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Performance of Track-to-Track Association Algorithms Based on Mahalanobis Distance

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Abstract: In multi-sensor tracking system, the track-to-track association problem is to determine whether a set of local tracks from different sensor systems are represent the same target. This problem is usually formulated as a binary hypothesis test, and the most common statistics is defined as the squared Mahalanobis distance (SMD) between the kinematic state estimates of two tracks. In this paper, three types of SMD algorithms are investigated, i.e., the SMD algorithm, the cumulative SMD algorithm, and the Discrete Wavelet Transform (DWT) algorithm which can be regarded as a generalized SMD ratio algorithm. The first one can be looked as singlescan algorithm, and the rest two are multiscan approaches. From another viewpoint, the first two are time domain algorithms, and the last one is a transform domain algorithm. The probability distribution functions of statistics defined by these algorithms have been discussed under the assumption that the estimates errors are independent across time. The Operating Characteristic Function is used to describe association performance. It shows that the multiscan algorithm performs better than the singlescan algorithm. As to multiscan algorithms, the DWT algorithm is superior to time domain algorithm. But better algorithm is more sensitive to the residual bias because the statistic based on SMD of target state estimates is directly contaminated by the bias. *Copyright © 2013 IFSA.*

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1. Introduction

In hierarchical multi-sensor tracking systems, each sensor system processes sensor data from its own sensors to generate and maintain single-sensor or locally-fused tracks, and a central fusion center that fuses the sensor tracks into a set of system tracks. The track-to-track association (T2TA) problem is to determine the pairing of sensor level tracks that correspond to the same true target from which the sensor level tracks originated. It must be resolved before these tracks can be fused.

A number of algorithms have been proposed to resolve T2TA problem. In the simplest scenario of two tracks source from two independent tracking systems, T2TA is usually formulated as a binary hypothesis testing problem, and the most classical test statistic is defined as the squared Mahalanobis distance (SMD)

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of the difference of kinematic state estimates [1]. Bar-Shalom [2] has pointed out that the errors of state estimates belong to the same target are not independent whereas the two tracking systems are independent because of the same process noise and revised the statistic formula. Ref. [3] has analyzed the performance of this algorithm, but the method is complex. Recently, some efforts are focused on developing novel statistic, such as feature-aided association, position topology association [4, 5]. Others are concerned with multiple targets assignment cost or join registration and association [6, 7]. All these algorithms are hard to exactly analyze the performance.

All approaches mentioned above belong to singlescan algorithms which just deal with the current track data. From the perspective of hypothesis testing, multiscan algorithm which uses historical information as while as the current data should have more test power. Ref. [8] directly extends singlescan SMD algorithm to multiscan algorithm by using cumulative sum of multiframe SMD as the statistic. Ref. [9] demonstrates this algorithm can not reach the theoretical performance because estimation errors are dependent across time. Ref. [10] presents an exact algorithm for calculating the test statistic accounting for the crosscorrelation.

We mainly investigate the theoretical test power of T2TA algorithms based on SMD in this paper. Three types of SMD algorithms are discussed in detail, i.e., the SMD algorithm [1, 2], the cumulative SMD algorithm [8], and the Discrete Wavelet Transform (DWT) algorithm [11, 12] which can be regarded as a generalized SMD ratio algorithm. The first one can be looked as singlescan algorithm, and the rest two are multiscan approaches. From another viewpoint, the first two are time domain algorithms, and the DWT algorithm is a transform domain algorithm. The distributions of testing statistics have been discussed. The Operating Characteristic Function (OCF) is used to compare the performance of different algorithms.

This work is based on the following assumptions: 1) sensors measure targets independently; 2) all tracks are updated synchronously; 3) all tracks have same length and 4) estimate errors of track states are independent across time in the multiscan algorithms.

The rest paper is organized as follows. The notations and the hypothesis test model as well as the performance metrics are discussed in Section 2. In Section 3, we look in sight the singlescan test use SMD as the statistics, and discuss some methods to deal with dependent error and residual bias. Section 4 describes two multiscan algorithms, the cumulative SMD test and the DWT ratio test, and compares the performance under typical tracking scenarios. Conclusions are in Section 5.

2. Notations and the Hypothesis Test

Consider the simplest T2TA problem to decide whether two tracks from two independent tracking systems i and j represent the same target. Let $\mathbf{x}_k^i$ and $\mathbf{x}_k^j$ denote the n-dimensional true target states at time instant k, $\hat{\mathbf{x}}_k^i$ and $\hat{\mathbf{x}}_k^j$ denote the corresponding state estimates, $\mathbf{P}_k^i$ and $\mathbf{P}_k^j$ denote the covariance matrix.

Denote the difference of the two estimates as

$$\hat{\boldsymbol{\delta}}_k^{ij} = \hat{\mathbf{x}}_k^i - \hat{\mathbf{x}}_k^j . \tag{1}$$

This is the estimate of the difference of the true states

$$\boldsymbol{\delta}_k^{ij} = \mathbf{x}_k^i - \mathbf{x}_k^j . \tag{2}$$

Suppose all the history estimates and covariance matrices until time k are available at the fusion center. The hypothesis of whether the two tracks have the same origin is

- H_0 : $\boldsymbol{\delta}_k^{ij} = \mathbf{0}$ at any time k ;
- H_1 : $\boldsymbol{\delta}_k^{ij} \neq \mathbf{0}$.

The statistics, denoted by D, is defined as functions of SMD in all algorithms discussed in this paper. The test threshold T can be chosen to satisfy

$$\mathrm{P}\{D > T \mid \mathrm{H}_0\} = \alpha , \tag{3}$$

where, P{} represent the probability, α is a given level of significance.

Then the test is

- Accept H_0 , if $D < T$;
- Accept H_1, otherwise.

To evaluate the performance of a T2TA algorithm, we define four metrics, probability of correct association P_{CA}, probability of false dismissal P_{FD}, probability of correct reject P_{CR} and probability of false association P_{FA}, as following,

$$\begin{aligned} P_{\mathrm{CA}} &= P\{D \le T \mid \mathrm{H}_0\} \\ P_{\mathrm{FD}} &= P\{D > T \mid \mathrm{H}_0\} \\ P_{\mathrm{CR}} &= P\{D > T \mid \mathrm{H}_1\} \\ P_{\mathrm{FA}} &= P\{D \le T \mid \mathrm{H}_1\} \end{aligned} . \tag{4}$$

Obviously, P_{FD} is the type I error probability (or α–error) of the track association hypothesis test, and P_{FA} is the type II error probability (or β–error). We have

$$P_{\mathrm{CA}} + P_{\mathrm{FD}} = 1,\ \ P_{\mathrm{FA}} + P_{\mathrm{CR}} = 1 . \tag{5}$$

By use Eq. (3) and Eq. (4), P_{CA} and P_{FD} can be obtained directly as

$$P_{\mathrm{CA}} = 1 - \alpha \, , P_{\mathrm{FD}} = \alpha . \tag{6}$$

To calculate P_{FA} and P_{CR}, we must have a specific alternative hypothesis, i.e. we must give hypothesis H_1 a specific assumption that there is a distance $\boldsymbol{\delta}_k^{ij} \equiv \mathbf{v}_k \neq 0$ between the two targets, then we can calculate the probability by integral calculus of the probability density function (PDF).

A more convenient tool to evaluate performance is the OCF. The OCF of a hypothesis test is the probability that the test fails to reject H_0 as a function of θ, where θ is the unknown parameter. It is defined as [13]

$$\beta(\theta) = P_\theta(\text{test fails to reject } \mathrm{H}_0), \tag{7}$$

and the plot of OCF is the so-called OC curve.

By using OCF, P_{FA} and P_{CR} are evaluated as

$$P_{\mathrm{FA}} = \beta(\theta), \ P_{\mathrm{CR}} = 1-\beta(\theta). \tag{8}$$

and P_{CR} is also called the test power.

Two important distributions, the noncentral chi-square distribution and the noncentral F-distribution, will be used when discuss the statistic's distribution.

If statistic D is a noncentral chi-square random variable with noncentrality parameter U and n degrees of freedom, denoted by $D \sim \chi_n^2(U)$, its characteristic function is [14]

$$\phi_D(jt) = (1-2jt)^{-n/2} \exp\left\{\frac{jtU}{1-2jt}\right\}, \tag{9}$$

and the probability can be evaluated by

$$\mathrm{P}\{D<T\} = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\sin(tT/2)}{t}\phi_D(jt)e^{-jtT/2}dt. \tag{10}$$

If statistic D is a noncentral F-distributed random variable, denoted by $D \sim F(n,m;U)$, which is defined as

$$D = \frac{X/n}{Y/m}. \tag{11}$$

where $X \sim \chi_n^2(U)$ and $Y \sim \chi_m^2(0)$, and its PDF is [14]

$$p_D(x) = \sum_{k=0}^{\infty} \frac{e^{-U/2}(U/2)^k}{\mathrm{B}(\frac{m}{2},\frac{n}{2}+k)k!}\left(\frac{n}{m}\right)^{\frac{n}{2}+k} \times \left(\frac{m}{m+nx}\right)^{\frac{n+m}{2}+k} x^{\frac{n}{2}-1+k}, \tag{12}$$

when $x \geq 0$ and zero otherwise. The term B() in Eq. (12) is beta function.

3. Singlescan Test and Performance

3.1 SMD Test

Suppose the estimates error of different sensor are independent, Gaussian, and unbiased, the SMD association algorithm define statistic as following

$$D_{\mathrm{SMD}} \triangleq (\hat{\boldsymbol{\delta}}_k^{ij})^{\mathrm{T}} \mathbf{P}_k^{-1} \hat{\boldsymbol{\delta}}_k^{ij}, \tag{13}$$

where

$$\mathbf{P}_k = \mathbf{P}_k^i + \mathbf{P}_k^j. \tag{14}$$

Under H_0,

$$\hat{\boldsymbol{\delta}}_k^{ij} \sim N(\mathbf{0}, \mathbf{P}_k), \ D_{\mathrm{SMD}} \sim \chi_n^2. \tag{15}$$

Given the significance level α, we can get the threshold

$$T_{\mathrm{SMD}} = \chi_{n,\alpha}^2 \tag{16}$$

and P_{CA} can be obtained directly by Eq.(6).

Define the normalized squared distance at time instant k as

$$U_k \triangleq \mathbf{v}_k^{\mathrm{T}} \mathbf{P}_k^{-1} \mathbf{v}_k. \tag{17}$$

Under H_1,

$$\hat{\boldsymbol{\delta}}_k^{ij} \sim N(\mathbf{v}_k, \mathbf{P}_k), \ D_{\mathrm{SMD}} \sim \chi_n^2(U_k). \tag{18}$$

Then P_{FA} can be evaluated by Eq. (9) and Eq. (10).

3.2. Dependent Errors

Eq. (13) and Eq. (14) assumes that state estimate errors in system i are independent of those from system j. This is not true because of the same process noise when estimates belong to the same target. If that is the case, then Eq. (14) must be replaced with [2]

$$\mathbf{P}_k = \mathbf{P}_k^i + \mathbf{P}_k^j - \mathbf{P}_k^{ij} - \mathbf{P}_k^{ji}, \tag{19}$$

where $\mathbf{P}_k^{ij}$ and $\mathbf{P}_k^{ji}$ are the cross covariances due to common process noise.

3.3. Residual Bias

Consider the case where $\hat{\boldsymbol{\delta}}_k^{ij}$ has been corrected for bias, but some residual bias still remains. It may arise from maneuver or transformation biases between two systems for tracks on the same target.

Let $\mathbf{b}_k \equiv \mathrm{E}\{\hat{\boldsymbol{\delta}}_k^{ij}\}$ denote the residual bias at time instant k. Define the normalized squared bias as

$$W \triangleq \mathbf{b}_k^{\mathrm{T}} \mathbf{P}_k^{-1} \mathbf{b}_k , \tag{20}$$

Under H_0,

$$D_{\mathrm{SMD}} \sim \chi_n^2(W) . \tag{21}$$

In practice, the residual bias is unknown in most occasions. If we ignore the unknown residual bias and still use central chi-square distribution threshold to make the test decision, P_{CA} doesn't subject to Eq. (6) and is degraded severely.

If ignored the residual bias, Fig. 1 shows P_{CA} decrease rapidly as W increases especially when n is small. This is coincident with the result in Ref. [3]. Thus it is necessary to remove the effects of biases in the SMD.

In recent literature [15], approaches are discussed to deal with the existence of residual bias as well as covariance jitter in Mahalanobis distance. It shows the bias problem is significant. So we introduce two methods to correct residual bias when calculating SMD.

Case I: $\mathbf{b}_k$ is exactly known. Eq. (13) can be substituted by

$$D_{\mathrm{SMD}} \triangleq (\hat{\boldsymbol{\delta}}_k^{ij} - \mathbf{b}_k)^{\mathrm{T}} \mathbf{P}_k^{-1} (\hat{\boldsymbol{\delta}}_k^{ij} - \mathbf{b}_k) . \tag{22}$$

Under H_0,

$$D_{\mathrm{SMD}} \sim \chi_n^2 , \tag{23}$$

and under H_1,

$$D_{\mathrm{SMD}} \sim \chi_n^2(U_k) , \tag{24}$$

where

$$U_k = (\mathbf{v}_k - \mathbf{b}_k)^{\mathrm{T}} \mathbf{P}_k^{-1} (\mathbf{v}_k - \mathbf{b}_k) . \tag{25}$$

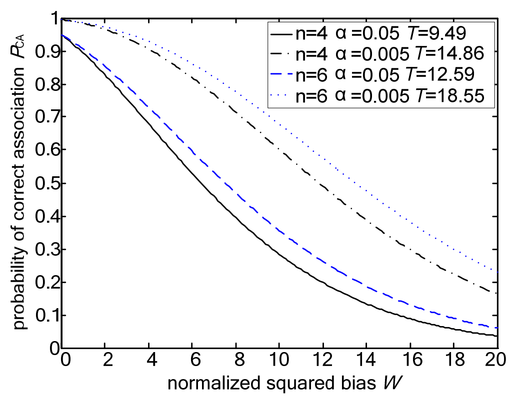

Fig. 1. The probability of correct association P_{CA} is deteriorated by the ignored residual bias W.

Case II: $\mathbf{b}_k$ is unknown, Gaussian but can be estimated. Suppose $\hat{\boldsymbol{\delta}}_k^{ij}$ and $\hat{\mathbf{b}}_k$ are independent, and $\hat{\mathbf{b}}_k$ is a linear unbiased estimator of $\mathbf{b}_k$, with known covariance matrix $\mathbf{Q}_k$, and

$$\hat{\mathbf{b}}_k \sim N(\mathbf{b}_k, \mathbf{Q}_k) , \tag{26}$$

then, Eq. (13) can be replaced with [15]

$$D_{\mathrm{SMD}} \triangleq (\hat{\boldsymbol{\delta}}_k^{ij} - \hat{\mathbf{b}}_k)^{\mathrm{T}} (\mathbf{P}_k + \mathbf{Q}_k)^{-1} (\hat{\boldsymbol{\delta}}_k^{ij} - \hat{\mathbf{b}}_k) . \tag{27}$$

Under H_0,

$$D_{\mathrm{SMD}} \sim \chi_n^2 , \tag{28}$$

because

$$\hat{\boldsymbol{\delta}}_k^{ij} - \hat{\mathbf{b}}_k \sim N(0, \mathbf{P}_k + \mathbf{Q}_k) . \tag{29}$$

Under H_1, since

$$\hat{\boldsymbol{\delta}}_k^{ij} - \hat{\mathbf{b}}_k \sim N(\mathbf{v}_k - \hat{\mathbf{b}}_k, \mathbf{P}_k + \mathbf{Q}_k) , \tag{30}$$

yield

$$D_{\mathrm{SMD}} \sim \chi_n^2(U_k) , \tag{31}$$

where the normalized squared distance U_k is

$$U_k = (\mathbf{v}_k - \hat{\mathbf{b}}_k)^{\mathrm{T}} (\mathbf{P}_k + \mathbf{Q}_k)^{-1} (\mathbf{v}_k - \hat{\mathbf{b}}_k) . \tag{32}$$

In practice, it is assumed that the two tracking systems can estimate the residual bias and its covariance independently. Denote the residual bias estimates as

$$\hat{\mathbf{b}}_k^i = \hat{\mathbf{x}}_k^i - \mathbf{x}_k^i , \hat{\mathbf{b}}_k^j = \hat{\mathbf{x}}_k^j - \mathbf{x}_k^j , \tag{33}$$

and the covariance matrices as

$$\mathbf{Q}_k^i = \mathrm{cov}(\hat{\mathbf{b}}_k^i) , \mathbf{Q}_k^j = \mathrm{cov}(\hat{\mathbf{b}}_k^j) . \tag{34}$$

In Eq. (27), the $\hat{\mathbf{b}}_k$ and $\mathbf{Q}_k$ can be calculated as following

$$\hat{\mathbf{b}}_k = \hat{\mathbf{b}}_k^i + \hat{\mathbf{b}}_k^j ,\ \mathbf{Q}_k = \mathbf{Q}_k^i + \mathbf{Q}_k^j . \tag{35}$$

4. Multiscan Test and Performance

4.1. Cummulative SMD Test

An intuitional method to extend singlescan test to multiscan test is to use cumulative sum of SMD over the most recent m frames of track data. It is called cumulative SMD (CSMD) algorithm or sliding window algorithm [10]. The test statistic is defined as [8]

$$D_{\text{CSMD}} \triangleq \sum_{l=k-m+1}^{k} (\hat{\boldsymbol{\delta}}_l^{ij})^{\text{T}} \mathbf{P}_l^{-1} \hat{\boldsymbol{\delta}}_l^{ij} . \tag{36}$$

Assuming the estimates errors of different sensor are independent, Gaussian, unbiased, and all estimation errors are independent across time, under H_0,

$$D_{\text{CSMD}} \sim \chi^2_{nm} , \tag{37}$$

and under H_1,

$$D_{\text{CSMD}} \sim \chi^2_{nm}(U), \tag{38}$$

where

$$U = \sum_{l=k-m+1}^{k} U_l = \sum_{l=k-m+1}^{k} \mathbf{v}_l^{\text{T}} \mathbf{P}_l^{-1} \mathbf{v}_l . \tag{39}$$

Obviously, SMD test is the special case of CSMD test while $m = 1$.

It must be noted that those assumptions, such as unbiased, independent across time, are not true in practice. The approaches to eliminate the effects of common process noise and residual bias have been discussed above. Ref. [12] shows a method to calculate the exact statistic accounting for the crosscorrelation.

The hypothesis test of T2TA problem is a test about the expectation of the true states distance, and the test parameter $\boldsymbol{\delta}_k^{ij}$ is a vector. The OCF can not be directly used to evaluate the performance of different algorithms in this occasion, and a scalar variable should be introduced to act as the test parameter.

We define the mean normalized squared distance $\bar{U}$ as

$$\bar{U} \triangleq \frac{1}{m} \sum_{l=k-m+1}^{k} U_l , \tag{40}$$

and the hypothesis test of T2TA problem can be transformed to

- $H_0 : \bar{U} = \mathbf{0}$;
- $H_1 : \bar{U} \neq \mathbf{0}$.

Now, the OCF of CSMD test can be defined as a function of $\bar{U}$, that is

$$\begin{aligned} \beta(\bar{U}) &= \text{P}_{\bar{U}}\{\text{test fails to reject H}_0\} \\ &= \text{P}_{\bar{U}}\{D_{\text{CSMD}} < \chi^2_{nm,\alpha}(m\bar{U})\} \end{aligned} , \tag{41}$$

Given $\alpha = 0.05$, $n = 2$ (suppose target states only contain 2-D position data), Fig. 2 shows three typical OCF curves of CSMD algorithm for m = 1, 2, and 4. While $\bar{U} = 9$, $\beta(\bar{U})$ = 0.2293, 0.0569 and 0.0027 respectively. Given $\beta(\bar{U}) = 0.2$, $\bar{U}$ = 9.7, 6.0, and 3.7.

To understand these data, we just discuss a special case that the covariance matrix is constant across time, i.e.,

$$\mathbf{P}_l = \mathbf{P} \equiv diag(\sigma^2, \sigma^2) , \tag{42}$$

yield,

$$\begin{aligned} \bar{U} &= \frac{1}{m} \sum_{l=k-m+1}^{k} \mathbf{v}_l^{\text{T}} \mathbf{P}_l^{-1} \mathbf{v}_l \\ &= \frac{1}{\sigma^2} \left(\frac{1}{m} \sum_{l=k-m+1}^{k} \mathbf{v}_l^{\text{T}} \mathbf{v}_l \right), \\ &= \frac{1}{\sigma^2} \bar{D}_{\text{E}} \end{aligned} \tag{43}$$

where $\bar{D}_{\text{E}}$ is the average squared Euclid distance.

If the Euclid distance between the two targets is 3σ constantly, i.e., $\bar{U} = 9$, then P_{FA} = 22.9 %, 5.69 %, and 0.27 % respectively with m = 1, 2 and 4.

On the other hand, to guaranty $P_{\text{FA}} \leq 0.2$, the Euclid distance between two targets should no less than 3.11σ, 2.45σ and 1.76σ for m = 1, 2, 4 respectively. It means the performance is better while window size increasing.

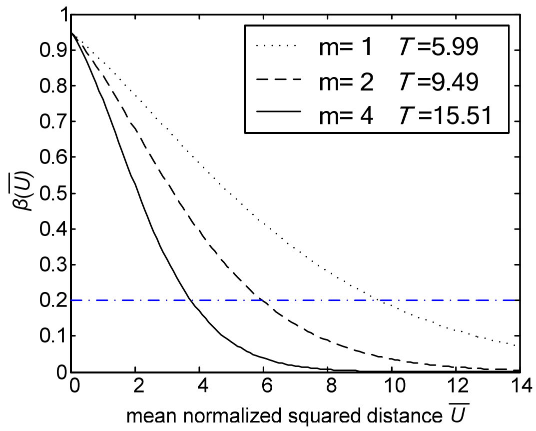

Fig. 2. Theoretical OCF curves of CSMD test ($\alpha = 0.05$, $n = 2$).

4.2. DWT Ratio Test

CSMD algorithm can be regarded as time domain approach. Ref. [11] presents a T2TA algorithm in transform space using DWT, and the formal formula of statistic is deduced in Ref. [12]. The essential idea of this algorithm is that deterministic signal and noise exhibit different features in wavelet domain, I.I.D. Gaussian white noise series still keep white with identical statistical properties, but the signal energy is concentrated mostly in the lower scale space.

The matrix form of 2^J frames track estimate differences, regarded as n-dimension time series, is denoted by

$$\Delta^{ij} = [\hat{\boldsymbol{\delta}}_{k-m+1}^{ij}, \hat{\boldsymbol{\delta}}_{k-m+2}^{ij}, \cdots, \hat{\boldsymbol{\delta}}_{k}^{ij}]^{\mathrm{T}} \in \mathbb{R}^{2^J \times n}. \quad (44)$$

The orthonormal DWT matrix **W** can be divided into J_0+1 submatrices [16]:

$$\begin{cases} \mathbf{W} = \left[\mathbf{W}_1^{\mathrm{T}}, \ldots, \mathbf{W}_{J_0}^{\mathrm{T}}, \mathbf{V}_{J_0}^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{R}^{2^J \times 2^J}, J_0 = 1, \ldots, J \\ \mathbf{W}_j = \left[\boldsymbol{w}_{j,1}, \boldsymbol{w}_{j,2}, \ldots, \boldsymbol{w}_{j,2^{J-j}}\right]_{j=1,\ldots,J_0}^{\mathrm{T}} \\ \mathbf{V}_{J_0} = \left[\boldsymbol{v}_1, \ldots, \boldsymbol{v}_{2^{J-J_0}}\right]^{\mathrm{T}} \end{cases}, \quad (45)$$

where $\{\boldsymbol{w}_{j,k},\ k = 1, \ldots, 2^{J-j}\}$ and $\{\boldsymbol{v}_k,\ k = 1, \ldots, 2^{J-J_0}\}$ are the orthonormal bases of the j-th wavelet space and the J_0-th scale space respectively. When $J_0 = J$, **W** is a standard DWT. When $J_0 < J$, **W** is a partial DWT. Both of them are orthonormal transformation.

Assuming the covariance matrix is constant across time, i.e., $\mathbf{P}_k \equiv \mathbf{P}$, the ratio of energy in high resolution wavelet space to energy in lower resolution scale space can be defined as the statistical test [12],

$$D_{\mathrm{DWT}} \triangleq \frac{2^{J_0-1}\,\mathrm{tr}\left[\mathbf{V}_{J_0}\Delta^{ij}\mathbf{P}^{-1}(\mathbf{V}_{J_0}\Delta^{ij})^{\mathrm{T}}\right]}{\mathrm{tr}\left[\mathbf{W}_1\Delta^{ij}\mathbf{P}^{-1}(\mathbf{W}_1\Delta^{ij})^{\mathrm{T}}\right]}, \quad (46)$$

where, tr() is trace function of square matrix, the recommend value of J_0 is $J-1$.

According to Eq. (46), DWT algorithm can be regarded as a generalized CSMD method, which test is a ratio between two cumulative SMDs after partial orthonormal transforming.

Ref. [11] shows that, under H_0,

$$D_{\mathrm{DWT}} \sim F(2^{J-J_0}n, 2^{J-1}n), \quad (47)$$

and under H_1, suppose all of deterministic signal energy can be concentrated to the lower scale space, then

$$D_{\mathrm{DWT}} \sim F(2^{J-J_0}n, 2^{J-1}n; \sum_{l=k-2^J+1}^{k} \mathbf{v}_l^{\mathrm{T}}\mathbf{P}_l^{-1}\mathbf{v}_l). \quad (48)$$

The OCF of DWT test is also defined as a function of $\bar{U}$, that is

$$\beta(\bar{U}) = \mathrm{P}_{\bar{U}}\{D_{\mathrm{DWT}} < F_\alpha(2^{J-J_0}n, 2^{J-1}n; m\bar{U})\}, \quad (49)$$

where the mean normalized squared distance $\bar{U}$ is defined in Eq. (40) with $m = 2^J$.

To check out the practical performance of DWT algorithm, according to Ref. [11], simulations has been done in scenario of two parallel flying targets and two cross flying targets respectively, see Fig. 3 and Fig. 4. Suppose there are two sensors, each sensor only track and report one target of these two. We want to know whether CSMD and DWT algorithm can distinguish these two tracks are different targets. Other scenarios may be taken into consideration but these two are convenient to calculate.

Targets velocity S is 1 Ma constantly. The track reporting periods of both systems are 1 s. Covariance matrices are

$$\mathbf{P}^i = diag(\sigma_1^2, \sigma_1^2),\ \mathbf{P}^j = diag(\sigma_2^2, \sigma_2^2), \quad (50)$$

where

$$\sigma_1 = 100,\ \sigma_2 = 300. \quad (51)$$

In the *parallel* scenario (Fig. 3), Δd^2 is the constant squared Euclid distance between two targets. For a given $\bar{U}$, Δd^2 can be calculated as

$$\begin{aligned} \Delta d^2 &= (\sigma_1^2 + \sigma_2^2)\bar{U} \\ &= 10^5\bar{U} \end{aligned}. \quad (52)$$

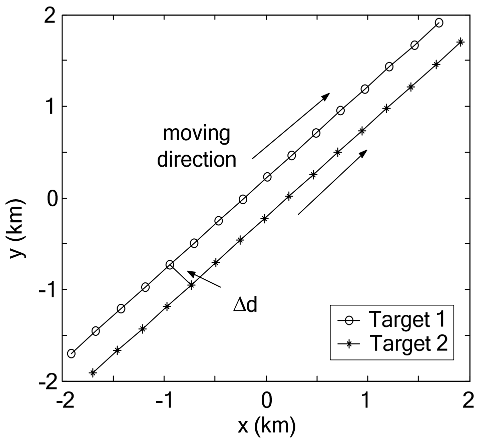

Fig. 3. Trajectories of two parallel moving targets.

In the *cross* scenario (Fig. 4), one target crosses the other at the middle of the track trajectory, the angle between the two trajectories is denoted by θ.

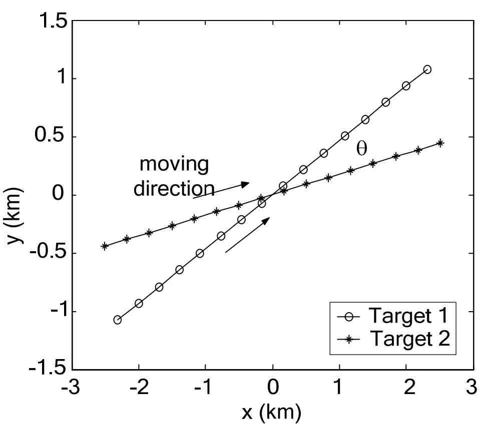

Fig. 4. Trajectories of two crossing targets.

Given θ, yield

$$\begin{aligned}\bar{U} &= \frac{1}{m(\sigma_1^2+\sigma_2^2)}\sum_{l=1}^{m}\mathbf{v}_l^{\mathrm{T}}\mathbf{v}_l \\ &= \frac{S^2(1-\cos\theta)}{4(\sigma_1^2+\sigma_2^2)}\left(0.5^2+1.5^2+\ldots+7.5^2\right). \\ &= \frac{42.5\times S^2(1-\cos\theta)}{(\sigma_1^2+\sigma_2^2)}\end{aligned} \quad (53)$$

According Eq.(53), θ is a function of $\bar{U}$,

$$\theta = arc\cos\left[1-\frac{(\sigma_1^2+\sigma_2^2)\bar{U}}{42.5S^2}\right]. \quad (54)$$

It should be mentioned that in this *cross* scenario $\bar{U}$ is limited by

$$\bar{U} \le 85S^2/(\sigma_1^2+\sigma_2^2), \quad (55)$$

and when the two targets moving face to face $\bar{U}$ reach the maximum value.

Fig. 5 is theoretical OCF curves of CSMD and DWT test and simulation results. The 'DWT' curve is the theoretical OCF curves of DWT algorithm under the assumption of perfect signal energy shrinkage effect, which means all signal energy is concentrated in the J_0-th scale space and only noise is left in wavelet space. The 'SimP' curve is simulation results of two closely parallel tracks using DWT algorithm based on the average of 1000 Monte Carlo runs, and the 'SimC' curve is of two closely cross tracks. The simulation parameters are selected as $\alpha = 0.05$, $n = 2$, $m = 16$, $J = 4$, $J_0 = 3$.

In Fig. 5, it shows DWT algorithm is superior to CSMD algorithm both in theoretic and simulation's performance. While $\bar{U} = 1$, the P_{FA} of the four curves are 46.5 %, 33.8 %, 20.4 % and 20 % respectively. The DWT algorithm is perfect to distinguish two parallel tracks because of the perfect energy concentrating effect of DWT. Unfortunately, the DWT algorithm is worse than CSMD algorithm if regard $\bar{U}$ as residual bias.

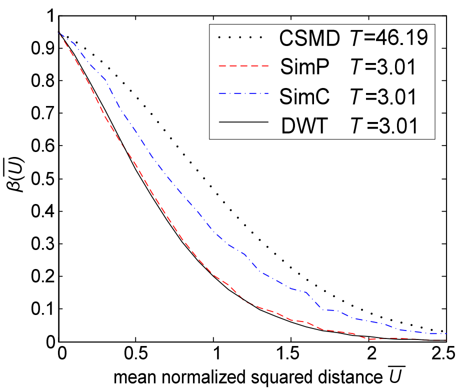

Fig. 5. Theoretical OCF curves of CSMD and DWT test and simulation results ($\alpha = 0.05$, $n = 2$, $m = 16$, $J = 4$, $J_0 = 3$).

5. Conclusions

In this paper, T2TA algorithms based on SMD have been revisited. The performance of singlescan SMD algorithm, multiscan cumulative SMD algorithm and a generalized SMD ratio algorithm based on DWT has been compared using binary hypothesis test. Some techniques about residual bias removing have also been discussed for calculating SMD. Under the assumption of estimates errors independent across time, the following can be concluded according to the performance comparison of those algorithms.

1) Multiscan algorithm performs better than singlescan algorithm.

2) The performance of a test statistic based on distance in some transform domain, such as Discrete Wavelet Transform, may be improved than time domain approaches.

3) The OCF curves also imply that better algorithm is more sensitive to residual bias. The reason is that the bias directly contaminates the testing statistics—SMD. Using bias removing technique or developing other measure, such as target feature, attribute and/or topology, combined with kinematic states to describe track similarity, maybe more suitable to resolve T2TA problem.

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