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METHODS FOR NUMERICAL CALCULATION OF PARAMETERS PERTAINING TO THE MICROSCOPIC FOLLOWING-THE-LEADER MODEL OF TRAFFIC FLOW: USING THE FAST SPLINE TRANSFORMATION

Nina Kuzhel[1], Andrii Bieliatynskyi[2], Olegas Prentkovskis[3], Iryna Klymenko[4], Šarūnas Mikaliūnas[5], Olena Kolganova[6], Svitlana Kornienko[7], Volodymyr Shutko[8]

[1]Dept of Air Transportation Management, National Aviation University, Kosmonavta Komarova ave 1, 03680 Kyiv, Ukraine
[2]Dept of Airport Reconstruction and Automobile Roads, National Aviation University, Kosmonavta Komarova ave 1, 03680 Kyiv, Ukraine
[3]Dept of Transport Technological Equipment, Vilnius Gediminas Technical University, Plytinės g. 27, LT-10105 Vilnius, Lithuania
[4]Dept of Economics, National University of Transport, Suvorova str. 1, 01010 Kyiv, Ukraine
[5]Dept of Railway Transport, Vilnius Gediminas Technical University, J. Basanavičiaus g. 28, LT-03224 Vilnius, Lithuania
[6]Dept of Information Security, National Aviation University, Kosmonavta Komarova ave 1, 03680 Kyiv, Ukraine
[7]Dept of Aviation Radioelectronic Complexes, National Aviation University, Kosmonavta Komarova ave 1, 03680 Kyiv, Ukraine
[8]Dept of Electronics, National Aviation University, Kosmonavta Komarova ave 1, 03680 Kyiv, Ukraine

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Abstract. As known in transport engineering, civil engineering, transport planning and mathematics, traffic flow is the study of interactions between vehicles, drivers and infrastructure (including highways, signage and traffic control devices), with the aim of understanding and developing an optimal road network with efficient movement of traffic and minimal traffic congestion problems. The presented paper discusses a small part of a traffic flow study – the development of the methodology for assessing the speed and acceleration of a car during the column movement following-the-leader, based on a new mathematical method. Two methods – (1) the numerical calculation of the first derivative, i.e. speed of the car movement; (2) the numerical calculation of the second derivative, i.e. acceleration of the car movement – were developed, using the fast spline transformation. In the future, parameters obtained with the help of two new methods, can be used to solve complex transportation problems, such as: (1) control of traffic flows; (2) organisation of harmonised work of traffic lights; (3) analysis of psycho-physiological condition of a driver, etc.

Keywords: traffic flow, car speed, car acceleration, column movement, following-the-leader model, fast spline transformation, numerical calculation.

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1. Topicality

Depending on road loading, some characteristic modes of transport flows were distinguished, connecting them with the concept of movement convenience levels.

Dense or sated flow (a level of movement convenience) is the most difficult structural form of traffic flow, which is characterised by identical speeds and approximately identical distances between cars moving one after another, without the overtaking possibility, which means that movement of each car in the flow is connected with actions of the front car. Movement speed sharply decreases and jams are possible at road intervals with

Corresponding author: Nina Kuzhel
E-mail: *kuzhelnina@ukr.net*

deteriorated conditions. This results in strained driving conditions.

Movement in a dense car flow requires special attention and high concentration. In this instance, keeping in a lane becomes especially urgent, as it is the main condition for fast and safe movement in the traffic flow. Additional obstacles and inconveniences to other drivers are often created by frequent and useless change of lanes, which often leads to road accidents.

Driving in a dense car flow requires choosing the speed, which depends on the speed of the flow movement. The distinctive feature of driving under limited conditions is that drivers get tired faster than usual and rather frequently lose control over themselves, tending to overtake vehicles moving in front. Most road accidents occur due to attempts to change a lane filled with passing cars.

Driving in a dense car flow requires the ability to keep a safe distance from the car moving in front. While choosing the distance, it is necessary to take into consideration conditions of the road pavement, developed traffic situation, technical conditions of own car, and compare the speed of own car with the average speed of the traffic flow. Besides, there is no need to keep too big of a distance as it may encourage other drivers to get in front.

2. Analysis of Researches and Publications

To solve problems related to the reduction of the number of road traffic accidents (especially car collision), it is necessary to undertake a detailed analysis of interaction between cars that move in the same direction.

Traffic flow problems can be analysed in three ways, which correspond to the three main scales of observation in physics:

- *Microscopic scale.* At the most basic level, every vehicle is considered as an individual. An equation – usually an ordinary differential equation – can be written for each. Cellular automation models can also be used, where the road is divided into cells, each of which either contains a moving car, or is empty. The Nagel–Schreckenberg model is a simple example of such model. As cars interact, it can model collective phenomena such as traffic jams. Microscopic traffic flow models simulate single vehicle–driver units, so the dynamic variables of models represent microscopic properties such as the position and speed of single vehicles. For example, microscopic models investigated in researches, which were carried out by scientists Knorr (2013); Sheu (2013); Vaiana *et al.* (2013); Kerner, Klenov (2010, 2006); Hawas, Hameed (2009); Yeo (2008); Kerner *et al.* (2006), etc.;
- *Macroscopic scale.* Similar to models of fluid dynamics, it is considered useful to employ a system of partial differential equations, which balance laws for some gross quantities of interest; e.g. the density of vehicles or their mean speed. A macroscopic traffic flow model is a mathematical model that formulates the relationships between traffic flow characteristics such as density, flow, mean speed of a traffic flow, etc. Such models are conventionally arrived at by integrating microscopic traffic flow models and converting the single-entity level characteristics to comparable system level characteristics. For example, macroscopic models investigated in researches, which were carried out by scientists Ngoduy (2013, 2012); Velasco, Saavedra (2008); Helbing *et al.* (2001); Zhang *et al.* (1997), etc.;
- *Mesoscopic (kinetic) scale.* The third, intermediate possibility, is to define a function $f(t,x,V)$, which expresses the probability of having a vehicle at time t in position x, which runs at a speed V. This function, following methods of statistical mechanics, can be computed using an integral-differential equation such as the Boltzmann equation. Mesoscopic models combine the properties of both microscopic and macroscopic simulation models. These models simulate individual vehicles, but describe their activities and interactions based on aggregate (macroscopic) relationships. For example, mesoscopic (kinetic) models investigated in researches, which were carried out by scientists Lu *et al.* (2013); Belloquid *et al.* (2012); Chiu *et al.* (2010); Bonzani, Mussone (2009); Tosin (2009); Bonzani *et al.* (2008); Coscia *et al.* (2007), etc.

The engineering approach to analysis of traffic flow problems is primarily based on empirical analysis (i.e. observation and mathematical curve fitting). One major reference used by American planners is the Highway Capacity Manual (2010). This recommends modelling traffic flows based on the whole travel time across a link using a delay/flow function, including the effects of queuing. This technique is used in many US traffic models and, for example, in the *Saturn* (*http://www.saturnsoftware.co.uk*) model in Europe, etc.

In European countries, a hybrid empirical approach to traffic design is used, combining macro-, micro-, and mesoscopic features. Rather than simulating a steady state of flow for a journey, transient 'demand peaks' of congestion are simulated. These are modelled by using small 'time slices' across the network throughout the working day or weekend. Typically, the origins and destinations for trips are first estimated and a traffic model is generated before being calibrated by comparing the mathematical model with observed counts of actual traffic flows, classified by the type of vehicle. 'Matrix estimation' is then applied to the model to achieve a better match to observed link counts before any changes, and the revised model is used to generate a more realistic traffic forecast for any proposed scheme. The model would be run several times (including a current baseline, an 'average day' forecast based on a range of economic parameters and supported by sensitivity analysis) in order to understand the implications of temporary blockages or incidents around the network. From the models, it is possible to total the time taken for all drivers of different types of vehicles on the network and, thus, deduce average fuel consumption and emissions.

To better represent traffic flow, relationships were established between three main characteristics – (1) flow; (2) density; (3) speed. These relationships help in planning, design, and operations of roadway facilities.

The bases of the mathematical modelling of traffic laws were established in 1910–1912 by the Russian scientist G. D. Dubelir (1910, 1912). The first attempt to generalise mathematical researches of traffic flows and present them in the form of an independent section of the applied mathematics was made by American scientist F. A. Haight (1963, 1966). Because of studies on high-density traffic flows and special experiments carried out by American experts, the theory of following-the-leader was proposed, mathematical expression of which is the microscopic model of traffic flow. It is called microscopic, because it considers an element of the flow, a pair of vehicles following one another. The model displays the laws of the complex 'driver–car–road–environment', particularly, the psychological aspect of car control. It considers that actions of a driver moving in a dense traffic flow are conditioned by the change in the speed of the leading car and the distance to it.

Problems related with traffic flow are considered in researches of domestic and foreign scientists, such as Beliatynskii and Kuzhel (2010); Beliatynskii *et al.* (2011); Beljatynskij *et al.* (2009); Junevičius (2011); Kuzhel (2011); Sil'yanov (1977); Lobanov, Sil'yanov (1974); Lobanov (1975); Haight (1963, 1966); Buslaev *et al.* (2012); Siegel, Coeymans (2005), etc.

The theory of traffic flow, such as following-the-leader, involves the development of theory for simplified dynamic models. It is based on hypothesis on existence of a law regarding the interaction of cars that follow one after another at a short distance. The differential equation of the theory of traffic flow, such as following-the-leader, is received from the initial condition that all cars move in a column at a distance that is required by traffic regulations. Then, coordinates of the position of the n and (n+1) cars can be described using the relation:

$$x_{n+1} = x_n + \left(l_0 + t_p \cdot v_n\right) + l_{n+1}, \tag{1}$$

where: x_n, x_{n+1} – coordinates of the back and front cars; l_0 – minimum distance between the stationary cars; $t_p \cdot v_n$ – distance between the cars that are established depending on the speed of the movement; l_{n+1} – length of the car; n – serial number of the car.

Differentiating the previous equation by time, the following is received:

$$\frac{dx_{n+1}}{dt} = \frac{dx_n}{dt} + t_p \cdot \frac{dv_n}{dt}, \tag{2}$$

where: n = 1, 2, 3.

This equation can be expressed in terms of speed in the following form:

$$v_{n+1} = v_n + t_p \cdot \frac{dv_n}{dt}; \tag{3}$$

$$v_{n+1} - v_n = t_p \cdot \frac{dv_n}{dt}; \tag{4}$$

$$\frac{dv_n}{dt} = \frac{1}{t_p} \cdot \left(v_{n+1} - v_n\right), \tag{5}$$

where: $\frac{dv_n}{dt}$ – acceleration of the back car; v_n, v_{n+1} – speeds of the back and front cars; l_p – time of the driver response.

This rule can be expressed in terms of speed in the following form:

$$\frac{d^2 x_n}{dt^2} = \frac{1}{t_p} \cdot \left(\frac{dx_{n+1}}{dt} - \frac{dx_n}{dt}\right). \tag{6}$$

3. Statement of the Research Task

To investigate this model of movement following-the-leader with real objects, it is necessary to process data on movement of connected objects (for example, by means of GPS-receivers), which are received with errors (errors caused by inaccuracy of measuring equipment).

It is also required to find the first and the second derivative of the 'noise' traffic diagrams of the objects, which correspond to the speeds and accelerations of the car movement. Therefore, the task to develop the mathematical method of the estimation of the parameters of movement, which would allow minimising the specified errors, was assigned.

4. Statement of the Basic Material

Methods for numerical calculation of the function derivative, which is observed against the casual errors of the test data, are based on smoothing of this function by polynoms of the best root-mean-square approach, Fourier series, splines (Korn, G. A., Korn, T. M. 2000). Then, the further finding of the derivative itself is carried out analytically.

That is, the task is assigned: to numerically calculate the derivative of function (for speed definition):

$$F\left(t\right) = \frac{dY\left(t\right)}{dt}, \tag{7}$$

where: $Y\left(t\right)$ – route (coordinate) of the car, which is observed against the errors.

Let on the interval $\left[0, T\right]$ at points $t = \left\{t_i\right\}_{i=1}^{N}$ the values $Y = \left\{y_i\right\}_{i=1}^{N}$ of some discrete time function be defined. They correspond to the (not yet estimated) reports of the derivative $F = \left\{f_i\right\}_{i=1}^{N}$ at points $t = \left\{t_i\right\}_{i=1}^{N}$.

Then Y and Y would be connected with the relations:

$$F = P \cdot Y; \tag{8}$$

$$Y = Q \cdot F, \tag{9}$$

where: P, Q – operators of the differentiation and integration accordingly.

Let's consider that values of a derivative F are described by the local cubic Hermitian spline (Korn, G. A., Korn, T. M. 2000):

$$S_3 = X \cdot A, \quad (10)$$

where: X – planning matrix; $A = \{a_j\}_{j=0}^{r}$ – vector of the parameter estimation (ordinates of points of the spline area patching). Such spline belongs to C^1 – class of the continuously differentiated functions.

Then $Y = Q \cdot X \cdot A$, the matrix is designated through $W = Q \cdot X$ with the dimension $N \cdot (r+1)$, which consists of the integrated local functions of the spline form.

The condition of the minimum of the standard deviation is to be fulfilled:

$$\sum_{i=1}^{N}\left(y_i - \sum_{j=0}^{r} w_{ij} \cdot a_j \right)^2 = \min, \; j = \overline{0,r}. \quad (11)$$

This condition is satisfied by the decision of the system of the normal equations:

$$(Y - W \cdot A)^T \cdot (Y - W \cdot A) = \min; \quad (12)$$

$$W^T \cdot W \cdot A = W^T \cdot Y; \quad (13)$$

$$A = \left(W^T \cdot W\right)^{-1} \cdot W^T \cdot Y. \quad (14)$$

The obtained vector of the estimated parameters $A = \{a_j\}_{j=0}^{r}$ completely defines the spline $S_3 = X \cdot A$. It should be noted that the matrixes W^T and $\left(W^T \cdot W\right)^{-1}$ do not depend on the input parameters and can be preliminarily calculated. Thus, using the time reports of the initial function $Y = \{y_i\}_{i=1}^{N}$, a spline approximation S_3 of the derivative F of this function can be quickly found without preliminary calculation of reports of the derivative $F = \{f_i\}_{i=1}^{N}$.

Values of the local cubic Hermitian spline (Korn, G. A., Korn, T. M. 2000) at the arbitrary point are calculated by formula:

$$S(t) = a_{j-1} \cdot {}^1x(t) + a_j \cdot {}^2x(t) + a_{j+1} \cdot {}^3x(t) + a_{j+2} \cdot {}^4x(t) \quad \text{for } t \in \left[t_{u_j}, t_{u_{j+1}}\right], \quad (15)$$

where: a_j – value of the ordinates of patching units of the spline areas; ${}^kx(t)$ – local shape functions, whose discrete values complete the columns of the planning matrix X and are calculated by formulas:

$${}^1X_{ij} = -\frac{h_j^2 \cdot x_{ij} \cdot (1 - x_{ij})^2}{h_{j-1} \cdot (h_{j-1} + h_j)}, \; j = \overline{2,r}, \; i = \overline{1 + m_{j-1}, m_j}; \quad (16)$$

$${}^2X_{i1} = 1 - x_{i1} - \frac{h_1 \cdot x_{i1}^2 \cdot (1 - x_{i1})}{h_1 + h_2}, \; i = \overline{1, m_1}; \quad (17)$$

$${}^2X_{ij} = 1 - x_{ij} - \frac{h_j \cdot x_{ij}^2 \cdot (1 - x_{ij})}{h_j + h_{j+1}} + \frac{h_j \cdot x_{ij} \cdot (1 - x_{ij})^2}{h_{j-1}},$$

$$j = \overline{2, r-1}, \; i = \overline{1 + m_{j-1}, m_j}; \quad (18)$$

$${}^2X_{ir} = 1 - x_{ir} - \frac{h_r \cdot x_{ir} \cdot (1 - x_{ir})^2}{h_{r-1}}, \; i = \overline{1 + m_{r-1}, m_r}; \quad (19)$$

$${}^3X_{i1} = x_{i1} - \frac{h_1 \cdot x_{i1}^2 \cdot (1 - x_{i1})}{h_2}, \; i = \overline{1, m_1}; \quad (20)$$

$${}^3X_{ij} = x_{ij} - \frac{h_j \cdot x_{ij}^2 \cdot (1 - x_{ij})}{h_{j+1}} - \frac{h_j \cdot x_{ij} \cdot (1 - x_{ij})^2}{h_{j-1} + h_j},$$

$$j = \overline{2, r-1}, \; i = \overline{1 + m_{j-1}, m_j}; \quad (21)$$

$${}^3X_{ir} = x_{ir} - \frac{h_r \cdot x_{ir} \cdot (1 - x_{ir})^2}{h_{r-1} + h_r}, \; i = \overline{1 + m_{r-1}, m_r}; \quad (22)$$

$${}^4X_{ij} = -\frac{h_j^2 \cdot x_{ij}^2 \cdot (1 - x_{ij})}{h_{j+1} \cdot (h_j + h_{j+1})}, \; j = \overline{1, r-1},$$

$$i = \overline{1 + m_{j-1}, m_j}; \quad (23)$$

$$x_{ij} = \frac{x_i - \tilde{x}_{j-1}}{h_j}; \; h_j = \tilde{x}_j - \tilde{x}_{j-1};$$

$$j = \overline{1, r}; \; x_i \in \left[\tilde{x}_{j-1}, \tilde{x}_j\right];$$

$$j = \overline{1, r-1}; \; x_i \in \left[\tilde{x}_{r-1}, \tilde{x}_r\right];$$

$$m_j = \sum_{u=1}^{j} K_u, \; j = \overline{1, r};$$

$$m_{-1} = m_0 = 0; \; m_r = N,$$

where: K_u – number of the reports on the interval u.

The number of multiplication operations, addition, necessary for calculation of fast spline approximation of the function derivative, which was observed:

$$M = N \cdot (r+1) + (r+1)^2. \quad (24)$$

Let's compare the quality of the offered method for numerical calculation of the function derivative, which is observed against the casual errors of the test data, with the classical method (smoothing by spline function and the further analytical finding of the derivative itself).

As an example, the function $y(t) = 10 \cdot t^3 + n(t)$ is taken on the interval $[0, 2]$ at points with the step 1/32 (Fig. 1), where $n(t)$ – Gaussian signal-independent noise. The derivative of the deterministic bases is $y'(t) = 30 \cdot t^2$ accordingly (Fig. 1). The calculations were performed in *MatLab*.

For 64 reports of the initial function and 16 patching units of the spline, the following results were received:

- standard deviation of the input Gaussian signal-independent noise changed from 0.2 to 1.1 (Fig. 2);
- at the same time, the standard deviation of the theoretical derivative of the derivative, numerically calculated by the classical method, changed from 2.40 to 3.48 (Fig. 2);
- standard deviation of the theoretical derivative of the derivative, numerically calculated by the offered method, changed from 0.53 to 2.87 (Fig. 2).

In a similar way, it is also possible to calculate the fast spline approximation of the second derivative (car acceleration).

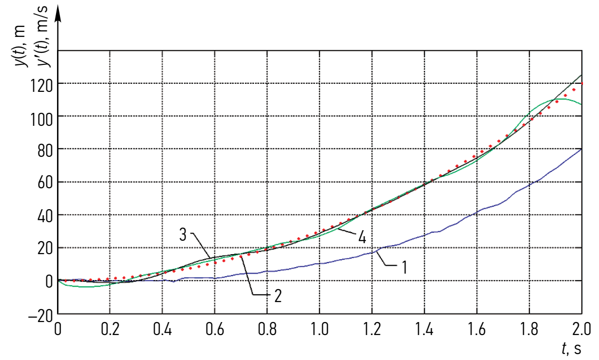

Fig. 1. The derivatives (car speeds), numerically calculated by the classical and offered methods from the input signal with the standard deviation $\sigma\left[n(t)\right]=0.5$ of input Gaussian signal-independent noise: 1 – input signal with the noise to differentiation; 2 – theoretical derivative; 3 – derivative, numerically calculated by the offered method; 4 – derivative, numerically calculated by the classical method

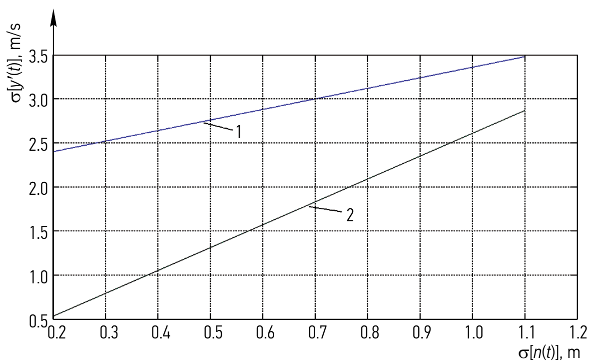

Fig. 2. Standard deviations of the theoretical derivative (car speed), numerically calculated by using the classical (1) and offered (2) methods

That is, the task is assigned: to numerically calculate the second derivative of the function (for acceleration definition):

$$Z(t)=\frac{d^2Y(t)}{dt^2}, \tag{25}$$

where: $Y(t)$ – route (coordinate) of the car, which is observed against the errors.

Let on the interval $\left[0,T\right]$ at points $t=\left\{t_i\right\}_{i=1}^{N}$ the values $Y=\left\{y_i\right\}_{i=1}^{N}$ of some discrete time function be defined. They correspond to the (not yet estimated) reports of the derivative $F=\left\{f_i\right\}_{i=1}^{N}$ at points $t=\left\{t_i\right\}_{i=1}^{N}$, and (not yet estimated) reports of the second derivative $Z=\left\{z_i\right\}_{i=1}^{N}$ at points $t=\left\{t_i\right\}_{i=1}^{N}$.

Then, Y and Z would be connected with the relations:

$$Z=V\cdot Y; \tag{26}$$

$$Y=L\cdot Z, \tag{27}$$

where: V, L – operators of the double differentiation and double integration accordingly.

Let's consider that values of the second derivative Z are described by the local cubic Hermitian spline (Korn, G. A., Korn, T. M. 2000):

$$S_3=X\cdot A, \tag{28}$$

where: X – planning matrix; $A=\left\{a_j\right\}_{j=0}^{r}$ – vector of parameter evaluation (ordinates of points of spline area patching). Such spline belongs to C^1 – class of the continuously differentiated functions.

Then $Y=L\cdot X\cdot A$, the matrix is designated through $G=L\cdot X$, with the dimension $N\cdot(r+1)$, which consists of the integrated local functions of the spline form.

The condition of the minimum of the standard deviation is to be fulfilled:

$$\sum_{i=1}^{N}\left(y_i-\sum_{j=0}^{r}g_{ij}\cdot a_j\right)^2=\min,\ j=\overline{0,r}. \tag{29}$$

This condition is satisfied by the decision of the system of the normal equations:

$$(Y-G\cdot A)^T(Y-G\cdot A)=\min; \tag{30}$$

$$G^T\cdot G\cdot A=G^T\cdot Y; \tag{31}$$

$$A=\left(G^T\cdot G\right)^{-1}\cdot G^T\cdot Y. \tag{32}$$

The obtained vector of the estimated parameters $A=\left\{a_j\right\}_{j=0}^{r}$ completely defines the spline $S_3=X\cdot A$. It should be noted that the matrixes G^T and $\left(G^T\cdot G\right)^{-1}$ do not depend on the input parameters and can be preliminary calculated. Thus, using the time reports of the initial function $Y=\left\{y_i\right\}_{i=1}^{N}$, a spline approximation S_3 of the second derivative Z of this function can be found quickly without preliminary calculation of the reports of the second derivative $Z=\left\{z_i\right\}_{i=1}^{N}$ and the first derivative $F=\left\{f_i\right\}_{i=1}^{N}$.

Values of the local cubic Hermitian spline (Korn, G. A., Korn, T. M. 2000) at the arbitrary point are calculated by formula:

$$S(t)=a_{j-1}\cdot{}^{1}x(t)+a_j\cdot{}^{2}x(t)+a_{j+1}\cdot{}^{3}x(t)+a_{j+2}\cdot{}^{4}x(t)\quad \text{for } t\in\left[t_{u_j},t_{u_{j+1}}\right], \tag{33}$$

where: a_j – value of the ordinates of patching units of the spline areas; ${}^{k}x(t)$ – local shape functions, discrete values of which complete the columns of the planning matrix X and are calculated by formulas(16)–(23), stated above.

The number of operations of multiplication, addition, necessary for calculation of fast spline approximation of the second derivative of function, which was observed:

$$M=N\cdot(r+1)+(r+1)^2. \tag{34}$$

Let's compare the quality of the offered method of the numerical calculation of the second derivative of function, which is observed against the casual errors of the test data, with the classical method (smoothing by spline function and the further analytical finding of the derivative itself).

As an example, the function $y(t)=10\cdot t^3+n(t)$ is taken on the interval $\left[0,2\right]$ at points with the step

1/32 (Fig. 3), where $n(t)$ – Gaussian signal-independent noise. The second derivative of the deterministic bases is $y''(t) = 60 \cdot t$, accordingly (Fig. 3).

For 64 reports of the initial function and 16 patching units of the spline, the following results were received:

- standard deviation of the input Gaussian signal-independent noise changed from 0.02 to 0.05 (Fig. 4);
- at the same time, the standard deviation of the second theoretical derivative of the second derivative, numerically calculated by using the classical method, changed from 2.72 to 12.45 (Fig. 4);
- standard deviation of the second theoretical derivative of the second derivative, numerically calculated by using the offered method, changed from 1.25 to 3.25 (Fig. 4).

While comparing the quality of the offered method for numerical calculation of the first and second derivatives (speed and acceleration of the cars accordingly) of function, which was observed against the casual errors of the test data, with the classical method (smoothing of this function by a spline and the further analytical finding of the derivative itself), it was established that the errors of the numerical calculation of the derivative of function, which were observed against the casual errors of the test data, of the offered method were lower than the errors of the numerical calculation of the same derivative of the classical method. It is indicated by the lower standard deviation.

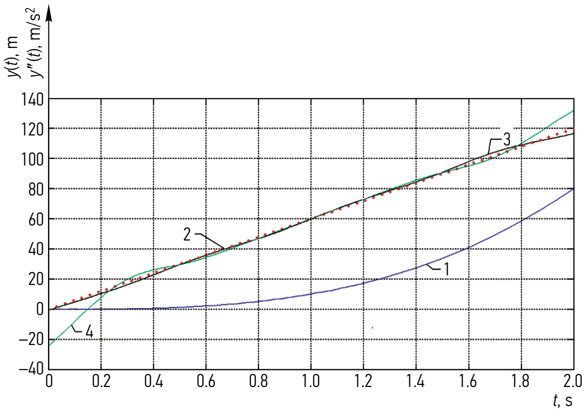

Fig. 3. Second derivatives (car accelerations), numerically calculated by using the classical and offered methods from the input signal with the standard deviation $\sigma\left[n(t)\right] = 0.05$ of input Gaussian signal-independent noise: 1 – input signal with the noise to differentiation; 2 – theoretical second derivative; 3 – second derivative, numerically calculated by using the offered method; 4 – second derivative, numerically calculated by using the classical method

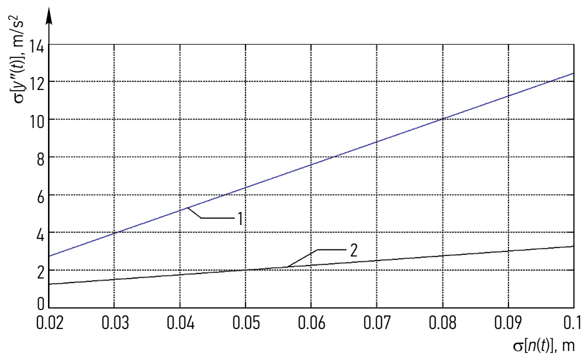

Fig. 4. Standard deviations of the theoretical second derivative (car acceleration), numerically calculated by using the classical (1) and offered (2) methods

Conclusions

1. The method for numerical calculation of the derivative (speed of the car movement) was developed using the fast spline transformation (the fast spline approximation of the derivative was constructed, which was calculated from the initial function without the previous estimation of the derivative of this function itself).
2. The method for numerical calculation of the second derivative (acceleration of the car movement) was developed, using the fast spline transformation (the fast spline approximation of the second derivative was constructed, which was calculated from the initial function without the previous estimation of the first and second derivative of this function).
3. The quality of the offered methods for numerical calculation of the derivative of function, which was observed against the casual errors of the test data, was compared with the classical method (smoothing of this function by a spline and the further analytical finding of the derivative itself).
4. Thus, the error of the numerical calculation of the derivative of function, which was observed against the casual errors of the test data, of the offered method were lower than the errors of the numerical calculation of the same derivative of the classical method.

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