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## FUZZY BELIEF STRUCTURE BASED VIKOR METHOD: AN APPLICATION FOR RANKING DELAY CAUSES OF TEHRAN METRO SYSTEM BY FMEA CRITERIA

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**Abstract.** Public transport is a critical part of civilization in this decade. The amount of money invested and the criticality of transferring people in an acceptable time and without any conflict made it a challenging problem for managers, especially in metropolises. Absolutely, making effective decisions in this area requires considering different aspects. Waiting time is a key criterion in apprising quality of public transport. In this paper, a real world case study of ranking causes of delay in Tehran (Iran) metro system is solved by developing multi attribute group decision-making *VIšeKriterijumska Optimizacija I KOmpromisno Rešenje* (in Serbian, VIKOR) method under uncertainty, where this uncertainty is captured by Fuzzy Belief Structures (FBS). The obtained results are then compared with a previously proposed Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method with FBSs. The results show that human related issues, along with the problems related to line and transportation system are the most important causes of delay. The obtained results of the problem seem acceptable for decision makers.

**Keywords:** group decision-making; VIKOR; fuzzy belief structure; center of gravity; failure mode and effect analysis.

### Introduction

Finding an ideal decision is the pursuit of many scholars and practitioners in different fields (Tzeng, Huang 2011). Multiple Criteria Decision-Making (MCDM) is a field of operations research, which deals with evaluating and choosing the best alternative based on several criteria. Yoon and Hwang (1995) classified MCDM problems into two main categories: Multiple Attribute Decision-Making (MADM) and Multiple Objective Decision-Making (MODM), the former applies in evaluation type problems while the latter is suitable for design type problems (Simon (1977) classified decision problems into selection and design problems).

The aim of MCDM methods is to provide a logical and scientific framework of dealing with risky decisions. One of the main challenges of 21st century is urbanization growth (Martine 2005). As stated by UN, today 54% of the world's population lives in urban areas (UN 2015). Considering this growing urbanization phenomena, transport plays a crucial role in urban management

(UN 2010). As Bainster (1996), and Gifford and Steg (2007) believed, transport holds major economic and social importance in improving expected standards of consumption and quality of life.

Considering the above challenges, transportation system is one of the implicational areas of MADM (Bagočius *et al.* 2014; Elevli 2014; Šateikienė *et al.* 2015). An MADM problem can be defined as follow (Zimmerman 1987): Suppose that we have a nonempty and finite set of decision alternatives, that their desirability will be judged according to a finite set of goals, attributes or criteria. The aim of MADM is to determine an optimal alternative having the highest degree of desirability in respect to all relevant goals.

The real world decision-making process is a common problem for employing the uncertainty phenomena. In fact, the required knowledge to formalize a decision-making problem are usually subjected to uncertainty. Dubois and Prade (1982), and Klir (1987) introduced ambiguity and vagueness as two types of uncertainty in



real-world problems. While ambiguity refers to unspecific choice between alternatives, vagueness deals with situations where there are not any sharp boundaries among some domain of interest (Inuiguchi *et al.* 2000).

This undeniable uncertainty is widely known and accepted in decision-making, especially – in MADM problems. Presence and acceptance of uncertainty requires a formal framework to be analyzed. Fuzzy set theory, introduced by Zadeh (1965), is one of the common frameworks in handling uncertainties (Liu, Lin 2006). Bellman and Zadeh (1970) initially applied the concept of fuzzy sets in decision-making problems. Fuzzy sets assign a membership value to each elements of a set. In ordinal fuzzy sets, this membership values are exact numbers. Some scholars criticized the ordinal fuzzy sets due to crispness of its membership values (Grattan-Guinness 1976). Therefore, some extensions are given to the concept of membership functions or values.

Zadeh (1975) introduced type-2 fuzzy sets where membership function itself is a fuzzy set. As an extension, interval type-2 fuzzy sets considered membership functions as closed intervals (Liang, Mendel 2000). Simplicity of operations over intervals caused more interesting about application of interval type-2 fuzzy sets in MADM. Wang *et al.* (2012) have found the interval type-2 fuzzy sets a very useful mean to depict the information in decision-making process. They used it for a group decision-making problem in order to calculate the attributes weights and aggregated decision matrix. Chen (2013) used the interval type-2 trapezoidal fuzzy numbers to determine the alternatives ratings and the importance of various criteria. Baležentis and Zeng (2013) extended the MULTIMOORA method based upon fuzzy type-2 sets and generalized interval-valued trapezoidal fuzzy numbers. This paper was proposed to aggregate the group decision-making in human resource management. The application of type-2 fuzzy numbers is illustrated in Maldonado *et al.* (2014), which proposed a design of multi objective genetic optimization problem.

Atanassov (1986) extended the ordinal fuzzy sets by attending a non-membership value  $\nu$  beyond the classic membership value  $\mu$ , and called the corresponding set as Intuitionistic Fuzzy Set (IFS). In a classic fuzzy set  $\nu = 1 - \mu$ , while in an IFS,  $\nu + \mu \leq 1$ . In an ordinal IFS, the  $\mu$  and  $\nu$  values are determined as crisp numbers. Latter, Atanassov and Gargov (1989) developed IFS to Interval-Valued Intuitionistic Fuzzy Sets (IVIFSs) where membership and non-membership degrees are expressed as closed intervals. There is a planetary of researches done over extending MADM techniques under IFS and IVIFS environments (Zhou *et al.* 2013; Razavi Hajiagha *et al.* 2015; Zavadskas *et al.* 2015; Tan *et al.* 2014; Chen 2015; Hashemi *et al.* 2016).

Fuzzy Belief Structure (FBS) is another extension of ordinal fuzzy sets. Initially, the Belief Structure (BS) introduced by Yang and Singh (1994), and Yang and Sen (1994) as an evidential reasoning approach for solving MADM problems. In a FBS model, the linguistic variables of evaluation grades are considered as crisp values.

Yang *et al.* (2006) introduced the FBS where evaluation grades are expressed as fuzzy numbers. In fact, an FBS is a combination of fuzzy set theory and the evidence combination rule of the Dempster–Shafer theory and therefore is a powerful way of dealing with uncertainty. Jiang *et al.* (2011) applied the FBS model to extend the TOPSIS method for group decision-making. Vahdani *et al.* (2014) also applied the above TOPSIS and FBS method in the Failure Mode and Effect Analysis (FMEA) for ranking the risk causes.

The aim of this paper is to propose an extension of VIKOR method under FBS. VIKOR method is a compromise based method for ranking alternatives (Opricovic 1998). This method is applied for MADM problems with irrelevant and conflicting attributes (Opricovic, Tzeng 2004). As the founder of VIKOR method, Opricovic extended the fuzzy VIKOR method and employed it in some applications (Opricovic 2007, 2011). Vahdani *et al.* (2010) proposed the interval type-2 VIKOR method (they applied the interval-valued fuzzy sets with a similar definition of interval type-2 fuzzy sets). The method is also developed in intuitionistic fuzzy (Devi 2011) and interval valued intuitionistic fuzzy (Park *et al.* 2011) environments. The idea of VIKOR method is also extended to solve multi-objective linear programming problems (Razavi Hajiagha *et al.* 2014). In this paper, it is supposed that a group of experts participate in decision-making process, who expresses their evaluations in decision matrix based on FBS models, i.e. they determine their belief degrees' regard to fuzzy evaluation grades of each alternatives with respect to each criteria. Considering the advantages of FBS in modeling data uncertainty by aggregating fuzzy evaluation grades and belief degrees and combining these advantages with the VIKOR method's advantage in determining a compromise solution of a decision-making problem could have made the FBS-VIKOR method as an appealing method of solving Multiple Attribute Group Decision-Making (MAGDM) problems.

The rest of paper is organized as follows. The VIKOR method is briefly introduced in Section 1. Then, the required concepts of FBS models are overviewed in Section 2. The proposed method of VIKOR under FBS is explained in Section 3. A real world case study is then solved by using the proposed method in Section 4. Finally, the paper is concluded.

## 1. VIKOR

A decision-making problem can be formulated in the form of a decision matrix consist of the following elements (Yu 1990):

- the set of alternatives  $A = \{A_1, A_2, \dots, A_m\}$ ;
- the set of criteria  $X = \{X_1, X_2, \dots, X_n\}$ ;
- the outcome or decision matrix  $D = [x_{ij}]$ , which element  $x_{ij}$  represents performance of alternative  $A_i$  based on the criterion  $X_j$ , for each  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ ;
- the vector  $W = \{w_1, w_2, \dots, w_n\}$ , where  $w_j \geq 0, j = 1, 2, \dots, n$  illustrates the weight or importance of criterion  $X_j$ .

An MCDM method to rank a set of alternatives based on conflicting criteria so-called VIKOR is introduced to be employed in practical problems. This method is based on closeness to the Ideal solution. Presume a set of  $m$  alternatives and  $n$  criteria, where the rating of each alternative  $A_i$  against to a criterion  $X_j$  is denoted as  $f_{ij}$ . This method includes the following steps:

**Step 1.** Determine the best  $f_i^+$  and worst  $f_i^-$  rating for all criteria as follow:

$$f_j^+ = \max_i f_{ij}; \quad (1)$$

$$f_j^- = \min_i f_{ij}. \quad (2)$$

In this regard, the ideal and anti-ideal candidates are respectively as  $(f_1^+, f_2^+, \dots, f_n^+)$  and  $(f_1^-, f_2^-, \dots, f_n^-)$ .

**Step 2.** Calculate the average  $S_i \in [0,1]$  and the worst group  $R_i \in [0,1]$  scores for the alternative  $A_i$ :

$$S_i = \sum_{j=1}^n w_j \frac{(f_j^+ - f_{ij})}{(f_j^+ - f_j^-)}; \quad (3)$$

$$R_i = \max_j \left( w_j \frac{(f_j^+ - f_{ij})}{(f_j^+ - f_j^-)} \right), \quad (4)$$

where:  $w_j$ ,  $j = 1, 2, \dots, n$  indicates the relative importance weights of the criterion  $j$ , which experts have determined.

**Step 3.** Calculate the  $Q_i$  for  $i = 1, 2, \dots, m$  as:

$$Q_i = v \frac{(S_i - S^+)}{(S^- - S^+)} + (1-v) \frac{(R_i - R^+)}{(R^- - R^+)}, \quad (5)$$

where:

$$S^+ = \min_i S_i; \quad (6)$$

$$S^- = \max_i S_i;$$

$$R^+ = \min_i R_i;$$

$$R^- = \max_i R_i. \quad (7)$$

Here  $v$  indicates the weight of maximum group utility. It can be determined as:

- $v > 0.5$  - selected by majority;
- $v = 0.5$  - consensus;
- $v < 0.5$  - veto.

**Step 4.** Sort the  $S$ ,  $R$  and  $Q$  values by ascending order to rank each on. There would be 3 lists of ranking illustrated as:  $S_{[0]}$ ,  $R_{[0]}$  and  $Q_{[0]}$ .

**Step 5.** Indicate the smallest  $Q_i$  value related to alternative  $j_1$  as a compromise solution if:

- $C_1$  - the alternative  $j_1$  has an acceptable advantage:  $Q_{[2]} - Q_{[1]} \geq DQ$ , where  $DQ = \frac{1}{m-1}$  and  $m$  - the number of alternatives;
- $C_2$  - the alternative  $j_1$  is stable within the decision-making process - it is also the best ranked in  $S_{[0]}$  or  $Q_{[0]}$ .

If one of these conditions is not satisfied, then the solution is as follow:

- alternative  $j_1$  and  $j_2$ , where  $Q_{j_2} = Q_{[2]}$  if  $C_2$  is not satisfied;
- alternatives  $j_1, j_2, \dots, j_k$  if  $C_1$  is not satisfied, alternative  $j_k$  is determined by  $Q_{[k]} - Q_{[1]} < DQ$  for maximum  $k$ , where  $Q_{j_k} = Q_{[k]}$ .

## 2. Fuzzy Belief Structure (FBS)

BS is a distributed assessment scheme where belief degrees are attained to different evaluation grades. Suppose that the evaluation grade of criteria consist a standard with  $N$  grades  $\{H_1, H_2, \dots, H_N\}$  and assume that is preferred to  $H_{k+1}$ . A BS for the assessment of criterion  $c$  can be represented as the following distribution:

$$S(c) = \{(H_n, \beta_n), n=1, 2, \dots, N\}, \quad (8)$$

where:  $\beta_n$  is the belief degree of the grade  $H_n$  in the evaluation, that  $\beta_n \geq 0$ ,  $n = 1, 2, \dots, N$  and  $\sum_{j=1}^n \beta_n \leq 1$ .

Eq. (8) means that the criterion  $c_n$  is assessed with the belief degree  $\beta_n$  at the grade  $H_n$ . If  $\sum \beta_n = 1$ , the distribution is complete and if  $\sum_{j=1}^n \beta_n \leq 1$  - it is incomplete.

The evaluation grade  $\{H_1, H_2, \dots, H_N\}$  consists of crisp number in ordinal BS. However, the ambiguity and vagueness of real-world problems required evaluation grades to be represented by fuzzy numbers. In this case, the fuzzy evaluation grades deal with fuzziness or vagueness and the belief degrees handled incompleteness or ignorance.

An important concept of FBS is to measure the belief distance. Jiang *et al.* (2011) defined the distance between to FBS  $S_1$  and  $S_2$  as:

$$d_{BS}(S_1, S_2) = d_{BS}(B_1, B_2) = \left( \frac{1}{2} (B_1 - B_2) \tilde{S} (B_1 - B_2)^T \right)^{1/2}. \quad (9)$$

That  $B_1$  and  $B_2$  are the belief degree distributions of  $S_1$  and  $S_2$ , respectively. Also,  $\tilde{S} = [\tilde{s}_{ij}]$  represents the similarity matrix between fuzzy evaluations grades. If the utilities of each fuzzy evaluation grade are represented by trapezoidal fuzzy numbers, i.e.  $U(H_n) = (u_1^n, u_2^n, u_3^n, u_4^n)$ ,  $n = 1, 2, \dots, N$ . Then, the similarity between  $H_i = (u_1^i, u_2^i, u_3^i, u_4^i)$  and  $H_j = (u_1^j, u_2^j, u_3^j, u_4^j)$  is calculated as:

$$\tilde{s}_{ij} = 1 - \frac{\sum_{k=1}^4 |u_k^i - u_k^j|}{4}, \quad (10)$$

where:  $0 \leq \tilde{s}_{ij} \leq 1$ , since  $u_k^n \in [0,1]$ ,  $k = 1, \dots, 4$ . If  $H_n$  is a triangular fuzzy number, then  $u_2^n = u_3^n$  and the similarity measure is measured as the same. Jiang *et al.* (2011) proved that the distance measure in Eq. (9) is located between 0 and 1, and is a symmetric measure. This measure will be used in the proposed VIKOR based on FBS model.

### 3. VIKOR Method Based on FBSs

As discussed earlier, in this paper, the multi-attribute VIKOR method is developed when decision-making information is given based on the FBS models. At the next subsections, the fundamentals of the proposed algorithm are explained. Then, an algorithmic scheme is presented to summarize these explanations.

#### 3.1. Problem Definition and Data Preparation

The considered problem is a multi-attribute group decision-making problem. Suppose that a group of  $K$  decision makers, denoted by  $DM_k, k = 1, 2, \dots, K$  are gathered to evaluate a set of alternatives  $A = \{A_1, A_2, \dots, A_m\}$  based on a set of criteria  $X = \{X_1, X_2, \dots, X_n\}$ . Each decision maker has a specific importance according to his/her role in decision-making process. Suppose that decision makers importance are determined in the form of a vector  $W^D = \{w^1, w^2, \dots, w^K\}$ . In addition, the criteria weight vector  $W = \{w_1, w_2, \dots, w_n\}$  can be determined using methods like simple rating or group Analytic Hierarchy Process (AHP) (Yoon, Hwang 1995; Saaty 1988). Each decision maker individually completed his/her decision matrix  $D^k = [S(x_{ij}^k)]$ ,  $k = 1, 2, \dots, K$ , where  $S(x_{ij}^k)$  is an FBS model like:

$$S(x_{ij}^k) = \{(H_n, \beta_{n,ij}^k), n = 1, 2, \dots, N\}, \quad (10)$$

where:  $\beta_{n,ij}^k$  represents decision maker  $k$ 's assurance that the performance of alternative  $A_i, i = 1, 2, \dots, m$  is at evaluation grade  $H_n, n = 1, 2, \dots, N$  in criterion  $X_j, j = 1, 2, \dots, n$ . If  $\sum_{n=1}^N \beta_{n,ij}^k = 1$ , then the  $k$ th decision maker's evaluation over  $A_i$  alternative in  $X_j$  criterion is complete and if  $\sum_{n=1}^N \beta_{n,ij}^k < 1$ , it is incomplete.

#### 3.2. Constructing the Aggregated Decision Matrix

As an important step in group decision-making, it is required to aggregate the individual decision matrices in an aggregated one. This aggregation is carried out using the evidential reasoning approach (Yang, Xu 2002; Yang *et al.* 2006). Since the individual decision matrices  $D^k, k = 1, 2, \dots, K$  are fuzzy belief matrices, the aggregated decision matrix  $D$  will be a fuzzy belief matrix. It can be expressed as:

$$D = [S(x_{ij})] = D^1 \oplus D^2 \oplus \dots \oplus D^K, \quad (11)$$

where:  $S(x_{ij})$  represents the aggregated fuzzy belief performance of alternative  $A_i$  in criterion  $X_j$ . The FBS model  $S(x_{ij}) = \{(H_n, \beta_{n,ij}), n = 1, 2, \dots, N\}$  is constructed based on FBS models  $S(x_{ij}^k), k = 1, 2, \dots, K$ . Wang *et al.* (2006) proposed an analytic relation between the individual belief degrees  $\beta_{n,ij}^k, k = 1, 2, \dots, K$  with the aggregated belief degree of  $\beta_{n,ij}$ , as:

$$\beta_n^{ij} = \frac{\mu \cdot (a_1 - a_2)}{1 - \mu \cdot \left( \prod_{k=1}^K (1 - w_k^d) \right)}, \quad (12)$$

where:

$$\begin{aligned} a_1 &= \prod_{k=1}^K \left( w_k^d + 1 - w_k^d \sum_{n=1}^N \beta_{n,ij}^k \right); \\ a_2 &= \prod_{k=1}^K \left( 1 - w_k^d \sum_{n=1}^N \beta_{n,ij}^k \right); \\ \mu &= \left( \sum_{n=1}^N \prod_{k=1}^K \left( w_k^d \beta_{n,ij}^k + 1 - w_k^d \sum_{n=1}^N \beta_{n,ij}^k \right) - \right. \\ &\quad \left. (N-1) \prod_{k=1}^K \left( 1 - w_k^d \sum_{n=1}^N \beta_{n,ij}^k \right) \right)^{-1}. \end{aligned} \quad (13)$$

#### 3.3. Normalizing the Aggregated Decision Matrix

If the FBS models are incomplete individually, then obtained aggregated FBS model will be incomplete. Therefore, a normalizing stage is proposed to complete the aggregated FBS models.

In this paper, the normalization is done based on the method of Jiang *et al.* (2011). Let  $s_{ij} = \{(H_n, \beta_n^{ij}), n = 1, 2, \dots, N\}$  be an incomplete FBS. Then,  $N$  BS peak points are defined as:

$$\begin{aligned} SP(s_{ij}) &= \{(H_n, \beta_n^m), n = 1, 2, \dots, N\}, \\ m &= 1, 2, \dots, N, \end{aligned} \quad (14)$$

with:

$$\beta_n^m = \begin{cases} \beta_n + \beta_H, & m = n; \\ \beta_n, & m \neq n, \end{cases} \quad (15)$$

where:  $\beta_H = 1 - \sum_{n=1}^N \beta_n^{ij}$  is the degree of ignorance for incomplete  $s_{ij}$ . Then, the center of gravity of FBS model  $s_{ij}$  is defined as:

$$\begin{aligned} SC(s_{ij}) &= \frac{\sum_{m=1}^N SP(s_{ij})}{N} = \\ &= \left\{ \left( H_n, \frac{\sum_{m=1}^N \beta_n^m}{N} \right), n = 1, 2, \dots, N \right\}. \end{aligned} \quad (16)$$

Then, the complete  $SC(s_{ij})$  is used instead of incomplete  $s_{ij}$  in the pooled decision matrix  $X$ .

#### 3.4. Constructing the Fuzzy Belief Distance Matrix

Considering the decreasing nature of evaluation grades, i.e.  $H_{n+1} > H_n$ , the positive ideal  $f_j^+$  and negative ideal  $f_j^-$ , FBS models for each criterion  $C_j, j = 1, 2, \dots, n$  are defined as follows:

$$f_j^+ = \{(H_N, 1), (H_{N-1}, 0), \dots, (H_1, 0)\} \quad (17)$$

and:

$$f_j^- = \{(H_N, 0), (H_{N-1}, 0), \dots, (H_1, 1)\}. \quad (18)$$

Now, the  $S_i$  and  $R_i$  must be found applying Eqs (3) and (4), respectively. To compute these values, the dis-

tance between FBS models are used. The  $S_i$  is calculated as follow by considering Eqs (3) and (9):

$$S_i = \sum_{j=1}^n w_j \frac{d_{BS}(f_j^+, f_{ij})}{d_{BS}(f_j^+, f_j^-)} \tag{19}$$

The  $R_i$  is also defined as:

$$R_i = \max_j \left( w_j \frac{d_{BS}(f_j^+, f_{ij})}{d_{BS}(f_j^+, f_j^-)} \right) \tag{20}$$

Denominators of both Eqs (19) and (20) are fixed values, since the ideal and negative ideal solutions are similar, in the sense of FBS models, for all the criteria. This fixed denominator is determined as below using Eq. (9) as:

$$d_{BS}(f_j^+, f_j^-) = \left( \frac{1}{2} (B_j^+ - B_j^-) S (B_j^+ - B_j^-)^T \right)^{1/2} \tag{21}$$

The following relation is obtained for all values of  $j, j = 1, 2, \dots, n$  by applying the positive ideal and negative ideal belief degrees in Eqs (17) and (18) in Eq. (21):

$$d_{BS}(f_j^+, f_j^-) = \left( \frac{1}{2} ((S_{11} + S_{NN}) - (S_{N1} + S_{NN})) \right)^{1/2} \tag{22}$$

Similarly,  $d_{BS}(f_j^+, f_{ij})$  can be found by using Eq. (9) and implying the belief degrees of  $f_j^+$  and  $f_{ij}$ , respectively. By Computing distances for each alternative with each criterion, the following distance matrix can be constructed, where  $d_{BS}(f_j^+, f_{ij})$  is illustrated with  $d_{ij}^+$ :

$$D = \begin{bmatrix} d_{11}^+ & d_{12}^+ & \dots & d_{1n}^+ \\ d_{21}^+ & d_{22}^+ & \dots & d_{2n}^+ \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1}^+ & d_{m2}^+ & \dots & d_{mn}^+ \end{bmatrix} \tag{23}$$

Then, the matrix  $D$  is normalized by dividing it to  $d$ , and the normalized distance matrix  $ND = [nd_{ij}^+]$ , where  $nd_{ij}^+ = \frac{d_{ij}^+}{d}, i = 1, 2, \dots, m; j = 1, 2, \dots, n$ . At the next stage, the weighted normalize fuzzy belief distance matrix  $WND = [wnd_{ij}^+]$  is computed by multiplying each element's of matrix  $ND$  in its corresponding criterion importance, i.e.  $wnd_{ij}^+ = w_j d_{ij}^+, i = 1, 2, \dots, m; j = 1, 2, \dots, n$ .

### 3.5. Applying VIKOR Method

Since the weighted normalized fuzzy belief distance matrix is obtained, now the VIKOR method can be applied to solve the MAGDM problem based on FBSs. First, the values of  $S_i, i = 1, 2, \dots, m$  is computed for alternatives as:

$$S_i = \sum_{j=1}^n (wnd_{ij}^+) \tag{23}$$

Then, the values of  $R_i, i = 1, 2, \dots, m$  are calculated:

$$R_i = \max_j (wnd_{ij}^+) \tag{24}$$

defining:

$$\begin{aligned} S^+ &= \min_i S_i; \\ S^- &= \max_i S_i \end{aligned} \tag{25}$$

and:

$$\begin{aligned} R^+ &= \min_i R_i; \\ R^- &= \max_i R_i. \end{aligned} \tag{26}$$

Calculate the  $Q_i$  for  $i = 1, 2, \dots, m$  using Eq. (5), where the  $\nu$  multiplier is interpreted as in classic VIKOR method. Applying the common values of  $S^+ = R^+ = 0$ , and  $S^- = R^- = 1$  in Eq. (5), the following relation is obtained:

$$Q_i = \nu S_i + (1 - \nu) R_i \tag{27}$$

Sort the  $S, R$ , and  $Q$  values by ascending order to rank alternatives. There would be 3 lists of ranking illustrated as:  $S_{[0]}, R_{[0]}$  and  $Q_{[0]}$ . After determining these lists, by using step 5 of the VIKOR method in Section 1, the final ranking list will be obtained.

### 3.6. Schematic Algorithm

In this regard, the proposed methodology is illustrated in Fig. 1.

## 4. Case Study

In this regard, the proposed algorithm is applied in a problem of public transportation.

Tehran, the capital of Iran, with a population of more than 14 million, is 17th largest metropolitan of the world. According to formal reports, more than 18 million intra-urban trips are taken in a working day of Tehran. This magnitude of trips highlighted the importance of intra-urban transformation system. Tehran metro consist of 5 lines and about 90 stations around the city, covers more than 3 million daily trips in Tehran and therefore it plays an important role in Tehran's transformation system. Fig. 2 displays the Tehran's metro map.

The Metro system plays an important role in Tehran's intra-urban transportation system. Considering a uniform traveling rate in its 12-hour working day, a one-minute delay in this system means that 70 person-hour working time of people will be lost. In addition, this delay caused traffic in metro stations, which its handling will be a challenging task. Therefore, it is necessary to find the most important causes of metro delays in Tehran.

To this end, the FMEA approach is used. FMEA determines the most important risks based on Risk Priority Number (RPN) index. RPN is defined as  $RPN = Occurrence \cdot Severity \cdot Detection$ . In fact, the more important risk is one that has higher severity; its occurrence is high, or its detection is more difficult. The classic FMEA has some weaknesses as noted by Vahdani *et al.* (2014):

- traditional FMEA neglects the relative importance among occurrence, severity, and detection;
- different values of occurrence, severity, and detection may produce similar RPN values;
- the traditional FMEA neglect to human/expert knowledge.

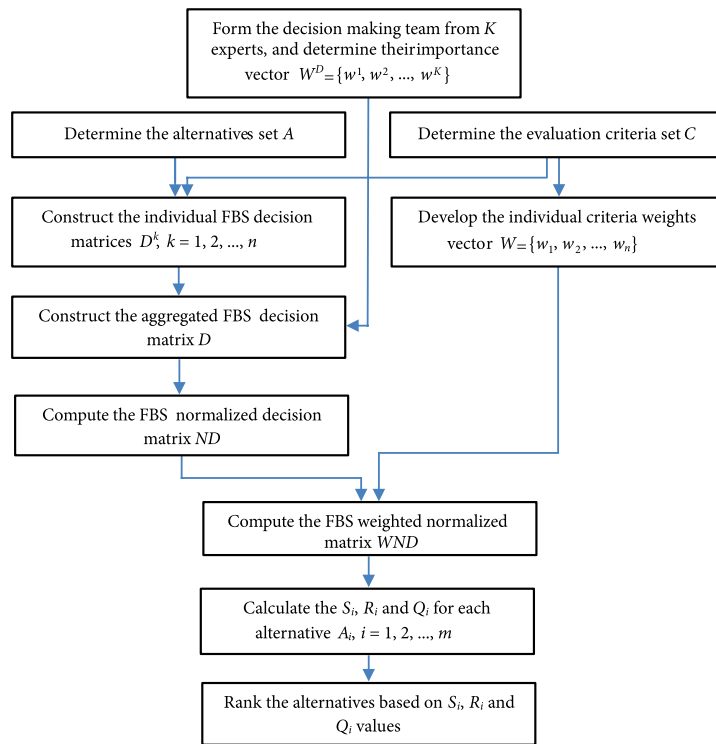


Fig. 1. The flowchart of proposed FBS-VIKOR method for MAGDM



Fig. 2. Tehran metro map

To avoid these weaknesses, the FMEA is combined with FBS based VIKOR method to evaluate the importance of delay causes. In fact, FMEA acts to define the critical criteria for ranking the causes of delay.

A team including 5 experts from Tehran Urban and Suburban Railway Operation Co is formed to find the most important delay causes in Tehran metro system. Considering the previous reports and documents, the team identified 11 factors as the main causes of delays:

- $A_1$  – the transportation system (including the train propulsion, brake system, pneumatic system, wagons door, computer accessories, and technical facilities);
- $A_2$  – power systems including the high pressure, distribution channel, transfer lines, etc.;
- $A_3$  – facilities including electrical and mechanical facilities;
- $A_4$  – control and sign unit involving the switch machine, rail circuit, interlocking, and software and hard ware of traffic control and trains safety system;
- $A_5$  – telecommunications;
- $A_6$  – line including the railway and switch;
- $A_7$  – buildings and stations;
- $A_8$  – passengers and other human factors;
- $A_9$  – unexpected events such as fire, smoke natural disasters, railway failure, and others;
- $A_{10}$  – leadership including the train leadership, supervisor of traffic control center, and maintenance;
- $A_{11}$  – traffic management in a way to indicate how to utilize the system.

These causes are evaluated by participating experts based on three criteria:

- *Severity*: occurring this cause, how much severe, based on delay time, will be the delays caused by it?
- *Occurrence*: how much probable is its occurrence?
- *Detection*: how much is it possible to detect the considered cause before its occurrence?

Using a group AHP method, prior to constructing individual matrices, the importance of criteria are determined as  $w_{severity} = 0.31$ ;  $w_{occurrence} = 0.41$ ;  $w_{detection} = 0.28$  and also  $W^D = (0.2, 0.2, 0.2, 0.2, 0.2)$ .

An FBS scale is also developed for assessing each delay cause about these criteria, as illustrated in Table 1. Decision makers express their beliefs about how percent they believe that, the severity, occurrence, or detection of a delay cause is hard, moderate, and/or simple.

Considering this FBS scale, individual fuzzy belief decision matrices including the evaluation of experts are shown in Table 2.

The next step is to form an aggregated decision matrix, constructing individual decision matrices, by applying Eqs (12) and (13), and decision makers weight vector  $W^D$ . This matrix  $D$  is formed as follows (Table 3).

Considering Table 3, except for  $S(x_{91})$  and  $S(x_{83})$ , all other aggregated FBS models are incomplete, therefore, a normalizing stage is necessary. Applying Eqs (15) and (16), the normalized FBS decision matrix will be obtained as illustrated in Table 4.

Table 1. FBS scale used for delay causes evaluation

Evaluation grade	Linguistic term	Utility
$H_1$	severe/hard	(0.5, 0.7, 0.9)
$H_2$	moderate	(0.3, 0.5, 0.7)
$H_3$	weak/simple	(0.1, 0.3, 0.5)

Table 2. FBS individual decision matrices

Alternatives	Experts	Severity	Occurrence	Detection
$A_1$	$E_1$	(0.70, 0.20, 0.10)	(0.20, 0.60, 0.20)	(0.30, 0.40, 0.30)
	$E_2$	(0.80, 0.05, 0.05)	(0.75, 0.05, 0.05)	(0.10, 0.40, 0.10)
	$E_3$	(0.80, 0.20, 0.00)	(0.80, 0.10, 0.10)	(0.80, 0.15, 0.05)
	$E_4$	(0.10, 0.65, 0.25)	(0.80, 0.10, 0.05)	(0.15, 0.60, 0.15)
	$E_5$	(0.20, 0.30, 0.50)	(0.80, 0.10, 0.10)	(0.60, 0.20, 0.10)
$A_2$	$E_1$	(0.60, 0.25, 0.15)	(0.70, 0.15, 0.15)	(0.10, 0.20, 0.60)
	$E_2$	(0.00, 0.05, 0.20)	(0.90, 0.00, 0.00)	(0.01, 0.01, 0.10)
	$E_3$	(0.90, 0.10, 0.00)	(0.80, 0.10, 0.10)	(0.10, 0.10, 0.80)
	$E_4$	(0.85, 0.05, 0.00)	(0.80, 0.05, 0.05)	(0.05, 0.05, 0.30)
	$E_5$	(0.40, 0.20, 0.10)	(0.80, 0.10, 0.10)	(0.10, 0.10, 0.80)
$A_3$	$E_1$	(0.20, 0.20, 0.50)	(0.10, 0.60, 0.20)	(0.10, 0.10, 0.80)
	$E_2$	(0.05, 0.05, 0.50)	(0.90, 0.05, 0.05)	(0.01, 0.01, 0.20)
	$E_3$	(0.70, 0.15, 0.15)	(0.70, 0.20, 0.10)	(0.20, 0.20, 0.60)
	$E_4$	(0.75, 0.10, 0.15)	(0.10, 0.50, 0.10)	(0.00, 0.10, 0.30)
	$E_5$	(0.80, 0.10, 0.10)	(0.30, 0.20, 0.10)	(0.10, 0.10, 0.10)
$A_4$	$E_1$	(0.60, 0.15, 0.15)	(0.50, 0.30, 0.20)	(0.75, 0.15, 0.05)
	$E_2$	(0.05, 0.50, 0.05)	(0.80, 0.050, 0.05)	(0.05, 0.60, 0.05)
	$E_3$	(0.50, 0.40, 0.10)	(0.30, 0.20, 0.50)	(0.30, 0.30, 0.30)
	$E_4$	(0.50, 0.20, 0.30)	(0.70, 0.20, 0.10)	(0.8, 0.15, 0.05)
	$E_5$	(0.50, 0.30, 0.20)	(0.80, 0.10, 0.10)	(0.7, 0.2, 0.1)
$A_5$	$E_1$	(0.80, 0.10, 0.10)	(0.05, 0.10, 0.85)	(0.05, 0.1, 0.85)
	$E_2$	(0.90, 0.05, 0.05)	(0.05, 0.05, 0.60)	(0.01, 0.01, 0.20)
	$E_3$	(0.80, 0.10, 0.10)	(0.10, 0.10, 0.80)	(0.10, 0.20, 0.70)
	$E_4$	(0.8, 0.00, 0.00)	(0.05, 0.05, 0.10)	(0.05, 0.05, 0.3)
	$E_5$	(0.60, 0.30, 0.10)	(0.80, 0.10, 0.10)	(0.70, 0.20, 0.10)
$A_6$	$E_1$	(0.70, 0.20, 0.10)	(0.15, 0.25, 0.60)	(0.15, 0.25, 0.60)
	$E_2$	(0.80, 0.00, 0.00)	(0.80, 0.01, 0.01)	(0.01, 0.01, 0.1)
	$E_3$	(0.95, 0.05, 0.00)	(0.50, 0.40, 0.10)	(0.40, 0.30, 0.30)
	$E_4$	(0.90, 0.00, 0.10)	(0.80, 0.15, 0.05)	(0.70, 0.20, 0.00)
	$E_5$	(0.90, 0.10, 0.00)	(0.90, 0.10, 0.00)	(0.50, 0.20, 0.00)
$A_7$	$E_1$	(0.85, 0.05, 0.10)	(0.05, 0.10, 0.85)	(0.05, 0.10, 0.85)
	$E_2$	(0.70, 0.10, 0.10)	(0.10, 0.10, 0.60)	(0.40, 0.10, 0.50)
	$E_3$	(0.80, 0.15, 0.05)	(0.20, 0.20, 0.60)	(0.30, 0.00, 0.07)
	$E_4$	(0.80, 0.10, 0.10)	(0.10, 0.40, 0.10)	(0.10, 0.40, 0.30)
	$E_5$	(0.70, 0.10, 0.00)	(0.60, 0.20, 0.00)	(0.30, 0.20, 0.10)
$A_8$	$E_1$	(0.60, 0.15, 0.25)	(0.70, 0.15, 0.15)	(0.70, 0.15, 0.15)
	$E_2$	(0.80, 0.05, 0.05)	(0.05, 0.50, 0.05)	(0.95, 0.03, 0.02)
	$E_3$	(0.95, 0.00, 0.05)	(0.60, 0.30, 0.10)	(0.70, 0.20, 0.10)
	$E_4$	(0.85, 0.05, 0.10)	(0.90, 0.05, 0.05)	(0.95, 0.05, 0.00)
	$E_5$	(0.50, 0.30, 0.20)	(0.90, 0.10, 0.00)	(0.90, 0.10, 0.00)
$A_9$	$E_1$	(0.10, 0.10, 0.80)	(0.80, 0.20, 0.00)	(0.00, 0.05, 0.95)
	$E_2$	(0.02, 0.02, 0.30)	(0.90, 0.05, 0.05)	(0.00, 0.02, 0.10)
	$E_3$	(0.20, 0.10, 0.70)	(0.97, 0.02, 0.01)	(0.15, 0.15, 0.70)
	$E_4$	(0.70, 0.30, 0.00)	(0.85, 0.10, 0.05)	(0.05, 0.05, 0.20)
	$E_5$	(0.10, 0.20, 0.70)	(1.00, 0.00, 0.00)	(0.10, 0.20, 0.40)
$A_{10}$	$E_1$	(0.40, 0.15, 0.45)	(0.60, 0.20, 0.10)	(0.40, 0.20, 0.20)
	$E_2$	(0.80, 0.05, 0.05)	(0.00, 0.05, 0.30)	(0.05, 0.05, 0.30)
	$E_3$	(0.80, 0.10, 0.10)	(0.30, 0.00, 0.70)	(0.20, 0.20, 0.60)
	$E_4$	(0.60, 0.15, 0.15)	(0.20, 0.40, 0.10)	(0.50, 0.30, 0.20)
	$E_5$	(0.00, 0.20, 0.70)	(1.00, 0.00, 0.00)	(0.60, 0.20, 0.20)
$A_{11}$	$E_1$	(0.50, 0.30, 0.20)	(0.70, 0.20, 0.10)	(0.10, 0.10, 0.80)
	$E_2$	(0.70, 0.15, 0.05)	(0.70, 0.15, 0.15)	(0.10, 0.25, 0.55)
	$E_3$	(0.90, 0.10, 0.00)	(0.40, 0.50, 0.10)	(0.30, 0.00, 0.70)
	$E_4$	(0.10, 0.10, 0.30)	(0.85, 0.10, 0.05)	(0.40, 0.10, 0.30)
	$E_5$	(0.10, 0.10, 0.80)	(0.40, 0.20, 0.10)	(0.30, 0.20, 0.00)

Table 3. Aggregated decision matrix

Alternatives	Severity	Occurrence	Detection
$A_1$	(0.5525, 0.2679, 0.1631)	(0.7252, 0.1612, 0.0830)	(0.4110, 0.3597, 0.1318)
$A_2$	(0.6284, 0.1213, 0.0778)	(0.8519, 0.0600, 0.0600)	(0.0691, 0.0890, 0.6086)
$A_3$	(0.5450, 0.1097, 0.2658)	(0.4538, 0.3133, 0.1032)	(0.0866, 0.1070, 0.4815)
$A_4$	(0.4615, 0.3073, 0.1510)	(0.6704, 0.1483, 0.1655)	(0.5654, 0.2650, 0.0976)
$A_5$	(0.8340, 0.0846, 0.0533)	(0.2047, 0.0756, 0.5511)	(0.1848, 0.1137, 0.4875)
$A_6$	(0.8948, 0.0524, 0.0283)	(0.6827, 0.1608, 0.1284)	(0.3899, 0.2020, 0.2062)
$A_7$	(0.8269, 0.0772, 0.0536)	(0.2036, 0.1940, 0.4712)	(0.2205, 0.1447, 0.5391)
$A_8$	(0.7955, 0.0862, 0.1036)	(0.6914, 0.1902, 0.0585)	(0.8844, 0.0773, 0.0382)
$A_9$	(0.2159, 0.1366, 0.5461)	(0.9350, 0.0503, 0.0147)	(0.0593, 0.0939, 0.5615)
$A_{10}$	(0.5573, 0.1173, 0.2762)	(0.4704, 0.1219, 0.2391)	(0.3745, 0.1896, 0.3083)
$A_{11}$	(0.5002, 0.1425, 0.2611)	(0.6635, 0.2061, 0.0847)	(0.2328, 0.1190, 0.5211)

Table 4. Normalized FBS decision matrix

Alternatives	Severity	Occurrence	Detection
$A_1$	(0.5580, 0.2734, 0.1686)	(0.7354, 0.1714, 0.0932)	(0.4435, 0.3922, 0.1643)
$A_2$	(0.6860, 0.1788, 0.1353)	(0.8613, 0.0694, 0.0694)	(0.1469, 0.1668, 0.6864)
$A_3$	(0.5715, 0.1362, 0.2923)	(0.4970, 0.3565, 0.1464)	(0.1949, 0.2153, 0.5898)
$A_4$	(0.4882, 0.3340, 0.1777)	(0.6757, 0.1536, 0.1707)	(0.5894, 0.2890, 0.1216)
$A_5$	(0.8434, 0.0940, 0.0626)	(0.2609, 0.1318, 0.6073)	(0.2561, 0.1850, 0.5589)
$A_6$	(0.9036, 0.0591, 0.0372)	(0.6921, 0.1701, 0.1378)	(0.4572, 0.2693, 0.2735)
$A_7$	(0.8410, 0.0913, 0.0677)	(0.2474, 0.2377, 0.5149)	(0.2524, 0.1766, 0.5710)
$A_8$	(0.8004, 0.0911, 0.1085)	(0.7114, 0.2101, 0.0785)	(0.8844, 0.0773, 0.0382)
$A_9$	(0.2497, 0.1704, 0.5799)	(0.9350, 0.0503, 0.0147)	(0.1544, 0.189, 0.6566)
$A_{10}$	(0.5736, 0.1337, 0.2926)	(0.5266, 0.1781, 0.2953)	(0.4170, 0.2321, 0.3508)
$A_{11}$	(0.5323, 0.1746, 0.2931)	(0.6787, 0.2214, 0.1000)	(0.2752, 0.1614, 0.5635)

The normalized FBS decision matrix is used to compute the distance matrix  $D$  by Eq. (23). Considering the fuzzy utilities of evaluation grades, the similarity matrix is first calculated:

$$S = \begin{bmatrix} 1 & 0.8 & 0.6 \\ 0.8 & 1 & 0.8 \\ 0.6 & 0.8 & 1 \end{bmatrix}.$$

The distance matrix  $D$  is computed as below by using  $S$ :

$$D = \begin{bmatrix} 0.2116 & 0.1255 & 0.2595 \\ 0.1529 & 0.0694 & 0.4897 \\ 0.2320 & 0.2343 & 0.4463 \\ 0.2423 & 0.1639 & 0.1915 \\ 0.0754 & 0.4278 & 0.4161 \\ 0.0462 & 0.1509 & 0.2718 \\ 0.0773 & 0.4078 & 0.4207 \\ 0.1016 & 0.1338 & 0.0544 \\ 0.4241 & 0.0298 & 0.4788 \\ 0.2313 & 0.2495 & 0.3040 \\ 0.2469 & 0.1505 & 0.4106 \end{bmatrix}.$$

Considering  $d_{BS}(f_j^+, f_j^-) = 0.6325$ , the normalized distance matrix  $ND$  is found by dividing elements of  $D$  into  $d_{BS}(f_j^+, f_j^-)$ . The obtained matrix is formed as:

$$ND = \begin{bmatrix} 0.3345 & 0.1984 & 0.4103 \\ 0.2418 & 0.1097 & 0.7743 \\ 0.3668 & 0.3704 & 0.7057 \\ 0.3831 & 0.2592 & 0.3028 \\ 0.1192 & 0.6764 & 0.6579 \\ 0.0731 & 0.2385 & 0.4298 \\ 0.1222 & 0.6448 & 0.6652 \\ 0.1606 & 0.2115 & 0.0861 \\ 0.6706 & 0.0471 & 0.7570 \\ 0.3657 & 0.3945 & 0.4811 \\ 0.3903 & 0.2379 & 0.6492 \end{bmatrix}.$$

Constructing the matrix  $ND$ , it remains to find the weighted normalized decision matrix  $WND$ , by multiplying each element of  $ND$  at its associated attributes weight. Then, the  $S_i$  and  $R_i$  values for each alternative is determined by Eqs (23) and (24). Defining  $S^+ = R^+ = 0$ , and  $S^- = R^- = 1$ , the  $Q_i$ ,  $i = 1, 2, \dots, 11$  values are computed using Eq. (27), for different values of  $\nu$ .



Table 5.  $S_i$ ,  $R_i$  and  $Q_i$  values for different  $\nu$

Alternative	$S_i$	$R_i$	$Q_i$										
			0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$A_1$	0.2999	0.1149	2	3	3	3	3	3	3	3	3	3	3
$A_2$	0.3367	0.2168	9	8	7	7	6	5	5	5	5	5	5
$A_3$	0.4632	0.1976	7	7	8	8	9	9	9	9	9	9	9
$A_4$	0.3098	0.1188	3	4	4	4	4	4	4	4	4	4	4
$A_5$	0.4985	0.2773	11	11	11	11	11	11	11	11	11	11	11
$A_6$	0.2408	0.1203	4	2	2	2	2	2	2	2	2	2	2
$A_7$	0.4885	0.2644	10	10	10	10	10	10	10	10	10	10	10
$A_8$	0.1606	0.0867	1	1	1	1	1	1	1	1	1	1	1
$A_9$	0.4392	0.2120	8	9	9	9	8	8	8	8	8	8	8
$A_{10}$	0.4098	0.1618	5	5	5	5	5	6	6	7	7	7	7
$A_{11}$	0.4003	0.1818	6	6	6	6	7	7	7	6	6	6	6

These values are computed and ranked at Table 5. According to this table, there are not any paradoxical ranking in different values of  $\nu$  and just some minor differences occur for these values. Using the mean of ranks as the aggregating methods (Yoon, Hwang 1995), the final ranking of delay causes is presented in Table 6, in the FBS-VIKOR column.

According to the results of Table 6, the three most important delay causes include passengers and other human factors, line including the railway and switch, the transportation system (including the train propulsion, brake system, pneumatic system, wagons door, computer accessories, and technical facilities). Therefore, some training and informative programs should be planned to resolve human related issues. In addition, it seems necessary to design and deploy an improved maintenance program for solving the problems related to line and transportation system. In this table, the results obtained by solving the problem using FBS-TOPSIS method of Jiang et al. (2011) and Vahdani et al. (2014) is also presented. As it is clear from Table 5 results, except for 6th and 7th ranks, all other alternatives obtained similar ranks in both methods. The Spearman's rank correlation between both methods is about 0.97.

Table 6. Final ranking and comparison

Alternatives	Rank	
	FBS-VIKOR	FBS-TOPSIS
$A_1$	3	3
$A_2$	6	5
$A_3$	9	9
$A_4$	4	4
$A_5$	11	11
$A_6$	2	2
$A_7$	10	10
$A_8$	1	1
$A_9$	8	8
$A_{10}$	5	7
$A_{11}$	7	6

**Conclusions**

Multi attribute group decision-making problems are a set of widely used procedures to choose the best alternative or to rank a set of alternatives based on a set of different and usually contrasting attributes. Vincke (1992) believed that this family of problems is usually difficult to solve. This difficulty is a result of nonexistence of a global optimal solution for these problems. On the other hand, often decision makers do not have complete information regard to required information for decision-making problems. This feature of uncertainty is formulated under different frameworks. One of the recent and strength frameworks of dealing with uncertainty is FBS. This well-defined approach constituted of an evaluation grade that is represented as fuzzy grades along with a belief degree assigned to each evaluation grade. In FBS, decision makers most specify their belief degree about each alternative performance regard to each attribute in every evaluation grade. Combination of fuzzy set theory and the evidence combination rule of the Dempster-Shafer theory made FBS as a powerful way of portraying human uncertainty.

In this paper, the VIKOR as an accepted MAGDM method is extended under the condition that decision makers express their judgments about alternatives performance regard to attributes in the form of FBS models. Using evidential reasoning approach, the individual FBS decision matrices of decision makers are aggregated in a single one. Then, the center of gravity method is applied to normalize the aggregated decision matrix. Distances of alternatives from these ideals are computed following the VIKOR method procedure by defining the positive ideal and negative ideal FBS models.

The main advantages of the proposed method can be stated as:

- providing a flexible framework for experts to state their judgment in a linguistic evaluation grade along with belief degrees;
- provide a method of group decision-making by aggregating different experts' opinion by considering their relative importance and weights;
- obtain a consensus based result for the decision-

making process which can be applied in different transportation decision-making problems and other areas.

An application of the proposed method is shown in ranking the causes of delay in Tehran metro system. Ranking criteria are defined according to FMEA approach. Then, five experts' opinion are gathered in the form of FBS model and the proposed FBS-based VIKOR method is applied. The obtained results showed that the most important delay causes are human related issues and line and transportation system. Finding the most important delay causes, a set of corrective actions can be designed to resolve the undesirable consequences. Comparison of the results with a previously proposed method represented a high consistency among the methods as a strengthening fact about the proposed FBS-VIKOR method. Future researches in this area can be directed toward extension of other decision-making methods, include outranking methods, under FBS information. In addition, researchers can concentrate on extension of mathematical operations of FBSs.

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