DETERMINATION OF PHYSICAL EQUATIONS USED IN PROCESS INDUSTRIES USING DIMENSIONAL ANALYSIS

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Abstract: *Technological process, however complex, can be broken down into a succession of distinct component processes, in which the input materials undergo changes in shape (mechanical processes), pressure, temperature, concentration, state of aggregation (physical processes) or structure molecular (chemical and biochemical processes). The unitary operations of most process phases in process industries are based on three fundamental processes: pulse transfer, heat transfer, and mass transfer.*

Key words: technological process, unitary operations, dimensional analysis.

1.Introduction

Process industries (chemical industry, petrochemical industry, metallurgical industry, food industry, pharmaceutical industry) are based on technological processes where raw materials (natural, synthetic, artificial) are transformed by a succession of mechanical, physical, chemical processes or biochemicals, in marketable finished products or in semi-finished products subsequently used as starting materials.

A technological process, however complex, can be broken down into a succession of distinct component processes, in which the input materials undergo changes in shape (mechanical processes), pressure, temperature, concentration, state of aggregation (physical processes) or structure molecular (chemical and biochemical processes).

The linear graphical representation of the processes from the entrance to the system to the exit of the finished products is called the technological flow. The graphic representation of the processes that make up the technological flow is called the technological scheme of the technological process (fig.1).

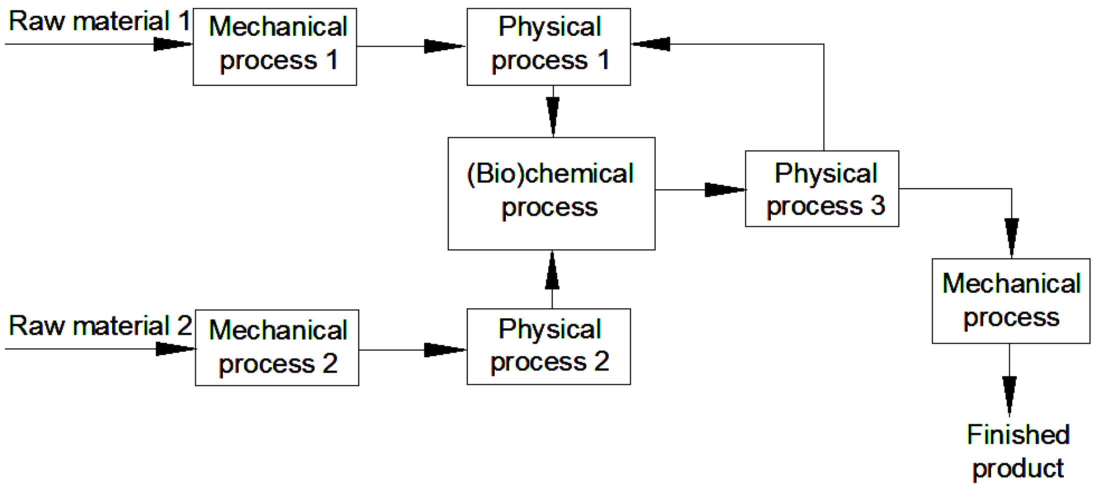

Fig.1. The diagrame of technological process

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This is based on the "black box" principle, each process being represented by a rectangle in which flows and material flows and energy flows. As a rule, in a technological process there are few chemical or biochemical processes, most of which are of a physical or mechanical nature.

The unitary operations of most process phases in process industries are based on three fundamental processes: pulse transfer, heat transfer, and mass transfer.

The study of these unitary operations is limited to the three fundamental processes of transfer, insisting on their mechanism, the phenomena in the boundary layer, the understanding of causes and primary effects that motivate and explain the peculiarities and usefulness of each unitary operation.

2. Verification of equations based on dimensional homogeneity

Dimensional analysis is based on the principle that any equation or relationship between variables must be dimensionally consistent (each term of the equation must have the same dimensions). For example, if an equation consists of a number of terms, each representing lengths, all of these terms will have to be of a length dimension.

The corollary of this principle is that if the whole equation is subdivided into any of its terms, all the terms remaining in the equation will have to be dimensionless. Using dimensional groups or numbers is very important in dealing with process engineering issues. Physical equations must be valid in any coherent system of measurement units, their dimensional verification being made in any coherent unit of measurement system.

3. Determining the general form of equations

The general form of a physical equation can be established either theoretically, by applying the basic laws of mathematics, physics and chemistry, or by experimental study in the case of complex phenomena or in the case of obtaining more general relationships or laws.

Dimensional analysis is an intermediate method between the theoretical and experimental methods that can be used to determine the correlations between the sizes that characterize a phenomenon or process in terms of numbers or dimensional groups.

The use of dimensional analysis in determining the shape of physical equations is possible due to their two basic properties: dimensional homogeneity and invariance of their shape when passing from a coherent unit of measurement system to another coherent unit of measure. It is thus possible to organize several sizes that interfere in a physical equation in a small number of non-dimensional groups, the numerical values of these groups, the numerical values of these groups being independent of the unit of measure system used.

4. Case study. Determination of equations in case of pressure difference

S-a constatat experimental că diferenţa de presiune ΔP, între extremităţile unei conducte prin care curge un fluid este o funcţie de diametrul conductei d, lungimea conductei l, viteza fluidului v, densitatea fluidului ρ şi vâscozitatea fluidului μ.

It has been found experimentally that the pressure difference ΔP between the ends of a fluid through which a fluid flows is a function of the diameter of the pipe d, the length of the pipe 1, the velocity of the fluid v, the density of the fluid ρ and the viscosity of the μ .

The pressure difference equation can be written:

$$\Delta P = f_1(d,l,v,\rho,\mu) \tag{1}$$

The form of the equation is unknown, but since any function can be developed in a series of powers, the function can be considered as the sum of a number of terms, each consisting of the product of the powers of the variables taken into account. The simplest form of such a relationship will be that in which only the first term of the power series is taken into account:

$$\Delta P = \text{const} \cdot d^{n_1} \cdot l^{n_2} \cdot v^{n_3} \cdot \rho^{n_4} \cdot \mu^{n_5} \quad (2)$$

For equation (2) to be dimensionally consistent, it is necessary that the term in the right-hand member has the same dimensions as the term in the left-hand member, so it will have to have the dimensions of a pressure.

Each variable in equation (2) can be expressed in terms of mass (M), length (L) and time (T). Dimensional we can appreciate:

$$\Delta P \equiv M \cdot L^{-1} \cdot T^{-2} \quad (3)$$
$$d \equiv L \quad (4)$$
$$l \equiv L \quad (5)$$
$$v \equiv L \cdot T^{-1} \quad (6)$$
$$\rho \equiv M \cdot L^{-3} \quad (7)$$
$$\mu \equiv M \cdot L^{-1} \cdot T^{-1} \quad (8)$$

Therefore,

$$M \cdot L^{-1} \cdot T^{-2} = L^{n_1} \cdot L^{n_2} \cdot (L \cdot T^{-1})^{n_3} \cdot (M \cdot L^{-3})^{n_4} \cdot (M \cdot L^{-1} \cdot T^{-1})^{n_5} \quad (9)$$

The condition of dimensional consistency must also be met by each of the fundamental variables: mass, length, time:

For M: $+1 = n_4 + n_5$ (10)
For L: $-1 = n_1 + n_2 + n_3 - 3n_4 - n_5$ (11)
For T: $-2 = n_3 - n_5$ (12)

The system of 3 equations with 5 unknowns (n_1, n_2, n_3, n_4, n_5) can be solved depending on any 2 of the 5 unknowns. Depending on n_2 and n_5, we obtain:

$$n_4 = 1 - n_5 \text{ (din ecuaţia M)} \quad (13)$$
$$n_3 = 2 - n_5 \text{ (din ecuaţia T)} \quad (14)$$

Substituting equations (13) and (14) in equation (11), we obtain:

$$-1 = n_1 + n_2 + (2 - n_5) - 3(1 - n_5) - n_5 \quad (15)$$

Meaning,

$$n_1 = -n_2 - n_5 \quad (16)$$

Returning and replacing in equation (2), we obtain:

$$\Delta P = const \cdot d^{-n_2-n_5} \cdot l^{n_2} \cdot v^{2-n_5} \cdot \rho^{1-n_5} \cdot \mu^{n_5} \quad (17)$$

or

$$\frac{\Delta P}{\rho v^2} = \text{const} \cdot \left(\frac{l}{d}\right)^{n_2} \cdot \left(\frac{\mu}{\rho v d}\right)^{n_5} \quad (18)$$

Since n_2 and n_5 are arbitrary constants, equation (18) can only be satisfied if the terms $\frac{\Delta P}{\rho v^2}$, $\frac{l}{d}$, $\frac{\mu}{\rho v d}$ are non-dimensional.

The $\frac{\rho v d}{\mu}$ group, known as the Raynolds number, is one of the most common in the fluid flow study. Based on this, we can estimate the flow type in a given geometry space.

In more general terms, equation (18) can be written:

$$\frac{\Delta P}{\rho v^2} = f_2\left(\frac{l}{d}, \frac{\rho v d}{\mu}\right) \quad (19)$$

Comparing equations (2) and (19) we find that a relationship between 6 variables was reduced to a relationship between only 3 dimensional groups.

5. Conclusions

Dimensional analysis is the domain that deals with establishing relationships between dimensional formulas of different physical sizes. Based on these relationships, we can sometimes determine approximate forms of valid laws in certain experimental situations. Even though formulas determined using dimensional analysis are only approximate, they can be of great help in simplifying the experiments to determine the correct form of those laws. Also, dimensional analysis can highlight dimensional reports of physical quantities, called criteria, which are used to characterize the predominance of a particular physical effect with respect to another.

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