International Journal of
Operations Research

International Journal of Operations Research Vol. 13, No. 4, 153−163 (2016)

Price Negotiation for Taxi Services under Private Customer Urgency

Ling-Chieh Kung[1] and Zong-Ting Chen

Department of Information Management, National Taiwan University
No. 1, Sec. 4, Roosevelt Rd., Taipei 10617, Taiwan

Received August 2016; Accepted November 2016

Abstract: We study whether the efficiency of the taxi industry can be improved by introducing price negotiation into the current pricing scheme. In particular, we examine a new business model for a taxi company to offer multiple contracts with various guaranteed travel times and prices when a customer books a taxi service. By considering the fact that customers can privately observe their urgency levels, we show that the proposed new pricing scheme in equilibrium induces higher taxi speed, serves more customers, and may enhances the utilities of the company and customers simultaneously.
Keyword — Games/group decisions, taxi services, price negotiation, price discrimination, adverse selection.

1. INTRODUCTION

In most countries, transportation services are regulated by the governments. The taxi service is not an exception. Even though in most regions there are private taxi companies hiring drivers to provide taxi services with various contracts, most governments still impose unified regulations to the taxi industry. One common regulation is fare control, typically imposed in the form of a standard pricing formula. Although the fare structures are different across cities, the pricing formulas generally consist of three components: an initial charge, a distance-based charge, and/or a delay-based charge. Some researchers argue that such a simple formula is not fair enough for all kinds of customers; see the references in Section 2. Another defect of this fare structure is its inflexibility and inefficiency. Seldom cities are capable of changing the taxi fee formula frequently enough to match supply and demand.

Unlike the inflexible regulated pricing formula, an "extremely flexible" pricing scheme is for a taxi driver and a customer to negotiate for the taxi fee. In this case, the driver makes some offers for the customer to select from, and the service will be provided only after a single price is agreed by everyone. In theory, negotiation is more flexible than a fixed pricing formulation and should be considered as a way to mitigate supply-demand mismatch. Uber's surge pricing mechanism, which dynamically adjust the current fare formula according to supply and demand, is somewhat applying the idea of price negotiation to match supply and demand. The price goes higher when demand is more than supply, and vice versa. However, some countries forbid this pricing scheme due to its information asymmetry problem. The reason is to protect customers. Because customers are less informed about the best route to the destination, they may be unable to estimate a reasonable taxi fee. If price negotiation is allowed, the passenger may then be cheated by the driver.

Technology advances motivate us to reconsider such forbiddance on taxi fee negotiation. Nowadays online route planning services, such as Google Maps, are popular and widely available to many people around the world. Between any pair of locations, one may easily find a few suggested routes whose distances are all close to that of a shortest route. There are even a lot of websites or mobile apps providing the service of estimating taxi payments.[2] People who use these services can have a reference of taxi fees and avoid being asked a too high price by the taxi driver. If the detriment of information asymmetry may be eliminated, price negotiation seems to be a more acceptable option and deserves more attentions and discussions.

This study makes an initial attempt to consider whether price negotiation may increase the efficiency of the taxi industry. It is admitted that individual negotiation between a taxi driver and a customer would still be to the disadvantage of the customer. Motivated by how Uber is responsible for determining its price to prevent individual negotiation, we will investigate how a taxi company, instead of a driver, may make offers to a customer for profit maximization. Our proposed contracting process works in the following way. Consider a customer who makes a call, go to the company's website, or uses a mobile app to book a taxi service. Once he enters his origin and destination, the company then does some calculations and then makes a few offers to him, each with a different price. If the

---

[1] Corresponding author's e-mail: lckung@ntu.edu.tw.

[2] Examples of this service include Numbeo (http://www.numbeo.com/taxi-fare/) and Taxi Fare Finder (http://www.taxifarender.com/), among others.

customer chooses any offer, he will pay the selected price after being sent to the destination regardless of the route, travel distance, or travel time. The customer makes a choice that he prefers the most.[3] We want to derive the equilibrium behaviors of the company and customer, study the profitability of the new business model, and examine whether the new model has the potential to benefit everyone.

If the company wants to offer multiple contracts with different prices for the customer to select, these contracts must be different in at least one additional dimension. In practice, customers are heterogeneous regarding how urgent they are to arrive the destinations. Therefore, in this study we assume the company will differentiate these offers with various guaranteed travel times. With multiple offers, an urgent customer may choose an expensive contract which guarantees to get to the destination in a short time, and a patient customer may choose a cheap contract with long travel time. In other words, the company price discriminate based on different service levels represented by travel times (or speeds) to screen customers' urgency level. To facilitate discussions, we will use metered pricing to denote pricing by the standard pricing formula and menu pricing to denote price negotiation between the company and customer. As we will demonstrate in this study, the ability of screening customers' hidden urgency levels is the main benefit of supplementing metered pricing with menu pricing.

The remainder of this study is organized as follows. Section 2 reviews some relevant works. In Section 3, we describe our model setting. We start our analysis by assuming that customer urgency is public in Section 4. Section 5 is then devoted to the analysis with private urgency. Section 6 concludes.

2. LITERATURE REVIEW

Pricing of most public transportation services in urban areas is often the responsibility of local governments, acting either individually or jointly through native agencies. Bladikas and Crowell (1985) categorize the public transportation services to three types: segment-based, meter-based, and unstable. According to their definitions, trains and taxis are mostly priced by meters mainly based on the travel distance. For all the three types of pricing, heavy users (e.g., passengers who travel to farther destinations) are charged higher price. In other words, the idea of price discrimination is applied in the pricing of most transportation services.

Price discrimination is a strategy so that a product or service is sold at different prices with distinct quantity, quality, or service level. This strategy has been proved to help the product or service provider in most industries. For transportation services, price discrimination plays the role of ensuring fairness. In most areas, price discrimination of the taxi industry is realized through a central-controlled pricing regulation. Typically, pricing control is to set up an upper bound or a lower bound of taxi fees. This regulation has been examined extensively by economists in different ways. Douglas (1972) proves that once quality differentiation is constrained, the taxi price generated by a competitive market may be inefficient and higher than the efficient price. De Vany (1975) considers more about customer waiting time and confirms that monopolistic-competition contention arises because of natural regulations in the taxi market. Cairns and Liston-Heyes (1996) show that pricing control and entry restriction are necessary for approaching social optimum when competition do not hold in the industry. Arnott (1996) proposes subsidization for achieving the social optimum. He proves that without subsidization taxi companies' income will not be enough to cover the empty time of taxis. This then causes inefficiency and reduces social welfare. Foerster and Gilbert (1979) and Shreiber (1975) use statistical approaches to study the taxi market. Foerster and Gilbert (1979) affirm that if taxi prices are independent of distance, the monopoly market will produce a lower level of taxi utilization. Shreiber (1975) demonstrates that without pricing control, customers will be unable to predict whether the next taxi will provide a lower price. Therefore, they may not refuse the contract provided by the first taxi. This is eventually gives taxi companies an incentive to provide higher prices. Based on these research, in this study we examine the effectiveness of metered pricing as regulated by the government. We go one step further to consider menu pricing as a remedy to the inflexibility of regulated metered pricing.

Some research discuss about the defects of the existing taxi fare structure and provide possible adjustments to solve the problem. Wong et al. (2008) construct a model to simulate an environment with multiple types of customers and taxis. They show how special taxis, such as handicapped-taxis, should make their taxi fee menus to differentiate different customers. Yang et al. (2010) use a nonlinear pricing model for taxi services to solve the problem that some taxi waiting queues with oversupply taxis. They identify a win-win situation created by a Pareto-improving policy inducing taxis to distribute more uniformly in the whole area. In this study, we also try to improve the current fare structure. Instead of using metered pricing only, we propose menu pricing as an alternative

[3] In practice, a customer should always have one final offer: to be priced based on the standard regulated pricing formula. If we include this option in the company's contract design problem, we will need to solve a contract design problem with type-dependent outside options, which is a well-documented analytically challenging problem. We will leave this problem for future studies and only compare pure price regulation and price negotiation in this study.

option.

3. MODEL.

We consider the relationship between a taxi company (she) who offers taxi services and a customer (he) who has reported his destination. Different customers have different degrees of urgency, which are typically hidden to taxi company. Therefore, we use θ to denote customer urgency or sensitivity of time. From the company's perspective, θ is a random variable with distribution F, density f, and support $[\theta_0, \theta_1]$. We assume that F and f satisfy the increasing failure rate (IFR) property, i.e., the inverse failure rate $\frac{1-F(\theta)}{f(\theta)}$ decreases in θ.[4] We follow the economics convention to call θ as the consumer's type. The company thus faces a typical adverse selection problem in which the company is the principal and the customer is the agent. One they agree on a pricing scheme to drive the customer to a given destination, the taxi driver determines her average speed v.[5] We assume that the travel distance is fixed to x and thus the travel time t, which is the indicator of service level in this trade, is solely determined by v according to $t = \frac{x}{v}$.

Players' utilities. The taxi company makes profit by completing the service and receiving a fee from the customer. Obviously, the company would hope the driver to drive fast to serve the next customer as soon as possible. However, driving slowly can also be beneficial as it takes more efforts and concentrations to maintain a high speed. Therefore, we model the utility function of the taxi company (as well as the taxi driver) as

$$U_D = p - K\ln(v) - Rt. \tag{1}$$

The first component of the company's utility is the payment p collected from the customer. The second component, $-K\ln(v)$, is the disutility of driving fast, which comes from the cost of exerting efforts to concentrate. We choose a strictly increasing and concave function to reflect the fact that the effort for driving is increasing in the average speed v with the marginal effort being decreasing. Here v is an exogenous scaling factor and the functional form $\ln(\cdot)$ is chosen for analytical tractability. Without loss of generality, we normalize $\ln(v)$ to be within 0 and 1, which requires v to be bounded by 1 and v. The last term $-Rt$ is to capture the opportunity cost for serving this customer, where t is the travel time and $R > 0$ is the exogenous average expected revenue per unit time.

The customer's utility also consists of three components. First, arriving the destination gives the customer an exogenous benefit $Q > 0$. The taxi fee p must then be subtracted from the benefit. Finally, the inconvenience of spending time brings a disutility θt, which increases as the travel takes more time (with a larger t) or as the customer is more impatient (with a larger θ). Collectively, we assume that the customer's utility is

$$U_c = Q - \theta t - p. \tag{2}$$

Metered pricing and Menu pricing. Nowadays most taxi services are regulated with metered pricing. One of the most commonly observed form of metered pricing contains three elements, a constant initial charge, a distance-based charge, and a delay-based charge. We adopt this form and assume that under metered pricing, the taxi fee will be given by[6]

$$p = \alpha + \beta(x - \hat{x})^+ + \gamma lt, \tag{3}$$

[4] This condition, adopted in the screening literature to rule out the bunching phenomenon (Laffont and Martimort, 2002), is satisfied by most usual distributions, including uniform, normal, logistic, chi-square, exponential and Laplace. See Bagnoli and Bergstrom (2005) for a more complete list.

[5] We assume that the taxi company has adopted some incentive alignment mechanism such as revenue sharing and two-part tariff to align the driver's and its own interests. Therefore, we will view the company and driver as an integrated entity.

[6] We follow the convention of optimization literature to use z^+ to denote $\max\{z, 0\}$.

where $\alpha \geq 0$, $\beta \geq 0$, and $\gamma \geq 0$ are exogenously determined by the government. While the initial fixed fee α is charged regardless of the travel distance or time, the distance-based charge $\beta(x-\hat{x})^{+}$ is determined by the travel distance above the predetermined initial distance $\hat{x}$. As both x and $\hat{x}$ are constants in this study, $\alpha+\beta(x-\hat{x})^{+}$ is a constant and will be denoted as η hereafter. The last component, the delay-based charge, is calculated based on the delay time, which is the amount of travel time during which the average speed is lower than a given level. By assuming that the length of the delay time is proportional to the total travel time t and is independent of the average speed v, we use $l \in (0,1)$ to denote the proportion of travel time that is considered delaying. The delay-based charge γlt is then the product of the delay charge per unit delay time γ and the delay time lt.[7] Throughout this study, we assume that $vR > \gamma l$ to ensure that the benefit of driving slowly to earn delay-based fees does not outweighs the opportunity cost of not serving the next customer.

Another possible way to price the taxi service is for the two players to determine a single price that will be affected by neither the travel distance nor the travel time. To persuade the customer to accept a predetermined price without worrying about the service, it is natural for the company to provide a guaranteed service level by guaranteeing the travel time. Therefore, the company offers a contract (p,t) that guarantees to arrive the destination by time t while charging a fixed fee p. As the customer's type θ, the time sensitivity, is private, the company's best strategy is to offer a menu of contracts $\{(p(\theta),t(\theta))\}$ for the customer to select from. This pricing mechanism is thus called menu pricing. Once being offered these contracts, the customer may select one or leave without being served to maximize his utility. We assume that the company has the ability of meeting her guaranteed travel time with no uncertainty and will not strategically break the promise to maintain her reputation.

Under metered pricing, the taxi fee is metered as $p=\eta+\gamma lt$. As $v \in [1,e]$ and $t=\dfrac{x}{v}$, the maximum value of t is 1, and the highest possible taxi fee is $\eta+\gamma lx$. Throughout this study, we assume that $Q=\eta+\gamma lx+\varepsilon$ for some $\varepsilon > 0$. This makes the most time-insensitive customer (whose $\theta=0$) be willing to accept metered pricing anyway. While this assumption is realistic, it also excludes unfair comparisons: Without this assumption, customers with high values of θ may never accept metered pricing, and menu pricing will be their only choice.

Timing. Under either metered pricing or menu pricing, we have a sequence of events. Under metered pricing, there are three stages: (1) θ is randomly determined and privately observed by the customer; (2) the customer decides whether to be served; (3) the company decides his average speed v. Under menu pricing, there are also three stages: (1) θ is randomly determined and privately observed by the customer; (2) the company offers a menu $\{(p(\theta),t(\theta))\}$; (3) the customer selects either a contract from the menu or leave.

4. PUBLIC UGENCY

To address our research questions, we start from a benchmark case in which the company can perfectly observe the customer urgency θ. Following the economics convention, we say that θ is public in this case. By examining the public urgency case first, we may exclude the adverse selection problem regarding θ and focus on the interaction between metered pricing and menu pricing. Our first step is to characterize company's optimal speed under menu pricing. We then consider the company's optimal menu design problem under metered pricing. Comparisons are made at the end.

4.1 Metered pricing

Suppose that the customer has agreed to be charged by the meter machine. In this case, the taxi fee p is determined according to the formula $p=\eta+\gamma lt$. The problem for the company to solve is

---

[7] To justify the assumption that the proportion $l \in (0,1)$ is not affected by the speed v, note that delays mainly resulted from red lights at intersections or traffic jams along roads. These delays really cannot be avoided by simply driving faster or slower. Also, a higher speed v in fact reduces the total delay time lt by reducing t.

$$\max_{v\in[1,e]} \eta + \gamma l \frac{x}{v} - K\ln(v) - R(\frac{x}{v}). \tag{4}$$

The objective function maximizes the company's utility, and the decision variable v is restricted within 1 and e. Note that the company here does not need to care about the customer's utility, because the customer has already chosen to be served in this way. In fact, if the customer anticipates a negative utility in equilibrium, he will not choose metered pricing, and the company will not have the chance of solving this problem. The solution to this problem is summarized in the following lemma.

Lemma 1. *Under metered pricing with public urgency, the equilibrium speed is*

$$v^* = \begin{cases} e & \text{if} \quad 0 \leq K \leq \frac{x}{e}(R-\gamma l) \\ \frac{(R-\gamma l)x}{K} & \text{if} \quad \frac{x}{e}(R-\gamma l) \leq K \leq x(R-\gamma l). \\ 1 & \text{if} \quad x(R-\gamma l) \leq K \end{cases} \tag{5}$$

The equilibrium price and customer's utility is and $U_C^T(\theta) = Q - \eta - (\theta + \gamma l)\frac{x}{v^*}$*, respectively.*

Proof. The first- and second-order derivatives of the utility function are $\frac{1}{v^2}(x(R-\gamma l) - Kv)$ and $\frac{1}{v^3}(Kv - 2x(R-\gamma l))$, respectively. These derivatives, as well as the fact that the stationary point and reflection point are both unique, shows that the function is quasi-concave with a unique maximizer, which is the stationary point $v_1 = \frac{x(R-\gamma l)}{K}$. If $v_1 < 1$, i.e., $x(R-\gamma l) < K$, the function is decreasing over $[1,e]$. Therefore, the optimal solution is 1. Similarly, if $v_1 > e$, which happens when $K < \frac{x(R-\gamma l)}{e}$, the function is increasing over $[1,e]$, and the optimal solution is e. Finally, when $1 < v_1 < e$, the optimal solution is the stationary point. This completes the proof. Q.E.D.

Lemma 1 shows that the equilibrium speed will be reduced when the cost of increasing speed K or the unit delay-based charge γ goes up. While this is trivial, this lemma does highlight the main incentives for the company to reduce the speed: To avoid the cost of exerting efforts and to collect delay-based revenue.

4.2 Menu pricing

Now consider the company's menu design problem. When θ is public, it is optimal for the company to offer a single contract (p,t) that guarantees a no-greater-than-t travel time and asks for a fixed fee p. To find the optimal contract, the company needs to maximize her utility while ensuring that the customer will choose to be served. It then follows that the company's contract design problem can be formulated as

$$\begin{aligned} &\max_{v\in[1,e],p\geq 0} p - K\ln(v) - R(\frac{x}{v}) \\ &\text{s.t. } Q - \theta(\frac{x}{v}) - p \geq 0, \end{aligned} \tag{6}$$

where $U_C^T(\theta)$ is the customer's equilibrium utility if he accepts metered pricing. The objective function maximizes the company's utility. The constraint ensures that the customer's utility upon choosing the contract will be nonnegative. The next lemma characterizes the optimal contract.

Lemma 2. *Under menu pricing with public urgency, the equilibrium pricing plan is given by*

$$(p^*, t^*) = \begin{cases} (Q - \theta(\frac{x}{e}), \frac{x}{e}) & \text{if} \quad 0 \leq K \leq \frac{x}{e}(R+\theta) \\ (Q - \frac{K\theta}{R+\theta}, \frac{K}{R+\theta}) & \text{if} \quad \frac{x}{e}(R+\theta) \leq K \leq x(R+\theta). \\ (Q - \theta x, x) & \text{if} \quad x(R+\theta) \leq K \end{cases} \tag{7}$$

Proof. First, note that the constraint must be binding at any optimal solution; otherwise the seller can increase p to improve her utility. Therefore, we have $p = Q - \theta(\frac{x}{v}) - U_C^+$. Plug in this into the objective function, the original problem reduces to

$$\max_{v \in [1,e]} Q - \theta(\frac{x}{v}) - U^+ - K\ln(v) - R(\frac{x}{v}). \tag{8}$$

The first- and second-order derivatives of the new objective function are $\frac{1}{v^2}(x(\theta+R) - Kv)$ and $\frac{1}{v^3}(Kv - 2x(\theta+R))$, respectively. Similar to the proof of Lemma 1, these derivatives show that the function is quasi-concave. The optimal speed can then be found by comparing 1, e, and the stationary point $v_2 = \frac{x(\theta+R)}{K}$. If $v_2 < 1$, i.e., $x(\theta+R) < K$, the function is decreasing over $[1,e]$, and the optimal speed is 1; if $v_2 > e$, i.e., $K < \frac{x(\theta+R)}{e}$, the function is increasing over $[1,e]$, and the optimal speed is e; otherwise the optimal speed is the stationary point $\frac{x(\theta+R)}{K}$. The optimal travel time t^* and price p^* can be found as $\frac{x}{v^*}$ and $Q - \theta(\frac{x}{v^*}) - U^+$, where v^* is the optimal speed, respectively. Q.E.D.

A few interesting findings may be obtained here. First, when K increases, t^* weakly increases, i.e., a higher cost encourages the company to offer a lower speed. On the contrary, when R increases, a larger opportunity cost motivates the company to offer a higher speed. t^* thus decreases. Unlike metered pricing, however, the customer's type θ now affects the optimal contract. In particular, when θ increases, t^* weakly decreases and p^* weakly increases. While the company must offer a higher speed to more urgent customers, as their willingness-to-pay is lower, they are charged a lower fee. Finally, the customer's equilibrium utility is always zero. By offering a contract, the company is able to extract all the surplus from the customer.

4.3 Comparisons

Our main focus of this study is on the comparison between metered pricing and menu pricing. Remarkably, switching to menu pricing enhances social welfare. To see this, the first step is to compare the equilibrium speeds under the two pricing schemes. By combining Lemmas 1 and 2, we may construct Table 1, which lists the equilibrium speeds under both scenarios and all possible conditions. A direct comparison shows that, under all possible conditions, switching to menu pricing always induces a higher equilibrium speed (unless the physical restriction on speed prevents that to happen). The increase in speed, as well as the fact that the company can extract all the surplus with menu pricing, shows that the use of metered pricing is simply a deviation from the efficient integrated scenario. It then follows that menu pricing indeed results in higher social welfare. We highlight these major findings in Proposition 1.

Ling-Chieh Kung, Zong-Ting Chen: *Price Negotiation for Taxi Services under Private Customer Urgency*
IJOR Vol. 13, No. 4, 153−163 (2016)

Table 1. Equilibrium speeds under public urgency

Condition	Metered pricing	Menu pricing
$0 \leq K \leq \frac{x}{e}(R-\gamma l)$	e	e
$\frac{x}{e}(R-\gamma l) \leq K \leq \frac{x}{e}(R+\theta)$	$\frac{(R-\gamma l)x}{K}$	e
$\frac{x}{e}(R+\theta) \leq K \leq x(R-rl)$	$\frac{(R-\gamma l)x}{K}$	$\frac{(R+\theta)x}{K}$
$x(R-rl) \leq K \leq x(R+\theta)$	1	$\frac{(R+\theta)x}{K}$
$x(R+\theta) \leq K$	1	1

Proposition 1. *Suppose the customer urgency is public. Compared to metered pricing, menu pricing results in weakly higher taxi speeds and social surplus in equilibrium. The increases are strict if* $\frac{x}{e}(R-\gamma l) \leq K \leq x(R+\theta)$.

One reason for menu pricing to induce a higher speed is the removal of delay-based charge. This removal provides the company incentives to offer a higher speed, which help increase social surplus. More importantly, the main benefit of menu pricing is the flexibility of taxi fee. When metered pricing is the only option, the company can do nothing but lose some very urgent customers because the metered price cannot be set to be low enough for them. However, by applying menu pricing, the company can charge a low price for urgent customers and therefore serve more customers. The market expansion benefit will be discussed more deeply the next section. In summary, the two benefits together increase social surplus and make the company-customer system more efficient.

Though social surplus can be increased by adopting menu pricing, the customer will always earn zero utility. This is because the company can observe a customer's urgency level and then specifically design a contract that takes away all the surplus from the customer. In the next section, we consider the more realistic situation with information asymmetry. We will show that the privatization of urgency level will protect the customer and earn him a positive utility.

5. PRIVATE URGENCY

In this section, we add the adverse selection problem regarding customer urgency back into our model. In this case, the company's menu can serve as a tool to screen the customer's type. We will proceed by first deriving the optimal speed under metered pricing, then constructing the optimal menu under menu pricing, and finally drawing implications.

5.1 Metered pricing

Suppose that the type-θ customer has chosen metered pricing. When the company needs to decide the speed, just like the case discussed in Section 4.1, she does not care about the customer's utility. In fact, the optimal speed v^* solved in Lemma 1 does not depend on θ at all. In other words, even if the company can observe the customer's type θ, this information does not affect the optimal speed. It then follows that the optimal speed with unobservable urgency is still v^*, the one adopted under public urgency.

5.2 Menu pricing

Now we consider the company's menu design problem under menu pricing. In contrast to the public urgency case, because now the company cannot observe the customer's type, her best strategy now is to design a menu of contracts for the customer to self-select and induce truthful revelation.

To formulate the menu design problem, let $(t(\theta), p(\theta))$ be the contract intended for the type-θ customer, which guarantees to send him to the destination in time $t(\theta)$ by charging him a fee $p(\theta)$. The company's problem is

$$\begin{aligned}
\max_{p(\theta)\geq 0, t(\theta)\in[\frac{x}{e},x]} \quad & \int_{\theta_0}^{\theta_1} \left(p(\theta) - K\ln(\frac{x}{t(\theta)}) - Rt(\theta) \right) f(\theta) d(\theta) \\
\text{s.t.} \quad & Q - \theta t(\theta) - p(\theta) \geq 0 \quad \forall \theta \in [\theta_0, \theta_1] \\
& Q - \theta t(\theta) - p(\theta) \geq Q - \theta t(\hat{\theta}) - p(\hat{\theta}) \quad \forall \theta, \hat{\theta} \in [\theta_0, \theta_1].
\end{aligned} \tag{9}$$

The objective function maximizes the company's expected utility (by incorrectly assuming that all customers will choose menu pricing). The first constraint is the individual rationality (IR) constraint to guarantee the customer's participation. The second constraint is the incentive compatibility (IC) constraint to ensure that, for any customer of type θ, reporting his true type θ is better than reporting any false type $\hat{\theta}$.

After solving the problem, we obtain the following lemma.

Lemma 3. *Suppose the speed limit constraints are omitted, the optimal menu that solves the problem in (1) satisfies, for all* $\theta \in [\theta_0, \theta_1]$,

$$t(\theta) = \frac{K}{R + \theta + \dfrac{F(\theta)}{f(\theta)}} \quad \text{and} \quad p(\theta) = Q - \theta t(\theta) - \int_{\theta}^{\theta_1} t(y) dy. \tag{10}$$

Proof. First, consider the IR constraint for $\theta < \theta_1$. We have

$$Q - \theta t(\theta) - p(\theta) \geq Q - \theta t(\theta_1) - p(\theta_1) > Q - \theta_1 t(\theta_1) - p(\theta_1) \geq 0, \tag{11}$$

where the first inequality is an IC constraint, and the last inequality is the IR constraint for θ_1. This implies that among all the IR constraints, only the one for the highest type θ_1 is not redundant. Now, let $U(\theta, \hat{\theta}) = Q - \theta t(\theta) - p(\hat{\theta})$ be the type-θ customer's utility once he reports $\hat{\theta}$ as his type and $U(\theta) = U(\theta, \theta)$ be the type-θ customer's equilibrium utility. We have $\frac{\partial}{\partial \theta} U(\theta, \hat{\theta}) \mid_{\hat{\theta}=\theta} = -t(\theta) \leq 0$, which implies that $U(\theta)$ is decreasing in θ, and $U(\theta) \geq U(\theta_1)$ for all θ. As $U(\theta_1)$ is bounded below by it IR constraint, this IR constraint for θ_1 must be binding at any optimal solution. Therefore, we have $U(\theta_1) = 0$, and the equality $U(\theta_1) = U(\theta) + \int_{\theta}^{\theta_1} [-t(u)] du$ then immediately implies that $U(\theta) = \int_{\theta}^{\theta_1} t(u) du$ for all θ. It then follows that $U(\theta) = Q - \theta t(\theta) - p(\theta) = \int_{\theta}^{\theta_1} t(u) du$. By replacing $p(\theta)$ in the objective function by the above equation and ignoring the IC constraints, the problem reduces to

$$\begin{aligned}
& \max_{t(\theta)\geq 0} \int_{\theta_0}^{\theta_1} \left\{ Q - \theta t(\theta) - \int_{\theta}^{\theta_1} t(u) du - K\ln(\frac{x}{t(\theta)}) - Rt(\theta) \right\} f(\theta) d(\theta) \\
= & \max_{t(\theta)\geq 0} \int_{\theta_0}^{\theta_1} \left\{ Q - \theta t(\theta) - \frac{F(\theta) t(\theta)}{f(\theta)} - K\ln(x) + K\ln(t(x)) - Rt(\theta) \right\} f(\theta) d(\theta),
\end{aligned} \tag{12}$$

where the equality comes from integration by parts. Pointwise optimization with respect to $t(\theta)$ then yields

$t(\theta) = \dfrac{K}{\theta + R + \dfrac{F(\theta)}{f(\theta)}}$. $p(\theta)$ can be obtained by using the equation above. The verification of the IC constraints follows directly from the monotonicity of $U(\theta)$ and is omitted. Q.E.D.

Suppose the speed limits are not constraining the company's decision, customers with different types should be offered different contracts. In particular, as $t(\theta)$ decreases in θ and $p(\theta)$ increases in θ, more urgent customers are served with higher speeds (smaller travel times) and charged higher fees. Because urgent customers are willing to pay more for higher speeds, it is to the company's best interest to differentiate the service levels (represented by the speeds) and price discriminates customers with heterogeneous urgency levels. We may also observe the well-documented downward distortion on $t(\theta)$ except for the lowest-type customer. To see this, note that $t(\theta) \leq \dfrac{K}{R+\theta}$ for all $\theta \geq \theta_0$, and $\dfrac{K}{R+\theta}$ is exactly the optimal speed without speed limits under public urgency (cf. Lemma 2). Cutting down the travel time for high-type customers discourages a low-type customer to mimic a high-type one (to utilize his low time sensitivity θ) and limits the information rent earned by the customer. Finally, the travel time is influenced by the cost of increasing the speed K and average expected revenue R in the same way as in the public urgency case.

When the presence of the speed limits, $t(\theta)$ may be restricted within $[\dfrac{x}{e}, e]$, and the optimal menu derived above should be adjusted accordingly. Let $t(\theta^l) = x$ and $t(\theta^h) = \dfrac{x}{e}$, where $t(\theta)$ is defined in Lemma 3. If $\theta^l \leq \theta_0 \leq \theta_1 \leq \theta^h$, the above menu will be feasible and thus optimal. If $\theta_0 \leq \theta^l$, all customers whose type $\theta \in [\theta_0, \theta^l]$ will be induced to choose the contract $(t(\theta^l), p(\theta^l))$. The company fails to differentiate these customers due to the lowest speed limit. If $\theta^h \leq \theta_1$, all customers whose type $\theta \in [\theta^h, \theta_1]$ will leave without being served. The company fails to serve these customers due to the highest speed limit.

5.3 Comparisons

First, we compare the equilibrium speeds under metered pricing and menu pricing. Because the equilibrium speed induced by menu pricing under public urgency is socially efficient, we also include it in the comparison summarized in the following proposition.

Proposition 2. *The equilibrium speed induced by metered pricing (under public or private urgency) is lower than that induced by menu pricing under public urgency, which is lower than that induced by menu pricing under private urgency.*

Proof. The proposition can be proved by a direct comparison among the equilibrium speeds derived in Lemmas 1, 2, and 3. Q.E.D.

Consider the equilibrium speeds not restricted by speed limits as an example. In this case, we have

$$\frac{(R-\gamma l)x}{K} < \frac{(R+\theta)x}{K} < \frac{(R+\theta+\dfrac{F(\theta)}{f(\theta)})x}{K} \tag{13}$$

for all $\theta \in [\theta_0, \theta_1]$, where the three speed are induced by metered pricing, induced by menu pricing under public urgency, and induced by menu pricing under private urgency. By recognizing that the second one is the socially efficient speed, it is clear that metered pricing makes the speed inefficiently low. This is due to the facts that metered pricing contains delay-based charge (which causes the term γl), and menu pricing enables price discrimination (which introduces the term θ). It is also clear that the adverse selection problem makes the speed inefficiently high (by bringing the term $\dfrac{F(\theta)}{f(\theta)}$ in). Despite that metered pricing and menu pricing both fail to induce the efficient speed, it can be concluded that the equilibrium speed is higher under menu pricing. This is the same as what we find under public urgency.

Because both equilibrium speeds are inefficient, one cannot easily determine whether social welfare is higher with metered pricing or menu pricing. Unfortunately, the complicated functional forms of the company's and customer's utility functions disallow us to make a clear comparison analytically. We therefore sort to numerical experiments to examine which pricing scheme can result in higher social welfare. It turns out that in most cases, menu pricing is more efficient then metered pricing. Note that this is the outcome of replacing metered pricing by menu pricing. In reality, what the company may do is to supplement metered pricing by inducing only part of the customers (intuitively, those of high or low values of θ) to choose menu pricing. This allows the company to better price discriminate (compared to offer no menu) while reducing the information rent earned by customers (compared to offer a menu to all customers). We are therefore optimistic about the mutual benefit of including menu pricing into the current taxi pricing scheme.

6. CONCLUSION

In this study, we consider the design of pricing schemes for taxi services. While pricing by the meter is widely adopted, we investigate whether allowing the taxi company to offer a menu for the customer to self-select can benefit both players. We construct a stylized model to address our research questions with a concentration on the informational issue caused by the unobservability of the customer's urgency level. Regardless of whether the urgency level is public or private, we show that menu pricing can always induce a higher taxi speed than metered pricing in equilibrium. Moreover, with the flexibility on setting the taxi fee, menu pricing allows the company to expand the market and customize the speeds offered to customers with different urgency levels. Our results suggest that menu pricing can benefit both taxi companies and customers and should be considered in the future.

Our study certainly has its limitations. First, we assumed a deterministic world with no uncertainty. If the travel time is subject to some randomness, the design of menu will become more complicated. This would be especially true if at least one player becomes risk averse. Adding time randomness into our model can help verify our findings. Second, it is assumed that the market is monopolized by a single company. Considering competition among taxi companies will definitely deliver new insights. Finally, the ideal travel distance is assumed to be common knowledge in this study. While this can be true in many cases as we discuss in the introduction, there are undoubtedly situations where the taxi company is more knowledgeable about the routes and ideal distance to the destination. The modeling and analysis of this issue calls for further investigation.

ACKNOWLEDGEMENT

We thank the area editor I-Hsuan Hong and two anonymous reviewers for their detailed comments and many valuable suggestions that significantly enhanced the quality of this work. All remaining errors are our own. We also thank the conference and seminar audience in National Tsing Hua University and National Chiao Tung University for their constructive comments. Finally, we thank Ying-Ju Chen, Chia-Wei Kuo, and Jun-Yu Yu for providing helpful guidance and Yi-An Lin and Pei-Jung Yang for formatting the manuscript before the publication.

REFERENCE

1. Arnott, R. (1993). Taxi travel should be subsidized. *Journal of Urban Economics*, 40(3): 316-333.
2. Bagnoli, M.T. (2005). Log-concave probability and its applications. *Economic Theory*, 26 (2): 445-469.
3. Bladikas, A.K., W.H. Crowell (1985). Pricing options for urban transportation modes. *Transportation Research Record*, 10(1012): 23-30.
4. Cairns, R.D. and Liston-Heyes, C. (1996). Competition and regulation in the taxi industry. *Journal of Public Economics*, 59(1): 1-15.
5. De Vany, A.S. (1975). Competition utilization under alternative regulatory restraints: an analysis of taxi markets. *The Journal of Political Economy*, 83(1): 83-94.
6. Douglas, G.W. (1972). Price regulation and optimal service standards: The taxicab industry. *Journal of Transport Economics and Policy*, 6(2): 116-127.
7. Foerster, J.F. and Gilbert, G. (1979). Taxicab deregulation: economic consequences and regulatory choices. *Transportation*, 8(4): 371-387.
8. Laffont, J. and Martimort, D. (2002). *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press, USA.
9. Loo, P.Y. Becky, Leung, S.Y. Betty, Wong, S.C., Yang, H. (2007). Taxi license premiums in Hong Kong: Can their fluctuations be explained by taxi as a mode of public transport? *International Journal of Sustainable Transportation*,

1(4): 249-266.

10. Shreiber, C. (1975). The economic reasons for price and entry regulation of taxicabs. *Journal of Transport Economics and Policy*, 9(25): 268-279.
11. Wong, K.I., Wong, S.C., Yang, H., Wu, J.H. (2008). Modeling urban taxi services with multiple user classes and vehicle modes. *Transportation Research Part B*, 42(10): 985-1007.
12. Yang, H., Ye, M., Tang, W.H., Wong, S.C. (2005). Regulating taxi services in the presence of congestion externality. *Transportation Research Part A*, 39(1): 17-40.
13. Yang, H., Ye, M., Tang, W.H., Wong, S.C. (2010). Nonlinear pricing of taxi services. *Transportation Research Part A*, 44(5): 337-348.