment interview were used over the span of six weeks. The twenty-four number talks were developed for students to invent, construct, and make sense of their own number strategies and their underlying key ideas. After six weeks of number talks, all students demonstrated an increase in accuracy, efficiency, and flexibility in their number calculations to 20. The case study data of two low-achieving, three average-achieving, and one high-level student reveals growth in their ability to articulate their thinking with an increase in their understanding of place value and number relationships.

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Chapter 1: Introduction

Context

"Indeed, math makes sense!" is the desired declaration, from students and teachers alike, that I long to hear in my work as a mathematics coach. Instead, what I often hear from teachers, parents, and the public in general, is that we need to go "back-to-basics" because students cannot automatically recall basic number facts. There is a belief that this "new math" is confusing students and that students simply need to memorize their basic number facts and follow standard algorithms to solve addition and subtraction problems. Memorization and the sole use of standard algorithms are in direct opposition however, to the reform movement in mathematics, born from the realization that computation skills alone do not ensure understanding of concepts behind required procedures (Battista, 1999).

Reform movement. In 1989, the National Council of Teachers of Mathematics (NCTM) called for a profound shift in instructional methods within mathematics education. *Principles and Standards for School Mathematics* (NCTM, 2000) reiterated and updated this call through various recommendations. Included were recommendations for students to "compute fluently" (p. 32), "apply and adapt a variety of appropriate strategies to solve problems" (p. 52), and "recognize reasoning and proof as fundamental aspects of mathematics" (p. 56). NCTM's (2000) Communication Standard (p. 128) stressed the importance of organization and consolidation of mathematical thinking and that it be communicated logically and plainly to their peers, teachers, and others; for students to analyze and evaluate their peers' mathematical thinking and strategies; and, to communicate mathematical ideas accurately.

In her foreword to *Intentional Talk: How to Structure and Lead Productive Mathematical Discussions* (Kazemi & Hintz, 2014), Franke stressed, "classroom conversations are crucial to mathematics learning," (p. vii) as endorsed in the high expectations of the Communication Standard (NCTM, 2000). Classroom conversation is where *number talks* come into play. For the purpose of this study, number talks are defined as "classroom conversations around purposefully crafted computation problems that are solved mentally" typically conducted in five to fifteen minutes (Parrish, 2014, p. xx). Number talks were developed by Ruth Parker and Kathy Richardson in the 1990s (Humphreys & Parker, 2015). A number talk is typically a time to explore different ways to solve one carefully generated number problem calculation such as 5 + 7. In this instance, the calculation is designed to elicit a range of strategies that can be discussed and compared for efficiency (e.g., counting three times, counting on, near doubles, or up through 10 and over). On the other hand, a *string* (sometimes also referred to as a number talk) is more specifically, a sequence of problem calculations posed to students in order to push them toward a particular strategy or key idea. For example: 5 + 7, 7 + 5, 7 + 3, 7 + 5, and 6 + 8. This string of problem calculations is posed one at a time with the hope that children would notice the commutative property in the answer to the first two calculations, then use the up to 10 and over in the next two, and finally, both methods in the final 'challenge' calculation.

It is important to note that number talks are different from math talk. Math talk is viewed as a more general "respectful but engaged conversation in which students can clarify their own thinking and learn from others through talk" (Chapin, O'Connor, & Anderson, 2003, p. 5). In short, math talk refers to a way to structure discourse about a particular math topic whereas a number talk is a carefully structured set of oral calculations that promotes *computational fluency.*

Personal ground. In my role as a Primary to Grade 3 mathematics coach (2015 to present), I visit numerous classrooms throughout our school board to assist teachers in mathematics instruction in order to enhance student achievement. One key area of focus has been on the development of number sense, including student's knowledge of number, number relations, and number operations (NCTM, 2000), enhanced through the use of number talks.

While math talk has always been an important instructional strategy in my twenty years of teaching, it was not until recently that I began to learn about number talks. I found myself wondering as a teacher, mathematics coach, and researcher whether or not I was meeting the call of the NCTM's (2000) Standards of Number and Operations, Problem Solving, Reasoning and Proof, and Communication. Furthermore, would the implementation of number talks increase students' abilities to solve problems with accuracy, efficiency, and flexibility as proponents argue (Parrish 2014; Russell, 2000)?

Purpose of the Study

The purpose of this mixed methods case study was to investigate the impact of daily number talks on the development of the mental math abilities of second graders within a reform-based classroom. A preassessment was administered to ascertain where students were located on Lawson's (2015) "Student Continuum of Numeracy Development: Addition and Subtraction" (p. 4). A postassessment was administered at the conclusion of six weeks. The results of the postassessment were compared with the preassessment results to determine the impact of daily number talks.

Research Questions

What is the impact of daily number talks on the development of mental math abilities of second graders within a reform-based classroom?

- Does the implementation of number talks increase students' ability to calculate with accuracy, efficiency, and flexibility?

- Does the implementation of number talks increase students' understanding of place value and number relationships?

Definitions

Within the context of this study of mental math calculations for addition and subtraction between 1 and 20 the following words have specific meanings:

- Accuracy is having the correct answer for a problem (Parrish, 2014).
- Efficiency is using an appropriate, expedient strategy for the problem (Parrish, 2014).
- Flexibility is using number relationships with ease in problems (Parrish, 2014).
- Place value is the value of a digit based on the position it occupies in a number (Small, 2013).
- Number relationships include four different types of relationships: spatial relationships; the relationships of more, less, and the same; anchors or benchmarks of 5 and 10; and part-part-whole relationships (Van de Walle & Lovin, 2006).

Significance of the Study

This research was a mixed methods case study of a class of Grade 2 students as they engaged in daily number talks. It provides insight into the development of young children's number sense and connections, mathematical thinking and reasoning, and range of mental computational skills. This study contributes information on the impact of daily number talks on students' accuracy, efficiency, and flexibility, in addition to the construction of new key ideas.

Contribution to the Mathematics Education Community

Teachers strive to provide students with learning environments which engage and inspire students to reach their full potential. Furthermore, teachers are interested in ways to improve instructional practices to enhance student achievement. While teachers use many instructional approaches, few are subjected to careful research documenting the efficacy of their use. This research contributes to determining the merits of a recent thrust in primary mathematics education: the use of number talks. Upon completion of the study, I will prepare an education session for teachers within our School Board. This research will contribute to professional development for educators and may result in improvements in student achievement for other classes as well.

Chapter 2: Literature Review

A Call for Reform

Over the past twenty-five years, there has been a significant paradigm shift in the scientific study of mathematics learning in both Canada and the United States (Battista, 1999). Concerns about the poor mathematics achievement and understanding of many children in traditional classrooms gave birth to the reform movement in mathematics instruction. Reformers implicated traditional rote instruction as the root cause of poor results and limited understanding. As researcher Michael Battista explained, "Reformers view mathematics as thinking and reasoning; they view teaching as involving and guiding students in the process of making sense of mathematical ideas" (p. 467). The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), along with other related documents from educators and researchers, called for the rethinking of mathematics instruction, content, and the nature of school mathematics.

Traditional mathematics instruction. In traditional mathematics instruction, the teaching focus is on students imitating mathematical procedures demonstrated by teachers with little to no understanding by students of what they are doing (Battista, 1999). Researcher James Hiebert's (1999) article, Relationships between Research and the NCTM Standards, fleshed out this claim about traditional mathematics. Hiebert concluded,

> Most characteristic of traditional mathematics teaching is the emphasis on teaching procedures, especially computation procedures. Little attention is given to helping students develop conceptual ideas, or even to connecting the procedures they are learning with concepts that show why they work. (p. 11)

Researchers Carpenter, Fennema, Franke, Levi, and Empson (2015) argued that the failure of traditional mathematics is partially caused by teachers failing to recognize and tap into the informal knowledge of children gained through personal experiences and natural strategies students use to solve problems.

Reform-based mathematics instruction. In comparison to traditional mathematics, reform mathematics focuses on problem solving and developing a conceptual understanding of mathematical concepts and ideas. Opportunities to read, write, and discuss mathematics are paramount, along with formulating and testing personal strategies by students in order to develop strong problem solving and mathematical reasoning skills (Battista, 1999; NCTM, 2000). Reformers believe students need not only to be able to recall facts from memory, but also have an ability to analyze and make sense of those basic number facts as well (Boaler, 2015).

Mathematical fluency, built on a solid foundation of conceptual understanding and strategies, is a long-term goal of mathematics education enabling students to apply mathematical reasoning to different situations (Battista, 1999). For this to happen, however, development of a mathematical mindset (Boaler, 2016) is paramount: "When students see mathematics as a set of ideas and relationships and their role as one of thinking about the ideas, and making sense of them, they have a mathematical mindset" (p. 34). Furthermore, as Boaler argued, mathematics is "a flexible conceptual subject that is all about thinking and sense making" (p. 35). In mathematics, therefore, it is essential for students to clarify and explain their work, to justify and defend their answers, to troubleshoot and revise their thinking, and to compare and connect similarities and differences among strategies (Kazemi & Hintz, 2014). In essence, "reasoning is central to the discipline of mathematics" (Boaler, 2016, p. 28).

Teaching and learning through constuctivism. Many of the instructional changes advocated by the NCTM's Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) and Everybody Counts: A Report to the Nation on the Future of Mathematics Education (Mathematics Science Education Board, Board of Mathematical Sciences, Committee on the Mathematical Sciences in the Year 2000, & National Research Council, 1989) "can best be understood from a constructivist perspective" (Clements & Battista, 1990, p. 6). Constructivism is a theory of learning, not a method of instruction (Clements, 1997). Regardless of the teaching style, students actively construct knowledge as best they can. Researcher Douglas Clements (1997) explains,

> There are times for many different types of constructing: time for "experiencing"; for "intuitive" learning; for learning by listening; for practice; and for conscious, reflective thinking. During these activities, students construct valuable, but different kinds of, knowledge. We need to balance these times to meet our goals for students. (p. 198)

Constructivist pedagogy strategies such as discovery learning and other hands-on approaches can be employed by teachers without resulting in the desired learning (Fosnot, 2005). Deep, conceptual learning occurs when there is a structrual shift in cognition. Therefore, from a constructivist perspective, teachers cannot direct learning to get all students to a certain "ah ha" at the end of a lesson. Notwithstanding, teaching is a planned activity.

Constructivists believe that knowledge is actively constructed in the mind of the learner as opposed to behaviorists who believe knowledge is passively received from the environment (Fosnot, 2005). When math educators draw on a constructivist rather than a behaviourist perspective in order to think about how they might transform and improve instruction, they suggest activities that are interactive and student-centered—which begin with students' thinking

(Clements, 1997; Van de Walle & Lovin, 2006). They believe learning also improves when assessment *for* learning occurs on a regular basis, not simply assessment *of* learning at the end of a unit of study (Van de Walle & Lovin, 2006). Furthermore, when students are actively involved in their learning, the process is as important as the product, whereby students construct their own knowledge as they personally try to make sense of situations (Clements, 1997; Van de Walle & Lovin, 2006). Battista (1999), for example, advocated for instruction which recognized and supported the personal construction of ideas as students "invent, test, and refine their own ideas" (p. 430) instead of blindly performing a procedure told to them. Unfortunately, as Battista (1994) further contended, "the prevailing view of educators and the public at large is that mathematics consists of set procedures and that teaching means telling students how to perform these procedures" (p. 463).

Working with Dutch mathematics educators and researchers, Hans Freudenthal, a renowned math educator and mathematician from the Netherlands, developed Realistic Mathematics Education (RME) in the early seventies employing the use of context and models (Fosnot, 2005) to promote children's mathematical development. Recognizing that children bring to school with them informal number sense, and self-discovered methods of computation (see for example Carpenter et al., 2015), RME begins with a context with which children are familiar. One of the central ideas of RME is that students "learn mathematics by developing and applying mathematical concepts and tools in daily-life problem situations that make sense to them" (Van Den Heuvel-Panhuizen, 2003, p. 9). Therefore, in the social context of the classroom, students are seen as active participants in the teaching-learning process.

In the late 1980s, mathematics educator Catherine Fosnot began working with Dutch educators, particularly Maarten Dolk, who drew upon Freudenthal and his work, to deepen their understanding of teaching and learning. Fosnot and Dolk (2001) argued that the process of learning happens through the construction of meaning where knowledge is constructed by the student, not 'discovered" as it is not situated outside of the mind of the learner. *Mathematizing* can be used to describe this process of constructing mathematical meaning from viewing a situation through a mathematical lens. Furthermore, Fosnot and Dolk contended that learning mathematics by doing is the best method of instruction, whereby "children are organizing information into charts and tables, noticing and exploring patterns, putting forth explanations and conjures, and trying to convince one another of their thinking—all processes that beg a verb form" (p. 4). When children truly engage in mathematizing—the activity of structuring, modelling, and interpreting one's world—they indeed become "young mathematicians at work" (Fosnot & Dolk, 2001, p. 25).

Additionally, reform-based (or constructivist informed) instruction focuses on the development of students' personal strategies and mathematical ideas (Clements & Battista, 1990) where the teacher actively guides and supports the students to construct their own mathematical thinking. Therefore, Clements and Battista (1990) suggested that students need opportunities to invent, test, and refine their thinking. The idea that children actively construct their own understanding is in direct opposition to those who believe students can learn by absorbing the teacher's knowledge through repetition or imitating computational procedures and rules. With an emphasis on conceptual understanding, reasoning, and problem solving, powerful mathematical thinkers develop when instruction focuses on, guides, and supports students construction of ideas (Battista, 1999).

Socio-constructivists believe learning is not an individual event but rather a social and conversational activity. According to Vygotsky (1978), a Soviet psychologist, social interactions play a fundamental role in cognitive development for language and discourse promotes thinking and develops reasoning. Teachers and more experienced peers can provide scaffolding for developing cognitive skills as students internalize collaborative conversations.

Regrettably, there is a widespread lack of fundamental understanding of the constructivist theory even among teachers, educational administrators, and professors of education (Battista, 1999). Constructivist learning is not “discovery learnings,” a method of teaching using manipulatives or cooperative learning. Rather, it is a theory of how students learn. In different contexts, students interpret, organize, and model various strategies and ideas based on their past constructed understanding in order to make sense of current experiences (Fosnot & Dolk, 2001). Piaget (1977) conceived of this process as assimilation, ‘to make similar.’ Throughout this process, learners *act on* information rather than simply taking it in; they interpret, infer, and organize new information to fit into their existing schema. Learning is accommodation of knowledge, building new ideas on old ideas. Accommodation is more substantial as it requires reshaping of existing schema. “This gets to the heart of constructivism” (Fosnot, 2005, p. 13).

Researchers Carpenter, Fennema, and Franke (1996) provided an excellent illustration of how theory and research can inform teaching and learning of early mathematics; they conceived of *Cognitively Guided Instruction* (CGI). The key to CGI is to provide teachers with research-based knowledge on the development of children’s mathematical thinking (cognition) and then allow teachers to decide how to apply that knowledge within the context of their teaching practice. Therefore, Franke and Kazemi (2001) suggest,

> teachers discuss CGI as a philosophy, a way of thinking about the teaching and learning of mathematics, not as a recipe, a prescription, or a limited set of knowledge. CGI teachers engage in sense-making around children’s thinking. They continually evaluate their

> understanding, adapt and build on their knowledge, and figure out how to make use of it in the context of their ongoing practice. (pp. 102-103)

Furthermore, CGI teachers provide opportunities for children to reconstruct and expand their existing knowledge by causing cognitive conflicts. This is indicative of a constructivist theory of learning. Does the theory of constructivism and the research on children's mathematical development (as delineated by Carpenter et al. [2015] in *Children's Mathematics: Cognitively Guided Instruction*) have implications for the learning of number facts?

Failure of traditional method of mathematics instruction. In 2008, Henry and Brown interviewed 275 first-grade students from nine schools in California during May and early June 2004. Based on their research they concluded that the repeated practice of rote memorization methods such as worksheets, flash cards, and timed tests led students to rely heavily on counting strategies. Likewise, the current state-approved textbooks encouraged the same thing rather than supporting the development of derived-fact strategies. While the majority of the students in the study failed to achieve success with a memorization standard, the researchers felt students could have mastered fact fluency with "a combination of derived-fact strategies and retrieval from long-term memory" (p. 180).

In similar fashion, driven by Piaget's theory of learning, Kamii and Dominick (1998) argued that teaching the standard algorithm approach is not only misguided but may, in fact, be detrimental to children's understanding of numbers. They noted research in the 1980s that concluded that children are more likely to make mistakes when trying to use standard algorithms without understanding and were thus more liable to produce incorrect answers than when employing their own strategies for problem solving. Supported by empirical research data, Kamii and Dominick (1998) even went so far as to say algorithms are harmful to children when taught as the sole method of calculation. The authors elaborated, “We have two reasons for saying that algorithms are harmful: (1) they encourage children to give up their own thinking, and (2) they ‘unteach’ place value, thereby preventing children from developing number sense” (p. 135). The researchers found that second and third graders in the “No algorithms” classes produced the highest percentage of correct answers (45 and 50 percent respectively); they produced more correct answers than all the fourth graders who were taught algorithms. Furthermore, when looking at incorrect answers, the “No algorithms” classes produced answers that were more reasonable than those students in the “Algorithms” class. Fourth graders, who had an additional year of algorithms, were even more unreasonable in their incorrect answers than those students in the third-grade “Algorithms” classes.

Similarly, Boaler (2016) argued against students learning math facts through mindless practice and speed drills. She reasoned that such an “approach to early learning about numbers can cause damage to students, [and can] make them think that being successful at math is about recalling facts at speed, and pushes them onto a procedural pathway that works against their development of a mathematical mindset” (p. 37). Furthermore, when math facts are learned in isolation and when students are led to believe that strong math students have quick recall of memorized facts, this can this lead to math anxiety. Based on evidence from studies conducted by brain researchers (see Delazer et al., 2005), Boaler (2016) concluded that conceptual mathematics activities designed to help students become proficient in basic fact recall is the optimal way to encourage both the learning of facts and the development of a mathematical mindset.

Fact learning: memorization versus from memory. Reformers suggest that teachers should be directed by the knowledge of how students learn mathematical concepts and skills and the developmental sequence of such (Clements & Battista, 1990; Battista, 1994; Carpenter et al., 2015), coupled with an accurate understanding of where students are in their development. This knowledge, in turn, guides their instruction to promote students' development with sense-making as an underlying goal. It is important, therefore, to be clear on the learning of math facts. There is little disagreement that fluency in basic number facts is essential (Henry & Brown, 2008). Rather, Henry and Brown (2008) contended the controversy surrounds "what constitutes basic facts fluency and how best to help children achieve this fluency" (p. 154).

The Common Core State Standards (2010) in the US de-emphasized the rote memorization of math facts and placed greater emphasis on numerical reasoning instead, thereby recognizing the difference between memorizing and remembering. In her forward to Newton's (2016) book entitled, *Math Running Records in Action: A Framework for Assessing Basic Fact Fluency in K-5*, Alison Mello, a K-8 Math Director and Math Consultant, elaborated on the difference between memorizing and remembering. She contended, "Memorization happens in a vacuum. It is an isolated experience" (p. xii). Rather than having students memorize basic number facts, the focus should be on enabling students to become strategic thinkers, because when we think actively about things we remember them. Based on documents from the National Research Council (2001, 2005, 2012), math educator Nicki Newton (2016) argued that "the teaching of the facts should be done in a way that focuses on structure of numbers, patterns, place value, properties and the relationship between the operations" (p. 13). Succinctly, students need to learn more than simple memorization of facts.

In the video, *Ignite Talk: "There IS a Difference"* (2014), K-5 math educator Graham Fletcher explained how memorization of number facts (which is void of strategy) is different from learning number facts from memory (which relies on strategies). Fluency built on learning number facts over time from memory enables students to develop efficient, accurate, and flexible ways of learning. Fletcher (2014) further elaborated that when strategies from memory are repeatedly practiced over and over again, they become automatic; he stated, “Automaticity of a strategy can appear to be memorization… but it’s not.” Rather, automaticity comes through learning, increasing efficiency through repetition and practice, leading to fluency and flexibility with numbers, applicable in other areas of mathematics as well. Math educator Van de Walle (1999) also supported purposeful practice for automaticity, arguing that “drill is appropriate when, a) the desired concepts have been meaningfully developed, b) flexible and useful procedures have been developed, and c) when there is a real need for speed and accuracy” (p. 9). Drill is also supported by Baroody (1985): “*once* children have the opportunity to find relationships in order to facilitate internalization and the automatic use of such knowledge” (p. 95, emphasis in original). In short, the purposeful practice of facts built on conceptual understanding is a key component in fact fluency whereby students are practicing to understand rather than simply practicing to memorize.

Based on Boaler’s (2015) research evidence, when students simply memorize their facts, they often do so without number sense which can lead to errors. If students forget a memorized fact, they lack strategies to figure out the answer. Additionally, “the more we emphasize memorization to students, the less willing they become to think about numbers and their relations, and to use and develop number sense” (Boaler, 2016, p. 40). In contrast to the simple memorization of facts, Fosnot and Uittenbogaard (2007a) argued that “when relationships are the focus, there are far fewer facts to remember, and big ideas like compensation, hierarchical inclusion, and part-whole relationships come into play” (p. 7). Finally, Lawson (2016) also supported fact learning through memory as opposed to memorization. She further concluded that "the potential learning that exists in the territory between direct modelling and memorization of facts is foundational for a great deal of later mathematics and for math fluency" (p. 2). Therefore, students need instructional programs which enable them to compute fluently (NCTM, 2000).

Reform Standard: Computational Fluency Built on Conceptual Understanding

The NCTM Standards (2000) describe computational fluency as "having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently" (p. 152). Understanding of place value, operational properties, and number relationship is foundational for *conceptual understanding* while the use of accurate, efficient, flexible strategies is characteristic of computational fluency.

Math educators Ebdon, Coakley, and Legnard (2003) contended that "computational fluency is NCTM's answer to knowing the basic number facts and understanding them" (p. 488). Furthermore, they argued that computational fluency, built upon an understanding of numerical relationships, enables students to rely on mathematical memory instead of memorization. When mathematical procedures are constructed by understanding interconnecting mathematical relationships, students will not forget them over the summer (Ebdon et al., 2003). Finally, computational fluency helps to reduce the cognitive load when students face more complex computational problems; further validation for why mental math (enhanced through number talks) is so vital (Thunder & Demchak, 2016).

Math educators O'Connell and SanGiovanni (2011) designed a program in fact acquisition, which capitalizes on utilizing early thinking strategies drawing on Thornton's pioneering research work in 1978 that emphasized how thinking strategies facilitate fact acquisition. O'Connell and SanGiovanni (2011) also drew on Fuson and Kwon's (1992) research on 18 middle-class first graders from two schools in Seoul, Korea. The students, who received explicit and sustained instruction in early number strategies structured by ten, rather than being drilled on basic facts using memorization-focused approaches, demonstrated high levels of competence in adding and subtracting single-digit and multi-digit numbers. This research supported O'Connell and SanGiovanni's (2011) goal of their fact acquisition program—students' automaticity of math facts based on understanding mathematical relationships.

Despite such research, there are still educators that focus on the end product of memorization (through traditional drill and practice) and fail to take into account a developmental approach to numerical fluency involving purposeful practice of math facts based on conceptual understanding (Ebdon et al., 2003; O'Connell & SanGiovanni, 2011; Thunder & Demchak, 2016). The latter, fluency approach to learning number facts, places emphasis on discovering efficient, effective derived fact strategies (Kling, 2011) essential in the development of mathematical proficiency (Baroody, 2006). Finally, mastery of addition and subtraction facts using a range of strategies occurs through stages of mathematical development.

Developmental Trajectory of Addition and Subtraction

Researchers have long maintained that students move through three general stages in mastering basic number facts for addition and subtraction. First, counting strategies, followed by reasoning strategies (relating unknown facts to known facts), ending in fact fluency (Baroody, 2006; Carpenter & Moser, 1984; Fosnot & Dolk, 2001; Garnett, 1992; Isaacs & Carroll, 1999). While researchers noted differences in the names or descriptions, the general progression was consistent and while mastery of basic number facts is the final stage of development (NCTM, 2000), research supports this involves more than simply rote memorization. Most learners will advance through this natural and progressive process of mathematical development (Guerrero &

Palomaa, 2011). Once again, however, focused attention is imperative in the transition stage from counting procedures to direct recall of facts from memory. Many researchers state that this transition stage is crucial for the development of conceptual understanding (Baroody, 2006; Baroody, Bajwa, & Eiland, 2009; Bezuk & Cegelka, 1995; Carnine & Stein, 1981; Carpenter & Moser, 1984; Garnett, 1992; Isaacs & Carroll, 1999; Lambert, Imm, & Williams, 2017; Lawson, 2016; Miller, Mercer, & Dillon, 1992; Newton, 2016; Steinberg, 1985; Thornton, 1978, 1990).

Carpenter and Moser (1984) conducted a 3-year longitudinal study of 88 students. While this research study focused on the developmental trajectory of children's addition and subtraction skills in first through third grade, Carpenter and Moser noted that traditional classroom instruction failed to support this path as teachers encouraged the jump from counting all, with manipulatives, to memorization of number facts. This instruction failed to take into account the extended time needed for children to develop a solid understanding of different strategies such as counting on, counting back, and derived-facts.

Thornton (1978) concluded, "curriculum and classroom efforts should focus more carefully on the development of strategy prior to drill on basic facts" (p. 226). Based on his research data, Thornton found that the use of thinking strategies focused on relationships as part of the teaching process resulted in more facts being learned after an eight-week instructional period than in classes where such aids to memory were not taught. Likewise, Carnine and Stein (1981) found that students learned a set of 24 facts with higher accuracy (84%) when instructed with a strategy for remembering facts than students with no aid for remembering (only 59% accuracy). Baroody (1985) stated that mastery of facts must "*include* discovering, labeling, and internalizing *relationships* [and] meaningful instruction (the teaching of thinking strategies) would probably contribute more directly to this process than drill approach alone" (p. 95, emphasis in original).

Thornton (1990) designed a conceptual framework for the learning of basic number facts. This framework included three major phases: "*Phase 1*: Understand the concept; *Phase 2*: Learn strategies or procedures to derive answers to unknown facts; and *Phase 3*: Practice so facts are memorized to the point of automatic recall" (p. 241, emphasis in original).

Steinberg (1985) studied the effect of teaching "noncounting, derived facts strategies in which the child uses a small set of known number facts to find or derive the solution to unknown number facts" (p. 337) to one second-grade class for an eight-week period with a focus on addition and subtraction facts. Steinberg found that children changed strategies from counting to using the derived facts strategies and were able to answer more of the fact problems within the two seconds allotted per problem than before instruction. It did not appear that counting on (e.g., to determine 5 + 3 a student would start at 5 and count up to 8 using their fingers to track the count) (Lawson, 2016) was a requirement for learning derived facts strategies.

Henry and Brown (2008) concluded, "Children who solve problems based on their developing understanding of counting are likely to build their understanding of number relationships and properties, and develop part-whole, or derived-fact, strategies that can be highly efficient in solving basic-fact problems" (p. 155). Derived-fact strategies then become tools children can use to solve multi-digit mental math problems.

Once again, it is important to note that "when elementary instruction jumps from count-all methods to memorized facts, the insightful period of development, whereby other strategies are used and developed, may become obscured" (Guerrero & Palomaa, 2011, p. 15). Another study of ninety-seven first graders in Flanders substantiated this claim, concluding that "children who are taught multiple reasoning strategies on sums over 20 are able to apply those strategies efficiently and adaptively on the basis of their individual strategy knowledge and skills"

(Torbeyns, Verschaffel, & Ghesquière, 2005, p. 18). Biddlecomb and Carr (2010) conducted a longitudinal study of the development of mathematics strategies and underlying counting schemes. They looked at student-generated strategies and commonly taught algorithms of 206 students over a three-year period, beginning in Grade 2. Their results supported the "importance of instruction in students' construction of strategies and the schemes that underlie those strategies" (p. 22).

Lawson (2015) identified four phases of strategy development: Direct Modelling & Counting, Counting More Efficiently & Tracking, Working with the Numbers, and Proficiency. She further discussed how the "Student Continuum of Numeracy Development: Addition and Subtraction" could be used to capture strategies used by students and their underlying key ideas (see Appendix A). Key ideas are "important mathematical properties or ideas that children construct as they work with different strategies" (Lawson, 2015, p. 3). Also, in her article entitled, "The Mathematical Territory Between Direct Modelling and Proficiency," Lawson (2016) advocated for a "guided-discovery" approach for learning facts as opposed to direct instruction or discovery mathematics. Referencing Van de Walle, Karp, Bay-Williams, McGarvey, and Folk (2015), Lawson contended:

> Students who work through and become competent using increasingly sophisticated strategies do so, not through direct instruction, but rather as a result of teachers posing well-constructed problems that elicit and work with these evolving strategies, augmented by extensive practice in different contexts. Once children have worked through these reasoning strategies, they can memorize whatever facts have not yet become automatic using targeted drills. (p. 4)

Other math educators (Buchholz, 2004, 2016; DiBrienza & Shevell, 1998; Ebdon et al., 2003; Fosnot & Uittenbogaard, 2007a, 2007b; Parrish, 2011, 2014; Russell, 2000; Scharton, 2004) have suggested a number of ways that teachers can support children's development of increasingly efficient strategies and progress in their continuum of development. One such way is *number talks*.

From direct instruction to active construction: the role of number talks. Since children's solution strategies change over time, and some may revert to less efficient strategies when faced with more challenging numbers, instruction should always aim to move students further along in their development. Focus should be on shifting children's strategies from counting to reasoning, to retrieval strategies thereby "building conceptual understanding and procedural fluency" (Parrish, 2014, p. xxvii). In addition to number talks supporting strategy development, they also may promote the development of mathematical mindsets (Boaler, 2016). Since mathematics is not about memorization as a method but rather thinking, sense-making, and the development of big ideas and connections, math educators believe that number talks may be an excellent technique to foster the development of student thinking and learning in these ways (Boaler, 2016; DiBrienza & Shevell, 1998; Ebdon et al. 2003; Fosnot & Dolk, 2001; Fosnot & Uittenbogaard, 2007a, 2007b; Franke & Kazemi, 2001; Parrish 2011, 2014; Russell, 2000).

During a number talk (or string [Fosnot & Uittenbogaard, 2007a, 2007b] or sequence [Lawson, 2015]), the teacher would pose a calculation on the board from a purposefully chosen number string of computational calculations. DiBrienza and Shevell (1998) define a number string as "a series of related but bare (devoid of context) computation problems that are specifically designed to elicit quick, efficient, and reliable strategies for computation from students… Number strings give students a chance to notice patterns and hone their computational skills in a constructivist way" (p. 21). The calculation questions "are designed to elicit specific strategies that focus on number relationships and number theory" (Parrish, 2014, p. 5) encouraging students to look at the numbers first, and then decide on which computation strategy to use (Fosnot & Uittenbogaard, 2007a). Strings are designed to "generate discussion on certain strategies or big ideas underlying an understanding of early number sense" (Fosnot & Uittenbogaard, 2007a, p. 6) (see Figure 1). Finally, each calculation in the number string is written horizontally to encourage place value thinking, because students often fail to take into account the magnitude of each digit and the corresponding place value of each digit when problems are written vertically (Parrish, 2014).

Working on the Basic Facts without Modeling D19[1] Using the Five- and Ten-Structures, Relating Addition and Subtraction, One More and One Less
10 + 4
14 – 10
14 – 9
14 – 4
8 + 8
16 – 8
16 – 9
16 – 7

Figure 1. Number string.

During number talks (or strings), students are asked to raise their thumb held to their chest when they have an answer (see Figure 2). While they are waiting, students are encouraged to think of another strategy to defend their answer and indicate so by raising their index finger along with their thumb. Teachers allow thinking time (with only a quiet form of acknowledgment when a student has an answer—their thumb raised with their fist to their chest). Thinking time ensures that the majority of the students have solved the problem. When most of

[1] *Minilessons for Early Addition and Subtraction: A Yearlong Resource* (Fosnot & Uittenbogaard, 2007a, p. 60).

the students have their thumb raised, the teacher asks students to share their answers. The teacher records both correct and incorrect answers without any verbal or physical expressions that would indicate agreement or disagreement of responses. All answers are accepted, respected, and considered, thereby creating a respectful, safe learning community. Students use the hand signal "me too" to indicate agreement with answers (and again when agreeing with strategies). With this hand signal, inspired by the American Sign Language, students raise their thumb and pinkie finger on one hand and rock their hand back and forth between the student they agree with and their own chest. The teacher then records student thinking on the board.

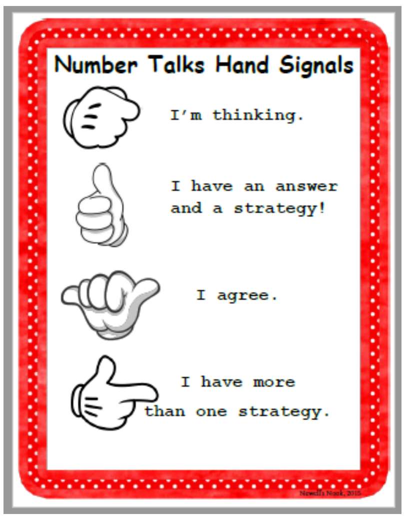


Figure 2. Number talks hand signals[2].

As a facilitator, questioner, and listener, the teacher tries to make sense of the development of each student's thinking in order to become better informed to make instructional decisions (Franke & Kazemi, 2001; Parrish, 2011). Furthermore, Franke and Kazemi (2001) advocated,

> Focusing on students' mathematical thinking remains a powerful mechanism for bringing pedagogy, mathematics, and student understanding together. As teachers struggle to make

[2] Number Talks Hand Signals (Newell's Nook, 2015).

> sense of their students' thinking and engage in practical inquiry, they elaborate how problems are posed, questions are asked, interactions occur, mathematical goals are accomplished, and learning develops. (p. 108)

Therefore, in a reform-oriented classroom, all members of the classroom community (including the teacher) are participating by "constructing their own knowledge and reflecting on and discussing this knowledge" (Hufferd-Ackles, Fuson, & Sherin, 2004, p. 83).

Thus, a successful number talk is rooted in communication (Parrish, 2011). Once answers are recorded, the students are then called upon to justify or defend their answers and prove their thinking to their peers while always remembering the ultimate question: "Does it make sense?" Parrish (2014) promoted, "Number talks can be a purposeful vehicle for making sense of mathematics; developing efficient computational strategies; communicating mathematically; and reasoning and proving solutions" (p. 15). As students think and reason as mathematicians during number talks, the authors contend that students develop computational fluency. Furthermore, when students are sharing and discussing, and when students are asked to make connections and look for relationships, they are indeed "doing mathematics." In doing so, Parrish (2011) argued students have the opportunity to do the following:

- **Clarify** thinking.
- **Investigate and apply** mathematical relationships.
- **Build** a repertoire of efficient strategies.
- **Make** decisions about choosing efficient strategies for specific problems.
- **Consider and test** other strategies to see if they are mathematically logical (p. 203, emphasis in original).

To reiterate, learning to examine strategies is a crucial part of number talks. As students discuss strategies they “analyze them, critique them, and find relationships among them” (Russell, 2000, p. 5). Over time, this process of explaining and justifying solutions becomes a habit, and further develops their ability to identify why some strategies will not work for certain problems.

Recognizing that the ability to explain and justify solutions takes time and practice to develop, educators Ebdon et al. (2003) coined the term “Mathematical Mind Journey (MMJ)” to describe number talks. They advocated for the development of a community mathematical classroom “where everyone’s ideas are respected and valued while students are empowered to work smarter, not just harder” (p. 486). In an MMJ classroom, students construct meaning “using what they know to solve what they do not know” (p. 487), thereby leading to mathematical sense and fluency. Ebdon et al. (2003) further advocated for encouraging student ownership of strategies enabling students to feel empowered as mathematical thinkers and learners.

Along the same premise, Fosnot and Uittenbogaard (2007a, 2007b) designed minilessons of strings to extend students’ mathematical thinking and developing number sense. Whether using the terms number talks, MMJ, or minilessons, the intent is the same: to help students build mental math and computational strategies (completed without pencil and paper) by supporting students to invent, construct, and make sense of their own number strategies and their underlying key ideas. Then, “by developing a repertoire of strategies, an understanding of the big ideas underlying why they work, and a variety of ways to model the relations, children are assembling powerful toolboxes for flexible and efficient computation” (Fosnot & Uittenbogaard, 2007a, p. 8). Moreover, as students give voice to their learning and understanding, the Content and Practice Standards (NCTM, 2000) become intertwined in purposeful and powerful ways.

Computational fluency is also enhanced through number talks as students focus on number relationships (Van de Walle & Lovin, 2006) and make use of those relationships. “The number 7, for example, is more than 4, two less than 9, composed of 3 and 4 as well as 2 and 5, is three away from 10, and can be quickly recognized in several patterned arrangements of dots” (p. 37) Accuracy (having the correct answer for a problem), efficiency (using an appropriate, expedient strategy for the problem), and flexibility (using number relationships with ease in problems) are the goals of number talks (NCTM, 2000; Parrish, 2014; Russell, 2000; Scharton, 2004).

Based on their experience, Fosnot and Dolk (2001) concluded that math talk proved instrumental in fostering the development of computational fluency along with mathematical ideas within a social community of mathematicians. Similarly, Torbeyns, Smedt, Ghesquière, and Verschaffel (2009) looked at various empirical research studies which supported the claim that children’s socio-mathematical environment influenced their strategy development as well. They concluded,

> the results of these studies indicate that children, who are encouraged by their socio-mathematical environment to acquire and apply diverse strategies, tend to spontaneously develop and use a variety of (self-invented) strategies, whereas their peers, who are instructed in a non-encouraging environment, seem not able to do so. (pp. 2-3)

This was further substantiated by Torbeyns et al.’s (2009) research results; they found “that younger lower achieving children’s strategy discovery does not come by itself but needs carefully designed instructional encouragement and support” (p. 13). Thus, how do teachers conduct a number talk in a manner that is conducive to learning the intended strategy?

Building a strong number talk community. Fosnot and Dolk (2001) argued that “a classroom becomes a community of learners [when students are] engaged in activity, discourse, and reflection" (p. 27). A social community of mathematicians prevails within a safe, risk-free environment. Such a community is a requisite for productive number talks (Parrish, 2011). If students are to discuss the merits of various strategies they need to feel comfortable with sharing their strategies, defending their thinking, challenging the thinking of other students, and investigating new strategies. As students and teachers seek to learn with understanding, acceptance and mutual respect must prevail.

In *Intentional Talk: How to Structure and Lead Productive Mathematical Discussions*, Kazemi and Hintz (2014) suggested norms of productive and nurturing learning environments and compiled a list.

In this class, we will do the following:

- Make sense of mathematics…;
- Keep trying even when problems are challenging…;
- Remember that it's okay to make mistakes and revise our thinking…;
- Share our mathematical ideas with our classmates…;
- Listen to understand someone else's ideas, give each other time to think…;
- Ask questions that help us better understand the mathematics…;
- Agree and disagree with mathematical ideas, not with each other…; and
- Remember that everyone has good mathematical ideas…. (pp. 19-20)

Kazemi and Hintz (2014) contended that it is important for all students to feel valued and acknowledged as sense-makers. The community will respond if high expectations are set in motion from the beginning. With these ingredients, over time, a vibrant number talk community may evolve (Kazemi & Hintz, 2014). Once a strong social community of mathematicians is established, can number talks lead to deeper mathematical understanding?

Number Talks: Perhaps a Path to Understanding

Christensen and Cooper (1991) conducted a study of 40 Grade 2 students to compare the effectiveness of direct instruction of cognitive strategies (direct instruction strategy group) compared to practice activities designed to facilitate invented strategies (practice of invented strategies group). Fifteen minutes of instruction was given each day for 12 weeks to both groups. Direct instruction was used to teach three strategy clusters: counting, near doubles, and near ten beginning with concrete materials moving to iconic pictures and then to abstract symbols. The succession of each lesson went from teacher-directed presentation and demonstration, to a discussion and guided practice, and concluded with independent practice. In comparison, the practice of invented strategies group was taught using drill-and-practice techniques such as flash cards, worksheets, written timed tests, and games. However, "it should be noted that practice was not conducted in a rote or meaningless way. Activities included opportunities for discovery, play, and independent exploration of number facts" (p. 367).

Christensen and Cooper (1991) hypothesized that the direct instruction strategy group would perform better than the practice of invented strategies group on written and oral tests; the data supported the opposite result. The researchers concluded, "Thus, it appears that practice led to invented strategies that were used more effectively than those acquired as a result of direct instruction" (pp. 368, 370). In short, the practice which led to the discovery of invented strategies, "produced more effective learning and more effective strategy use" (p. 363). It may be that children who had the opportunity to construct their own strategy rather than learning through direct instruction had greater success. It is interesting to note that half the children in both groups failed to use cognitive strategies at all during testing, instead they used counting strategies. In conclusion, the researchers suggested further investigation was warranted to determine if "greater strategy use would have been facilitated if the teacher had attempted to guide the student to discover the strategies during practice activities rather than explicitly teaching them" (p. 370) in the direct instruction strategy group. Posttesting also provided some evidence "to suggest that strategies invented during practice facilitated the transition to direct retrieval" (p. 370) in the 'practice of invented strategies' group. This self-invention of strategies is an integral component to number talks.

While not specifically focusing on number talks, Carpenter, Franke, Jacobs, Fennema, and Empson (1997) conducted a 3-year longitudinal study of invention and understanding in 82 children's multidigit addition and subtraction in Grades 1 through 3. They concluded that a high percentage (90%) of students indeed used invented strategies: "Students who used invented strategies before they learned standard algorithms demonstrated better knowledge of base-ten number concepts and were more successful in extending their knowledge to new situations than were students who initially learned standard algorithms" (p. 3). Moreover, there was a difference in the number of systematic errors between the invented-strategy groups and the algorithms groups, with the former group demonstrating significantly fewer errors. Carpenter et al. (1997) suggested the findings of this study were "consistent with the theoretical perspective (Hiebert, 1986; Hiebert & Carpenter, 1992) that supports the development of understanding before mastery of procedures" (p. 16). One final note of importance from this study is the fact that "the strategies were constructed in a social context in which students shared strategies with one another. However, none of the teachers made an explicit effort to teach a particular invented strategy or gave any one invented strategy a place of prominence" (p. 18). With this research in mind, one is left wondering about the effectiveness of using number talks as a guided-discovery approach as advocated by Lawson (2016) to increase number sense and connections, mathematical thinking and reasoning, and different mental computational skills. Unfortunately, while there is a lack of empirical research on this topic, many educators believe in the success of number talks or minilessons.

As a graduate student, Susan Scharton (2004) studied the development of students' understanding of arithmetic by teaching mathematics in a first-grade and second-grade classroom. She then compared the differences in understanding between students taught through more traditional methods versus students who had opportunities to come up with their own strategies to solve problems. Scharton concluded, when given the chance to create their own strategies, students become accountable to make sense of what they are doing. Furthermore, students deepened their own understanding along with other students' understanding when asked to explain and analyze their thinking in both oral and written communication. Moreover, Scharton suggested,

> exposure to a variety of computation strategies allows students access to methods that they may not have considered on their own. As their knowledge of different strategies grows, so does their computational flexibility. Through these opportunities, students can make sense of how and why arithmetic works. (p. 278)

In a similar fashion, when looking at MMJ (number talk) strategies in action within a second-grade classroom, educators Ebdon et al. (2003) concluded number talks were one way for teachers to help students progress up Fosnot and Dolk's (2001, 2016) *landscape of learning*. Based on the premise that learning is developmental, Fosnot & Dolk (2001) developed "The Early Number Sense, Addition and Subtraction Landscape" (Fosnot, 2016, p. 97) of learning which identified important landmarks of big ideas, strategies, and models.

Through number talks, Parrish (2011) suggested that students can "clarify thinking; investigate and apply mathematical relationships; build a repertoire of effective strategies; make decisions about choosing efficient strategies for specific problems; and consider and test other strategies to see if they are mathematically logical" (p. 203). Furthermore, Boaler (2016) advocated that number talks was the very best way to teach both number sense and math facts at the same time, as they enabled students to see the flexible and conceptual nature of math. Coupled with the fact that "students love to give their different strategies and are usually completely engaged and fascinated by the different methods that emerge" (p. 49), number talks may be invaluable.

Principal Jenny Nauman (2016) of Shields Elementary School in Lewes, Delaware shared her experience of a school-wide implementation of daily number talks in *Principal* (May/June 2016). She contended,

> Number talks have become a staple in our math instruction for numerous reasons. The technique is a great way to build mathematical fluency through conceptual understanding without the typical 'drill and kill.' Furthermore, the technique helps to build a classroom culture where students can make mistakes and share misconceptions. Lastly, number talks encourage student conversation because students are given the opportunity to share and explain their thinking verbally. (p. 40)

She further documented that after three years students met or exceeded the standard on the statewide assessment, as evidenced by an increase from 85 percent to 95 percent of third-, fourth-, and fifth-graders. Nauman also testified to "a huge difference on [their] universal math screener and STAR math results from fall to winter" (p. 41).

Educators DiBrienza and Shevell (1998) contended that children developed stronger number sense when allowed to invent their own strategies using number strings, and participated in number talks on a regular basis. Thorough conceptual understanding, along with efficiency in computation, required both time and effort. Buchholz (2004) recounted her experience of exploring strategies with students and testified that students invent their own strategies and proceed to "explain them with enthusiasm and pride.… To my students, equations were not just equations anymore; they were numbers they could manipulate in any way that made sense to them" (p. 365). Grade 2 teacher Buchholz also attested to the importance of daily strategy practice, stating:

> The more strategies we learned, the longer our Mental Math time took. Every minute was worth it. My students seemed to be picturing one another's strategies mentally. This combination of intense study of strategies and daily opportunity for practice added up to success. (p. 365)

Math educators DiBrienze and Shevell (1998) further argued, "Children who are given opportunities to explore and construct strategies will derive aesthetic pleasure in playing with numbers and searching for elegant solutions" (p. 25). Similarly, once students warm up their math brains through number talks, "the energy spills over into the next mathematical task… [and] it is exciting to watch students' mathematical understanding develop and to see students excited about numbers and to see teachers in awe of how their students are thinking (Ebdon et al., 2003, p. 488).

Summary and implication. In brief, many math educators (Buchholz, 2004, 2016; DiBrienza & Shevell, 1998; Ebdon et al., 2003; Fosnot & Uittenbogaard, 2007; Parrish, 2011, 2014; Russell, 2000; Scharton, 2004) claim that number talks (or minilessons) in which children “invent,” or construct, discuss, and apply a range of strategies to solve calculations improve their mathematical fluency, confidence, and achievement. Can number talks create a path to understanding?

Chapter 3: Methodology

Overview

In today's society, in order to be successful, students need a deep conceptual understanding of mathematics (Fosnot & Uittenbogaard, 2007b; NCTM, 2000). Part of this entails a solid understanding of number, number relations, and number operations, enabling students to estimate, calculate mentally, and judge the reasonableness of their results (NCTM, 2000). What types of instruction foster children's ability to calculate with mathematical fluency? Number talks (or calculation strings) have recently been suggested as one way to improve children's mathematical fluency. As a reminder, my research questions are: What is the impact of daily number talks on the development of mental math abilities of second graders within a reform-based classroom?

- Does the implementation of number talks increase students' abilities to solve problems with accuracy, efficiency, and flexibility?
- Does the implementation of number talks increase students' understanding of place value and number relationships?

Research Design

In order to best address the research questions, I used a mixed methods approach (Creswell, 2014). This included a qualitative embedded case study design which generated data. I subjected my qualitative codes to a quantitative analysis generating descriptive statistics on the various codes such as accuracy (correct, incorrect), strategy used (counting three times, etc.), mathematical phases (1 through 4), and so on. This "mixing" or blending of the data enabled me to glean a deeper understanding of the research questions.

As a wholistic approach, qualitative research involves discovery and thus "qualitative researchers are interested in understanding how people interpret their experiences, how they construct their worlds, and what meaning they attribute to their experiences" (Merriam & Tisdell, 2016, p. 6). This research process involves generating questions, collecting data from participants (generally in the natural setting), inductively to deductively analyzing data in order to build general themes, and interpretation of the data by the researcher (Creswell, 2014). It is also important to note that in qualitative research, words, collected and analyzed in multiple ways, are used as data (Merriam & Tisdell, 2016). Furthermore, "*the researcher is the primary instrument for data collection and analysis*" (Merriam & Tisdell, 2016, p. 16, emphasis in original).

This research project was a mixed methods study exploring the impact of daily number talks conducted by me on the mental math abilities of second graders in a reform-based classroom. As a mathematics coach with the Lakeridge District School Board (pseudonym), I had been afforded the opportunity to implement number talks in various classrooms throughout our board. Subsequently, I began searching for research-based evidence on the impact of number talks with the impetus of improving my teaching practice and enhancing student achievement. In my search of the literature, I found a gap in research-based evidence pertaining to the impact of number talks. This interest had led to research questions best approached through a mixed methods study where the focus was on discovery, insight, and understanding (Merriam & Tisdell, 2015). Further to the qualitative research paradigm, this study entailed interviewing as well as observing, and analyzing—central activities in qualitative research. As the researcher, I was the primary instrument for data collection and analysis.

A case study design enabled me to provide "an in-depth description and analysis" (Merriam & Tisdell, 2016, p. 57) of the impact of daily number talks on one classroom of learners bounded by time and activity. Overall class analysis was completed for pre-, mid-, and postassessments. I used an embedded case study design (Yin, 2009) where I focused on six students' mental math development over the course of 24 number talk lessons (see Figure 3) as a way to understand the change in thinking over time.

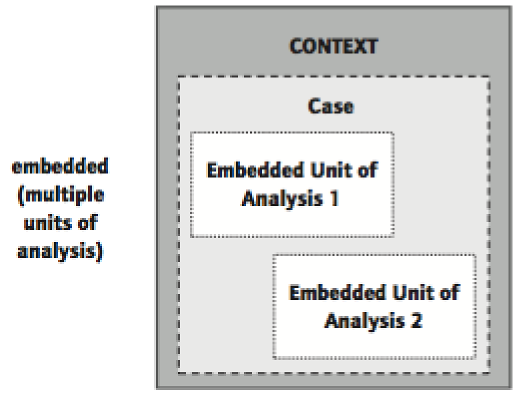


Figure 3. Embedded case study design[3].

The classroom teacher and I identified one high-level student, three average-achieving students, and two low-achieving students to be the focus of this study. The intention was to look at six students' conceptual understanding and computational fluency and to determine if and how this changed over the course of the study. These six students in three clusters (low-, average-, and high-achieving) constituted the three embedded analyses.

I used action research methods to conduct the case study given my concomitant focus on improving the quality of my teaching practice to enhance student achievement in conceptual understanding and computational fluency. "The goal of action research is to address a specific *problem* in a practice-based setting, such as a classroom, a workplace, a program, or an organization (Merriam & Tisdell, 2016, p. 4, emphasis in original). Also characteristic of action research, the design of the specific number talk lessons intervention was emergent. The design

[3] *Case Study Research: Design & Methods* (Yin, 2009, p. 46).

unfolded throughout the study based on the cycle of planning, acting, observing, and reflecting (Merriam & Tisdell, 2016). The first of 24 number talks was based on data from the in-depth preassessment interviews. Subsequent number talks were planned daily based on observations and reflections from evidence of students' mathematical thinking during number talks and midassessments. This cycle of planning, acting, observing, and reflecting (Merriam & Tisdell, 2016) was imperative to meet student needs throughout this process.

Research Sample

Since the purpose of the study was to "discover, understand, and gain insight (Merriam & Tisdell, 2016, p. 96) into the impact of number talks, the research sample was purposely chosen. The research took place at my home base school, Coastal Academy (pseudonym), within Lakeridge School District (pseudonym), comprised of two counties in Southwestern Nova Scotia. With a decline in population, both counties rely on tourism, along with fishing, shipping, aerospace, and software industries. This class of second graders, many of whom had low socioeconomic status, had a wide range of learning needs along with behavioural concerns and some mental health issues. Several students received support in mathematics outside of the classroom.[4] A full-time teacher assistant was assigned to a student diagnosed with Autism Spectrum Disorder and taking medication for Attention Deficit Hyperactivity Disorder (ADHD). A second student was on medication for ADHD but also had some social/behavioural concerns. Three additional students had severe behavioural/social issues. I chose this particular class based on being a Grade 2 reform-based classroom reflecting the purpose of my study. Since the Grade 2 teacher was a former colleague of mine for many years, and since I worked with these students previously in Grade 1 as a mathematics coach, I was both an insider and outsider to the

[4] No students received math support for basic number facts or strategies for the duration of this study.

community (Merriam & Tisdell, 2016). As a teacher involved in the delivery of daily number talks, I had an "insider" teacher's perspective and an "outsider" or researcher's perspective on teaching and student learning. As stated previously, the classroom teacher and I identified one high-level student, three average-achieving students, and two low-achieving students to be the focus of the case studies and therefore the data collection efforts.

Procedure

Ethics approval from Lakehead University and the principal of the school where the study took place was required (see Table 1). Since student data was used, introductory letters, along with consent forms, were necessary; the letters and consent forms were provided to the school principal (see Appendices B & C) and parents (see Appendices D & E). Furthermore, the classroom teacher also received an introductory letter and consent form (see Appendices F & G). Since the participants were young, second-grade children, verbal consent to participate was the only requirement (see Appendix H). I collected signed consent forms which are stored in a locked filing cabinet at the school where data collection took place. Pseudonyms were used for the name of the school, school board, and all students in order to maintain anonymity and confidentiality. Also, students were identified by a number only on all assessments and questionnaires. My data will be securely stored for a minimum of five years at Lakehead University as per ethics requirements policy.

Table 1

Procedure Timeline

Action Steps	Timeline
Ethics approval ▪ Lakehead University ▪ School Board ▪ Principal	March 2017
Introductory letters & consent forms ▪ Principal ▪ Parents ▪ Teacher	March 2017
Verbal consent ▪ Students	March 2017
Preassessment interviews	April 2017
Daily number talks	April – May 2017
Postnumber talk questionnaires	May 2017
Postassessment interviews	May 2017

I conducted a preassessment interview (see Appendix I) prior to facilitating daily number talks four times per week over a period of six weeks (thus 24 number talks). I used a general sequencing of daily number talks based on recommendations from the literature (see Table 2) with the understanding that this would be modified over the course of the study (based on preassessment results, daily observations, and students' work samples) (see Appendix J for detailed Daily Number Talk Planning Sheet).

Table 2

Number Talk Lessons

Visuals	Targeted facts
Use models for thinking	Small doubles & one-apart (near-doubles)
• Dot cards	Five-anchor
• Ten-frames	Sums of 10
• Rekenrek[5]	Subtracting from 10
• Open number line	Adding 10
	Subtracting 10 from a teen number
Use number sentences	Subtracting the ones from a teen number
	Larger doubles
	Half facts
	One-apart (near-doubles)
	Two-apart (doubles + 2)
	Make-10
	Back-through-10
	Up-through-10

At the conclusion of six weeks, a postassessment interview (see Appendix I) was given to evaluate student development of key ideas and strategies. In addition, students completed a postnumber talk questionnaire (see Appendix K) in order to gain understanding of their thoughts and opinions regarding number talks along with assessing student ability to name/explain strategies. All students completed questionnaires at the same time in their classroom.

Data Collection

Multiple sources were used for in-depth data collection (Merriam & Tisdell, 2016) including student interviews, observations, video recordings of number talks, and documents including pre-, mid, and postassessments completed by all students, along with a reflective journal which I completed (see Table 3). During number talks, I recorded students' strategies on chart paper acknowledging each student's strategy by his or her name. However, at the end of

[5] A Rekenrek is "a calculating frame consisting of two rows of ten beads with two sets of five beads in each row" (Fosnot & Uittenbogaard, 2007a, p. 5).

each number talk, I reviewed the videotape and re-wrote all students' strategies neatly and accurately on new chart paper. Photos of the re-written chart paper were taken at the end of each day to document the strategies used by students. Each student was assigned a number that was used as a means to identify his or her work; the students did not know their numbers. Student number was the only identification of students on all of the chart paper photos. Pre-, mid-, and postassessments and the postquestionnaires were labeled with each student's assigned number after it had been photocopied. Photocopies of midassessments were made so original work could be handed back to students.

Table 3

Data Collection

	Timeline: Number talks		
Data Source	Pre-	During	Post-
Preassessment interview	✓		
Observations in reflective journal		✓	
Video recordings of daily number talks		✓	
Samples of student work		✓	
Photos of chart paper (documenting strategies used by students during each number talk)		✓	
Postnumber talk questionnaire			✓
Postassessment interview			✓

All running records were coded and Lawson's (2015) "Student Continuum of Numeracy Development: Addition and Subtraction" (p. 4) (see Appendix A) were used to identify each student's phase of development. It was expected that students would employ a wider variety of strategy use with more accuracy, efficiency, and flexibility on the postassessment.

Data was also gathered from video recordings of number talks and strategy use by students, in addition to my own reflective journal entries. There was a two-fold purpose of video recordings: firstly, to answer the research questions, and secondly, for use in future professional development for educating teachers. My reflective journal entries (completed after viewing video recordings of number talks on a daily basis) encompassed my personal observations from each number talk and student strategy use, and so forth. Daily reflections also enabled me to determine next steps for each number talk (plan, act, observe, reflect) (Merriam & Tisdell, 2016, p. 235).

Data Analysis

Since "data analysis is the process of making sense out of the data" (Merriam & Tisdell, 2016, p. 202), I simultaneously collected and analyzed the data (see Table 4 for a summary of data sources and analyses). Nonetheless, the analysis became more intense as the project continued and once data collection was complete. All student interviews and reflective journal entries were placed into chronological order, thereby creating a beginning, middle, and ending, similar to a traditional storytelling model. This enabled me to paint a picture of student development over time.

Table 4

Summary of Data Sources and Analyses

Data source	Type of analysis
Pre- and postassessment results	Coded correct/incorrect, fact type, strategy use, and phase; generated numbers for correctness, strategy use, and phase
Video recordings	Summarized to analyze explanations of strategy use and coded correct/incorrect, fact type, strategy use, and phase; generated numbers for correctness, strategy use, and phase
Photocopied student work (midassessments)	Collected examples of independent work solving problems and coded correct/incorrect, fact type, strategy use, and phase; generated numbers for correctness, strategy use, and phase
Postquestionnaires	Coded opinions of students and their feelings regarding number talks
Reflective journal	Daily observations written and reviewed for impact of number talks

Once the data was ordered, Stage 1 of the analysis process began. Rather than starting with a theory (Creswell, 2014), I reviewed my data from a general perspective (mainly involving inductive analysis) and I began to develop an overall sense of the number talk experience.

During Stage 2, after completing my reflective journal entries in Microsoft Word and then scanning pre-, mid-, and postassessments along with the postnumber talk questionnaires in Adobe, I transferred my data to ATLAS.ti (computer-assisted qualitative data analysis software) and began coding my data using a priori codes identified in Table 5. I had developed a priori codes from a combination of Lawson's (2015) "Student Continuum of Numeracy Development: Addition and Subtraction" (p. 4) and Nova Scotia's *Mathematics 2 Curriculum Guide Implementation Draft* (Department of Education and Early Childhood Development, 2013). The goal of data analysis was to find answers to my research questions. Following the initial coding, I then reexamined my codes and modified them several times to reflect more specific units of meaning (see Table 6; for a detailed list of emergent codes see Appendix L). I rearranged my codes into a network creating categories/themes, from which the answers to my research questions emerged. This visual model helped me to "*link*, the conceptual elements—the categories—together in some meaningful way" (Merriam & Tisdell, 2016, p. 216, emphasis in original).

Table 5

A Priori Codes

Categories	A priori codes
Accuracy	Correct; incorrect; automatic retrieval; wrong operation; self-corrected; attempted self-correction; prolonged thinking time; guessed
Addition Facts	Plus-zero facts; plus-one facts; plus-two facts; five-anchor facts; sum of 10 facts; make-10 facts; doubles facts; one-apart (near-doubles) facts; two-apart (doubles + 2) facts; plus-three facts; adding 10 to a number facts; compensation
Addition Strategies	Counting three times (counting all); finger counting on (direct modelling); counting on in head & tracking; counting on from the larger number; doubles or near doubles; make-10; adding 10; using known fact (adjusting); using known fact (compensation)
Subtraction Facts	Subtracting 0; subtracting 1; subtracting a number from itself; subtracting within 5; subtracting within 10; subtracting difference of 1; subtracting half facts; subtracting 10 from a number; subtracting ones digit from a teen number; back-through-10; up-through-10
Subtraction Strategies	Counting three times (counting all—direct modelling); finger counting back (direct modelling); counting back in head & tracking; finger counting up (direct modelling); counting up in head & tracking; used related fact (think-addition strategy); back-through-10; up-through-10
Phases (Efficiency)	Direct modelling & counting; counting more efficiently & tracking; working with the numbers; proficiency
Key Ideas	One-to-one correspondence; cardinality; part-whole relationship; hierarchical inclusion; commutative and associative properties; equivalence; unitizing; place value
Opinions	Better at mental math (mm); the same at mm; worse at mm; learned from classmates; did not learn from classmates; liked best; liked least
Impact	Accuracy; efficiency; flexibility, appropriate strategy use

Table 6

Sample of Emergent Codes

Category	Code	Definition
Answer (Accuracy)	Ans correct	answer is correct
	Ans correct_auto	answer is correct/automatic retrieval
	Ans correct_auto_sc	answer is correct/automatic retrieval/self-corrected
	Ans correct_sc	answer is correct/self-corrected
	Ans correct_asc_sc	answer is correct/attempted self-correction/self-corrected
	Ans correct_wo_sc	answer is correct/wrong operation/self-corrected
	Ans correct_wo_sc_auto	answer is correct/wrong operation/self-corrected/auto
	Ans incorrect	answer is incorrect
	Ans incorrect_auto	answer is incorrect/automatic retrieval
	Ans incorrect_asc	answer is incorrect/attempted self-correction
	Ans incorrect_wo	answer is incorrect/wrong operation
	Ans incorrect_dk	answer is incorrect/student did not know/could not solve

Stage 2 also involved transforming the raw data into tables and graphs—forms which aid in understanding and interpreting data. The tabulation of numbers through rearranging, ordering and manipulating the data enabled me to generate descriptive statistics such as correctness, strategy use, and phase development for both the second-grade class as a whole and the six case studies.

Stage 3 involved interpretation of all my findings and drawing conclusions from my results to answer the research questions. Stages 2 and 3 were comprised of both inductive and deductive analysis procedures because as Creswell (2014) suggests, a researcher should "build their patterns, categories, and themes from the bottom up by organizing the data into increasingly more abstract units of information….Then deductively, the [researcher will] look back at their data from the themes to determine if more evidence can support each theme" (p. 186). The descriptive statistics, represented in tables and graphs adjunct to the discussions, verify my conclusions. At Stage 3, I once again reviewed the existing literature in an attempt to find additional evidence to either substantiate or contradict my ideas.

In addition to coding daily number talks, student assessments, and questionnaires, these data sources were also analyzed. Since action research components are embedded within this case study, data analysis not only focused on what happened but also "*how* it happen[ed] over the course of the ongoing action research cycle of plan, act, observe, reflect" (Merriam & Tisdell, 2016, p. 235, emphasis in original). Thus, not only was data analysis involved in coding data and organizing it into themes based on what happened at the beginning and end of the six-week study, focus was also given to how this unfolded over the course of the study. As previously mentioned, "plan, act, observe, reflect," in fact, happened on a daily basis during the facilitation of number talks.

Validity, Reliability, and Ethics

Merriam and Tisdell (2016) state, "All research is concerned with producing valid and reliable knowledge in an ethical manner" (p. 237). While validity "hinges on the meaning of reality" (Merriam & Tisdell, 2016, p. 242), rather than being assessed on reality itself (which is holistic, multidimensional, and ever-changing), validity is concerned with whether or not the findings are *credible* given the data presented. In qualitative research, many researchers use the terms "*trustworthiness*, *authenticity*, and *credibility*" (Creswell, 2014, p. 201, emphasis in original) to address validity.

Reliability in qualitative research indicates consistency in the researcher's approach with different researchers and different projects, in essence, "the extent to which research findings can be replicated" (Merriam & Tisdell, 2016, p. 250). Reliability can become problematic in qualitative studies because human behaviour is never static, and one person's experiences are not necessarily more reliable than what someone else experiences. Therefore, Merriam and Tisdell

(2016) argue, "The more important question for qualitative research is *whether the results are consistent with the data collected*" (p. 251, emphasis in original). In short, given the data, the results should make sense. A study is dependable if the results are consistent with the data collected. In order to ensure consistency with the data all coding was documented and thereby scrutable. My supervisor co-coded five percent of the data for inter-rater reliability and increased consistency. Coding tables were generated linking codes to data in a way that enables other researchers to comprehend the analysis.

Generalizability is also an important consideration, which means that "what we learn in a particular situation we can transfer or generalize to similar situations subsequently encountered" (Merriam & Tisdell, 2016, p. 255). In short, as Merriam and Tisdell (2016) suggest, a "rich, thick description" (p. 257) enables transferability and credibility, therefore a thorough description of the setting and participants of the study are given along with a detailed account of the number talk lessons. Finally, an in-depth description of the findings with adequate evidence from data (including assessments, video recordings of number talks, and questionnaires) are provided.

Finally, to ensure confidentiality, I securely stored original data while completing the assignment. Data were then sent to my thesis supervisor at Lakehead University who will store the data on an external hard drive for a minimum of five years. Furthermore, I ensured anonymity through the use of pseudonyms for all names and places referenced in this research report.

Limitations of the Study

Some limitations of this study should be considered concerning the design of the study. While it is an "in-depth analysis… [this case study is] bounded by time and activity" (Creswell, 2014, p. 14), and it is clear that one Grade 2 class will not be representative of all second graders, enough descriptive details of the study's context are provided for readers to compare if the study 'fits' with their situation (Merriam & Tisdell, 2016). Furthermore, this study was not designed to make a comparison between two different methods of instruction for basic number facts in order to determine which method is superior. The purpose was to focus on the impact of number talks on one class.

Chapter 4: Results

Overview

The research project began with the preassessment interviews before the first day of number talks. Number talks are approximately fifteen to twenty-five-minute classroom conversations around a purposely designed sequence of computation problems that students solve mentally. I assessed the impact of daily number talks on the development of the mental math abilities of second graders through the analysis of their responses. The responses to all pre-, mid-, and postassessments were analyzed and coded. While all 22 students in this class participated in our daily number talks and completed all assessments, I excluded three students from the data set due to lack of parental consent. Nineteen students were assessed on seventy-seven preassessment items, two midassessment items after week two, four midassessment items after week four, and finally, the original seventy-seven items as a postassessment.

I administered the preassessment interviews one-on-one in a small room adjacent to the classroom. The purpose of each interview was to assess student accuracy, efficiency, and flexibility in both basic addition and subtraction facts. Each preassessment interview consisted of 42 addition questions and 35 subtraction questions. After the preassessment, the students participated in 24 video-recorded daily number talks. I administered two midassessments: one at the end of the second week of number talks and one at the end of the fourth week of number talks. The first midassessment consisted of two addition questions where I asked students to solve each problem with two different strategies (if possible). The second midassessment consisted of two addition and two subtraction questions. Following the 24 number talks, all 22 students completed the postquestionnaire and the postassessment interview.

The data from the pre-, mid-, and postassessment interviews were reviewed and analyzed to determine how the whole group performed. A total of six (two low-achieving, three average-achieving, and one high-achieving) students were selected, and the analysis of their data from the pre-, mid-, postassessments, along with data from the 24 video recordings were reported as individual case studies to determine how each student progressed during the research project.

Results of the Preassessment and Analysis

In the preassessment, students displayed greater accuracy on addition (96%) as opposed to subtraction (81%). These results are consistent with research which suggests students have more difficulty mastering subtraction facts than mastering addition facts (Steinberg, 1985; Thornton & Toohey, 1985). While the overall accuracy was 89% on the preassessment; accuracy is merely one aspect of computational fluency (NCTM, 2000; Newton, 2016; Parrish, 2014; Russell, 2000; Scharton, 2004). Since computational fluency is multi-dimensional (Roicki, 2017), students were challenged to become not only more accurate but also efficient and flexible when computing (Russell 2000). I was hoping that through our 24 number talks, number relationships would become the foundation for strategies used by students to help them remember basic facts as math educators suggest (Van de Walle & Lovin, 2006). What types of strategies did children use to solve the calculations? Were they increasingly efficient and flexible?

To analyze the calculation strategies according to efficiency or phase of development, I grouped the student solution strategies by phases similar to those used by researcher Alex Lawson (2015) in her "Student Continuum of Numeracy Development: Addition and Subtraction" (p. 4). In Phase 0 students either cannot explain their thinking, or they skip the question due to difficulty, or they answer incorrectly. Phase 1 includes direct modelling and counting; Phase 2 is counting more efficiently and tracking; Phase 3 involves working with the numbers, and Phase 4 is proficiency.

On the preassessment, students' use of counting strategies (Phases 1 & 2) was similar for addition (20%) and subtraction (23%). However, the percentage of students in Phase 0 was greater for subtraction at 20% versus addition at 4%. Additionally, students were using more efficient strategies (Phases 3 & 4) for 76% of addition questions and 58% of subtraction questions (see Table 7).

Table 7

Responses as a Percentage on Preassessment Grouped by Phase (N = 19)

	Phase 0	Phase 1	Phase 2	Phase 3	Phase 4
Addition (n = 798)	4	4	16	26	50
Subtraction (n = 665)	20	8	15	11	47

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Examining the pattern of accuracy of students in each phase, the second graders had errors when using counting strategies in both addition and subtraction. Errors in subtraction were marginally higher in Phases 1 and 2 (2% and 7 % respectively compared to 1% and 2% in Phases 1 and 2 for addition). Students were able to complete all addition questions but one. In contrast, students either skipped, couldn't explain their thinking, or automatically recalled incorrect answers with 5% of their subtraction facts.

Analysis by fact type. Examining the *fact type* allows us to determine whether difficulty of fact type influences students' proficiency (accuracy and efficiency). Mathematics educators Isaacs and Carroll (1999) contend that,

> …as children increase their proficiency at various strategies, they begin to remember the simplest facts. Knowing the simpler facts makes possible more efficient strategies for

> harder facts. Gradually, students master more and more efficient strategies and commit more and more facts to memory. At the end of the process, students can accurately and automatically produce all the basic number combinations. (p 509)

I drew on Lawson, Wark, Girardin and Cooper's (2017) unpublished fact assessment instrument to group the many facts I assessed under common structure headings (see Table 8). These groupings are based on common structure of the questions, not solution strategies; although in two instances, Make-10 and Subtracting Back/Up-Through-10, the groupings could be perceived as a suggested strategy.

Table 8

Facts Included on the Assessments Grouped by Fact Type

n + 1, n + 2	Sm D	Sm ND	A5a < 10	A5a > 10	n + 0
9 + 1	3 + 3	3 + 4	5 + 2	5 + 6	0 + 9
	4 + 4	4 + 5	5 + 3	5 + 7	5 + 0
	5 + 5[3]		5 + 4	5 + 8[2*]	8 + 0
				5 + 9	

Sum of 10	n + 10	Lg D	Lg ND	Make-10		Plus 3
2 + 8	10 + 2	6 + 6	6 + 7	3 + 9	7 + 5	3 + 6
3 + 7	10 + 4	7 + 7[2]	7 + 8	4 + 9	7 + 9	
4 + 6	10 + 5	8 + 8	8 + 9	6 + 8	8 + 6[**]	
	10 + 8	9 + 9	9 + 8[**]	6 + 9[**]	9 + 5	
				7 + 4	9 + 6	

S1	SD1	S5a ≤ 10	S5a > 10	n – 0	n – n
7 – 1	5 – 4	7 – 5	14 – 5	2 – 0	9 – 9
	9 – 8	8 – 3			
	12 – 11	10 – 5			

Sf10		S1T	S10T	SHF	SB/U10
10 – 2	10 – 7	12 – 2	14 – 10	12 – 6	14 – 8
10 – 4	10 – 8	13 – 3	15 – 10	14 – 7[2]	15 – 6[*]
10 – 6	10 – 9	14 – 4	17 – 10[2]	16 – 8	15 – 9
		17 – 7	19 – 10	18 – 9	16 – 7
					16 – 9[*]

Note. n + 1 = number plus one; n + 2 = number plus 2; Sm D = small double; Sm ND = small near double; A5a < 10 = adding 5 anchor less than 10; A5a > 10 = adding 5 anchor greater than ten; n + 0 = adding zero to a number; Sum of 10 = facts which equal 10; n + 10 = adding 10 to a number; Lg D = large double; Lg ND = large near double; Make-10 = make a 10 and add some more; Plus 3 = adding 3 to a number; S1 = subtracting one; SD1 = subtracting with a difference of 1; S5a ≤ 10 = subtracting 5 anchor less than or equal to 10; S5a > 10 = subtracting 5 anchor greater than 10; n – 0 = subtracting zero from a number; n – n = subtracting a number from itself; Sf10 = subtracting from 10; S1T = subtracting ones from a teen number; S10T = subtracting ten from a teen number; SHF = subtracting half facts; SB/U10 = subtracting back-through-ten or up-through-ten.

[2] Question *appeared twice* on both the pre- and postassessments.

[*] Question was *also* on midassessment.

[**] Question was *only* on midassessment.

Addition. The effect of fact types was evident as students had 100% accuracy on Plus 0, Plus 1, Small Doubles, Plus 10, and Plus 3 facts. All remaining addition fact types had an accuracy rate of 90% or higher (see Appendix M1). The percentage of students using more efficient strategies (Phases 3 & 4), was 100% for Plus 0, Plus 1, and Small Doubles, 97% for Large Doubles, and 92% for Plus 10 facts. On the other hand, students were using less efficient counting strategies for 68% of Plus 3 facts, 42% for Sum of 10 facts, 36% for 5 Anchor ≤ 10 facts, and 26% for Small Near Double, 5 Anchor > 10 and Make-10 facts (see Table 9).

Table 9

Correct Responses as a Percentage Grouped by Phases for each Fact Type on Addition Preassessment (N = 19)

	Phases	
Fact Type	1 & 2	3 & 4
Plus 0 (n = 57)	0	100
Plus 1 (n = 19)	0	100
Sm Doubles (n = 76)	0	100
Lg Doubles (n = 95)	2	97
Plus 10 (n = 76)	8	92
Lg Near Doubles (n = 57)	19	72
Sm Near Doubles (n = 38)	26	71
5 Anchor < 10 (n = 57)	26	67
Make-10 (n = 152)	26	64
5 Anchor > 10 (n = 95)	36	57
Sum of 10 (n = 57)	42	54
Plus 3 (n = 19)	68	32

Note. Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Examining fact types more finely, according to the four phases of development, all 19 students were proficient with all Plus 0, Plus 1, Small Doubles, and most Large Doubles facts. Only two students were proficient with the facts 5 + 4 and 5 + 7, one student for 5 + 8, and a different student for 4 + 9 and 9 + 6. No students were proficient at Large Near Doubles and out of 152 Make-10 questions, there were only two questions on which proficiency was demonstrated (see Appendix M2 for a detailed summary of specific facts and Appendix M3 for a detailed summary of correct/incorrect responses grouped by fact type and phase development). It was surprising there wasn't a higher percentage of proficiency in addition facts given that the curriculum outcome for Nova Scotia students in Grade 2 reads as follows: "Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts" (Department of Education and Early Childhood Development, 2013, p. 3). The preassessment was completed during the end of March/the beginning of April as students approached the end of their grade 2 year. The results of the subtraction preassessment interviews were even more out of step.

Subtraction. Students had 100% accuracy on Subtracting 1 facts only. There was a 95% accuracy rate on Subtracting 0 facts and Subtracting Ones from a Teen Number and an 89% accuracy rate on Subtracting a Number from Itself and a Difference of 1 facts. The more difficult fact types, with the greatest percentage of errors, were Subtracting Back-Through or Up-Through-Ten (38%), 5 Anchor > 10 (37%), and Subtracting Half facts (25%) (see Appendix N1). The percentage of students using more efficient strategies (Phases 3 & 4) was 100% for Subtracting 1, 95% for Subtracting 0, and 89% for Subtracting a Number from Itself facts. On the other hand, students were using less efficient counting strategies for 41% of Back-Through-Ten/Up-Through-Ten, 32% of 5 Anchor ≤ 10, and 27% of Subtracting from 10 facts (see Table 10).

Table 10

Correct Responses as a Percentage Grouped by Phases for each Fact Type on Subtraction Preassessment (N = 19)

	Phases	
Fact Type	1 & 2	3 & 4
Subtracting 1 (n = 19)	0	100
Subtracting 0 (n = 19)	0	95
Subtracting N from Itself (n = 19)	0	89
Subtracting Ones from a Teen # (n = 76)	20	75
Difference of 1 (n = 57)	16	74
Subtracting Half Facts (n = 95)	15	60
Subtracting 10 from a Teen Number (n = 95)	21	57
Subtracting from 10 (n = 114)	27	54
5 Anchor ≤ 10(n = 57)	32	54
5 Anchor > 10 (n = 19)	26	32
Subtracting B/U 10 (n = 95)	41	22

Note. Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Examining subtraction fact types more finely, there was only one question (7 – 1) on which all 19 students showed proficiency. Eighteen students were at Phase 4 for 2 – 0 and 17 students at Phase 4 for 9 – 9. Two students were proficient with the facts 10 – 7 and 16 – 7. Only one student was in Phase 4 for 14 – 5 and 15 – 6. There were no students in Phase 4 for 14 – 8, 15 – 9, or 16 – 9. (see Appendix N2 for a detailed summary of specific facts and Appendix N3 for a detailed summary of correct/incorrect responses grouped by fact type and phase development).

Summary. Students were more accurate on their preassessment when solving addition questions (96%) than subtraction questions (81%). While the percentage of students at Phase 4 was similar for both addition and subtraction (50% and 47% respectively), there was a greater percentage of students at Phase 3 for addition (26%) than subtraction (11%) (see Table 7). While all students were able to solve the addition problems, there were 32 subtraction problems (5%) on which students were incorrect and at Phase 0 (meaning they skipped the question, were unable to explain their thinking, or incorrectly recalled the answer). Results show students were not yet skillful in their accuracy, efficiency, and flexibility for the harder addition and subtraction fact types.

Results of the Midassessment and Analysis

Two midassessments were administered in which students were asked to compute six questions in total and record their mathematical thinking. The results can be found in Appendix O.

Summary. The midassessments were a different type of assessment considering students were asked to share their thinking strategies in writing. Researchers Kling and Bay-Williams (2014) argue that “writing provide[s] an excellent opportunity to assess flexibility and understanding of strategy selection and application” (p. 493). Similar to the preassessments, more errors were made on subtraction than addition questions, and there were more students in Phase 3 for addition questions than subtraction questions. While there were more errors made in Phase 3 on the second midassessment as opposed to no errors made in Phase 3 on the first midassessment, students were willing to take risks and try different strategies on the second midassessment as evidenced by only one question being skipped by one student due to difficulty on the second midassessment. Also of significance was the fact that only one student used cubes and counted all on the midassessments (15 – 6 and 16 – 9). Overall, Phase 2 (where students counted on/counted back) accounted for only 18% of strategy use. Greater detail on the mid-assessments will be given in the case studies.

Postassessments Results

Between the pre- and postassessments, overall accuracy increased slightly on addition (from 96% to 99% respectively) while accuracy on subtraction rose 17% (81% on the preassessment to 98% on the postassessment) (For a detailed analysis of the postassessments see Appendices P1 to P4).

Addition. There was little change in the easier fact types for addition (Plus 0, Plus 1, Small Doubles, and Large Doubles) as students were mostly proficient before number talks. More significant changes were evident when comparing the remaining fact types. By the postassessment, students in Phase 0 dropped to 2% for Make-10 facts and 1% for 5 Anchor > 10 facts, all remaining fact types were at 0% for Phase 0 by the postassessment. Inefficient counting strategies were replaced with more efficient strategies of working with numbers as indicated by the decrease in percentage of students in Phase 1 & 2 for each fact type to less than 20% except for Plus 3 facts where 37% of students were still using the counting on strategy (Phase 2). Furthermore, student proficiency increased on all of the harder fact types as well (see Table 11 and Figure 4; see Appendices Q1 & Q2 for a more detailed analysis).

Table 11

Responses as a Percentage Grouped by Phases for each Fact Type on Addition Pre- and Postassessments (N = 19)

	Phases					
	0		1 & 2		3 & 4	
Fact Type	Pre	Post	Pre	Post	Pre	Post
Plus 0 (n = 57)	0	0	0	0	100	100
Plus 1 (n = 19)	0	0	0	0	100	100
Sm Doubles (n = 76)	0	0	0	0	100	100
Lg Doubles (n = 95)	1	0	2	0	97	100
Plus 10 (n = 76)	0	0	8	3	92	97
5 Anchor < 10 (n = 57)	7	0	26	7	67	93
Lg Near Doubles (n = 57)	9	0	19	11	72	89
Sm Near Doubles (n = 38)	3	0	26	11	71	89
Sum of 10 (n = 57)	4	0	42	14	54	86
Make-10 (n = 152)	10	2	26	16	64	82
5 Anchor > 10 (n = 95)	7	1	36	19	57	80
Plus 3 (n = 19)	0	0	68	37	32	63

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

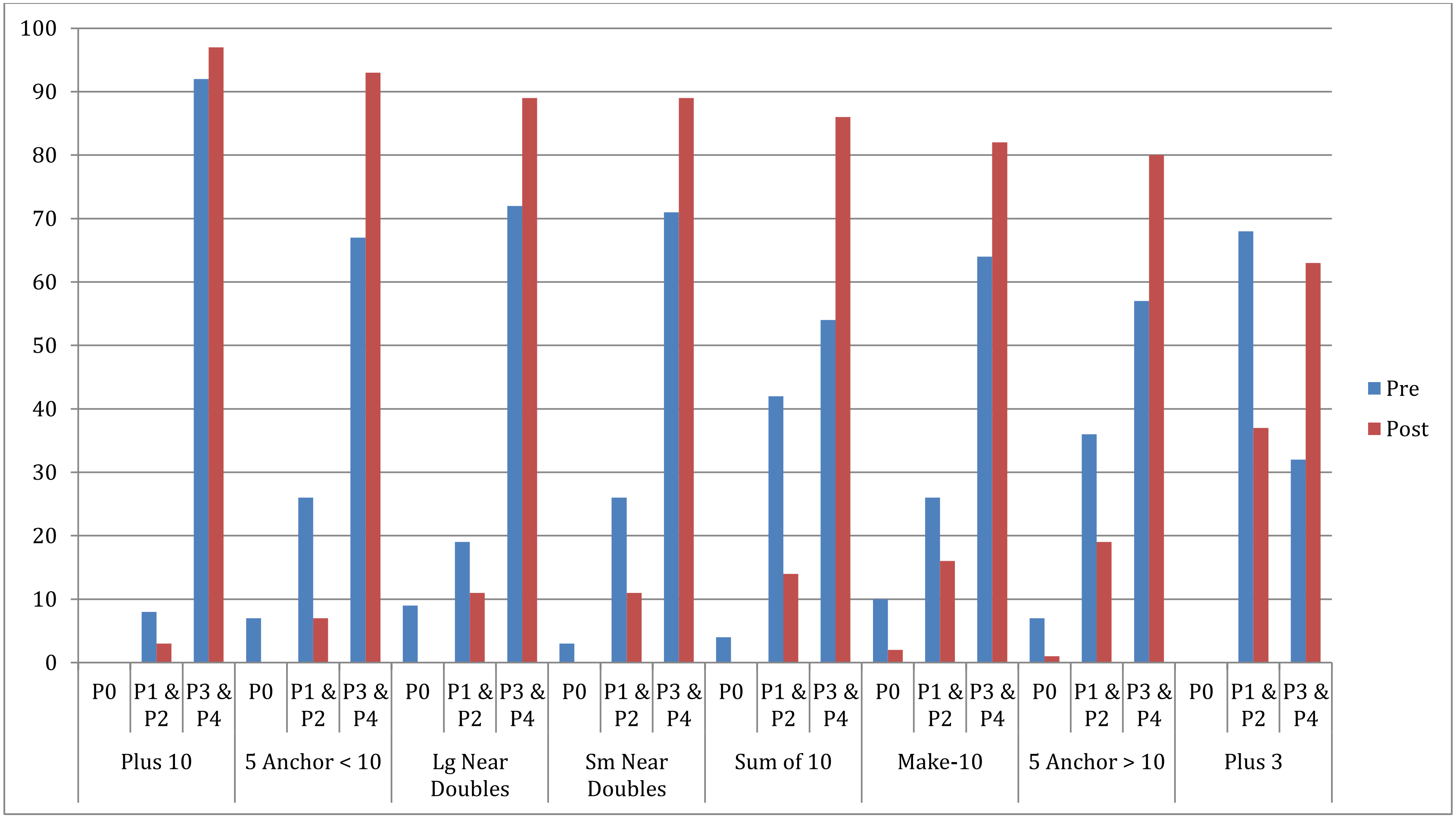


Figure 4. Responses as a percentage grouped by combined phases for each fact type on addition pre- and postassessments.

P0 = can't explain thinking/skipped due to difficulty/incorrect answer; P1 = direct modelling & counting; P2 = counting more efficiency & tracking; P3 = working with the numbers; P4 = proficiency.

Subtraction. Similarly, with subtraction, there were less significant changes in the easier fact types (Subtracting 1, Subtracting 0, and Subtracting a Number from Itself) with greater changes on the remaining more difficult fact types. Consistent, once again, with Steinberg (1985) and Thornton and Toohey's (1985) research, the findings suggests students have more difficulty mastering subtraction facts than mastering addition facts evidenced by the continued use of Counting All and Counting-Back/Counting-Up (Phase 2) strategies on the postassessment. Despite the continued use of inefficient counting strategies used by students, their instances were decreased and in many cases, replaced with more efficient strategies of working with the numbers. Furthermore, like addition, student proficiency increased on all of the harder fact types as well (see Table 12 and Figure 5; see Appendices Q3 and Q4 for a more detailed analysis). These postassessment results coincide with Christensen and Cooper's (1991) research results which suggested that student invented strategy practice facilitated the transition to automaticity. One is left to wonder, how did these changes transpire?

Table 12

Responses as a Percentage Grouped by Phases for each Fact Type on Subtraction Pre- and Postassessments (N = 19)

	Phases					
	0		1 & 2		3 & 4	
Fact Type	Pre	Post	Pre	Post	Pre	Post
Subtracting 1 (n = 19)	0	0	0	0	100	100
Subtracting 0 (n = 19)	5	0	0	0	95	100
Subtracting N from Itself (n = 19)	11	0	0	0	89	100
Difference of 1 (n = 57)	11	2	16	5	74	93
Subtracting Ones from a Teen # (n = 76)	5	0	20	9	75	91
Subtracting 10 from a Teen Number (n = 95)	22	2	21	12	57	86
Subtracting from 10 (n = 114)	19	3	27	14	54	83
5 Anchor > 10 (n = 19)	42	5	26	16	32	79
Subtracting Half Facts (n = 95)	25	4	15	19	60	77
5 Anchor ≤ 10(n = 57)	14	4	32	30	54	67
Subtracting B/U 10 (n = 95)	37	3	41	34	22	63

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

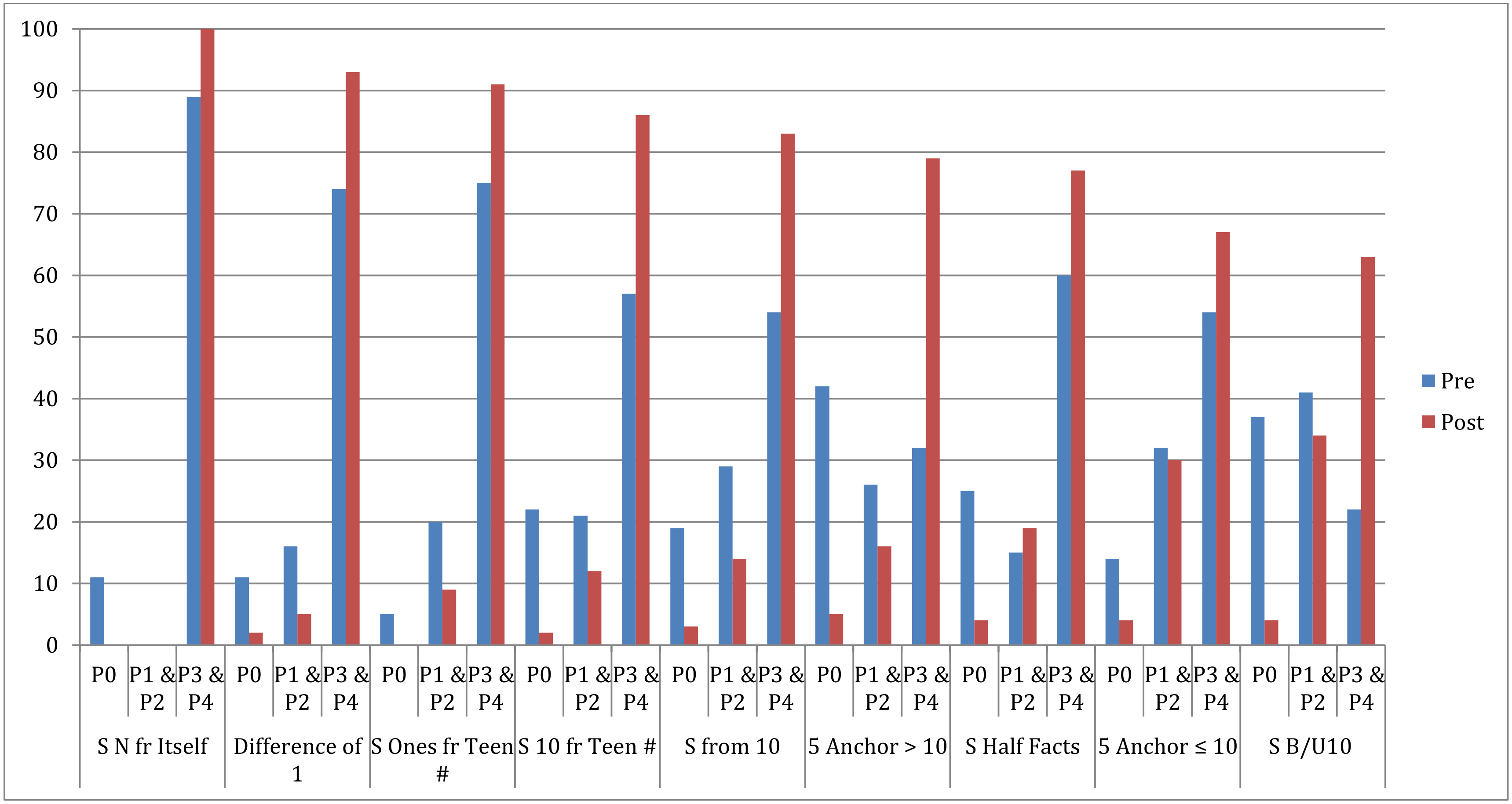


Figure 5. Responses as a percentage grouped by combined phases for each fact type on subtraction pre- and postassessments.

P0 = can't explain thinking/skipped due to difficulty/incorrect answer; P1 = direct modelling & counting; P2 = counting more efficiency & tracking; P3 = working with the numbers; P4 = proficiency.

Going Deeper: The Focus Students as Embedded Case Studies

Although the data collected through the pre-, mid-, and postassessment interviews provided an overall progression of student development, I now turn to each of the six focus students to explore in more depth the many ways in which their strategies and reasoning processes evolved throughout the research project.

The following individual case studies will discuss each student's progress through the research project. After the preassessment interviews, the six focus students and their classmates participated in 24 daily number talks where each lesson was designed to encourage strategy construction in an attempt to increase number sense and connections, mathematical thinking and reasoning, and different mental computational skills. The 24 lessons followed the sequence previously outlined in Table 2 (see Appendix J for detailed Daily Number Talk Planning Sheet). For each case study that follows, I will discuss the student's progression through number talks.

Case 1: Randy's progression through number talks.

Pre- and postassessments. Randy was a low-achieving student who made great progress during this research project. His accuracy on addition questions rose from 86% on the preassessment to 100% on the postassessment. In subtraction, he started further behind and there was an even more substantial increase—from 34% on the preassessment to 97% on the postassessment. Similarly, Randy demonstrated more efficient strategy use (Phases 3 and 4) for addition questions than for subtraction questions on both the pre- and postassessments; increasing from 60% to 91% for addition and from 23% to 46% for subtraction questions (see Table 13; see Appendices R1 to R4 for Randy's detailed Pre- and Postassessments).

Table 13

Responses as a Percentage on Randy's Pre- and Postassessments Grouped by Phase

	Phase 0		Phase 1		Phase 2		Phase 3		Phase 4	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Addition (n = 42)	14	0	10	0	17	10	19	31	41	60
Subtraction (n = 35)	71	3	0	45	6	6	0	3	23	43

Note. Phase 0 = can't explain thinking/skipped due to difficulty/ incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Addition. On the addition preassessment, Randy was 100% proficient on Plus 0, Plus 1, Small Doubles, Large Doubles, and Add 10 facts. By the postassessment, he also became proficient on Large Near Doubles, Add 3, and 5 Anchor > 10 facts. While Randy's responses were in Phases 0 to 2 on 40% of the facts for addition on the preassessment, this was significantly reduced to 9% on the postassessment. This 9% was accounted for by four out of the five 5 Anchor > 10 facts which Randy solved by counting on. By the postassessment, Randy was no longer demonstrating Phases 0 or 1 responses for any addition facts. For the remaining fact types (Small Near Doubles, Make-10, 5 Anchor < 10, and Sum of 10), by the postassessment, Randy was using more efficient strategies for 90% of the addition questions. Looking more closely at the addition fact types on the postassessment, Randy was in Phase 2 for 5 Anchor > 10 Facts. Randy solved one fact from this group (5 + 9) by using the Make-10 Strategy. It was evident he had this strategy under control since he used it accurately for 87% of the Make-10 fact questions with the remaining becoming proficient. Once again, Randy's gain in proficiency included the fact types Sum of 10, Subtracting from 10, and Add 10 to a Number which is pivotal in fact development. When students, like Randy, are then able to use these facts to Make-10 for example, these strategies have greater mathematical "legs" as Lawson (2015) argued.

Using 10 “can be used in many different situations, including situations that involve larger (decade) numbers” (p. 66) (see Table 14).

Table 14

Responses as a Percentage on Randy’s Addition Pre- and Postassessments Grouped by Phase and Fact Type

	Phase 0		Phase 1		Phase 2		Phase 3		Phase 4	
Fact Type	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Plus 0 (n = 3)	0	0	0	0	0	0	0	0	100	100
Plus 1 (n = 1)	0	0	0	0	0	0	0	0	100	100
Sm Doubles (n = 4)	0	0	0	0	0	0	0	0	100	100
Lg Doubles (n = 5)	0	0	0	0	0	0	0	0	100	100
Add 10 (n = 4)	0	0	0	0	0	0	0	0	100	100
Lg ND (n = 3)	33	0	0	0	67	0	0	0	0	100
Plus 3 (n = 1)	0	0	0	0	33	0	67	0	0	100
5 a > 10 (n = 5)	0	0	0	0	100	0	0	0	0	100
Sm ND (n = 2)	33	0	0	0	67	0	0	33	0	67
Make-10 (n = 8)	50	0	50	0	0	0	0	50	0	50
5 a < 10 (n = 3)	38	0	0	0	0	0	62	87	0	13
Sum of 10 (n = 3)	0	0	60	0	20	80	20	20	0	0

Note. Phase 0 = can’t explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Subtraction. On the preassessment, subtraction proved to be very difficult for Randy noted by a reduction in both accuracy and phase development between addition and subtraction. He was in Phase 0 for 71% of the subtraction questions on the preassessment (see Table 13). Randy skipped nine questions due to difficulty, inaccurately solved 12 questions and couldn’t explain his thinking, and accurately recalled two questions but couldn’t explain his thinking (see Table 15). Randy’s difficulty was also illustrated by his question posed on the presassessment, “Can we just not do the big ones?” (P166). [6]

[6] P means Primary Document within the ATLAS.ti program. The number indicates the specific document.

Table 15

Incorrect Responses on Randy's Subtraction Preassessment Grouped by Phase and Fact Type

	Incorrect: Phase			
	0	1	2	3
Diff of 1 (n = 3)	1	0	0	1
S 10 fr Teen (n = 5)	4	1	0	0
SHF (n = 5)	4	0	1	0
S fr 10 (n = 6)	3	0	0	0
5 a ≤ 10 (n = 3)	2	0	1	0
S B/U 10 (n = 5)	5	0	0	0

Note. Phase 0 = can't explain thinking/skipped due to difficulty/ incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

On the preassessment, Randy was 100% proficient at Subtracting 1, Subtracting 0, and Subtracting a Number from Itself (see Table 16). However, there was only one fact where he attempted to use a related fact (Phase 3 on 12 – 11), but his answer was incorrect. There was only one question which Randy attempted to use Counting All (Phase 1), but again, his answer was incorrect (see Table 15). I wrote on Randy's subtraction preassessment, "Really struggles to explain thinking. Found this very difficult" (P166). Randy's difficulty coincides with Lawson's (2015) finding that student's "ability to work with numbers in subtraction lags behind that of addition" (p. 21).

Table 16

Responses as a Percentage on Randy's Subtraction Pre- and Postassessments Grouped by Phase and Fact Type

	Phase 0		Phase 1		Phase 2		Phase 3		Phase 4	
Fact Type	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S1 (n = 1)	0	0	0	0	0	0	0	0	100	100
S0 (n = 1)	0	0	0	0	0	0	0	0	100	100
S N fr Itself (n = 1)	0	0	0	0	0	0	0	0	100	100
Diff of 1 (n = 3)	67	33	0	0	33	33	0	0	0	33
S 1s fr Teen (n = 4)	0	0	0	0	0	0	0	0	100	100
S 10 fr Teen (n = 5)	100	0	0	80	0	0	0	0	0	20
SHF (n = 5)	100	0	0	100	0	0	0	0	0	0
S fr 10 (n = 6)	67	0	0	0	17	0	0	0	17	100
5 a ≤ 10 (n = 3)	100	0	0	67	0	0	0	33	0	0
5 a > 10 (n = 1)	100	0	0	100	0	0	0	0	0	0
S B/U 10 (n = 5)	100	0	0	80	0	20	0	0	0	0

Note. Phase 0 = can't explain thinking/skipped due to difficulty/auto recall but incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

On the preassessment, when I asked Randy what subtraction meant, he answered, "You take away a number" (P167) and his strategy use was indicative of his understanding of subtraction as removal. This is an early and somewhat limited understanding of subtraction. When students understand subtraction only as *take away* it can be problematic (Fuson, 1986). When students are asked, for example, to solve a join-change-unknown problem (Carpenter et al., 1999) where nothing is being taken-away, if students do not understand subtraction also as *difference* they struggle. Whitacre, Schoen, Champagne, and Goddard (2016/2017) contend that, "Thinking about differences as distances between numbers (e.g., on a number line) can help students make important connections among the ideas of addition and subtraction" (p. 305). Students need to understand both models (removal and difference) for flexibility in solving subtraction problems.

By the postassessment, there was only one question (12 – 11) which Randy solved incorrectly. Not only was there an increase in accuracy, but there was also an increase in phase development during the study. Phase 0 decreased from 71% to 3 % while Phase 1 increased from 0% to 45%. While Randy was still using an inefficient strategy of counting all, for him, this was growth. Also notably, his proficiency increased from 23% on the preassessment to 43% on the postassessment (see Table 16). I wrote on Randy's postassessment, "More stamina! Tries to look at numbers and how they are related now" (P231). What accounted for his growth?

Midassessments and twenty-four number talks. Although Randy was able to accurately answer both questions on the first midassessment (8 + 6 and 9 + 8), he was only able to use one strategy (Near Doubles) to explain his thinking. Nevertheless, his strategy choice was both appropriate and efficient. By midassessment, Randy was able to accurately use a related fact to solve 5 + 8 recording "5 + 7 + 1 = 13" (P103). I believe Randy was trying to use relational thinking when solving the next question 6 + 9 when he recorded "5 + 8 + 1 = 14" (P103). However, he would have needed to add one more. Despite his incorrect answer, Randy was taking a risk and trying to use a more efficient strategy than counting all or counting on. It was evident Randy struggled with subtraction on the preassessment, nevertheless, on the midassessment he was able to solve 15 – 6 accurately but recorded his thinking as "15 – 7 – 1 = 9" (P103). On the final question, 16 – 9, Randy answered correctly, but his recorded strategy did not make sense: "15 – 6 – 5 = 7" (P103). Despite these errors, Randy did not skip the questions, nor did he record a counting strategy (see Table 17). While Randy was able to answer correctly, he repeatedly struggled to write his thinking. This is consistent with mathematics educator Marian Small's argument that written communication causes difficulty for some students due to the “elements of formality and symbolism that are not present in oral communication” (2013, p. 120).

Table 17

Correct/Incorrect Responses on Randy’s Midassessment Groups by Phase

	Correct		Incorrect
Question	Phase 0	Phase 3	Phase 3
8 + 6		d/nd	
8 + 6	can’t explain		
9 + 8		d/nd	
9 + 8	can’t explain		
5 + 8		urf	
6 + 9			Urf
15 – 6		urf*	
16 – 9		urf*	

Note. Phase 0 = can’t explain thinking/skipped due to difficulty/ incorrect answer; Phase 3 = working with the numbers; Phase 4 = proficiency; d/nd = doubles/near doubles; urf = using related fact.
* answer was correct but there was an error in recording the strategy.

Randy was a full participant during the number talks with 31 instances during our 24 number talks where Randy contributed to our classroom discussions. Although Randy correctly answered 14 questions during our number talks, there were six times when he self-corrected, one account of attempting to self-correct, and five incorrect answers. I recorded his thinking strategy on chart paper four times throughout our number talks (P31, P34, P37, P40). He used Phase 3 strategies for three out of these four instances and counted all only one time (interestingly on Day 13 when students were using individual Rekenreks). On the sixth day of our number talks, when Randy was trying to explain his thinking, he demonstrated perseverance in solving the question and initiated moving to the front of the room to show his thinking on the Rekenrek. He raised his arms in the air exclaiming, “Yes!” when correctly solving the problem with great pride and returned to his seat with a big smile on his face (P246). Despite the difficulties he displayed on the preassessment, he eagerly participated in our number talks. When asked on the Post

Number Talk Questionnaire what he liked best about number talks, he wrote, “When we figured out strategies” (P67). He also shared on his questionnaire that he felt he learned new strategies for solving addition and subtraction problems from his classmates because “number talks are special” (P67).

Case 2: Adam’s progression through number talks.

Pre- and postassessments. Adam was another low-achieving student who demonstrated significant growth during this research project. His accuracy on addition questions rose from 88% on the preassessment to 100% on the postassessment. In subtraction, he started further behind, and there was an even more substantial increase—from 51% correct on the preassessment to 100% on the postassessment. Like Randy, Adam demonstrated more efficient strategy use (Phases 3 and 4) for addition questions than for subtraction questions on both the pre- and postassessments; increasing from 69% to 88% for addition and from 26% to an astounding 97% for subtraction questions (see Table 18; see Appendices S1 to S4 for Adam’s detailed Pre- and Postassessments).

Table 18

Responses as a Percentage on Adam’s Pre- and Postassessments Grouped by Phase

	Phase 0		Phase 1		Phase 2		Phase 3		Phase 4	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Addition (n = 42)	12	0	2	0	17	12	17	21	52	67
Subtraction (n = 35)	48	0	20	0	6	3	0	8	26	89

Note. Phase 0 = can’t explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Addition. On the addition preassessment, Adam was 100% proficient on Plus 0, Plus 1, Small Doubles, Large Doubles, and Add 10 facts. By the postassessment, he also became proficient on Sum of 10, Small Near Doubles, and 5 Anchor < 10 facts. While Adam’s responses were in Phases 0 to 2 on 31% of the facts for addition on the preassessment, this was reduced to 12% (Phase 2) on the postassessment. This 12% accounted for five questions within Plus 3, 5 Anchor > 10 and Make-10 facts which Adam solved by counting on. Like Randy, by the postassessment, Adam was no longer demonstrating Phases 0 or 1 responses for any addition facts. For the remaining fact types (Large Near Doubles, 5 Anchor > 10, and Make-10), by the postassessment, Adam was using more efficient strategies for 75% for those fact types (56% Phase 3 and 19% Phase 4). Adam's gain in proficiency included Sum of 10 facts. He was then able to use this knowledge to more efficiently solve (Phases 3 and 4) 60% of 5 Anchor > 10 facts and 75% of Make-10 facts. Adam was on the cusp of mastery of Large Near Doubles facts with 67% in Phase 3. It is interesting to note that four out of the five errors Adam had on the addition preassessment happened when Adam was attempting to work flexibly with number (Phase 3). Although Adam was willing to take risks on the preassessment, he needed to deepen his understanding of how numbers are related in order to consistently work with numbers to compute with accuracy. The only other error was made when attempting to count all on 5 + 8 (see Table 19).

Table 19

Responses as a Percentage on Adam's Addition Pre- and Postassessments Grouped by Phase and Fact Type

	Phase 0		Phase 1		Phase 2		Phase 3		Phase 4	
Fact Type	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Plus 0 (n = 3)	0	0	0	0	0	0	0	0	100	100
Plus 1 (n = 1)	0	0	0	0	0	0	0	0	100	100
Sm Doubles (n = 4)	0	0	0	0	0	0	0	0	100	100
Lg Doubles (n = 5)	0	0	0	0	0	0	0	0	100	100
Add 10 (n = 4)	0	0	0	0	0	0	0	0	100	100
Sum of 10 (n = 3)	0	0	0	0	67	0	0	0	33	100
Sm ND (n = 2)	0	0	0	0	0	0	50	0	50	100
5 a < 10 (n = 3)	0	0	0	0	0	0	33	0	67	100
Lg ND (n = 3)	0	0	0	0	0	0	100	67	0	33
5 a > 10 (n = 5)	60	0	0	0	0	40	20	40	20	20
Make-10 (n = 8)	25	0	13	0	50	25	13	63	0	13
Plus 3 (n = 1)	0	0	0	0	100	100	0	0	0	0

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Subtraction. On the preassessment, subtraction also proved to be quite difficult for Adam noted by a reduction in both accuracy and phase development between addition and subtraction. He was in Phase 0 for 48% of the subtraction questions on the preassessment (see Table 18). Unlike Randy, Adam did not skip any questions due to difficulty. I wrote on Adam's preassessment, "Has trouble explaining strategies" (P7) along with "very careless counting with cubes. Seemed confused about subtraction" (P12). Adam tried to count all unsuccessfully on seven questions. When solving 13 – 3, he incorrectly counted up in his head. When solving 10 – 7, 10 – 9, 10 – 8, and 10 – 6, on each occasion he inaccurately tried to count back. This was interesting as it would have been more efficient to count back when solving 13 – 3 and count up when computing the subtract from 10 questions, yet Adam did the opposite. On four questions he mistakenly tried to use a related fact. For example, when asked: If your friend was having trouble solving 14 – 8, what would you tell your friend to do? Adam's reply was, "16 take away 9 is 7 so 8 is 1 less than 9 so 6. But, different start numbers. Oh yeah, it's 5 (P12) (see Table 20).

Table 20

Incorrect Responses on Adam's Subtraction Preassessment Grouped by Phase and Fact Type

	Incorrect: Phase			
	0	1	2	3
S N fr Itself (n = 1)	1	0	0	0
S 10 fr Teen (n = 5)	0	3	0	1
S B/U 10 (n = 5)	0	2	0	1
SHF (n = 5)	0	1	0	0
5 a > 10 (n = 1)	0	1	0	0
S fr 10 (n = 6)	0	0	4	1
S 1s fr Teen (n = 4)	0	0	1	0

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers

On the preassessment, Adam was 100% proficient only for Subtracting 1 and Subtracting 0. By the postassessment, this expanded to include Subtract a Number from Itself, Difference of 1, Subtracting Ones from a Teen, Subtracting Ten from a Teen, and Subtracting Half facts. On the preassessment, Adam was in Phases 0 to 2 for 74% of the subtraction questions. This was substantially reduced to 3% on the postassessment—on one occasion Adam counted back in his head correctly to solve 8 – 3. Adam worked with numbers (Phase 3) for 8% of the subtraction questions on the postassessment and for the remaining 89% of questions, Adam was now proficient (up from 26% on the preassessment) (see Table 21).

Table 21

Responses as a Percentage on Adam's Subtraction Pre- and Postassessments Grouped by Phase and Fact Type

	Phase 0		Phase 1		Phase 2		Phase 3		Phase 4	
Fact Type	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S1 (n = 1)	0	0	0	0	0	0	0	0	100	100
S 0 (n = 1)	0	0	0	0	0	0	0	0	100	100
S N fr Itself (n = 1)	100	0	0	0	0	0	0	0	0	100
Diff of 1 (n = 3)	33	0	67	0	0	0	0	0	0	100
S 1s fr Teen (n = 4)	25	0	50	0	0	0	0	0	25	100
S 10 fr Teen (n = 5)	80	0	0	0	0	0	0	0	20	100
SHF (n = 5)	20	0	0	0	0	0	0	0	80	100
S fr 10 (n = 6)	83	0	0	0	17	0	0	17	0	83
5 a > 10 (n = 1)	100	0	0	0	0	33	0	0	0	67
S B/U 10 (n = 5)	60	0	40	0	0	0	0	40	0	60
5 a ≤ 10 (n = 3)	0	0	33	67	33	0	0	33	33	0

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

There was one misunderstanding Adam displayed on the preassessment that is noteworthy. He incorrectly recalled 9 – 9 = 9. I was wondering if this was simply an error or a misunderstanding about subtraction. However, on the very next question 5 – 4, Adam answered, "5 – 5 = 5 so 5 – 4 = 3" (P11). By the postassessment, Adam's understanding of subtraction was developed, and his overall accuracy and phase development increase was significant. I wrote on Adam's postassessment, "Great job explaining strategies. Needs purposeful practice" (P9). How did Adam's growth transpire?

Midassessments and twenty-four number talks. Although Adam was able to accurately answer both questions on the first midassessment (8 + 6 and 9 + 8), he was only able to use one strategy (Near Doubles) to explain his thinking for 8 + 6. He was able to use Near Doubles in two different ways to compute 9 + 8 (see Figure 6). His strategy choices were both appropriate and efficient. Adam was able to accurately solve 5 + 8 on the second midassessment by recording counting on from the larger number. I believe Adam was thinking the sum of 6 + 9 was 15 but did not know how to record his thinking and recorded an incorrect sum of 14 (see Figure 6). On the preassessment for 15 – 6, Adam was able to use and explain Back-Through-Ten accurately. I believe he used this strategy again on the midassessment for 15 – 6 but missed recording the initial step of 15 – 5 = 10 and simply recorded 10 – 1 = 9 (see Figure 6). Once again, consistent with mathematics educator Marian Small's (2013) theory, written communication proved more difficult for Adam. On the final midassessment question, 16 – 9, Adam's answer of 5 was incorrect. I believe he was trying to use a known fact and adjusting (take away 10 and add one back). However, he made the initial error in 16 – 10 = 4 so when he added one more it made 5 (see Figure 6) Just like Randy, Adam was taking risks and trying out more sophisticated strategies which he witnessed his classmates using during number talks.

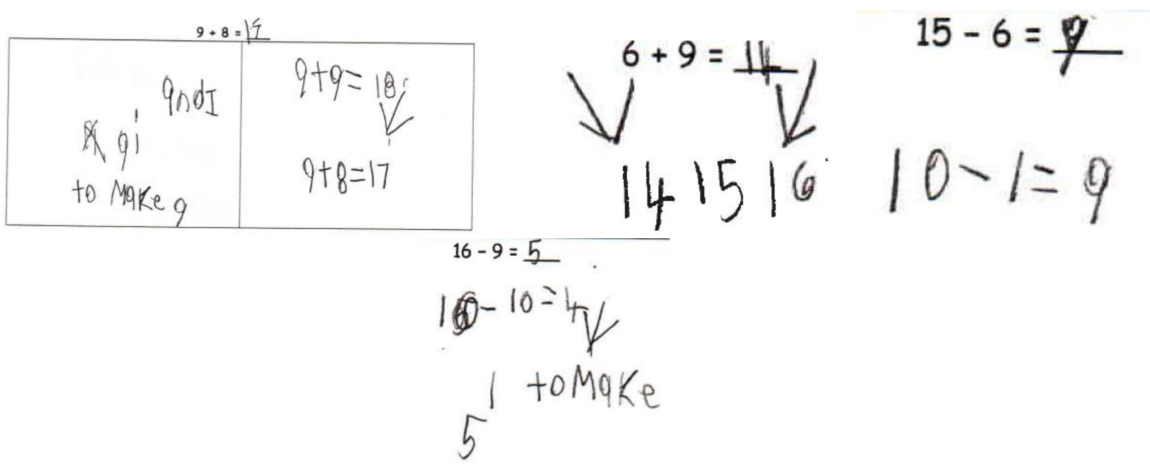


Figure 6. Adam's written communication on midassessments.

Adam was a full participant during the number talks[7] with 24 instances during our 24 number talks where Adam contributed to our classroom discussions. There was only one time when Adam self-corrected; otherwise, his answers were always correct. Adam never repeated or rephrased or expanded on another classmate's thinking, yet he stood out as a very active listener during our number talks. It was obvious he wanted to learn within this social community of mathematicians. I recorded his thinking strategy on chart paper eighteen times, 17 of which were in Phase 3, illustrating his growing ability to work flexibly with numbers. On Day 18 of Week 5, Adam tried to defend his answer of 8 when computing 17 – 9. At the time, I failed to understand the strategy he was trying to explain. Upon reflection, I believe that Adam was trying to split 17 into 7 and 10 in order to take from 10 (knowing that 10 – 9 = 1 which would be added to the 7 to get 8). However, this sophisticated strategy was very difficult for Adam to explain. When I asked Adam why he started with 10, he said, "I forget." Adam had heard this strategy used by another student during our number talks, and three days later Adam was able to use and explain this strategy successfully. On Day 21, when sharing his strategy for 12 – 9 Adam explained, "I know 10 take away 9 is 1, and we were at 12, so that would be 3." His classmate Fran was able to elaborate on Adam's thinking and explained to the rest of the class how he knew to add two. Adam eagerly engaged in number talks and stated on his Postnumber Talk Questionnaire that he was better in his ability to solve math problems in his head after doing number talks (P10) and his data corroborated this.

Cases 3 to 5: Average-achieving students (Oliver, Fran & Helen's) progression through number talks.

[7] Adam missed five number talks during week 3 due to his absence from school.

Pre- and postassessments. Oliver, Fran, and Helen were three average-achieving students who also demonstrated growth during this research project. Their accuracy on addition questions rose from 93% and 95% for Oliver and Fran respectively on the preassessment, to 100% accuracy for both Oliver and Fran on the postassessment. Helen was unique regarding her accuracy on the addition preassessment which was 100%, yet one error (7 + 5 = 11) on her use of a near doubles strategy resulted in 98% accuracy on the postassessment. In subtraction, both Oliver and Fran started further behind, and there was an even more substantial increase—from 69% and 91% respectively on the preassessment, to 100% accuracy for both Oliver and Fran on the postassessment. Again, Helen was unique in the fact that she was 100% accurate on all subtraction questions on both assessments. Overall, these three average achieving students were similar to the low-achieving students as they demonstrated more efficient strategy use (Phases 3 and 4) for addition questions than for subtraction questions on preassessment. By the postassessment, however, demonstration of more efficient strategy use (Phases 3 and 4) were very similar for both addition (average 94%) and subtraction (average 93%) (see Table 22; for detailed pre- and postassessments see Appendices T1 to T4 for Oliver, U1 to U4 for Fran, and V1 to V4 for Helen).

Table 22

Responses as a Percentage on Average-Achieving Students' Pre- and Postassessments Grouped by Phase

	Phase 0		Phase 1		Phase 2		Phase 3		Phase 4	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Addition (n = 42)										
Oliver	7	0	0	0	19	10	33	33	41	57
Fran	5	0	0	0	29	5	12	36	55	60
Helen	0	2	10	0	12	2	26	17	52	79
Subtraction (n = 35)										
Oliver	31	0	0	0	17	14	11	11	40	74
Fran	9	0	0	0	11	6	14	9	66	86
Helen	0	0	20	0	11	0	0	17	69	83

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Addition. On the addition preassessment, all three average students were 100% proficient on Plus 0, Plus 1, Small Doubles, and Add 10 facts. Plus 3 facts would be added to this list for both Fran and Helen. By the postassessment, Oliver and Fran also became proficient on Large Doubles; Fran and Helen became proficient on Sum of 10 and 5 Anchor < 10 facts as well. While Fran's responses were in Phases 0 to 2 on 34% of the facts for addition on the preassessment, this was reduced to 5% (Phase 2) on the postassessment. This 5% was accounted for by two questions, (3 + 4 and 5 + 8), which Fran solved by counting on from the larger number. While Oliver's responses were in Phases 0 to 2 on 26% of the facts for addition on the preassessment, this was reduced to 10% (Phase 2) on the postassessment. This 10% was accounted for by four questions (3 + 4, 3 + 6, 3 + 7, and 7 + 4) which Oliver solved by counting on from the larger number. While Helen's responses were in Phases 0 to 2 on 22% of the facts for addition on the preassessment, this was reduced to 4% on the postassessment. This 4% was accounted for by two questions: Helen solved 7 + 4 by counting on from the larger number, and she made an error on 7 + 5 when she tried to use a near doubles strategy. Out of the three average achieving students, Helen's one error on 7 + 5 was the only response in Phase 0 or 1 for any addition facts on the postassessment. By the postassessment, Oliver was using more efficient strategies for 90% of the addition questions while Fran and Helen were using more efficient strategies for 96 % of the addition questions (see Tables 23 to 25).

Table 23

Responses as a Percentage on Oliver's Addition Pre- and Postassessments Grouped by Phase and Fact Type

	Phase 0		Phase 1		Phase 2		Phase 3		Phase 4	
Fact Type	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Plus 0 (n = 3)	0	0	0	0	0	0	0	0	100	100
Plus 1 (n = 1)	0	0	0	0	0	0	0	0	100	100
Sm Doubles (n = 4)	0	0	0	0	0	0	0	0	100	100
Add 10 (n = 4)	0	0	0	0	0	0	0	0	100	100
Lg Doubles (n = 5)	20	0	0	0	0	0	0	0	80	100
Sum of 10 (n = 3)	0	0	0	0	100	33	0	0	0	67
Sm ND (n = 2)	0	0	0	0	0	50	50	0	50	50
Lg ND (n = 3)	0	0	0	0	0	0	100	67	0	33
5 a < 10 (n = 3)	0	0	0	0	33	0	67	67	0	33
Make-10 (n = 8)	13	0	0	0	25	13	63	63	0	25
5 a > 10 (n = 5)	20	0	0	0	20	0	60	100	0	0
Plus 3 (n = 1)	0	0	0	0	100	100	0	0	0	0

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Table 24

Responses as a Percentage on Fran's Addition Pre- and Postassessments Grouped by Phase and Fact Type

	Phase 0		Phase 1		Phase 2		Phase 3		Phase 4	
Fact Type	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Plus 0 (n = 3)	0	0	0	0	0	0	0	0	100	100
Plus 1 (n = 1)	0	0	0	0	0	0	0	0	100	100
Sm Doubles (n = 4)	0	0	0	0	0	0	0	0	100	100
Add 10 (n = 4)	0	0	0	0	0	0	0	0	100	100
Plus 3 (n = 1)	0	0	0	0	0	0	0	0	100	100
Sum of 10 (n = 3)	0	0	0	0	33	0	0	0	67	100
Lg Doubles (n = 5)	0	0	0	0	0	0	0	20	100	80
Sm ND (n = 2)	0	0	0	0	0	50	0	0	100	50
5 a < 10 (n = 3)	0	0	0	0	33	0	33	33	33	67
Make-10 (n = 8)	25	0	0	0	50	0	25	75	0	25
Lg ND (n = 3)	0	0	0	0	67	0	33	100	0	0
5 a > 10 (n = 5)	0	0	0	0	80	20	20	80	0	0

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Table 25

Responses as a Percentage on Helen's Addition Pre- and Postassessments Grouped by Phase and Fact Type

	Phase 0		Phase 1		Phase 2		Phase 3		Phase 4	
Fact Type	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Plus 0 (n = 3)	0	0	0	0	0	0	0	0	100	100
Plus 1 (n = 1)	0	0	0	0	0	0	0	0	100	100
Sm Doubles (n = 4)	0	0	0	0	0	0	0	0	100	100
Add 10 (n = 4)	0	0	0	0	0	0	0	0	100	100
Plus 3 (n = 1)	0	0	0	0	0	0	0	0	100	100
5 a < 10 (n = 3)	0	0	0	0	0	0	33	0	67	100
Sum of 10 (n = 3)	0	0	0	0	33	0	0	0	67	100
Lg Doubles (n = 5)	0	0	40	0	0	0	0	0	60	100
Sm ND (n = 2)	0	0	0	0	0	0	50	0	50	100
5 a > 10 (n = 5)	0	0	20	0	40	0	20	60	20	40
Make-10 (n = 8)	0	13	0	0	25	13	75	25	0	50
Lg ND (n = 3)	0	0	33	0	0	0	67	67	0	33

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Subtraction. On the preassessment, subtraction proved to be more difficult for Oliver noted by a reduction in both accuracy and phase development between addition and subtraction. Oliver had 31% incorrect answers on the preassessment while Fran was at 9% and Helen was at 0% (see Table 26). On the preassessment, all three average students were 100% proficient for Subtracting 1, Subtracting 0, Subtracting a Number from Itself, Subtracting 10 from a Teen. Fran and Helen were also 100% proficient for Difference of 1 and Subtracting 1 from a Teen facts on the preassessement. Oliver also mastered these facts by the postassessment. Furthermore, by the postassessment, all three average students achieved proficiency with 5 Anchor > 10 facts. While Fran had 100% proficiency on Subtracting Half facts on the preassessment, Helen mastered these facts by the postassessment. On the other hand, by the postassessment, Oliver was using more efficient strategies (Phases 3 and 4) for only 40% of the Subtracting Half facts. Likewise, with

Subtracting from 10 facts, by the postassessment both Fran and Helen showed 100% proficiency while Oliver demonstrated use of more efficient strategies (Phases 3 and 4) for 88% of these facts. By the postassessment, Helen had mastered the 5 Anchor ≤ 10 facts while Oliver used more efficient strategies (Phases 3 and 4) for 66% of these facts and Fran was at 33%. (see Tables 27 - 29). What transpired between the assessments?

Table 26

Incorrect Responses on Oliver's Subtraction Preassessment Grouped by Phase and Fact Type

	Incorrect: Phase			
	0	1	2	3
SHF (n = 5)	0	0	2	1
S fr 10 (n = 6)	1	0	0	1
5 a ≤ 10 (n = 3)	0	0	2	0
5 a > 10 (n = 1)	0	0	0	1
S B/U 10 (n = 5)	0	0	1	2

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Table 27

Responses as a Percentage on Oliver's Subtraction Pre- and Postassessments Grouped by Phase and Fact Type

	Phase 0		Phase 1		Phase 2		Phase 3		Phase 4	
Fact Type	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S1 (n = 1)	0	0	0	0	0	0	0	0	100	100
S 0 (n = 1)	0	0	0	0	0	0	0	0	100	100
S N fr Itself (n = 1)	0	0	0	0	0	0	0	0	100	100
S 10 fr Teen (n = 5)	0	0	0	0	0	0	0	0	100	100
S 1s fr Teen (n = 4)	0	0	0	0	0	0	25	0	75	100
Diff of 1 (n = 3)	0	0	0	0	33	0	0	0	67	100
5 a > 10 (n = 1)	100	0	0	0	0	0	0	0	0	100
S B/U 10 (n = 5)	60	0	0	0	40	0	0	20	0	80
S fr 10 (n = 6)	33	0	0	0	33	17	0	33	33	50
SHF (n = 5)	60	0	0	0	0	60	40	0	0	40
5 a ≤ 10 (n = 3)	67	0	0	0	0	33	33	33	0	33

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Table 28

Responses as a Percentage on Fran's Subtraction Pre- and Postassessments Grouped by Phase and Fact Type

	Phase 0		Phase 1		Phase 2		Phase 3		Phase 4	
Fact Type	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S1 (n = 1)	0	0	0	0	0	0	0	0	100	100
S 0 (n = 1)	0	0	0	0	0	0	0	0	100	100
S N fr Itself (n = 1)	0	0	0	0	0	0	0	0	100	100
Diff of 1 (n = 3)	0	0	0	0	0	0	0	0	100	100
S 1s fr Teen (n = 4)	0	0	0	0	0	0	0	0	100	100
S 10 fr Teen (n = 5)	0	0	0	0	0	0	0	0	100	100
SHF (n = 5)	0	0	0	0	0	0	0	0	100	100
S fr 10 (n = 6)	0	0	0	0	17	0	50	0	33	100
5 a > 10 (n = 1)	100	0	0	0	0	0	0	0	0	100
S B/U 10 (n = 5)	40	0	0	0	20	0	40	60	0	40
5 a ≤ 10 (n = 3)	0	0	0	0	67	67	0	0	33	33

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Table 29

Responses as a Percentage on Helen's Subtraction Pre- and Postassessments Grouped by Phase and Fact Type

	Phase 0		Phase 1		Phase 2		Phase 3		Phase 4	
Fact Type	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S1 (n = 1)	0	0	0	0	0	0	0	0	100	100
S 0 (n = 1)	0	0	0	0	0	0	0	0	100	100
S N fr Itself (n = 1)	0	0	0	0	0	0	0	0	100	100
Diff of 1 (n = 3)	0	0	0	0	0	0	0	0	100	100
S 1s fr Teen (n = 4)	0	0	0	0	0	0	0	0	100	100
S 10 fr Teen (n = 5)	0	0	0	0	0	0	0	0	100	100
5 a ≤ 10 (n = 3)	0	0	0	0	0	0	0	0	100	100
SHF (n = 5)	0	0	20	0	20	0	0	0	60	100
S fr 10 (n = 6)	0	0	33	0	33	0	0	0	33	100
S B/U 10 (n = 5)	0	0	60	0	20	0	0	100	20	0
5 a > 10 (n = 1)	0	0	100	0	0	0	0	100	0	0

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Midassessments and twenty-four number talks. All three average-achieving students were able to correctly answer 8 + 6 on the first midassessment and use two different strategies to explain their thinking. While Fran used two Phase 3 strategies (see Figure 7), both Oliver and Helen used one Phase 3 strategy first and then resorted to counting on for their second strategy. Oliver's recorded strategy for solving 8 + 6 was interesting to consider. I believe Oliver was employing the Make-10 strategy and took 2 from the 6 to give it to the 8 to make 10, but had difficulty correctly recording this on paper (see Figure 8). While Helen was able to use two different strategies to solve 9 + 8 on the first midassessment, Fran was only able to use one strategy. Oliver was incorrect when solving 9 + 8 and could not explain his thinking.

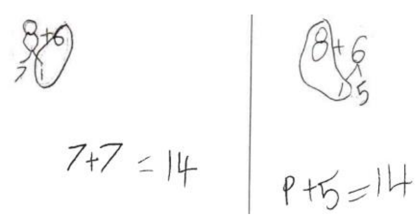


Figure 7. Fran's written communication on midassessment.

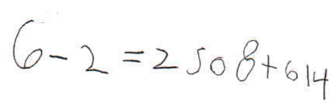


Figure 8. Oliver's written communication on midassessment.

On the second midassessment, all three average students were able to accurately and efficiently (Phase 3) solve 5 + 8 and 6 + 9. To solve 15 – 6, Fran used a related fact, Helen counted back, and Oliver used the wrong operation and counted on from 15 to incorrectly answer 21. All three students incorrectly answered 16 – 9 on the second midassessment. Oliver inaccurately tried to count back, while Fran and Helen inaccurately tried to use a Phase 3 strategy. In was interesting to compare Helen's thinking when solving 6 + 9 (answered correctly) to 16 – 9 (answered incorrectly) where she tried to use a similar strategy (see Figure 9). The day following the midassessment, I recorded her thinking on chart paper to share with the class during a number talk, and I asked the students if they thought this strategy would work. After discovering that some students agreed and some students disagreed, we had a productive whole class discussion. In the end, the students agreed that an efficient strategy to solve 16 – 9 would be to subtract 10 from 16 (not from 15 as Helen had recorded on her midassessment) to get to 6, and then add one back on, to get to 7.

Figure 9. Helen's written communication on midassessment.

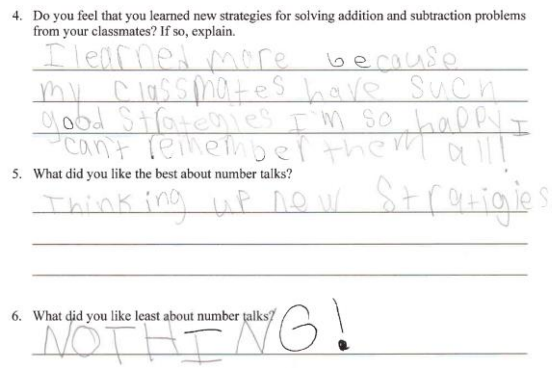
All three average students were full participants during the number talks. There were 44 instances during our 24 number talks where Oliver contributed to our classroom discussions; 42 instances for Helen, and 36 instances for Fran. Fran and Helen's answers were always correct during our number talks, and there were no accounts of self-correcting. Oliver contributed four incorrect answers and self-corrected twice.

Oliver had no problem sharing with the class when and how he had revised his thinking (P259). Without prompting, Oliver was able to make connections with previous questions, demonstrating relational thinking (P245). Furthermore, he was able to make connections with other students' strategies (P246) and rephrase his classmates' thinking (P263). By the end of our 24 number talks, Oliver took ownership of his adjusting strategy when computing questions such as 12 – 9 and was proud of his efficiency and accuracy giving a cheer such as "Boom-shack-a-lack-a-boom!" when sharing this within our community of learners (P279). Nineteen times I recorded Oliver's strategy on chart paper demonstrating Phase 3 strategy use for all but two questions where he used the Back-Through-10 strategy directly modelling with his individual Rekenrek.

Helen was one of the most enthusiastic participants in our number talks. Like Oliver, she was able to make connections with previous questions (P244, P256, P263, P266, P269) and between classmates' strategies (P266), and was able to repeat, rephrase, or elaborate on students' strategies (P269, P272, P310). During the number talks, Helen demonstrated her understanding of the commutative property of addition, sharing with the class how you can "switch around" addition facts (P244). She also understood the inverse relationship between addition and subtraction and could articulate how she thought of fact families when solving questions (P253, P256, P258, P263). Helen was able to articulate her thinking well as evidenced on Day 17 of our number talks. We began our number talk by reviewing strategies shared on the chart paper the previous day for solving 14 – 9 and 15 – 8. Students were asked to share with their elbow partner a strategy they felt was efficient and then explain why. Sharing with the whole class, Helen stated, "I was thinking of Quinn's [strategy] because I really like the way, about how he splits numbers into two and he tries to get himself to that friendly ten because then it's super easy to get to the destination of what you want to get to." Helen began using "Quinn's" Back-Through-10 strategy throughout our remaining number talks (P269, P276, P278). I recorded Helen's strategies 23 times on chart paper; four times she used a Phase 2 strategy (once solving 13 – 8 with the Back-Through-10 strategy on her individual Rekenrek), while the remaining 19 questions were solved with more efficient (Phase 3) strategies. Helen hated to miss a day of number talks, and after missing a day, she informed the class she "did math strategies the whole time [she] was gone!" (P269).

Similar to Oliver and Helen, Fran was able to make connections with previous questions (P248, P250) and between classmates' strategies (P17), and was able to repeat, rephrase, or elaborate on students' strategies (P253, P255, P263, P272, P278). Like Helen, Fran demonstrated her understanding of the inverse relationship between addition and subtraction (P245, P258, P262, P270). I recorded Fran's strategies 16 times on chart paper; three times she used a Phase 2 strategy (once using the Back-Through-10 strategy on her individual Rekenrek to compute 16 - 7), while the remaining 13 questions were solved with more efficient (Phase 3) strategies.

On the Postnumber Talk Questionnaire, Fran commented that number talks are "good for your brain" (P57) and this was evidenced by her growth. In fact, all three average students felt they were better at mental math after number talks when answering the postquestionnaire. When asked on the postquestionnaire if she learned new strategies, Fran answered, "Yes, because I love math." Oliver stated that what he liked best about number talks were the "strategies" and answered "nothing" when asked what he liked least about number talks. Helen's enthusiasm for number talks was also evidenced by her responses on the postquestionnaire (see Figure 10). Since she was so happy [and excited] about number talks, she acknowledge that she couldn't remember all of the strategies when asked in questions 1 and 2 to name/explain as many addition and subtraction strategies as she could.


4. Do you feel that you learned new strategies for solving addition and subtraction problems from your classmates? If so, explain.

I learned more because my classmates have such good strategies I'm so happy I can't remember them all

5. What did you like the best about number talks?

Thinking up new Stratigies.

6. What did you like least about number talks?

NOTHING!

Figure 10. Helen's responses on the postnumber talk questionnaire.

Case 6: Betty's progression through number talks.

Pre- and postassessments. Betty was a high-achieving student who also demonstrated growth during this research project. This growth, however, was not due to an increase in accuracy; Betty was 100% accurate on all assessments. Betty's growth was demonstrated in her increase in more efficient strategy use (Phases 3 and 4) for both addition and subtraction questions (see Table 30; see Appendices W1 to W4 for Betty's detailed pre- and postassessments), and her contributions made to our social community of mathematicians during our number talks.

Table 30

Responses as a Percentage on Betty's Pre- and Postassessments Grouped by Phase

	Phase 2		Phase 3		Phase 4	
	Pre	Post	Pre	Post	Pre	Post
Addition (n = 42)	2	0	36	7	62	93
Subtraction (n = 35)	9	0	20	11	71	89

Note. Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Addition. On the preassessment, Betty was in Phase 2 for 2% of the addition questions and was in Phase 3 for 36% of the questions. By the postassessment, Phase 2 was reduced to 0% and

Phases 3 was reduced to 7%. Between the two assessments, Phase 4 increased from 62% to 93% (see Table 30).

Looking more closely at addition, there were only four addition fact types which Betty had not mastered: Sum of 10, 5 Anchor > 10, Make-10, and Large Near Doubles facts. She counted on for one fact (3 + 7) and the remaining 36% of facts Betty was able to accurately and efficiently work with numbers to solve each question. By the postassessment, Betty used more efficient strategies to solve 3 + 7, 5 + 7, and 5 + 8 (reducing Phase 3 from 36% on the preassessment to 7% on the postassessment). She was able to recall from memory all remaining addition facts (see Tables 31).

Table 31

Responses as a Percentage on Betty's Addition Pre- and Postassessments Grouped by Phase and Fact Type

	Phase 2		Phase 3		Phase 4	
Fact Type	Pre	Post	Pre	Post	Pre	Post
Plus 0 (n = 3)	0	0	0	0	100	100
Plus 1 (n = 1)	0	0	0	0	100	100
Sm Doubles (n = 4)	0	0	0	0	100	100
Lg Doubles (n = 5)	0	0	0	0	100	100
Add 10 (n = 4)	0	0	0	0	100	100
Sum of 10 (n = 3)	33	0	33	33	33	67
Sm ND (n = 2)	0	0	0	0	100	100
5 a < 10 (n = 3)	0	0	0	0	100	100
Lg ND (n = 3)	0	0	0	0	100	100
5 a > 10 (n = 5)	0	0	60	40	40	60
Make-10 (n = 8)	0	0	100	0	0	100
Plus 3 (n = 1)	0	0	100	0	0	100

Note. Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Subtraction. On the preassessment, Betty was in Phase 2 for 9 % of the subtraction questions and was in Phase 3 for 20% of the questions. By the postassessment, Phase 2 was reduced to 0% and Phase 3 was reduced to 11%. Between the two assessments, Phase 4 increased from 71% to 89% (see Table 30).

Looking more closely at subtraction, Betty was 100% proficient for Subtracting 1, Subtracting 0, Subtracting a Number from Itself, Subtracting Ones from a Teen, and Subtracting 10 from a Teen facts. By the postassessment, this expanded to include Subtracting Half Facts, Subtracting from 10, and 5 Anchor ≤ 10 facts. On the preassessment, Betty used counting strategies for 10 – 7, 15 – 9, and 16 – 9 and was in Phase 3 for 7 subtraction facts. By the postassessment, Betty no longer used any counting strategies and was in Phase 3 for only 4 facts: 14 – 5, 14 – 8, 15 – 6, and 16 – 9; all remaining subtraction facts were mastered (see Table 32). Considering Betty's accuracy and efficiency, she made an interesting comment during the postassessment. I wrote on her subtraction postassessment, "Betty commented that she doesn't really like flashcards with the class or Around the World [a whole class game] because they go too fast." Betty was a student who thought slowly and deeply about math. Researcher Jo Boaler (2016) argues that students who think slowly and deeply can be "put off mathematics" (p. 30) when mathematics is presented as a speed race through timed math tests, flashcards or other activities where students compete against the clock.

Table 32

Responses as a Percentage on Betty's Subtraction Pre- and Postassessments Grouped by Phase and Fact Type

	Phase 2		Phase 3		Phase 4	
Fact Type	Pre	Post	Pre	Post	Pre	Post
S1 (n = 1)	0	0	0	0	100	100
S0 (n = 1)	0	0	0	0	100	100
S N fr Itself (n = 1)	0	0	0	0	100	100
Diff of 1 (n = 3)	0	0	0	0	100	100
S 1s fr Teen (n = 4)	0	0	0	0	100	100
S 10 fr Teen (n = 5)	0	0	0	0	100	100
SHF (n = 5)	0	0	20	0	80	100
S fr 10 (n = 6)	17	0	17	0	66	100
5 a > 10 (n = 1)	0	0	33	0	67	100
5 a ≤ 10 (n = 3)	40	0	60	60	0	40
S B/U 10 (n = 5)	0	0	100	100	0	0

Note. Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Midassessments and twenty-four number talks. Consistent with the accuracy Betty displayed on the preassessment, she was 100% accurate on all midassessment questions. Furthermore, her strategy choices were both appropriate and efficient, and she was able to clearly reflect her thinking strategies in writing (see Figure 11).

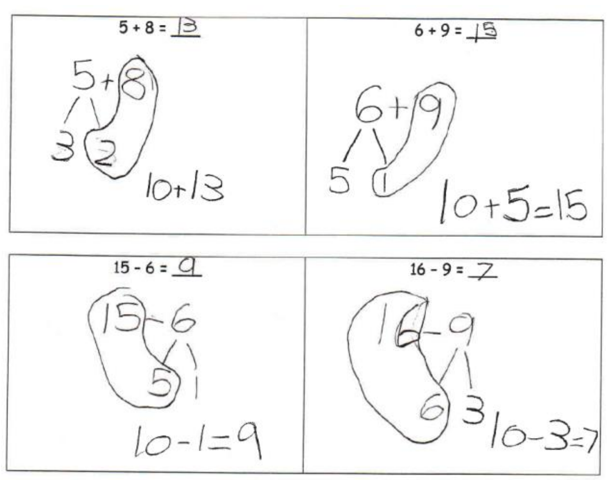


Figure 11. Betty's written communication on midassessment.

Betty was a very active participant during the number talks with 66 instances during our 24 number talks where Betty contributed to our classroom discussions. There were only three times when Betty self-corrected; otherwise, her answers were always correct. I recorded her thinking strategy on chart paper twenty-five times, and she consistently used Phase 3 strategies. It was clearly evident that Betty could work flexibly with numbers and she delighted in sharing her strategies with her classmates. Throughout the study, Betty grew in confidence, and she looked for connections between her strategy use and those of her classmates. She would make comments such as, "It's kinda like Mary's…" (P240). She was also able to make connections between addition and subtraction use by students. For example, when another student solved 10 – 2 using a Think-Addition strategy, Betty made the connection and said, "But he did it in plus" (P244). Later on, during our number talks, Betty was able to use more sophisticated language and actually named the Think-Addition strategy (D276). Betty was also able to repeat or rephrase her classmates' strategies and over time (P244, P253, D276), she began to add on to their thinking (P273, P276). Every day we began our number talks by reading our target, "I can compute with efficiency, accuracy, and flexibility." Not only did Betty aspire to achieve this goal for herself, over time, Betty even encouraged her classmates to use more efficient strategies. On the last day of our number talk lessons, students were solving the problem 12 – 9. A student suggested counting up from 9 as a strategy. Betty added on to this thinking and suggested using the Up-Through-Ten strategy: "Instead of just counting up, you could just make 10 and then just add 2 more, and 1 + 2 is 3" (P53, P278) (see Figure 12).

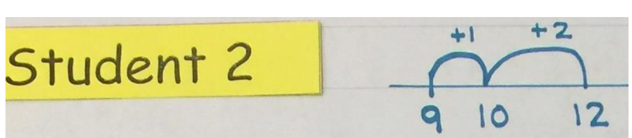


Figure 12. Chart paper showing Betty's strategy.

If another student made an error, without prompting, Betty would try to understand their thinking and figure out why their strategy needed to be revised (P63, P272, P273). Betty was readily able to decompose numbers, and she began using this terminology to explain her thinking during our number talks (P264). Betty, like other students in class, began to take ownership of her strategies. Betty called taking 10 away and adjusting by adding one more for questions such as 17 – 9 “her” strategy (P267, P274). An interesting exchange happened on Week 6 Day 22 between Betty (who “owned” the compensation strategy) and Quinn (who “owned” the Back-Through-10 strategy). Quinn wanted to use Betty’s strategy to solve 12 – 8 since someone else had already shared his Back-Through-Ten strategy.

Quinn: “Since I had no choice… so I knew like Betty did her strategy, so I did hers. But I don’t know how she does that.”

Myself: “Oh, interesting.”

Betty: “I can help!”

Myself: “Awesome; help him!”

Betty: “He makes a 10 out of the 8, and he takes away 10 which would be 2 [more], and then you add 2 more.”

Quinn: “I thought you add 1 more. That’s what you usually do.”

Betty: “But it’s 8 this time, not 9.”

Quinn: “Ooohhh.”

Betty always actively engaged in number talks and stated on her Postnumber Talk Questionnaire that she was better in her ability to solve math problems in her head after doing number talks (P54) writing, “I learned a lot of new strategies.” Her favourite part of number talks was sharing and the games. Her least favourite part of number talks was “when it ends.”

Chapter 5: Discussion

The general purpose of this study was to explore the following research question: what is the impact of daily number talks on the development of mental math abilities of second graders within a reform-based classroom? Although many math educators (Buchholz, 2004, 2016; DiBrienza & Shevell, 1998; Ebdon et al., 2003; Fosnot & Uittenbogaard, 2007; Parrish, 2011, 2014; Russell, 2000; Scharton, 2004) claim that number talks (or minilessons) in which children “invent,” or construct, discuss, and apply a range of strategies to solve calculations improve their mathematical fluency, confidence, and achievement, there is a lack of research-based evidence to support their effectiveness. This study addresses that gap. In support of their claims, the findings indicate that even in a short six-week period, number talks can foster growth in both computational fluency and conceptual understanding for all students.

Accuracy, Efficiency, and Flexibility

More specifically I asked: Does the implementation of number talks increase students’ ability to calculate with accuracy, efficiency, and flexibility?

First I found that students were already mostly accurate in their addition calculations and improved where they were not. They became measurably more accurate in their more challenging subtraction calculations.

Second, for both addition and subtraction, I found an increase in efficiency between the pre- and postassessments. Student can move from inefficient counting strategies (Phases 1 and 2) to more efficient strategies of working with numbers (Phase 3) which can then lead to proficiency (Phase 4). Students can demonstrate greater growth in harder fact types in both addition and subtraction as the majority of students were proficient with the easier fact types

(Plus 0, Plus 1, Small Doubles, Large Doubles, Subtracting 1, Subtracting 0, and Subtracting a Number from Itself) prior to number talks.

Third, I found, along with many math educators (Buchholz, 2004, 2016; DiBrienza & Shevell, 1998; Ebdon et al., 2003; Fosnot & Uittenbogaard, 2007; Parrish, 2011, 2014; Russell, 2000; Scharton, 2004), that students' conceptual understanding enhances as their accuracy, efficiency, and flexibility increases (NCTM, 2000). Between the pre- and postassessments, overall accuracy increased slightly on addition (from 96% to 99% respectively) while accuracy on subtraction rose 17% (81% on the preassessment to 98% on the postassessment).

Finally, I found an increase in students' flexibility as demonstrated by their progression through Lawson's (2015) "Student Continuum of Numeracy Development: Addition and Subtraction" (p. 4), As Parrish (2014) suggests, I found that as students show an increase in their "ability to use number relationships with ease in computation" (p. 5) their flexibility in working with numbers is amplified which was also Scharton's (2004) experience.

Place Value and Number Relationships

The final research question was: Does the implementation of number talks increase students' understanding of place value and number relationships? Consistent with other researchers (Baroody, 1985; Baroody et al., 2009), I found that Grade 2 are able to recognize and discuss patterns and number relationships. Through doing so, they can develop greater number sense as well, foundational to fact mastery (Lawson, 2016; O'Loughlin, 2007; Van de Walle & Lovin, 2006). For example, knowing how numbers are related to ten helps students use more efficient strategies (which leads to proficiency) when computing Make-10 facts; when solving 8 + 6, knowing that 8 is 2 away from 10, students can take 2 from 6 to make (8 + 2) + 4 = 14. They know that they can decompose the 6 and re-associate the 2 with 8 to make 10 and add the remaining 4. They know this will give the same sum. To master this strategy, students must also understand and explain what happens when you add 10 to a number—which leads to the construction of place value understanding. By the postassessment, accuracy was 97% or greater on fact types involving 10 (Sum of 10, Plus 10, Make-10, Subtracting from 10, Subtracting Ones from a Teen, Subtracting 10 from a Teen, and Subtracting Back-Through or Up-Through 10 facts) (see addition section and Table 11 on p. 55). By the postassessment, I also found that students can use more efficient strategies (Phases 3 and 4) for these fact types as well. As students grew in their understanding that numbers can be taken apart and put back together again using the10-anchor (the key idea of the associative property) they also knew that one group of ten is also two groups of five and other combinations to make a sum of ten (the key idea of hierarchical inclusion). Findings support that students can move in their development to see that a set of ten ones can be perceived as a single entity. For example, instead of recognizing teen numbers as ten ones and some more, the second graders can understand numbers 11 through 19 as one ten and some more. Ten has a significant role in our base ten system and students can understand that the position of digits represents different values.

I also found that students can develop the knowledge of the power of 10 as a reference or anchor point for both addition and subtraction throughout our number talks. Over the course of this study, similar again to Baroody et al.'s (2009) research, I found more and more students able to break numbers apart (decompose) and put numbers back together again (compose). For example, when computing 8 + 5, a student may decompose the 5 into 2 and 3 and then compose 10 (Make-10) using the 8 and 2. The student would then add 10 and the remaining 3 to find 13. Also in support of Baroody et al.'s (2009) research, findings suggest that "the concepts of composition and decomposition are central to inventing other reasoning strategies" (p. 73). I also found students can adjust across the equal sign. For example, if a student knows 6 + 7 = 13, they can reason that 6 + (7 + 1) = (13 + 1).

The results indicate that all students improved their computational fluency (NCTM, 2000; Parrish, 2014; Russell, 2000). We remember things we think about and work with, when we connect this knowledge with previous knowledge, the connecting becomes the thinking which we are more likely to remember (Baroody et al., 2009); I found this to be true as students discover, share, and critique strategies through number talks. The findings suggest a strategies-based approach to fact learning is successful for second graders. Van de Walle and Lovin (2006) explain, “The strategy provides a mental path from fact to answer. Soon the fact and answer are ‘connected’ as the strategy becomes almost unconscious” (p. 97). Christensen and Cooper (1991) also found that children who have the opportunity to construct with their own strategy rather than learning through direct instruction have greater success. Guided discovery of strategies is an integral component to number talks.

Summary of Major Findings in Case Studies

Low-achieving students can achieve significant growth through number talks with an increase in accuracy, efficiency, flexibility, and stamina. While these students require additional time for mastery of the more difficult fact types, they are on the same path, but at a rate suited to their individual abilities. Researchers have found that students with learning disabilities are “*delayed* and not cognitively different from that of normally achieving children” (Woodward & Montague, 2002, p. 94) when learning number facts (Goldman, Pellegrino, & Mertz, 1988; Putnam, deBettencourt, & Leinhardt, 1990; Russell & Ginsburg, 1984). Average-achieving students can display growth through number talks but often have greater accuracy and efficiency to start with than their low-achieving peers. Average-achieving students are able to make connections with previous questions and between classmates' strategies. They are able to repeat, rephrase, or elaborate on students' strategies as well. High-achieving students also display growth and can still actively engage in number talks as they "play with numbers" and think deeply about number relationships and as they try to understand other students' mathematical thinking; all of which offer satisfaction. Over time, these students can also encourage their classmates to try more efficient strategies. For all students, number talks can build their confidence and develop more robust reasoning. In turn, confidence can lead to "higher levels of motivation, engagement, and achievement" (Boaler, 2016, p. 145).

Children self–corrected during the talks. Self-correction shows that students can understand and try to make sense of their mistakes; they are able to catch them and revise their thinking accordingly. Researchers have found self-correction and sense-making to be a hallmark of high achieving classrooms (Geist, 2000). Students who self-correct are obviously monitoring their thinking and are aiming for accuracy. Confidence grew as students caught and corrected their mistakes; as Boaler (2016) argues, "If we believe that we can learn, and that mistakes are valuable, our brains grow to a greater extent when we make a mistake" (p. 13). Students are actively learning when they self-correct and taking ownership of their learning as well. Instead of being teacher-directed, students ask themselves, "Does my answer make sense?"

Over time, students take ownership of their strategies—*they* are the ones doing the math as Van de Walle and Lovin (2006) suggest. Students can explain their strategies with enthusiasm and pride, which was also found by Buchholz (2004). This is exciting to see as Fosnot (2016) explains the significance of ownership in student development.

> Ownership is critical in building self-confidence and a positive growth mindset. Ultimately it is ownership of the mathematics that will promote and result in solving problems with

> tenacity and confidence. We are mentoring, working *with* these young students as *they* do mathematics. The mood and tenor are collaborative—it is a conversation that flows back and forth. (p. 42, emphasis in original)

Within a community of learners, there is diversity in thinking, yet all contributions are valued. A student's ability to articulate their thinking can evolve as they connect to and build on the thinking of others. On the preassessment, there were ten instances where students skipped the question due to difficulty and 25 instances where students were unable to explain their thinking. By the postassessment, no questions were skipped and there was only one instance where a student incorrectly recalled a fact and did not explain their thinking.

Conclusions

I did not ask the question: What kinds of things did I do that resulted in this change? However, I can speculate that creating a classroom culture of respect, enthusiasm, and a community of learners aided in the success of number talks. Kazemi and Hintz's (2014) norms for doing mathematics (see list on page 25) were reviewed with students and posted on the wall by the carpet where we gathered for our daily number talks. Throughout this research project, I referred to all students as mathematicians. I set clear expectations from the start—all students were to be active, responsible participants. Changing the power structure in the classroom, I gave up sole authority. I was a facilitator of learning, and the students themselves became the innovators of mathematical strategies (Flynn, 2017; Lambert et al., 2017). Within this collaborative community of mathematicians, a culture was created whereby students were not only expected to share their thinking, but to listen to one another, and learn to analyze and reflect on what others had to say (Small, 2013). As students shared, listened, questioned and critiqued the reasoning of others, they shared the responsibility of learning as well (Flynn, 2017), all within a safe environment. Furthermore, as students invented strategies and deepened their understanding, not only did they develop as mathematicians, they developed a positive growth mindset as well (Fosnot, 2016). I think that these factors contribute to a productive and nurturing classroom environment as advocated for by Kazemi and Hintz (2014).

The findings support the claim that learning is a social process that requires language and discourse which promotes thinking and develops reasoning (Vygotsky, 1978). An essential part of number talks is indeed communication. Consistent with Torbeyns et al.'s (2009) research, I found students needed instructional encouragement and support with this strategy-discovery approach. Communication skills (oral, written, and symbolic) had to be modelled and cultivated (Small, 2013). This included using talk moves (revoicing, repeating, reasoning, adding on, wait time, and turn-and-talk). While I began by modeling talk moves, over time, the students themselves initiated talk moves. Putting thoughts into words, or recording thinking strategies as on the midassessments, pushed students to clarify their thinking and their communication skills evolved.

Written communication provides a permanent record, and a different sort of insight into student thinking than oral communication alone provides as mathematical thinking became visible. Furthermore, written communication forces students to slow down and reflect on each step of their thinking "providing sensory feedback as the hand is engaged in the writing, fostering better memory of the material" (Small, 2013, p. 126). Thus, as students are allowed to practice these communication skills, they become confident math thinkers, and growth in all forms of communication becomes evident. For example, throughout this study, there were increased amounts of student-to-student talk. Also, students were able to feel they were making progress as they stated in their written communication, on their oral communication in our daily number talks, and as we engaged in “gallery walks” reviewing the chart papers where I recorded students’ computation strategies. Recording student thinking on chart paper allowed students to see one another’s mental thinking step-by-step also allowing for reflection. There is an overall feeling of mathematics learning through these intertwined processes of communication and reflection as recommended in the Standards (NCTM, 2000).

Our target was reviewed daily, “I can compute with accuracy, efficiency, and flexibility.” My experience as an educator of twenty years is that students often believe that the answer is the most important part and they are often conditioned to describe their thinking only when the answer is incorrect. While accuracy was never compromised during our number talks, students quickly began to realize that our conversations went well beyond getting the correct answer. All students were empowered to think for themselves, and all students’ thinking was valued. As students discover strategies, they take ownership of them, and strategies are named after them. This supports Van de Walle and Lovin’s (2006) argument that “a strategy is most useful to students when it is theirs, built on and connected to concepts and relationships they already own” (p. 96).

Computational fluency was encouraged through thoughtfulness and sense-making as opposed to speed (as advocated by Boaler [2016]; and Flynn, [2017]) and perseverance was valued. Through our number talks, students were allowed to compute in ways that made sense to them, and student thinking was validated as students shared within their community of learners. As students looked for similarities and differences between strategies, crucial connections were established which led to conceptual understanding. This supports Van de Walle and Lovin’s (2006) claim that understanding is all about connections. Furthermore, sharing of strategies and mathematical thinking collectively can build a greater understanding (Flynn, 2017) as sharing allows students to solidify and consolidate their thinking. All mathematicians were encouraged to try and were praised for the efforts daily as we ended each number talk with a cheer to celebrate success within our community of learners (cheers were included in the Daily Number Talk Planning Sheet; see Appendix J).

On many occasions during number talks, we would end with a *challenge question* which students enjoyed (e.g., in the string 5 + 7, 7 + 5, 7 + 3, and 7 + 5, the challenge question would be 6 + 8). Their engagement and delight supports DiBrienze and Shevell's (1998) claim that "children who are given opportunities to explore and construct strategies will derive aesthetic pleasure of playing with numbers and searching for elegant solutions" (p. 25). Gaining mastery over easier facts, persevering through challenges, seeing classmates' success, and experiencing success themselves, all persuaded students to have greater confidence and self-efficacy. This, in turn, elevated students' mood as collectively they formed a stronger sense of commitment to learning and a positive growth-mindset toward mathematics. Higher motivation led to greater effort which led to an increase in student achievement. Randy, the student who was unable to answer countless subtraction questions on the preassessment, wrote on his Postnumber Talk Questionnaire that "Number talks are special" (P67).

Students knew that I was invested in number talks and I articulated to them that number talks were the best part of my day (P244)[8]. Likewise, students were invested in the number talks; rather than managing behaviours, I was able to spend my energy on facilitating and learning from the discussion. Once again, clear expectations from the start can help to establish a vibrant community of learners where students actively participate in their learning within a safe, risk-

[8] As a reminder to students, I often wore my button which read, "Number talks–the best part of my day."

free environment (Parrish, 2011). I found that an increase in academic engagement can lead to a decrease in disruptive behaviours.

The beauty of number talks is that they provide multiple entry points for a range of learners allowing for access and equity. With multiple entry points and the sharing of different strategies ranging in sophistication, numbers talks are beneficial for all members of the learning community. Baroody et al., 2009 found this to be true as well concluding, “Giving all children the opportunity to explore number and their relations can be beneficial to their mathematical thinking and learning” (p. 77). While some students are constructing their understanding of combinations of ten, for example, others are looking for generalizations and the relationship between numbers (Flynn, 2017). Each student is able to work from their own understanding, use strategies that make sense to them, justify their thinking, and ask questions to deepen their learning. Periodically allowing students to turn and talk with their elbow partner also promotes further access and equity as all students are able to voice their thinking. Each student benefits in his or her own way as they grow in their ability to think and reason mathematically. Access and equity are further encouraged through the use of models. Similar to math educators Lambert et al.’s (2017) finding, models are a valuable tool used to help students visualize mathematical relationships, again providing wider access to the mathematical ideas being discussed.

I firmly believe, as Boaler (2016) argues, that number talks is the very best strategy to teach both number sense and math facts at the same time for they enable students to see the flexible and conceptual nature of math. This is coupled with the fact that “students love to give their different strategies and are usually completely engaged and fascinated by the different methods that emerge” (p. 49). I also firmly believe, as O’Loughlin (2007) suggests, “These experiences will help my students in the future as they consider more complex mathematical relationships and continue becoming competent mathematicians" (p. 138).

Finally, while many math educators have advocated for the use of number talks as a means of strengthening children's mathematical fluency and understanding this study offers some of the first empirical data that supports these claims.

Considerations for Further Research

While this case study allowed for an in-depth analysis it was bounded by time and activity (Crewsell, 2014). One Grade 2 class is not representative of all second graders yet enough detail is provided to readers to determine if this study "fits" with their situation (Merriam & Tisdell, 2016). To build on the finding of this study, the same intervention could be used with a few modifications. Future studies could pursue larger numbers, include a control group, and a delayed postassessment. Furthermore, video recorded student interviews would provide a richer source of data.

Lastly, fostering what Carpenter, Franke, and Levi (2003) call relational thinking through true-false number sentences could be a next step with number talks. When students are able to think relationally, rather than performing all the calculations, they can solve number sentences by focusing on the relationship between the numbers in the equation instead. To clarify and consolidate their understanding, students could also write their own number sentences providing them with the template found in Figure 13 (Carpenter, et al., 2003; Molina & Ambrose, 2006).

___ + ___ = ___ + ___,

___ - ___ = ___ - ___, or

___ + ___ = ___ - ___.

Figure 13. Template for students to write number sentences.

In turn, thinking relationally about numbers during number talks should make fact learning easier and more robust.

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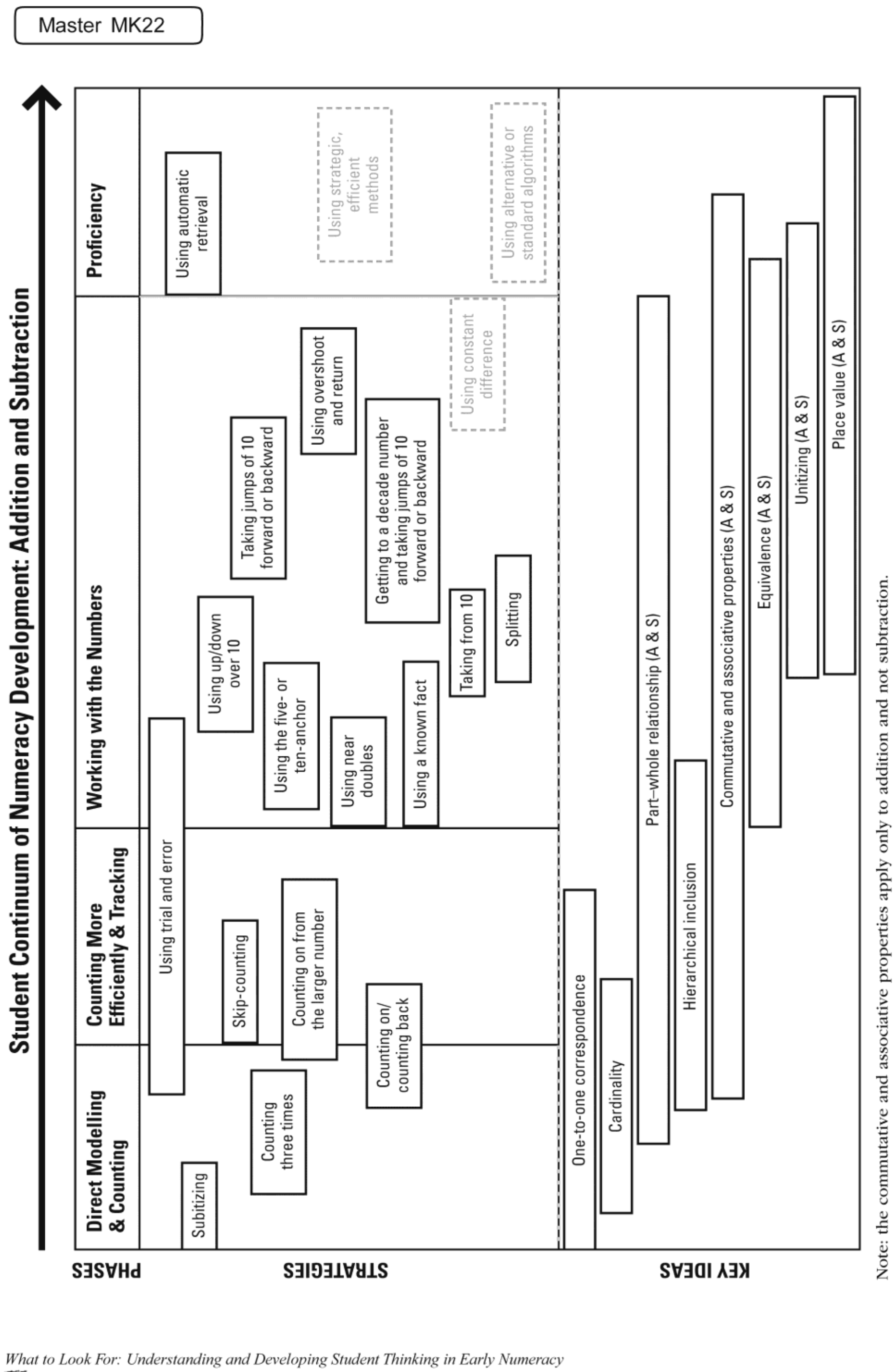
Appendices

Appendix A: Student Continuum of Numeracy Development: Addition and Subtraction[9]



What to Look For: Understanding and Developing Student Thinking in Early Numeracy

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[9] *What to Look For: Understanding and Developing Student Thinking in Early* Numeracy (Lawson, 2015, p. 4).

Appendix B: Principal Letter

(to be printed on university letterhead)

March 24, 2017

Dear [Principal's Name],

Thank you for considering participation in this study. My goal for my Master of Education thesis is to investigate the impact of daily number talks on the development of addition and subtraction strategies. The title of my study is *The impact of daily number talks on the development of mental math abilities of second graders within a reform-based classroom.* Presently, there is very little research evidence on the impact of number talks.

In order to gather the information needed for the study, I will be conducting daily number talks for six weeks in [Teacher's] Grade 2 classroom. Number talks are approximately fifteen-minute classroom conversations around purposefully crafted computation problems that students solve mentally. I will complete a pre- and postassessment (Math Running Record) on each student to determine what they have learned over the course of the number talk lessons. [Teacher] will have access to the assessments. Some samples of students' work will be collected. Also, with permission, some groups of students will be videotaped so that I will be able to listen carefully to how they have solved the problems. Conversations may be transcribed and quoted anonymously in my final project in order to illustrate strategies used. My supervisor, Dr. Lawson, or I may also make use of some of the edited classroom footage and student work samples for professional development of teachers.

This research will not take away from the normal learning environment in the classroom, and there is no apparent risk. If parents choose not to have a child participate, the child will still be engaged in the math lessons. The only difference is that his or her data will not be used. If parents give permission for a child to participate, the child will also be asked whether he or she is willing to take part in this research.

I hope [Teacher] and her students will participate for the duration of the study; however, as the Principal, you may withdraw your permission at any time, for any reason, without penalty, as participation is entirely voluntary. I do not anticipate any negative consequences as a result of participation in this study.

The [Name of] School Board, [Name of] School, [Teacher] and her students will not be identified in any written publication, including my master's thesis, possible journal articles or conference presentations. If video data is used for professional development, the students will be identified by first name only, however, if students use the teacher's surname it may be revealed. The raw data that is collected will be securely stored at Lakehead University for a minimum of five years after completion of the project. A report of the research will be available upon request.

The research project has been approved by Lakehead University Research Ethics Board and a letter of support has been obtained from the [Name of] School Board. If you have any questions related to the ethics of the research and would like to speak to someone outside of the research team, please contact Sue Wright at the Research Ethics Board at 807-343-8283 or research@lakeheadu.ca.

You are welcome to contact me at 902-521-5257 or stewara@gnspes.ca if you have any questions concerning this research project. I would be pleased to speak with you.

If you agree to participate in the study, please sign the attached letter of consent and return it to me. Please keep this letter in case you would like to contact any of us.

Sincerely,

Angela Stewart

Mrs. Angela Stewart
Master of Education Student
Lakehead University
902-521-5257
stewara@gnspes.ca

Dr. Alex Lawson, Ph.D.
Thesis Supervisor
Lakehead University
807-343-8720
alawson@lakeheadu.ca

Ms. Sue Wright
Research Ethics Board
Lakehead University
807-343-8283
research@lakeheadu.ca

Appendix C: Principal Consent Form

(to be printed on letterhead)
Principal Consent Form

I ____, agree to participation in the study with
(Principal's Name/please print)

Angela Stewart as described in the attached letter.

I understand that:

1. [Teacher] and her students will be videotaped in the classroom environment as part of the research.
2. Their participation is entirely voluntary and I can withdraw permission at any time, for any reason, with no penalty.
3. There is no apparent danger of physical or psychological harm.
4. In accordance with Lakehead University policy, the raw data will remain confidential and securely stored at Lakehead University for a minimum of five years.
5. The [Name of] School Board, [Name of] School, [Teacher] and her students will remain anonymous in any written publication resulting from the research project.
6. The video clips of the classroom or student work may be included in Professional Development for teachers conducted by Angela Stewart or Dr. Lawson. If [Teacher] appears in video clips, she may be identified by surname. If students appear in the video clips, they will be identified by first name only.

☐ I initial this box to give permission for [Teacher] and her students to appear in video clips which may be used for Professional Development purposes, as outlined above in 6.

If you approve of participation in my study, please complete this page and return it to me.

____
Name of Principal (please print)

____ ____________________________

Signature of Principal (please print) Date

Appendix D: Parent Letter

(to be printed on letterhead)

March 24, 2017

Dear Parents/Guardians of Potential Participant,

My name is Angela Stewart and I am a P-3 Mathematics Coach for the [Name of] School Board. I am also working on my Master of Education degree at Lakehead University. In partial fulfillment of my degree, I am conducting a research study called *The impact of daily number talks on the development of mental math abilities of second graders within a reform-based classroom.* This is an invitation for your child to participate in this study. Participation is optional. The purpose of this research is to gain a better understanding of the impact of daily number talks (mental math) on the mathematical development of students and how this can be used to assess and plan for instruction.

In my role as a mathematics coach, I have been leading number talks and assessing students in various classrooms throughout the board. However, your child's class has been selected to allow me to conduct a formal study of the impact of number talks on student development. While I am inviting all 22 students in [Teacher's] class to participate in this study, given the large number of students, I will follow six children in particular to assess any changes in their thinking over the course of the study. I will select the six students in consultation with their classroom teacher.

As part of the study, I will be conducting daily number talks for six weeks during the spring of 2017. Number talks are approximately fifteen-minute classroom conversations around math problems that students solve mentally. Data will be collected from multiple sources including student interviews, video recordings of number talks, and a reflective journal where I will record daily observations regarding number talk lessons. I will also complete a Math Running Record on each student to determine their mathematical thinking. Furthermore, students will complete a questionnaire at the end of the six weeks of number talks. The purpose of the questionnaire will be for me to gain understanding of each student's thoughts and opinions regarding number talks along with assessing their ability to name and explain strategies. All students will complete questionnaires at the same time in their classroom. Photos of the whiteboard will be taken at the end of each number talk to document the strategies used by students. At times, student work may be completed on notepaper or chart paper. Some student work completed in class will be collected, but only data from the six students selected will be used in the research. Photocopies of student work will be made so original work can be handed back to students; photos will be taken of all work on chart paper.

If you choose for your child to participate in this study, their participation in number talks will be videotaped allowing me to listen carefully as students explain strategies used to solve addition and subtraction questions. Their conversations may be transcribed and quoted anonymously in my final project in order to illustrate their use of strategies. My supervisor, Dr. Lawson, or I may also make use of some of the edited classroom footage, video recordings, and/or work samples for professional development of teachers.

Your child will not be identified in any written publication, including my master's thesis, possible journal articles, however, if video data is used for professional development, your child may be identified by first name only (therefore, anonymity and confidentiality would not be maintained). The raw data that is collected will be securely stored at Lakehead University for a minimum of five years. Participation in this study is voluntary, and you may withdraw the use of your child's data at any time until the thesis is completed, for any reason, without penalty. The Lakehead University Research Ethics Board, the [Name of] School Board, and the Principal of [Name of] School have approved the research project. If you have any questions related to the ethics of the research and would like to speak to someone outside of the research team, please contact Sue Wright at the Research Ethics Board at 807-343-8283 or research@lakeheadu.ca.

This research will not take away from the normal learning environment in the classroom, and there is no risk to any of the students involved. This research is simply being conducted to document the impact of daily number talks. If you choose not to have your child participate, he or she will still be engaged in the daily number talks, I will still complete two Math Running Records on each child, and all students will complete the questionnaire. The only difference is that if students opt out of the study, their data will not be used. If you give permission for your child to participate, your child will also be asked whether he or she is willing to take part in this research.

You are welcome to contact me at [School's phone number] or stewara@gnspes.ca if you have any questions concerning this research project. I would be very pleased to speak with you.

If you agree to allow your child to participate in the study, please sign the attached letter of consent and return it to [Teacher] at the school. Please keep this letter in case you would like to contact any one of us.

Sincerely,

Angela Stewart

Mrs. Angela Stewart
Master of Education Student
Lakehead University
[School's phone number]
stewara@gnspes.ca

Dr. Alex Lawson, Ph.D.
Thesis Supervisor
Lakehead University
807-343-8720
alawson@lakeheadu.ca

[Name of Principal]
[Name of] School
[School's phone number]
[Principal's e-mail address]

Ms. Sue Wright
Research Ethics Board
Lakehead University
807-343-8283
research@lakeheadu.ca

Appendix E: Parent/Guardian Consent Form

(to be printed on letterhead)
Parent/Guardian Consent Form

I DO give permission for my son/daughter, ____,
(Student's Name/please print)

to participation in the study with Angela Stewart as described in the attached letter.

I understand that:
1. My child will be videotaped in the classroom environment as part of the research.
2. My child's participation is entirely voluntary, and I can withdraw permission at any time, for any reason, with no penalty.
3. There is no apparent danger of physical or psychological harm.
4. In accordance with Lakehead University policy, the raw data will remain confidential and securely stored at Lakehead University for a minimum of five years.
5. All participants will remain anonymous in any publication resulting from the research project.
6. The video clips of the classroom or student work may be included in Professional Development for teachers conducted by me or Dr. Lawson. If my child appears in the video clips, he or she will be identified by first name only (therefore anonymity and confidentiality will not be maintained).

☐ I initial this box to give permission for my child to appear in video clips which may be used for Professional Development purposes, as outlined above in 6.

7. I can receive a summary of the project, upon request, following the completion of the project, by calling or writing, or by providing my address or email address below.

Please keep the introductory letter on file should you have any further questions. If you agree to let you child take part in the study, please complete this page and have your child return it [Teacher].

____

Name of Parent/Guardian (please print)

____ ____________________________

Signature of Parent/Guardian Date

Address or email address (if you would like a summary of the findings):

____

Appendix F: Teacher Letter

(to be printed on letterhead)

March 24, 2017

Dear [Teacher's Name],

Thank you for considering participation in this study. My goal for my Master of Education thesis is to investigate the impact of daily number talks on the development of addition and subtraction strategies. The title of my study is *The impact of daily number talks on the development of mental math abilities of second graders within a reform-based classroom.*

In order to gather the information needed for the study, I will be conducting daily number talks for six weeks in your classroom if you agree to participate. Number talks are approximately fifteen-minute classroom conversations around purposefully crafted computation problems that students solve mentally. The students will complete a pre- and postassessment to determine what they have learned over the course of the number talk lessons. You will have access to the assessment results. Some samples of students' work will be collected. Some of the lessons may be videotaped, or I made do a video recording of the number talks. Also, with permission, some groups of students will be videotaped so that I will be able to listen carefully to how they have solved the problems. Conversations may be transcribed and quoted anonymously in my final project in order to illustrate strategies used. My supervisor, Dr. Lawson, or I may also make use of some of the edited classroom footage and student work samples for professional development of teachers.

If you agree to participate, as part of the project, you will need to: distribute and collect cover letters and permission forms from parents/guardians; collect student work; and, allow time for me to complete testing. I will ensure that all resources needed for each lesson will be provided. I hope that you will participate for the duration of the study; however, you may withdraw at any time, for any reason, without penalty, as your participation is entirely voluntary. I do not anticipate any negative consequences as a result of participation in this study.

You and your students will not be identified in any written publication, including my master's thesis, possible journal articles or conference presentations. If video data is used for professional development, your students will be identified by first name only, but if children use your surname, it may be revealed. The raw data that is collected will be securely stored at Lakehead University for a minimum of five years after completion of the project. A report of the research will be available upon request. I can be reached at 902-521-5257 or stewara@gnspes.ca. if you have any questions or concerns.

The research project has been approved by the Lakehead University Research Ethics Board, the [Name of] School Board, and the Principal of [Name of] School. If you have any questions related to the ethics of the research and would like to speak to someone outside of the research team, please contact Sue Wright at the Research Ethics Board at 807-343-8283 or research@lakeheadu.ca.

If you agree to participate in the study, please sign the attached letter of consent and return it to me.

Sincerely,

Angela Stewart

Mrs. Angela Stewart
Master of Education Student
Lakehead University
902-521-5257
stewara@gnspes.ca

Dr. Alex Lawson, Ph.D.
Thesis Supervisor
Lakehead University
807-343-8720
alawson@lakeheadu.ca

Ms. Sue Wright
Research Ethics Board
Lakehead University
807-343-8283
research@lakeheadu.ca

Appendix G: Teacher Consent Form

(to be printed on letterhead)
Teacher Consent Form

I ____, do agree to participation in the study with
(Teacher's Name/please print)

Angela Stewart as described in the attached letter.

I understand that:

1. I may be videotaped in the classroom environment as part of the research.
2. My participation is entirely voluntary, and I can withdraw permission at any time, for any reason, with no penalty.
3. There is no apparent danger of physical or psychological harm.
4. In accordance with Lakehead University policy, the raw data will remain confidential and securely stored at Lakehead University for five years and then destroyed.
5. I will remain anonymous in any publication resulting from the research project.
6. The video clips of the classroom or student work may be included in Professional Development for teachers conducted by Angela Stewart or Dr. Lawson. If I appear in the video clips, I may be identified by surname.

☐ I initial this box to give my permission to appear in video clips which may be used for Professional Development purposes, as outlined above in 6.

____

Name of Third Party (please print)

____ ____________________________

Signature of Third Party Date

____

Name of Teacher (please print)

____ ____________________________

Signature of Teacher Date

Appendix H: Script for Student Consent

Script for Student Consent

The following will be read aloud to the class by the classroom teacher prior to the start of the research project:

"As many of you are aware, Mrs. Stewart is a Primary to Grade 3 math coach with our school board. She is also a graduate student at Lakehead University where she is learning even more about math! Mrs. Stewart is doing a study in one of her courses and is wondering if each of you would like to participate. Over the next six weeks, Mrs. Stewart will be leading number talks. During number talks, Mrs. Stewart will ask you some questions where you will share, justify, or defend your answers. Before we begin number talks and after six weeks of number talks, Mrs. Stewart will meet with each of you separately to complete a Math Running Record. During her time with us, she will collect some of your work, and if you would like to participate, she may videotape you as you share your strategies. Mrs. Stewart and I want to help other teachers learn about number talks so we might share with other teachers, some of the video clips of you. If at any time you do not want to be recorded or would rather not have your work shared, please tell Mrs. Stewart or me, and we will make sure that does not happen."

Appendix I: Pre- and Postassessment[10]

Addition Running Record **Part 1: Strategy Level, Accuracy, and Efficiency**			
3 + 3	a wo sc asc pth g f c ca co coh coln d/nd m10 a10 kfa kfc	AD	1 2 3 4
4 + 5	a wo sc asc pth g f c ca co coh coln d/nd m10 a10 kfa kfc	AD1	1 2 3 4
5 + 3	a wo sc asc pth g f c ca co coh coln d/nd m10 a10 kfa kfc	A5a	1 2 3 4
5 + 8	a wo sc asc pth g f c ca co coh coln d/nd m10 a10 kfa kfc	A5a AM10	1 2 3 4
0 + 9	a wo sc asc pth g f c ca co coh coln d/nd m10 a10 kfa kfc	A0	1 2 3 4
3 + 7	a wo sc asc pth g f c ca co coh coln d/nd m10 a10 kfa kfc	AS10	1 2 3 4
10 + 4	a wo sc asc pth g f c ca co coh coln d/nd m10 a10 kfa kfc	A10	1 2 3 4
7 + 7	a wo sc asc pth g f c ca co coh coln d/nd m10 a10 kfa kfc	AD	1 2 3 4
5 + 6	a wo sc asc pth g f c ca co coh coln d/nd m10 a10 kfa kfc	AD1	1 2 3 4
7 + 5	a wo sc asc pth g f c ca co coh coln d/nd m10 a10 kfa kfc	AD2 AM10	1 2 3 4
9 + 6	a wo sc asc pth g f c ca co coh coln d/nd m10 a10 kfa kfc	AM10 AC	1 2 3 4
8 + 9	a wo sc asc pth g f c ca co coh coln d/nd m10 a10 kfa kfc	AD1 AM10 AC	1 2 3 4
4 + 9	a wo sc asc pth g f c ca co coh coln d/nd m10 a10 kfa kfc	AM10 AC	1 2 3 4
3 + 6	a wo sc asc pth g f c ca co coh coln d/nd m10 a10 kfa kfc	A3	1 2 3 4
Accuracy & Efficiency: ✓ – Correct ✗ – Incorrect a – Automatic retrieval wo – Wrong operation sc – Self-corrected asc – Attempted self-correction pth – Prolonged thinking time g – Guessed (Ask student to prove it) **Model:** f – fingers; c – cubes **Strategies:** ca – Counting three times (counting all – direct modelling) co – Counting on (direct modelling) coh – Counting on in head & tracking coln – Counting on from the larger number d/nd – Doubles or near doubles m10 – Make-10 a10 – Adding 10 kfa – Using known fact (adjusting) kfc – Using known fact (compensation)		**Addition Facts:** AD – Doubles Facts AD1 – One-Apart (Near-Doubles) Facts AD2 – Two-Apart (Doubles + 2) Facts A5a – Adding 5-Anchor Facts A0 – Plus-Zero Facts AS10 – Sum of 10 Facts A10 – Adding 10 to a Number Facts AM10 – Make-10 Facts AC – Adding Compensation A3 – Plus-Three Facts	**Phases:** 1 – Direct Modelling & Counting 2 – Counting More Efficiently & Tracking 3 – Working with the Numbers 4 – Proficiency

[10] Adapted from *Math Running Records in Action: A Framework for Assessing Basic Fact Fluency in Grades K-5* (Newton, 2016, pp. 43, 46, 50, 97, 101).

<table>
<tr><th colspan="5">Part 2: Addition Flexibility Assessment</th></tr>
<tr>
<td>Small Doubles & One-Apart (Near-Doubles) Facts

What strategy would you use to solve these facts?
4 + 4 =

5 + 5 =

3 + 4 =

AD & AD1</td>
<td>Five-Anchor Facts to 10

What strategy would you use to solve these facts?
5 + 2 =

5 + 4 =

A5a</td>
<td>Five-Anchor Facts over 10

What strategy would you use to solve these facts?
5 + 7 =

5 + 9 =

A5a</td>
<td>Plus-Zero Facts

8 + 0 =
5 + 0 =
What happens when you add zero to a number?

A0</td>
<td>Sum of 10 Facts

2 + 8 =
9 + 1 =
4 + 6 =
5 + 5 =
What do you notice about these facts?

AS10</td>
</tr>
<tr>
<td>Adding 10 Facts

10 + 2 =
10 + 8 =
10 + 5 =
What do you do when you add 10 and a number?

A10</td>
<td>Larger Doubles Facts

8 + 8 =
6 + 6 =
9 + 9 =
7 + 7 =
What kind of facts are these?

AD</td>
<td>One-Apart (Near-Doubles) Facts

If a friend did not know how to solve 6 + 7, what would you tell them to do?

What about 7 + 8?

AD1</td>
<td>Two-Apart Facts (Doubles + 2)

If a friend did not know how to solve these facts, what would you tell them to do?
7 + 9 =

6 + 8 =

AD2</td>
<td>Make-10 Facts

What strategy do you use to solve these problems?
5 + 8 =

7 + 4 =

AM10</td>
</tr>
<tr><td colspan="5">Compensation
If a friend did not know how to solve these facts, what would you tell them to do?
9 + 5 =   3 + 9 =
AC</td></tr>
<tr><td colspan="5">Comments:</td></tr>
</table>

Subtraction Running Record
Part 1: Strategy Level, Accuracy, and Efficiency

2 – 0	a wo sc asc pth g f c ca fcb cbh fcu cuh urf b10 u10	S0	1 2 3 4
7 – 1	a wo sc asc pth g f c ca fcb cbh fcu cuh urf b10 u10	S1	1 2 3 4
9 – 9	a wo sc asc pth g f c ca fcb cbh fcu cuh urf b10 u10	SN	1 2 3 4
5 – 4	a wo sc asc pth g f c ca fcb cbh fcu cuh urf b10 u10	Sw5	1 2 3 4
8 – 3	a wo sc asc pth g f c ca fcb cbh fcu cuh urf b10 u10	Sw10	1 2 3 4
10 – 2	a wo sc asc pth g f c ca fcb cbh fcu cuh urf b10 u10	Sf10	1 2 3 4
12 – 11	a wo sc asc pth g f c ca fcb cbh fcu cuh urf b10 u10	SD1	1 2 3 4
14 – 7	a wo sc asc pth g f c ca fcb cbh fcu cuh urf b10 u10	SHF	1 2 3 4
17 – 10	a wo sc asc pth g f c ca fcb cbh fcu cuh urf b10 u10	S10T	1 2 3 4
13 – 3	a wo sc asc pth g f c ca fcb cbh fcu cuh urf b10 u10	S1T	1 2 3 4
16 – 7	a wo sc asc pth g f c ca fcb cbh fcu cuh urf b10 u10	B10	1 2 3 4
15 – 9	a wo sc asc pth g f c ca fcb cbh fcu cuh urf b10 u10	U10	1 2 3 4

Accuracy & Efficiency:

✓ – Correct
✗ – Incorrect
a – Automatic retrieval
wo – Wrong operation
sc – Self-corrected
asc – Attempted self-correction
pth – Prolonged thinking time
g – Guessed
Model: f – fingers; c - cubes

Strategies:

ca – Counting three times (counting all – direct modelling)
cb – Counting back (direct modelling)
cbh – Counting back in head & tracking
cu – Counting up (direct modelling)
cuh – Counting up in head & tracking
urf – Used related fact (think-addition strategy)
b10 – Back-through-10
u10 – Up-through-10

Comments:

Subtraction Facts:

SO – Subtracting 0
S1 – Subtracting 1
SN – Subtracting a number from itself
Sw5 – Subtracting within 5
Sw10 – Subtracting within 10
Sf10 – Subtracting from 10
SD1 – Subtracting difference of 1
SHF – Subtracting half facts
S10T – Subtracting 10 from a number
S1T – Subtracting ones digit from a teen number
B10 – Back-through-10
U10 – Up-through-10

Phases:

1 – Direct Modelling & Counting
2 – Counting More Efficiently & Tracking
3 – Working with the Numbers
4 – Proficiency

<table>
<tr><th colspan="4">Part 2: Subtraction Flexibility Assessment</th></tr>
<tr>
<td>Subtracting from 10

10 – 4 =
10 – 7 =
10 – 9 =
10 – 8 =
10 – 6 =

What do you do to solve these problems?

Sf10</td>
<td>Think-Addition (Subtracting with a Difference of 1 or 2)

7 – 5 =
9 – 8 =

SD1/SD2</td>
<td>Half Facts

16 – 8 =
14 – 7 =
18 – 9 =
12 – 6 =
10 – 5 =

What do you notice about these facts?

SHF</td>
<td>Subtracting 10 from a Teen Number

19 – 10 =
15 – 10 =
17 – 10 =
14 – 10 =

What do you do when you take 10 from a teen number?

S10T</td>
</tr>
<tr>
<td>Subtracting the Ones Digit from a Teen Number

14 – 4 =
17 – 7 =
12 – 2 =

What do you think to do when you solve these problems?

S1T</td>
<td>Back-Through-10

If your friend was having trouble solving 15 – 6, what would you tell your friend to do?

What about 14 – 5?

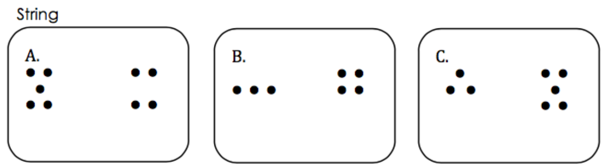
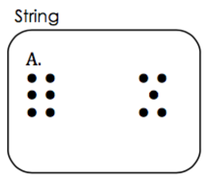
B10</td>
<td colspan="2">Up-Through-10

If your friend was having trouble solving 16 – 9, what would you tell your friend to do?

What about 14 – 8?

U10</td>
</tr>
<tr><td colspan="4">Comments:</td></tr>
</table>

Appendix J: Daily Number Talk Planning Sheet

Week 1	
Day 1 April 10, 2017 Dot Cards NT[11], p. 99 Absent:	**Doubles +/- 1 or 2**   Cheer: Fireworks
Day 2 April 11, 2017 Rekenrek Minilessons[12], p. 25 Absent: Kent Denise	**B3 Using the Five- and Ten-Structures** 5 on the bottom 8 on the bottom 5 on the top, 4 on the bottom 5 on the top, 5 on the bottom 5 on the top (skipped) 6 on the bottom 7 on the top, 5 on the bottom Cheer: Fireworks
Day 3 April 12, 2017 Rekenrek Minilessons, p. 26 Absent: Kent Ugo did not participate	**B5 Using the Five-Structure, Compensation, Make Ten** 8 on the top, 2 on the bottom 9 on the top, 1 on the bottom 7 on the top, 3 on the bottom 8 on the top, 4 on the bottom 9 on the top, 6 on the bottom 8 on the top, 5 on the bottom Cheer: Fireworks
Day 4 April 13, 2017 Rekenrek Minilessons, pp. 29-30 Absent: Kent	**B10 & B11 Using the Five- and Ten-Structures, Relating Addition, and Subtraction** **How many beads on the top are missing to complete the ten?** 3 on the top 7 on the top 2 on the top

[11] The first day of number talks I used the following resource: Parrish, S. (2014). *Number talks: Helping children build mental math and computation strategies*. Sausalito, CA: Math Solutions.
[12] For all remaining number talks, I used the following resource: Fosnot, C. T., & Uittenbogaard, W. (2007a). *Minilessons for early addition and subtraction: A yearlong resource*. Orlando, FL: Harcourt School Publishers.

	6 on the top 9 on the top 5 on the top

Week 2	
Day 5 April 18, 2017 Rekenrek Minilessons, p. 30 Absent: Kent	**B12 Using the Five- and Ten-Structures, Relating Addition, and Subtraction** **How many beads on the top are missing to make 10?** 4 on the top 6 on the top 3 on the top 7 on the top 2 on the top 8 on the top 5 on the top Flashcards for review: 5 + 5 = 3 + 7 = 4 + 6 = 2 + 8 = 1 + 9 = Missing addends: __ + 5 = 10 1 + __ = 10 __ + 7 = 10 4 + __ = 10 2 + __ = 10 10 – 5 = 10 – 6 = 10 – 9 = 10 – 7 = 10 – 8 = **B13 Combinations that Make Ten, Compensation, Making Ten** 9 on the top, 2 on the bottom 8 on the top, 3 on the bottom Cheer: Sign Language Applause
Day 6 April 19, 2017 Rekenrek Minilessons, pp. 32-33 *Reinforce with Bridge-to-Ten Strategy Cards from The Box of Facts Absent: Kent	**B16 Compensation, Making Ten** **Look quickly! What do you see? How do you see it?** 10 on the top, 7 on the bottom 9 on the top, 8 on the bottom 10 on the top, 2 on the bottom 9 on the top, 3 on the bottom 9 on the top, 6 on the bottom 3 on the top, 8 on the bottom 8 on the top, 7 on the bottom (skipped)

Quinn Ian did not participate	Bridge-to-Ten Strategy Cards (Ten-Frames) 9 and 8 9 and 3 9 and 6 Cheer: Text "WOW" Game: Salute (What to Look For, pp. 167-168) 10 minutes
Day 7 April 20, 2017 Using Ten-Frame Dot Cards instead of Rekenrek Minilessons, p. 34 Absent:	**B18 Compensation, Making Ten** Bridge-to-Ten Strategy Cards (Ten-Frames) **Look quickly! What do you see? How do you see it?** 10 and 4 **What happens when you add 10 to a one-digit number?** 9 and 5 10 and 3 9 and 4 9 and 6 4 and 8 8 and 7 (also showed this on the Rekenrek) Whole class: **How many do you see? (using the Rekenrek)** Show 15. Take away 5 Show 18. Take away 8 Show 19. Take away 10 Show 11. How can I make 20? Show 20. Take away 5 Model 5 + 10 = 15 15 – 5 = 15 15 – 10 = 5 Cheer: High Five (since it's National High Five Day!)
Day 8 April 21, 2017 Rekenrek Minilessons, pp. 41-42 Absent:	**B34 Using the Five- and Ten-Structures, Assessment** Rekenrek **Show the image for a few seconds, write the problem, and then cover the rack.** 3 on the top, 5 on the bottom 4 on the top, 5 on the bottom 7 on the top, 5 on the bottom 7 on the top, 8 on the bottom 9 on the top, 7 on the bottom 9 on the top, 6 on the bottom 9 on the top, 5 on the bottom Cheer: A Round of Applause Gallery Walk—looking at all of the different strategies students have shared so far. Assessment: Show two different strategies for solving each problem: 8 + 6 and 9 + 8.

Week 3	
Day 9 April 24, 2017 Think-Addition Subtraction Strategy Cards from The Box of Facts Absent: Adam	**How many dots are missing?** 5 + __ = 10 __ + 9 = 10 __ + 3 = 6 2 + __ = 10 __ + 7 = 14 8 + __ = 16 __ + 6 = 12 4 + __ = 10 __ + 9 = 18 __ + 9 = 11 Challenge question (no visual): 15 – 9 = Cheer: Snap & Cheer
Day 10 April 25, 2017 Rekenrek Minilessons, p. 51 Absent: Mary Adam	**Partial Use of the Rekenrek** **D1 Using the Five- and Ten-Structures, Relating Addition, and Subtraction** Rekenrek **Start off with a quick image, and establish the total number of beads shown. Write it down and then write the remainder of the expression.** **How many beads do you see?** 2 on the top, 5 on the bottom; **How many now?** 7 + 8 7 on the top, 8 on the bottom; **How many now?** 15 – 7 Cheer: Snap & Cheer
Day 11 April 26, 2017 Rekenrek Minilessons, p. 51 Absent: Mary Adam	**D1 Using the Five- and Ten-Structures, Relating Addition, and Subtraction** Rekenrek **Start off with a quick image, and establish the total number of beads shown. Write it down and then write the remainder of the expression.** 7 on the top, 9 on the bottom; **How many now?** 16 – 5 8 on the top, 9 on the bottom; **How many now?** 17 – 10 Cheer: The Golf Clap
Day 12 April 27, 2017 Rekenrek Minilessons, p. 52 Absent: Adam	**D2 Using the Five- and Ten-Structures, Relating Addition, and Subtraction** Rekenrek **Start off with a quick image, and establish the total number of beads shown. Write it down and then write the remainder of the expression.** 3 on the top, 5 on the bottom; **How many now?** 8 + 6 8 on the top, 6 on the bottom; **How many now?** 14 - 8 • Also, show this with Missing-Addend Subtraction Cards and Bridge-to-Ten Strategy Cards from The Box of Facts **Using the Missing-Addend Subtraction Cards: What is the missing**

	part? 9 + __ = 15 **Using the Back-to-Ten Strategy Cards:** 16 – 9 Cheer: Sign Language Applause Game: Salute (What to Look For, pp. 167-168) 15 minutes
Day 13 April 28, 2017 Rekenrek Minilessons, p. 54 Absent: Adam Betty Ian	**Individual Rekenreks D6 Using the Five- and Ten-Structures, Relating Addition and Subtraction** **Write down the problem and ask students to use their own Rekenrek.** (In the discussion, have students describe how they set up the numbers on their rack, what the result is, and how they figured out their answers.) 5 + 6 11 – 6 13 – 9 4 + 9 16 – 7 Cheer: Cowboy cheer (one finger in the air and circle it around like a lasso while saying "YEEHAW!") Games: Make 10 (What to Look For, pp. 171-172) & Addition War (What to Look For, p. 176) 15 minutes

Week 4	
Day 14 May 2, 2017 Rekenrek Minilessons, p. 54 Absent: Pam	**Individual Rekenreks D5 Using the Five- and Ten-Structures, Relating Addition and Subtraction** **Write down the problem and ask students to use their own Rekenrek.** (In the discussion, have students describe how they set up the numbers on their rack, what the result is, and how they figured out their answers.) 14 – 6 6 + 8 13 – 8 Cheer: Rain Cheer
Day 15 May 3, 2017 Absent:	**Working with the Basic Facts without Modelling, Missing Addends** 8 + ___ = 13 9 + ___ = 17 Cheer: Two finger clap
Day 16 May 4, 2017 Absent: Ian	**Working with the Basic Facts without Modelling** 7 + ___ = 16 2 + ___ = 11 14 – 9 = ___ 15 – 8 = ___ Cheer: Raise the Roof (Good Thinking!)

Day 17 May 5, 2017 Absent: Ian Fran Ugo	**Working with the Basic Facts without Modelling** 14 – 5 = ___ Cheer: Raise the Roof (Good Thinking!)

Week 5	
Day 18 May 8, 2017 Absent: Helen Ian Fran (left early)	**Working with the Basic Facts without Modelling** 16 – 7 = ___ 13 – 9 = ___ 17 – 9 = ___ Cheer: Na-na-na Cheer Before students return to their seats, do addition flash cards quickly. Then have students review their work from Friday in small groups.
Day 19 May 9, 2017 Absent: Ian Randy Quinn arrived late	Warm up with subtracting from 10 flashcards. **Working with the Basic Facts without Modelling** *12 – 11 = *13 – 3 = *7 – 5 = *9 – 8 = 16 – 8 = 15 – 9 = * Quick review, just one strategy verbally. Cheer: Round of Applause
Day 20 May 10, 2017 Absent: Ian Mary	Warm up with addition flash cards. **Working with the Basic Facts without Modelling** 13 – 9 = *Focus on one question and show the counting back strategy. Ask students to turn and talk with their elbow partner for a more efficient strategy. Cheer: Cowboy Cheer Game: Steal the Bundle
Day 21 May 11, 2017 Absent: Pam	Warm up with addition flash cards. **Working with the Basic Facts without Modelling** 12 – 9 = 11 – 7 = 12 – 4 = 14 – 7 = 17 – 8 = Cheer: Fireworks

	Game: Piggy Bank War

Week 6	
Day 22 May 15, 2017 Minilessons, p. 54 Absent: Helen Pam Ian left part way through	**Working with the Basic Facts without Modelling** Show a strategy for 16 – 9. Ask, "Will this strategy work?" *18 – 1 *18 – 17 *18 – 10 *18 – 8 18 – 9 *17 – 10 17 – 9 13 – 5 12 – 8 * Quick review, just one strategy verbally. Cheer: Rain Cheer
May 16, 2017	School cancelled ☹
Day 23 May 17, 2017 Absent: Ugo Pam Ian	**Working with the Basic Facts without Modelling** 15 – 6 13 – 4 Cheer: Text the word "W-O-W"
Day 24 May 18, 2017 Absent: Randy Larry	**Working with the Basic Facts without Modelling** 7 + 9 6 + 8 15 – 6 14 – 5 16 – 9 14 – 8 Cheer: Raise the Roof, Good Thinking

Appendix K: Postnumber Talk Questionnaire[13]

1. Name/explain as many addition strategies as you can.

____

____

2. Name/explain as many subtraction strategies as you can.

____

____

3. Are you better, the same, or worse in your ability to solve math problems in your head after doing number talks? ______________________

4. Do you feel that you learned new strategies for solving addition and subtraction problems from your classmates? If so, explain.

____

____

5. What did you like the best about number talks?

____

____

6. What did you like least about number talks?

____

____

[13] Adapted from "The Impact of Regular Number Talks on Mental Math Computation Abilities" (Johnson & Partlo, 2014, p. 38).

Appendix L: Emergent Codes

Category	Code	Definition
Answer (Accuracy)	Ans correct	answer is correct
	Ans correct_auto	answer is correct/automatic retrieval
	Ans correct_auto_sc	answer is correct/automatic retrieval/self-corrected
	Ans correct_sc	answer is correct/self-corrected
	Ans correct_asc_sc	answer is correct/attempted self-correction/self-corrected
	Ans correct_wo_sc	answer is correct/wrong operation/self-corrected
	Ans correct_wo_sc_auto	answer is correct/wrong operation/self-corrected/auto
	Ans incorrect	answer is incorrect
	Ans incorrect_auto	answer is incorrect/automatic retrieval
	Ans incorrect_asc	answer is incorrect/attempted self-correction
	Ans incorrect_wo	answer is incorrect/wrong operation
	Ans incorrect_dk	answer is incorrect/student did not know/could not solve
Facts - Addition	AFact_A1/2	n + 1 or n + 2 (up to 10) facts
	AFact_AsD/ND	adding: small doubles & near-doubles facts
	AFact_A5a≤10	adding: five-anchor facts to 10
	AFact_A5a>10	adding: five-anchor or ten-anchor facts over 10
	AFact_A0	n + 0 = n or 0 + n = n facts
	AFact_As10	sum of 10 facts
	AFact_A10	n + 10 or 10 + n facts
	AFact_AlgD/ND	adding: large doubles & near-doubles facts
	AFact_AM10	adding: make-10 facts
	AFact_A3	adding: plus-three facts
Facts - Subtraction	SFact_SD1/2	subtracting difference of 1 or 2
	SFact_S1	subtracting 1 (n – 1)
	SFact_S5a≤10	subtracting: five-anchor facts to 10
	SFact_S5a>10	subtracting: five-anchor or ten-anchor facts over 10
	SFact_S0	subtracting 0 (n – 0)
	SFact_SN	subtracting a number from itself (n – n = 0)
	SFact_Sf10	subtracting from 10
	SFact_S10T	subtracting 10 from a number (n – 10)
	SFact_S1T	subtracting ones digit from a teen number
	SFact_SHF	subtracting half facts
	SFact_SB/U10	subtracting: back-through-10 or up-through 10
Phase 0 (Efficiency)	Stgy PHASE 0	can't explain thinking/skipped due to difficulty/incorrect answer
	Stgy 00_auto x	student automatically recalled fact, but answer is incorrect
	Stgy 00_can't explain thinking	student couldn't explain their thinking
	Stgy 00_skipped	student skipped the question stating they found it too difficult

Category	Code	Definition
Phase 1 (Efficiency)	Stgy PHASE 1	direct modelling & counting
	Stgy 01_ca	counting three times (counting all); direct modelling *addition:* student fully represents the problem with objects, then counts the objects to find a solution *subtraction:* student fully represents the problem, starting with the whole, separating what is taken away, and then counting what is left
	Stgy 01_co_dm	counting on: student fully represents the problem with objects, then counts on from the one set of objects to find a solution; direct modelling
	Stgy 01_coln_dm	counting on larger number: student fully represents the problem with objects, then counts on from the larger set of objects to find a solution; direct modelling
	Stgy 01_b10 with concrete support	back-through-ten strategy using manipulatives
Phase 2 (Efficiency)	Stgy PHASE 2	counting more efficiently & tracking
	Stgy 02_cb	counting back; counting more efficiently and tracking
	Stgy 02_co	counting on; counting more efficiently and tracking
	Stgy 02_coln	counting on from the larger number; counting more efficiently and tracking
	Stgy 02_cu	counting up; counting more efficiently and tracking
Phase 3 (Efficiency)	Stgy PHASE 3	working with the numbers
	Stgy 03_5/10 anchor	using the five- or ten-anchor; working with the numbers (operating on or with the numbers)
	Stgy 03_b10	back-through-ten; working with the numbers (operating on or with the numbers)
	Stgy 03_d/nd	using doubles or near doubles; working with the numbers (operating on or with the numbers)
	Stgy 03_u10	up-through-ten; working with the numbers (operating on or with the numbers)
	Stgy 03_urf	using related fact; working with the numbers (operating on or with the numbers) *addition:* using known fact—adjusting (when finding the sum, if you add to or subtract from an addend, then the same amount must be added to or subtracted from the sum); compensation (taking an amount from one number and "giving" it to the other number results in the same sum) *subtraction:* using a related fact (think-addition strategy) or student knows a nearby fact and adjusts
	Stgy 03_using10	using up/down over 10; working with the numbers (operating on or with the numbers) addition: making 10 subtraction: back-through-10 or up-through-10

Category	Code	Definition
Phase 4 (Efficiency)	Stgy PHASE 4	proficiency
	Stgy 04_prof	proficiency: student can automatically recall fact but also understands the relationships within each fact and can find the answer another way if they momentarily forget the fact
Opinions	Opn mm_better	student feels they are better in their ability to solve math problems in their head after doing number talks
	Opn mm_same	student feels they are the same in their ability to solve math problems in their head after doing number talks
	Opn mm_worse	student feels they are worse in their ability to solve math problems in their head after doing number talks
	Opn learned_yes	student feels they learned from their classmates
	Opn learned_no	student feels they did not learn from their classmates
	Opn liked_best	what student liked the best about number talks
	Opn liked_least	what student liked the least about number talks
Documents	Doc pra00	document: preassessment week 00 day 00
	Doc mia08	document: midassessment (1) week 02 day 08
	Doc mia17	document: midassessment (2) week 04 day 17
	Doc poq24	document: postquestionnaire week 06 day 24
	Doc poa24	document: postassessment week 06 day 25
	Doc chp	document: chart paper
	Doc vid	document: video

Appendix M1: Correct Responses as a Percentage on Addition Preassessment Grouped by Fact Type (N = 19)

Fact Type	% Correct
Plus 0 (n = 57)	100
Plus 1 (n = 19)	100
Sm Doubles (n = 76)	100
Plus 10 (n = 76)	100
Plus 3 (n = 19)	100
Lg Doubles (n = 95)	99
Sm Near Doubles (n = 38)	97
Sum of 10 (n = 57)	96
5 Anchor < 10 (n = 57)	93
5 Anchor > 10 (n = 95)	93
Lg Near Doubles (n = 57)	91
Make-10 (n = 152)	90

Appendix M2: Correct/Incorrect Responses on Addition Preassessment Grouped by Specific Question and Phase (N = 19)

	Correct: Phase				Incorrect: Phase			
	1	2	3	4	0	1	2	3
Plus 0								
0 + 9				19				
5 + 0				19				
8 + 0				19				
Plus 1								
9 + 1				19				
Sm Doubles								
3 + 3				19				
4 + 4				19				
5 + 5				19				
5 + 5				19				
Lg Doubles								
6 + 6				19				
7 + 7	1			17	1			
7 + 7				19				
8 + 8	1			18				
9 + 9				19				
Plus 10								
10 + 2		2		17				
10 + 4		1		18				
10 + 5		1		18				
10 + 8		2		17				
Sum of 10								
2 + 8	1	4		14				
3 + 7	1	9		7			1	1
4 + 6	1	8	1	9				
Sm Near Doubles								
3 + 4		7	7	4				1
4 + 5	1	2	4	12				
5 Anchor < 10								
5 + 2		6		13				
5 + 3		6	4	7				2
5 + 4		3	12	2			1	1
Plus 3								
3 + 6		13	2	4				

	Correct: Phase				Incorrect: Phase			
	1	2	3	4	0	1	2	3
5 Anchor > 10								
5 + 6	1	2	9	7				
5 + 7	3	7	7	2				
5 + 8	3	3	9	1		1	1	1
5 + 8	3	4	10				1	1
5 + 9	3	5	9				1	1
Make-10								
3 + 9		5	14					
4 + 9	2	5	11	1				
6 + 8		2	13			1		3
7 + 4	1	5	12				1	
7 + 5	1	5	9				2	2
7 + 9	1	3	14					1
9 + 5		4	14					1
9 + 6	2	4	8	1		1	3	
Lg Near Doubles								
6 + 7	1	4	14					
7 + 8		2	16				1	
8 + 9	2	2	11			2	2	

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix M3: Correct/Incorrect Responses as a Percentage on Addition Preassessment Grouped by Phase (N = 19)

	Correct: Phase				Incorrect: Phase			
Fact Type	1	2	3	4	0	1	2	3
Plus 0 (n = 57)	0	0	0	100	0	0	0	0
Plus 1 (n – 19)	0	0	0	100	0	0	0	0
Sm Doubles (n = 76)	0	0	0	100	0	0	0	0
Lg Doubles (n = 95)	2	0	0	97	1	0	0	0
Plus 10 (n = 76)	0	8	0	92	0	0	0	0
Sum of 10 (n = 57)	5	37	2	53	0	0	2	2
Sm ND (n = 38)	3	24	29	42	0	0	0	3
5a < 10 (n = 57)	0	26	28	39	0	0	2	5
Plus 3 (n = 19)	0	68	11	21	0	0	0	0
5 a > 10 (n = 95)	14	22	46	11	0	1	3	3
Make-10 (n = 152)	5	22	63	1	0	1	4	5
Lg ND (n = 57)	5	14	72	0	0	4	5	0

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix N1: Correct Responses as a Percentage on Subtraction Preassessment (N = 19)

Fact Type	% Correct
Subtracting 1 (n = 19)	100
Subtracting 0 (n = 19)	95
Subtracting Ones from a Teen # (n = 76)	95
Subtracting N from Itself (n = 19)	89
Difference of 1 (n = 57)	89
5 Anchor ≤ 10(n = 57)	86
Subtracting from 10 (n = 114)	82
Subtracting 10 from a Teen Number (n = 95)	78
Subtracting Half Facts (n = 95)	75
5 Anchor > 10 (n = 19)	63
Subtracting B/U 10 (n = 95)	62

Appendix N2: Correct/Incorrect Responses on Subtraction Preassessment Grouped by Specific Question and Phase (N = 19)

	Correct: Phase					Incorrect: Phase			
	0	1	2	3	4	0	1	2	3
S 1									
7 – 1					19				
S 0									
2 – 0					18	1			
S N from Itself									
9 – 9					17	2			
Diff of 1									
5 – 4		1	1		15	1			1
9 – 8		2	2		13	1		1	
12 – 11		1	2		14				2
S 1s fr a Teen									
12 – 2			2	1	16				
13 – 3			5	4	8			2	
14 – 4		3	3	1	12				
17 – 7		2		1	14			2	
S 10 fr a Teen									
14 – 10		1	3	1	12	2			
15 – 10		2	3	1	10	2	1		
17 – 10		2	2	2	8		2	2	1
17 – 10		2	2		11	2		1	1
19 – 10		3			9	2	1	4	
S Half Facts									
12 – 6				2	13	1		2	1
14 – 7		1	1	2	10	1		3	1
14 – 7		1	3	2	11	1	1		
16 – 8		2	4	2	3	2	3	3	
18 – 9		1	1	2	10	2	1	2	
S fr 10									
10 – 2	1	1	5	2	9				1
10 – 4		2	8	4	4				1
10 – 6		1	3	5	3	1		3	3
10 – 7		2	5	2	2	3		4	1
10 – 8			1	4	11	1		1	1
10 – 9			3	1	14			1	
5 Anchor ≤ 10									
7 – 5		2	7	3	4	1		1	1
8 – 3			8	3	4			3	1
10 – 5			1	4	13	1			

	Correct: Phase					Incorrect: Phase			
	0	1	2	3	4	0	1	2	3
5 Anchor > 10									
14 – 5	1	3	2	5	1		1	2	4
S B/U 10									
14 – 8		1	4	4		1	1	3	5
15 – 6		3	3	7	1	1	1	1	2
15 – 9		5	6	1		1		3	3
16 – 7		3	2	4	2	1	3	2	2
16 – 9		6	6	2		1		3	1

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix N3: Correct/Incorrect Responses as a Percentage on Subtraction Preassessment Grouped by Phase (N = 19)

	Correct: Phase					Incorrect: Phase			
Fact Type	0	1	2	3	4	0	1	2	3
S1 (n = 19)	0	0	0	0	100	0	0	0	0
S 0 (n = 19)	0	0	0	0	95	5	0	0	0
S N from Itself (n = 19)	0	0	0	0	89	11	0	0	0
Diff of 1 (n = 57)	0	7	9	0	74	4	0	2	5
S 1s fr a Teen # (n = 76)	0	7	13	9	66	0	0	5	0
S 10 fr a Teen # (n = 95)	0	11	11	4	53	8	4	7	2
S Half Facts (n = 95)	0	5	9	11	49	7	5	11	2
S fr 10 (n = 114)	1	5	22	16	38	4	0	8	6
5 A ≤10(n = 57)	0	4	28	18	37	4	0	7	4
5 A > 10 (n = 19)	5	16	11	26	5	0	5	11	21
S B/U 10 (n = 95)	0	19	22	19	3	5	5	13	14

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix O: Correct/Incorrect Responses on Midassessments Grouped by Phase (N = 19)

	Correct: Phase					Incorrect: Phase		
	0	1	2	3	4	0	2	3
8 + 6 (n = 38)								
Adam	1			1				
Betty				2				
Denise				2				
Ellen	1			1				
Fran				2				
Helen			1	1				
Ian				2				
Jack	1			1				
Kent	1			1				
Larry				2				
Mary				2				
Oliver			1	1				
Pam			1	1				
Quinn				2				
Randy	1			1				
Steve	1			1				
Ugo	2							
Victor				2				
Walter			1	1				
9 + 8 (n = 38)								
Adam				2				
Betty				2				
Denise				2				
Ellen	2							
Fran	1			1				
Helen			1	1				
Ian				2				
Jack	1			1				
Kent	1			1				
Larry						2		
Mary				2				
Oliver						2		
Pam	1		1					
Quinn	1			1				
Randy	1			1				
Steve				2				
Ugo	2							
Victor				2				
Walter				2				

	Correct: Phase					Incorrect: Phase		
	0	1	2	3	4	0	2	3
5 + 8 (n = 19)								
Adam			1					
Betty				1				
Denise				1				
Ellen			1					
Fran				1				
Helen				1				
Ian			1					
Jack						1		
Kent								1
Larry			1					
Mary			1					
Oliver				1				
Pam			1					
Quinn				1				
Randy				1				
Steve				1				
Ugo				1				
Victor				1				
Walter				1				
6 + 9 (n = 19)								
Adam						1		
Betty				1				
Denise				1				
Ellen				1				
Fran				1				
Helen				1				
Ian			1					
Jack	1							
Kent							1	
Larry				1				
Mary				1				
Oliver				1				
Pam			1					
Quinn				1				
Randy								1
Steve				1				
Ugo				1				
Victor					1			
Walter				1				

	Correct: Phase					Incorrect: Phase		
	0	1	2	3	4	0	2	3
15 – 6 (n = 19)								
Adam				1				
Betty				1				
Denise							1	
Ellen	1							
Fran				1				
Helen			1					
Ian			1					
Jack	1							
Kent			1					
Larry				1				
Mary								1
Oliver							1	
Pam			1					
Quinn				1				
Randy				1				
Steve				1				
Ugo		1						
Victor				1				
Walter				1				
16 – 9 (n = 19)								
Adam								1
Betty				1				
Denise			1					
Ellen			1					
Fran								1
Helen								1
Ian			1					
Jack	1							
Kent							1	
Larry				1				
Mary							1	
Oliver							1	
Pam			1					
Quinn				1				
Randy				1				
Steve				1				
Ugo		1						
Victor								1
Walter								1

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix P1: Correct/Incorrect Responses on Addition Postassessment Grouped by Question and Phase (N = 19)

	Correct: Phase			Incorrect: Phase		
	1	2	3	4	2	3
Plus 0						
0 + 9				19		
5 + 0				19		
8 + 0				19		
Plus 1						
9 + 1				19		
Sm Doubles						
3 + 3				19		
4 + 4				19		
5 + 5				19		
5 + 5				19		
Lg Doubles						
6 + 6				19		
7 + 7				19		
7 + 7				19		
8 + 8			2	17		
9 + 9				19		
Plus 10						
10 + 2				19		
10 + 4				19		
10 + 5		1		18		
10 + 8		1		18		
Sum of 10						
2 + 8		1		18		
3 + 7		5	1	13		
4 + 6		2	1	16		
Sm Near Doubles						
3 + 4		3	5	11		
4 + 5		1	2	16		
5 Anchor ≤10						
5 + 2			1	18		
5 + 3		1	4	14		
5 + 4		3	8	8		
Plus 3						
3 + 6		7	1	11		

	Correct: Phase				Incorrect: Phase	
	1	2	3	4	2	3
5 Anchor > 10						
5 + 6		2	7	10		
5 + 7		3	9	6	1	
5 + 8		7	10	2		
5 + 8		3	14	2		
5 + 9		3	7	9		
Make-10						
3 + 9		2	5	12		
4 + 9		3	3	13		
6 + 8		3	14	2		
7 + 4		6	7	5		1
7 + 5		2	13	3		1
7 + 9		2	12	5		
9 + 5		3	7	9		
9 + 6		3	4	11		
Lg Near Doubles						
6 + 7		3	13	3		
7 + 8		1	15	3		
8 + 9	1	1	10	7		

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix P2: Correct/Incorrect Responses as a Percentage on Addition Postassessment Grouped by Phase (N = 19)

	Correct: Phase				Incorrect: Phase	
Fact Type	1	2	3	4	2	3
Plus 0 (n = 57)	0	0	0	100	0	0
Plus 1 (n = 19)	0	0	0	100	0	0
Sm Doubles (n = 76)	0	0	0	100	0	0
Lg Doubles (n = 95)	0	0	2	98	0	0
Plus 10 (n = 76)	0	3	0	97	0	0
Sum of 10 (n = 57)	0	14	4	82	0	0
Sm N Doubles (n = 38)	0	11	18	71	0	0
5 Anchor < 10 (n = 57)	0	7	23	70	0	0
Plus 3 (n = 19)	0	37	5	58	0	0
5 Anchor > 10 (n = 95)	0	19	49	31	1	0
Make-10 (n = 152)	0	16	43	39	1	1
Lg Near Doubles (n = 57)	2	9	67	23	0	0

Note. Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix P3: Phase by Individual Questions on Subtraction Postassessment (N = 19)

	Correct: Phase				Incorrect: Phase		
	1	2	3	4	0	2	3
S 1							
7 – 1				19			
S 0							
2 – 0				19			
S N from Itself							
9 – 9				19			
Diff of 1							
5 – 4		2		17			
9 – 8		1		18			
12 – 11				18			1
S 1s fr a Teen							
12 – 2		1		18			
13 – 3		1		18			
14 – 4		3		16			
17 – 7		2	1	16			
S 10 fr a Teen							
14 – 10				18		1	
15 – 10	1	1		17			
17 – 10	3	1		15			
17 – 10	1		1	16		1	
19 – 10	3	1		15			
S Half Facts							
12 – 6	2	1	1	14			1
14 – 7	2	2	1	14			
14 – 7	2	1	2	14			
16 – 8	3	3		11		1	1
18 – 9	1	1		16	1		
S fr 10							
10 – 2		2	1	16			
10 – 4	1	2	3	11		1	1
10 – 6		3	3	12			1
10 – 7	3	3	2	11			
10 – 8		1		18			
10 – 9		1		18			
5 Anchor ≤10							
7 – 5	3	5	3	6		2	
8 – 3		8	4	7			
10 – 5	1			18			
5 Anchor > 10							
14 – 5	2	1	5	10		1	

	Correct: Phase				Incorrect: Phase			
	1	2	3	4	0	2	3	Total
S B/U 10								
14 – 8	3	3	10	1			2	19
15 – 6	2	2	4	11				19
15 – 9	3	4	7	4			1	19
16 – 7	2	6	1	10				19
16 – 9	4	3	8	4				19

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix P4: Correct/Incorrect Responses as a Percentage on Subtraction Postassessment Grouped by Phase (N = 19)

	Correct: Phase				Incorrect: Phase		
Fact Type	1	2	3	4	0	2	3
S1 (n = 19)	0	0	0	100	0	0	0
S 0 (n = 19)	0	0	0	100	0	0	0
S N from Itself (n = 19)	0	0	0	100	0	0	0
Diff of 1 (n = 57)	0	5	0	93	0	0	2
S 1s fr a Teen # (n = 76)	0	9	1	89	0	0	0
S 10 fr a Teen # (n = 95)	8	3	1	85	0	2	0
S Half Facts (n = 95)	11	8	4	73	1	1	2
S fr 10 (n = 114)	4	11	8	75	0	1	2
5 A ≤10(n = 57)	7	23	12	54	0	4	0
5 A > 10 (n = 19)	11	5	26	53	0	5	0
S B/U 10 (n = 95)	15	19	32	32	0	0	3

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix Q1: Responses as a Percentage Grouped by Phases for each Fact Type on Addition Pre- and Postassessments (N = 19)

	Phase									
	0		1		2		3		4	
Fact Type	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Plus 0	0	0	0	0	0	0	0	0	100	100
Plus 1	0	0	0	0	0	0	0	0	100	100
Sm Doubles	0	0	0	0	0	0	0	0	100	100
Lg Doubles	1	0	2	0	0	0	0	2	97	98
Plus 10	0	0	0	0	8	3	0	0	92	97
Sum of 10	4	0	5	0	37	14	2	4	53	82
Sm ND	3	0	3	0	24	11	29	18	42	71
5 Anchor < 10	7	0	0	0	26	7	28	23	39	70
Plus 3	0	0	0	0	68	37	11	5	21	58
Make-10	10	2	5	0	22	16	63	43	1	39
5 Anchor > 10	7	1	14	0	22	19	46	49	11	31
Lg ND	9	0	5	2	14	9	72	67	0	23

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix Q2: Graph of Responses as a Percentage Grouped by Phase ad Fact Type on Addition Pre- and Postassessments

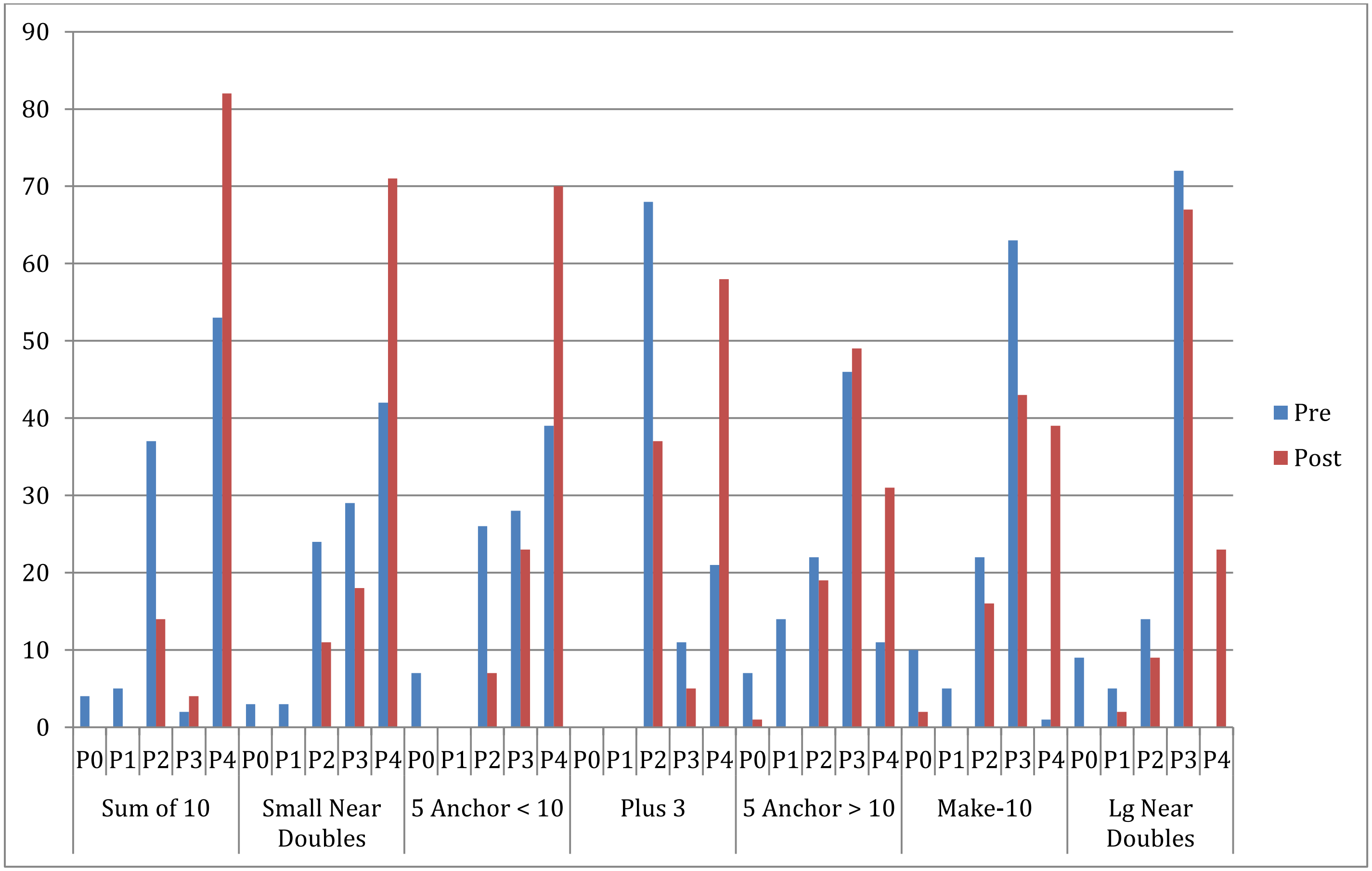


Figure 14. Responses as a percentage grouped by phases for each fact type on addition pre- and postassessments.

P0 = can't explain thinking/skipped due to difficulty/incorrect answer; P1 = direct modelling & counting; P2 = counting more efficiency & tracking; P3 = working with the numbers; P4 = proficiency.

Appendix Q3: Responses as a Percentage Grouped by Phases for each Fact Type on Subtraction Pre- and Postassessments (N = 19)

	Phase									
	0		1		2		3		4	
Fact Type	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Subtracting 1	0	0	0	0	0	0	0	0	100	100
Subtracting 0	5	0	0	0	0	0	0	0	95	100
S N from Itself	11	0	0	0	0	0	0	0	89	100
Difference of 1	11	2	7	0	9	5	0	0	74	93
S 1s fr Teen #	5	0	7	0	13	9	9	1	66	89
S 10 fr Teen #	22	2	11	8	11	3	4	1	53	85
S from 10	19	3	5	4	22	11	16	8	38	75
S Half Facts	25	4	5	11	9	8	11	4	49	73
5 Anchor ≤ 10	14	4	4	7	28	23	18	12	37	54
5 Anchor > 10	42	5	16	11	11	5	26	26	5	53
S B/U10	37	3	19	15	22	19	19	32	3	32

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix Q4: Graph of Responses as a Percentage Grouped by Phase and Fact Type on Subtraction Pre- and Postassessments

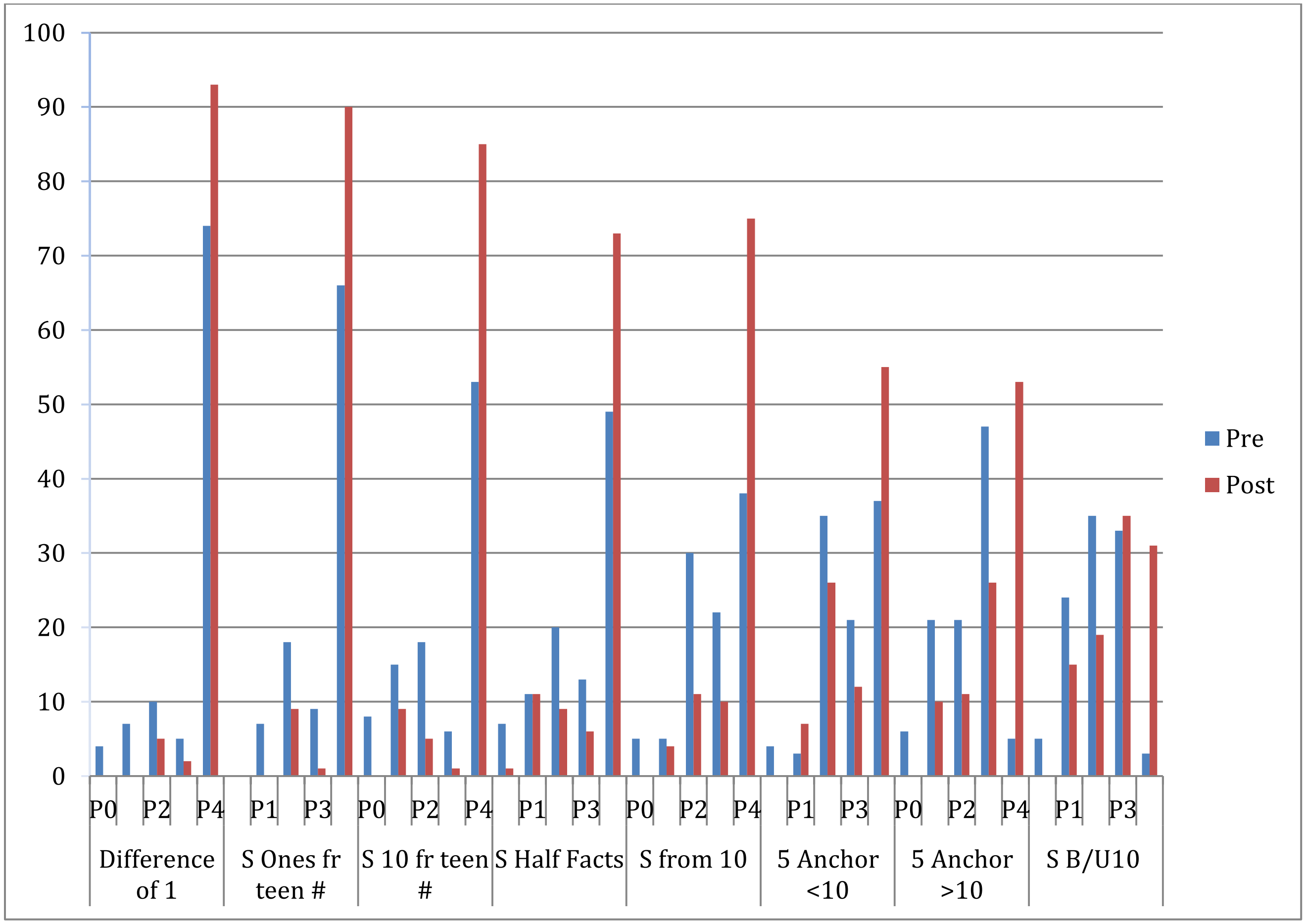


Figure 15. Responses as a percentage grouped by phases for each fact type on subtraction pre- and postassessments.

P0 = can't explain thinking/skipped due to difficulty/incorrect answer; P1 = direct modelling & counting; P2 = counting more efficiency & tracking; P3 = working with the numbers; P4 = proficiency.

Appendix R1: Responses on Randy's Addition Preassessments Grouped by Question and Phase

	Correct: Phase				Incorrect: Phase		
Fact Type/Question	1	2	3	4	1	2	3
Plus 0							
0 + 9				1			
5 + 0				1			
8 + 0				1			
Plus 1							
9 + 1				1			
Sm Doubles							
3 + 3				1			
4 + 4				1			
5 + 5				1			
5 + 5				1			
Lg Doubles							
6 + 6				1			
7 + 7				1			
7 + 7				1			
8 + 8				1			
9 + 9				1			
Add 10							
10 + 2				1			
10 + 4				1			
10 + 5				1			
10 + 8				1			
Sum of 10							
2 + 8		1					
3 + 7						1	
4 + 6		1					
Sm Near Doubles							
3 + 4							1
4 + 5	1						
5 Anchor < 10							
5 + 2		1					
5 + 3		1					
5 + 4						1	
Plus 3							
3 + 6		1					
5 Anchor > 10							
5 + 6	1						
5 + 7	1						
5 + 8			1				
5 + 8		1					
5 + 9	1						
Make 10							
3 + 9			1				
4 + 9			1				
6 + 8					1		
7 + 4						1	
7 + 5						1	
7 + 9			1				
9 + 5			1				
9 + 6			1				
Lg Near Doubles							
6 + 7		1					
7 + 8			1				
8 + 9			1				
Total (n = 42)	4	7	8	17	1	4	1

Note. Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix R2: Responses on Randy's Addition Postassessments Grouped by Question and Phase

	Correct: Phase		
Fact Type/Question	2	3	4
Plus 0			
0 + 9			1
5 + 0			1
8 + 0			1
Plus 1			
9 + 1			1
Sm Doubles			
3 + 3			1
4 + 4			1
5 + 5			1
5 + 5			1
Lg Doubles			
6 + 6			1
7 + 7			1
7 + 7			1
8 + 8			1
9 + 9			1
Add 10			
10 + 2			1
10 + 4			1
10 + 5			1
10 + 8			1
Sum of 10			
2 + 8			1
3 + 7			1
4 + 6			1
Sm Near Doubles			
3 + 4			1
4 + 5		1	
5 Anchor < 10			
5 + 2			1
5 + 3			1
5 + 4		1	
Plus 3			
3 + 6			1
5 Anchor > 10			
5 + 6	1		
5 + 7	1		
5 + 8	1		
5 + 8	1		
5 + 9		1	
Make 10			
3 + 9			1
4 + 9		1	
6 + 8		1	
7 + 4		1	
7 + 5		1	
7 + 9		1	
9 + 5		1	
9 + 6		1	
Lg Near Doubles			
6 + 7		1	
7 + 8		1	
8 + 9		1	
Total (n = 42)	4	13	25

Note. Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix R3: Responses on Randy's Subtraction Preassessment Grouped by Question and Phase

	Correct: Phase			Incorrect: Phase			
Fact Type/Question	0	2	4	0	1	2	3
S 1							
7 – 1			1				
S 0							
2 – 0			1				
S N from Itself							
9 – 9			1				
Diff of 1							
5 – 4		1					
9 – 8				1			
12 – 11							1
S 1s from a Teen							
12 – 2			1				
13 – 3			1				
14 – 4			1				
17 – 7			1				
S 10 from a Teen							
14 – 10				1			
15 – 10				1			
17 – 10				1			
17 – 10					1		
19 – 10				1			
S Half Facts							
12 – 6						1	
14 – 7				1			
14 – 7				1			
16 – 8				1			
18 – 9				1			
S from 10							
10 – 2	1						
10 – 4		1		1			
10 – 6				1			
10 – 7				1			
10 – 8							
10 – 9			1				
5 Anchor ≤ 10							
7 – 5				1			
8 – 3						1	
10 – 5				1			
5 Anchor > 10							
14 – 5	1						
S B/U 10							
14 – 8				1			
15 – 6				1			
15 – 9				1			
16 – 7				1			
16 – 9				1			
Total (n = 35)	2	2	8	19	1	2	1

Note. Phase 0 = can't explain thinking/skipped due to difficulty/ incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix R4: Responses on Randy's Subtraction Postassessment Grouped by Question and Phase

	Correct: Phase				Incorrect: Phase
Fact Type/Question	1	2	3	4	3
S 1					
7 – 1				1	
S 0					
2 – 0				1	
S N from Itself					
9 – 9				1	
Diff of 1					
5 – 4		1			
9 – 8				1	
12 – 11					1
S 1s from a Teen					
12 – 2				1	
13 – 3				1	
14 – 4				1	
17 – 7				1	
S 10 from a Teen					
14 – 10				1	
15 – 10	1				
17 – 10	1				
17 – 10	1				
19 – 10	1				
S Half Facts					
12 – 6	1				
14 – 7	1				
14 – 7	1				
16 – 8	1				
18 – 9	1				
S from 10					
10 – 2				1	
10 – 4				1	
10 – 6				1	
10 – 7				1	
10 – 8				1	
10 – 9				1	
5 Anchor ≤ 10					
7 – 5	1				
8 – 3			1		
10 – 5	1				
5 Anchor > 10					
14 – 5	1				
S B/U 10					
14 – 8	1				
15 – 6	1				
15 – 9	1				
16 – 7		1			
16 – 9	1				
Total (n = 35)	16	2	1	15	1

Note. Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix S1: Responses on Adam's Addition Preassessment Grouped by Question and Phase

	Correct: Phase				Incorrect: Phase	
	1	2	3	4	1	3
Plus 0						
0 + 9				1		
5 + 0				1		
8 + 0				1		
Plus 1						
9 + 1				1		
Sm Doubles						
3 + 3				1		
4 + 4				1		
5 + 5				1		
5 + 5				1		
Lg Doubles						
6 + 6				1		
7 + 7				1		
7 + 7				1		
8 + 8				1		
9 + 9				1		
Add 10						
10 + 2				1		
10 + 4				1		
10 + 5				1		
10 + 8				1		
Sum of 10						
2 + 8				1		
3 + 7		1				
4 + 6		1				
Sm Near Doubles						
3 + 4			1			
4 + 5				1		
5 Anchor < 10						
5 + 2				1		
5 + 3				1		
5 + 4			1			
Plus 3						
3 + 6		1				
5 Anchor > 10						
5 + 6				1		
5 + 7			1			
5 + 8					1	
5 + 8						1
5 + 9						1
Make 10						
3 + 9		1				
4 + 9		1				
6 + 8						1
7 + 4	1					
7 + 5						1
7 + 9			1			
9 + 5		1				
9 + 6		1				
Lg Near Doubles						
6 + 7			1			
7 + 8			1			
8 + 9			1			
Total (n = 42)	1	7	7	22	1	4

Note. Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix S2: Responses on Adam's Addition Postassessment Grouped by Question and Phase

	Correct: Phase		
Fact Type	2	3	4
Plus 0			
0 + 9			1
5 + 0			1
8 + 0			1
Plus 1			
9 + 1			1
Sm Doubles			
3 + 3			1
4 + 4			1
5 + 5			1
5 + 5			1
Lg Doubles			
6 + 6			1
7 + 7			1
7 + 7			1
8 + 8			1
9 + 9			1
Add 10			
10 + 2			1
10 + 4			1
10 + 5			1
10 + 8			1
Sum of 10			
2 + 8			1
3 + 7			1
4 + 6			1
Sm Near Doubles			
3 + 4			1
4 + 5			1
5 Anchor < 10			
5 + 2			1
5 + 3			1
5 + 4			1
Plus 3			
3 + 6	1		
5 Anchor > 10			
5 + 6			1
5 + 7		1	
5 + 8	1		
5 + 8	1		
5 + 9		1	
Make 10			
3 + 9		1	
4 + 9			1
6 + 8		1	
7 + 4	1		
7 + 5	1		
7 + 9		1	
9 + 5		1	
9 + 6		1	
Lg Near Doubles			
6 + 7		1	
7 + 8		1	
8 + 9			1
Total (n = 42)	5	9	28

Note. Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix S3: Responses on Adam's Subtraction Preassessment Grouped by Question and Phase

	Correct: Phase			Incorrect: Phase			
	1	2	4	0	1	2	3
S 1							
7 – 1			1				
S 0							
2 – 0			1				
S N from Itself							
9 – 9				1			
Diff of 1							
5 – 4							1
9 – 8	1						
12 – 11	1						
S 1s from a Teen							
12 – 2			1			1	
13 – 3							
14 – 4	1						
17 – 7	1						
S 10 from a Teen							
14 – 10			1				
15 – 10					1		
17 – 10					1		
17 – 10							1
19 – 10					1		
S Half Facts							
12 – 6			1				
14 – 7			1				
14 – 7			1				
16 – 8					1		
18 – 9			1				
S from 10							
10 – 2							1
10 – 4		1					
10 – 6						1	
10 – 7						1	
10 – 8						1	
10 – 9						1	
5 Anchor ≤10							
7 – 5	1						
8 – 3		1					
10 – 5			1				
5 Anchor > 10							
14 – 5					1		
S B/U 10							
14 – 8							1
15 – 6					1		
15 – 9	1						
16 – 7					1		
16 – 9	1						
Total (n = 35)	7	2	9	1	7	5	4

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix S4: Responses on Adam's Subtraction Postassessment Grouped by Question and Phase

	Correct: Phase		
	2	3	4
S 1			
7 – 1			1
S 0			
2 – 0			1
S N from Itself			
9 – 9			1
Diff of 1			
5 – 4			1
9 – 8			1
12 – 11			1
S 1s from a Teen			
12 – 2			1
13 – 3			1
14 – 4			1
17 – 7			1
S 10 from a Teen			
14 – 10			1
15 – 10			1
17 – 10			1
17 – 10			1
19 – 10			1
S Half Facts			
12 – 6			1
14 – 7			1
14 – 7			1
16 – 8			1
18 – 9			1
S from 10			
10 – 2			1
10 – 4			1
10 – 6		1	
10 – 7			1
10 – 8			1
10 – 9			1
5 Anchor ≤10			
7 – 5			1
8 – 3	1		
10 – 5			1
5 Anchor > 10			
14 – 5			1
S B/U 10			
14 – 8		1	
15 – 6			1
15 – 9		1	
16 – 7			1
16 – 9			1
Total (n = 35)	1	3	31

Note. Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix T1: Responses on Oliver's Addition Preassessment Grouped by Question and Phase

	Correct: Phase			Incorrect: Phase		
	2	3	4	0	2	3
Plus 0						
0 + 9			1			
5 + 0			1			
8 + 0			1			
Plus 1						
9 + 1			1			
Sm Doubles						
3 + 3			1			
4 + 4			1			
5 + 5			1			
5 + 5			1			
Lg Doubles						
6 + 6			1			
7 + 7				1		
7 + 7			1			
8 + 8			1			
9 + 9			1			
Add 10						
10 + 2			1			
10 + 4			1			
10 + 5			1			
10 + 8			1			
Sum of 10						
2 + 8	1					
3 + 7	1					
4 + 6	1					
Sm Near Doubles						
3 + 4		1				
4 + 5			1			
5 Anchor < 10						
5 + 2	1					
5 + 3		1				
5 + 4		1				
Plus 3						
3 + 6	1					
5 Anchor > 10						
5 + 6		1				
5 + 7	1					
5 + 8		1				
5 + 8		1				
5 + 9					1	
Make 10						
3 + 9		1				
4 + 9		1				
6 + 8						1
7 + 4		1				
7 + 5	1					
7 + 9	1					
9 + 5		1				
9 + 6		1				
Lg Near Doubles						
6 + 7		1				
7 + 8		1				
8 + 9		1				
Total (n = 42)	8	14	17	1	1	1

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix T2: Responses on Oliver's Addition Postassessment Grouped by Question and Phase

	Correct: Phase		
Fact Type	2	3	4
Plus 0			
0 + 9			1
5 + 0			1
8 + 0			1
Plus 1			
9 + 1			1
Sm Doubles			
3 + 3			1
4 + 4			1
5 + 5			1
5 + 5			1
Lg Doubles			
6 + 6			1
7 + 7			1
7 + 7			1
8 + 8			1
9 + 9			1
Add 10			
10 + 2			1
10 + 4			1
10 + 5			1
10 + 8			1
Sum of 10			
2 + 8			1
3 + 7	1		
4 + 6			1
Sm Near Doubles			
3 + 4	1		
4 + 5			1
5 Anchor < 10			
5 + 2		1	
5 + 3		1	
5 + 4			1
Plus 3			
3 + 6	1		
5 Anchor > 10			
5 + 6		1	
5 + 7		1	
5 + 8		1	
5 + 8		1	
5 + 9		1	
Make 10			
3 + 9		1	
4 + 9			1
6 + 8		1	
7 + 4	1		
7 + 5		1	
7 + 9		1	
9 + 5		1	
9 + 6			1
Lg Near Doubles			
6 + 7		1	
7 + 8		1	
8 + 9			1
Total (n = 42)	4	14	24

Note. Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix T3: Responses on Oliver's Subtraction Preassessment Grouped by Question and Phase

	Correct: Phase			Incorrect: Phase		
	2	3	4	0	2	3
S 1						
7 – 1			1			
S 0						
2 – 0			1			
S N from Itself						
9 – 9			1			
Diff of 1						
5 – 4			1			
9 – 8			1			
12 – 11	1					
S 1s from a Teen						
12 – 2			1			
13 – 3		1				
14 – 4			1			
17 – 7			1			
S 10 from a Teen						
14 – 10			1			
15 – 10			1			
17 – 10	1					
17 – 10			1			
19 – 10			1			
S Half Facts						
12 – 6						1
14 – 7					1	
14 – 7		1				
16 – 8					1	
18 – 9		1				
S from 10						
10 – 2	1					
10 – 4	1					
10 – 6						1
10 – 7				1		
10 – 8			1			
10 – 9			1			
5 Anchor ≤ 10						
7 – 5					1	
8 – 3					1	
10 – 5		1				
5 Anchor > 10						
14 – 5						1
S B/U 10						
14 – 8	1					
15 – 6						1
15 – 9					1	
16 – 7						1
16 – 9	1					
Total (n = 35)	6	4	14	1	5	5

Note. Phase 0 = can't explain thinking/skipped due to difficulty/incorrect answer; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix T4: Responses on Oliver's Subtraction Postassessment Grouped by Question and Phase

	Correct: Phase		
	2	3	4
S 1			
7 – 1			1
S 0			
2 – 0			1
S N from Itself			
9 – 9			1
Diff of 1			
5 – 4			1
9 – 8			1
12 – 11			1
S 1s from a Teen			
12 – 2			1
13 – 3			1
14 – 4			1
17 – 7			1
S 10 from a Teen			
14 – 10			1
15 – 10			1
17 – 10			1
17 – 10			1
19 – 10			1
S Half Facts			
12 – 6			1
14 – 7	1		
14 – 7	1		
16 – 8	1		
18 – 9			1
S from 10			
10 – 2		1	
10 – 4	1		
10 – 6			1
10 – 7		1	
10 – 8			1
10 – 9			1
5 Anchor ≤10			
7 – 5	1		
8 – 3		1	
10 – 5			1
5 Anchor > 10			
14 – 5			1
S B/U 10			
14 – 8		1	
15 – 6			1
15 – 9			1
16 – 7			1
16 – 9			1
Total (n = 35)	5	4	26

Note. Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix U1: Responses on Fran's Addition Preassessment Grouped by Question and Phase

	Correct: Phase			Incorrect: Phase	
Fact Type	2	3	4	2	3
Plus 0					
0 + 9			1		
5 + 0			1		
8 + 0			1		
Plus 1					
9 + 1			1		
Sm Doubles					
3 + 3			1		
4 + 4			1		
5 + 5			1		
5 + 5			1		
Lg Doubles					
6 + 6			1		
7 + 7			1		
7 + 7			1		
8 + 8			1		
9 + 9			1		
Add 10					
10 + 2			1		
10 + 4			1		
10 + 5			1		
10 + 8			1		
Sum of 10					
2 + 8			1		
3 + 7	1				
4 + 6			1		
Sm Near Doubles					
3 + 4			1		
4 + 5			1		
5 Anchor < 10					
5 + 2			1		
5 + 3	1				
5 + 4		1			
Plus 3					
3 + 6			1		
5 Anchor > 10					
5 + 6		1			
5 + 7	1				
5 + 8	1				
5 + 8	1				
5 + 9	1				
Make 10					
3 + 9	1				
4 + 9	1				
6 + 8		1			
7 + 4	1				
7 + 5	1				
7 + 9		1			
9 + 5					1
9 + 6				1	
Lg Near Doubles					
6 + 7		1			
7 + 8	1				
8 + 9	1				
Total (n = 42)	12	5	23	1	1

Note. Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix U2: Responses on Fran's Addition Postassessment Grouped by Question and Phase

	Correct: Phase		
Fact Type	2	3	4
Plus 0			
0 + 9			1
5 + 0			1
8 + 0			1
Plus 1			
9 + 1			1
Sm Doubles			
3 + 3			1
4 + 4			1
5 + 5			1
5 + 5			1
Lg Doubles			
6 + 6			1
7 + 7			1
7 + 7			1
8 + 8		1	
9 + 9			1
Add 10			
10 + 2			1
10 + 4			1
10 + 5			1
10 + 8			1
Sum of 10			
2 + 8			1
3 + 7			1
4 + 6			1
Sm Near Doubles			
3 + 4	1		
4 + 5			1
5 Anchor < 10			
5 + 2			1
5 + 3		1	
5 + 4			1
Plus 3			
3 + 6			1
5 Anchor > 10			
5 + 6		1	
5 + 7		1	
5 + 8	1		
5 + 8		1	
5 + 9		1	
Make 10			
3 + 9			1
4 + 9		1	
6 + 8		1	
7 + 4		1	
7 + 5		1	
7 + 9		1	
9 + 5			1
9 + 6		1	
Lg Near Doubles			
6 + 7		1	
7 + 8		1	
8 + 9		1	
Total (n = 42)	2	15	25

Note. Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix U3: Responses on Fran's Subtraction Preassessment Grouped by Question and Phase

	Correct: Phase			Incorrect	
	2	3	4	2	3
S 1					
7 – 1			1		
S 0					
2 – 0			1		
S N from Itself					
9 – 9			1		
Diff of 1					
5 – 4			1		
9 – 8			1		
12 – 11			1		
S 1s from a Teen					
12 – 2			1		
13 – 3			1		
14 – 4			1		
17 – 7			1		
S 10 from a Teen					
14 – 10			1		
15 – 10			1		
17 – 10			1		
17 – 10			1		
19 – 10			1		
S Half Facts					
12 – 6			1		
14 – 7			1		
14 – 7			1		
16 – 8			1		
18 – 9			1		
S from 10					
10 – 2	1				
10 – 4		1			
10 – 6		1			
10 – 7		1			
10 – 8			1		
10 – 9			1		
5 Anchor ≤ 10					
7 – 5	1				
8 – 3	1				
10 – 5			1		
5 Anchor > 10					
14 – 5					1
S B/U 10					
14 – 8		1			
15 – 6					1
15 – 9				1	
16 – 7	1				
16 – 9		1			
Total (n = 35)	4	5	23	1	2

Note. Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix U4: Responses on Fran's Subtraction Postassessment Grouped by Question and Phase

	Correct: Phase		
	2	3	4
S 1			
7 – 1			1
S 0			
2 – 0			1
S N from Itself			
9 – 9			1
Diff of 1			
5 – 4			1
9 – 8			1
12 – 11			1
S 1s from a Teen			
12 – 2			1
13 – 3			1
14 – 4			1
17 – 7			1
S 10 from a Teen			
14 – 10			1
15 – 10			1
17 – 10			1
17 – 10			1
19 – 10			1
S Half Facts			
12 – 6			1
14 – 7			1
14 – 7			1
16 – 8			1
18 – 9			1
S from 10			
10 – 2			1
10 – 4			1
10 – 6			1
10 – 7			1
10 – 8			1
10 – 9			1
5 Anchor ≤ 10			
7 – 5	1		
8 – 3	1		
10 – 5			1
5 Anchor > 10			
14 – 5			1
S B/U 10			
14 – 8		1	
15 – 6			1
15 – 9		1	
16 – 7			1
16 – 9		1	
Total (n = 35)	2	3	30

Note. Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix V1: Responses on Helen's Addition Preassessment Grouped by Question and Phase

	Correct: Phase			
Fact Type	1	2	3	4
Plus 0				
0 + 9				1
5 + 0				1
8 + 0				1
Plus 1				
9 + 1				1
Sm Doubles				
3 + 3				1
4 + 4				1
5 + 5				1
5 + 5				1
Lg Doubles				
6 + 6				1
7 + 7				1
7 + 7	1			
8 + 8	1			
9 + 9				1
Add 10				
10 + 2				1
10 + 4				1
10 + 5				1
10 + 8				1
Sum of 10				
2 + 8				1
3 + 7		1		
4 + 6				1
Sm Near Doubles				
3 + 4			1	
4 + 5				1
5 Anchor < 10				
5 + 2				1
5 + 3				1
5 + 4			1	
Plus 3				
3 + 6				1
5 Anchor > 10				
5 + 6				1
5 + 7		1		
5 + 8	1			
5 + 8			1	
5 + 9		1		
Make 10				
3 + 9			1	
4 + 9		1		
6 + 8			1	
7 + 4			1	
7 + 5			1	
7 + 9			1	
9 + 5			1	
9 + 6		1		
Lg Near Doubles				
6 + 7			1	
7 + 8			1	
8 + 9	1			
Total (n = 42)	4	5	11	22

Note. Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix V2: Responses on Helen's Addition Postassessment Grouped by Question and Phase

	Correct: Phase			Incorrect: Phase
	2	3	4	3
Plus 0				
0 + 9			1	
5 + 0			1	
8 + 0			1	
Plus 1				
9 + 1			1	
Sm Doubles				
3 + 3			1	
4 + 4			1	
5 + 5			1	
5 + 5			1	
Lg Doubles				
6 + 6			1	
7 + 7			1	
7 + 7			1	
8 + 8			1	
9 + 9			1	
Add 10				
10 + 2			1	
10 + 4			1	
10 + 5			1	
10 + 8			1	
Sum of 10				
2 + 8			1	
3 + 7			1	
4 + 6			1	
Sm Near Doubles				
3 + 4			1	
4 + 5			1	
5 Anchor < 10				
5 + 2			1	
5 + 3			1	
5 + 4			1	
Plus 3				
3 + 6			1	
5 Anchor > 10				
5 + 6			1	
5 + 7		1		
5 + 8		1		
5 + 8		1		
5 + 9			1	
Make 10				
3 + 9			1	
4 + 9			1	
6 + 8		1		
7 + 4	1			
7 + 5				1
7 + 9		1		
9 + 5			1	
9 + 6			1	
Lg Near Doubles				
6 + 7		1		
7 + 8		1		
8 + 9			1	
Total (n = 42)	1	7	33	1

Note. Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix V3: Responses on Helen's Subtraction Preassessment Grouped by Question and Phase

	Correct: Phase		
	1	2	4
S 1			
7 – 1			1
S 0			
2 – 0			1
S N from Itself			
9 – 9			1
Diff of 1			
5 – 4			1
9 – 8			1
12 – 11			1
S 1s from a Teen			
12 – 2			1
13 – 3			1
14 – 4			1
17 – 7			1
S 10 from a Teen			
14 – 10			1
15 – 10			1
17 – 10			1
17 – 10			1
19 – 10			1
S Half Facts			
12 – 6			1
14 – 7			1
14 – 7		1	
16 – 8	1		
18 – 9			1
S from 10			
10 – 2	1		
10 – 4	1		
10 – 6		1	
10 – 7		1	
10 – 8			1
10 – 9			1
5 Anchor ≤ 10			
7 – 5			1
8 – 3			1
10 – 5			1
5 Anchor > 10			
14 – 5	1		
S B/U 10			
14 – 8		1	
15 – 6	1		
15 – 9	1		
16 – 7			1
16 – 9	1		
Total (n = 35)	7	4	24

Note. Phase 1 = direct modelling & counting; Phase 2 = counting more efficiency & tracking; Phase 4 = proficiency.

Appendix V4: Responses on Helen's Subtraction Postassessment Grouped by Question and Phase

	Correct: Phase	
	3	4
S 1		
7 – 1		1
S 0		
2 – 0		1
S N from Itself		
9 – 9		1
Diff of 1		
5 – 4		1
9 – 8		1
12 – 11		1
S 1s from a Teen		
12 – 2		1
13 – 3		1
14 – 4		1
17 – 7		1
S 10 from a Teen		
14 – 10		1
15 – 10		1
17 – 10		1
17 – 10		1
19 – 10		1
S Half Facts		
12 – 6		1
14 – 7		1
14 – 7		1
16 – 8		1
18 – 9		1
S from 10		
10 – 2		1
10 – 4		1
10 – 6		1
10 – 7		1
10 – 8		1
10 – 9		1
5 Anchor ≤10		
7 – 5		1
8 – 3		1
10 – 5		1
5 Anchor > 10		
14 – 5	1	
S B/U 10		
14 – 8	1	
15 – 6	1	
15 – 9	1	
16 – 7	1	
16 – 9	1	
Total (n = 35)	6	29

Note. Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix W1: Responses on Betty's Addition Preassessment Grouped by Question and Phase

	Correct: Phase		
Fact Type	2	3	4
Plus 0			
0 + 9			1
5 + 0			1
8 + 0			1
Plus 1			
9 + 1			1
Sm Doubles			
3 + 3			1
4 + 4			1
5 + 5			1
5 + 5			1
Lg Doubles			
6 + 6			1
7 + 7			1
7 + 7			1
8 + 8			1
9 + 9			1
Add 10			
10 + 2			1
10 + 4			1
10 + 5			1
10 + 8			1
Sum of 10			
2 + 8			1
3 + 7	1		
4 + 6		1	
Sm Near Doubles			
3 + 4			1
4 + 5			1
5 Anchor < 10			
5 + 2			1
5 + 3			1
5 + 4			1
Plus 3			
3 + 6			1
5 Anchor > 10			
5 + 6			1
5 + 7		1	
5 + 8			1
5 + 8		1	
5 + 9		1	
Make 10			
3 + 9		1	
4 + 9		1	
6 + 8		1	
7 + 4		1	
7 + 5		1	
7 + 9		1	
9 + 5		1	
9 + 6		1	
Lg Near Doubles			
6 + 7		1	
7 + 8		1	
8 + 9		1	
Total (n = 42)	1	15	26

Note. Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix W2: Responses on Betty's Addition Postassessment Grouped by Question and Phase

	Correct: Phase	
Fact Type	3	4
Plus 0		
0 + 9		1
5 + 0		1
8 + 0		1
Plus 1		
9 + 1		1
Sm Doubles		
3 + 3		1
4 + 4		1
5 + 5		1
5 + 5		1
Lg Doubles		
6 + 6		1
7 + 7		1
7 + 7		1
8 + 8		1
9 + 9		1
Add 10		
10 + 2		1
10 + 4		1
10 + 5		1
10 + 8		1
Sum of 10		
2 + 8		1
3 + 7	1	
4 + 6		1
Sm Near Doubles		
3 + 4		1
4 + 5		1
5 Anchor < 10		
5 + 2		1
5 + 3		1
5 + 4		1
Plus 3		
3 + 6		1
5 Anchor > 10		
5 + 6		1
5 + 7	1	
5 + 8	1	
5 + 8		1
5 + 9		1
Make 10		
3 + 9		1
4 + 9		1
6 + 8		1
7 + 4		1
7 + 5		1
7 + 9		1
9 + 5		1
9 + 6		1
Lg Near Doubles		
6 + 7		1
7 + 8		1
8 + 9		1
Total (n = 42)	3	39

Note. Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix W3: Responses on Betty's Subtraction Preassessment Grouped by Question and Phase

	Correct: Phase		
	2	3	4
S 1			
7 – 1			1
S 0			
2 – 0			1
S N from Itself			
9 – 9			1
Diff of 1			
5 – 4			1
9 – 8			1
12 – 11			1
S 1s from a Teen			
12 – 2			1
13 – 3			1
14 – 4			1
17 – 7			1
S 10 from a Teen			
14 – 10			1
15 – 10			1
17 – 10			1
17 – 10			1
19 – 10			1
S Half Facts			
12 – 6			1
14 – 7			1
14 – 7			1
16 – 8		1	
18 – 9			1
S from 10			
10 – 2			1
10 – 4			1
10 – 6		1	
10 – 7	1		
10 – 8			1
10 – 9			1
5 Anchor ≤ 10			
7 – 5		1	
8 – 3			1
10 – 5			1
5 Anchor > 10			
14 – 5		1	
S B/U 10			
14 – 8		1	
15 – 6		1	
15 – 9	1		
16 – 7		1	
16 – 9	1		
Total (n = 35)	3	7	25

Note. Phase 2 = counting more efficiency & tracking; Phase 3 = working with the numbers; Phase 4 = proficiency.

Appendix W4: Responses on Betty's Subtraction Postassessment Grouped by Question and Phase

	Correct: Phase	
	3	4
S 1		
7 – 1		1
S 0		
2 – 0		1
S N from Itself		
9 – 9		1
Diff of 1		
5 – 4		1
9 – 8		1
12 – 11		1
S 1s from a Teen		
12 – 2		1
13 – 3		1
14 – 4		1
17 – 7		1
S 10 from a Teen		
14 – 10		1
15 – 10		1
17 – 10		1
17 – 10		1
19 – 10		1
S Half Facts		
12 – 6		1
14 – 7		1
14 – 7		1
16 – 8		1
18 – 9		1
S from 10		
10 – 2		1
10 – 4		1
10 – 6		1
10 – 7		1
10 – 8		1
10 – 9		1
5 Anchor ≤ 10		
7 – 5		1
8 – 3		1
10 – 5		1
5 Anchor > 10		
14 – 5	1	
S B/U 10		
14 – 8	1	
15 – 6	1	
15 – 9		1
16 – 7		1
16 – 9	1	
Total (n = 35)	4	31

Note. Phase 3 = working with the numbers; Phase 4 = proficiency.