



## SOME TECHNICAL ASPECTS OF THE RHEOLOGICAL PROPERTIES OF HIGH CONCENTRATION FINE SUSPENSIONS TO AVOID ENVIRONMENTAL DISASTERS

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**Abstract.** The behavior of slurries and suspensions made by mixing solid particles and liquids is very important for various industrial applications. The latest accidental failure events at tailings facilities (Kolontár, Baia Mare) have focused public interest into this field. Nowadays, environmental practice is turning to use dry deposition techniques or at least as high concentration slurries or pastes as possible, to avoid large spills in case of an accidental failure of an embankment. High concentration slurries are becoming widely accepted in many environmentally related operations such as underground backfilling or simple tailings deposition. However, the hydraulic transport of pastes or high density slurries requires higher energy input via pumps, and, in addition, the energy requirement or pressure loss calculation methods are also different because the rheology of pastes differs from that of dilute slurries. At the University of Miskolc, Institute of Raw Materials Preparation and Environmental Processing, Miskolc, Hungary, this topic has been studied for many decades. The fine suspension – coarse mixture flow model was introduced, and it has been determined that the flow behavior of fine suspensions made of solid particles smaller than a limit particle size can be measured and interpreted in almost the same way as for single phase clear liquids. Based on these measured rheological parameters of fine suspensions, the frictional energy loss can be calculated. The aim of this paper is to give a summary and data base about the rheological behavior of different industrial materials based on pilot scale hydraulic loop measurements. An Anton-Paar rotational viscometer and a tube viscometer with three measuring pipe sections, – developed by our institute – were used for testing. The results of measurements of various granular materials, such as sands, fly ashes, perlite, tailings and red mud are presented in connection with environmental applications. Based on these results, empirical relationships are presented, where the parameters are determined by simple function fitting into the data of measurements carried out at discrete concentration values. The rheology of fine suspensions of any concentration up to the measured maximum can be calculated by these relationships.

**Keywords:** fine suspension rheology, dense slurries, paste technology, environmental sciences.

### Introduction

The flow behavior of single phase or clear liquids is described by the constitutive equation (Govier, Aziz 1972 [2004]). The constitutive equation is the relation between the velocity of deformation (shear rate tensor) and the shear stress (stress tensor). Generally, it is a tensor equation. One dimensional constitutive equations (the relation between the shear rate and shear stress is analyzed only in one specific direction) are used for engineering applications. The constitutive equation of Newtonian fluids was determined by Newton, using flows between parallel plains – one plain was steady, one was moving at a constant velocity – and found that the flow velocity profile was linear, thus shear rate and shear stress are proportional and the proportionality factor between them is

the absolute viscosity. Newtonian fluids, such as air or water flow according to the Newtonian constitutive equation. There is a group of fluids in which flow behavior changes with shear time or the shear itself. At a constant shear rate, the resistance of the internal friction of some fluids decreases, and this is called thixotropy. Others have increasing resistance of internal friction over time, and these fluids are rheopectic. Viscoelastic liquids show elastic features, as they regain some of their original shape after the deformation forces cease; however they possess viscous properties as well. Usually, suspensions of fine solids in the practice of mechanical processing do not show elastic behavior, and their flow properties are generally constant over time; however in higher concentrations they behave as non-Newtonian fluids.

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Non-Newtonian fluids are viscous fluids, at constant shear rate the shear stress is also constant, but their flow cannot be described by the Newtonian constitutive equation. The following (Table 1) engineering flow models (nonphysical models) were applicable for the test materials used in our investigations.

Table 1. Engineering flow models have been used for all tests described in this paper

Flow model	Constitutive equation	Rheological parameters
Newtonian	$\tau = \mu \left( \frac{du}{dr} \right)$	$\mu$ – absolute viscosity
Bingham plastic	$\tau = \tau_0 + \eta \left( \frac{du}{dr} \right)$	$\tau_0$ – yield-stress $\eta$ – coefficient of rigidity
Power law $n > 1$ – pseudoplastic, $n < 1$ – dilatant	$\tau = K \left( \frac{du}{dr} \right)^n$	$K$ – consistency index $n$ – flow behavior index

The three most important devices for measuring rheology of single phase liquids are the falling sphere, capillary, and the rotational viscometer.

Mechanical processing deals with coarse disperse systems. The disperse part contains particles, bubbles and drops larger than  $0.1 \mu\text{m}$  dispersed in a solid, liquid or gas phase. If solid particles are mixed with liquid (dispersed, for example, in water), the description of the flow behavior of these mixtures is more difficult than the one for single phase fluids described earlier. What is flowing? The solid particle itself is not deforming and there is no shear stress inside it. Is only the liquid flowing? In resting liquids, dense particles larger than  $0.1 \mu\text{m}$  are settling down. Multiphase flow was early investigated by several authors (Newitt *et al.* 1955; Condolios, Chapus 1963; Wasp *et al.* 1977; Hill *et al.* 1986; Hanks 1980). Recently, scientific interest has been focused on suspension rheology related to wet grinding (He *et al.* 2004, 2006; Johnson *et al.* 2000; Matijasic *et al.* 2008). In the case of ultra-fine grinding, both of the two different approaches should be applied: colloid chemistry ( $<0.1 \mu\text{m}$ ), where physical-chemical surface phenomena are dominant and mechanical processing ( $>0.1 \mu\text{m}$ ) where mechanical forces are the most important. In the mechanical processing industry, the most important application is the flow of solid – liquid mixtures in pipelines, – namely hydraulic transport – and this basic phenomenon can definitely be described by the rheological behavior of the flowing materials. In Hungary, the biggest pipelines transport coal power plant fly-ashes, and this is our primary interest. According to the Tarján–Faitli fine suspension – coarse mixture flow model (Tarján, Faitli 1998; Böhm

*et al.* 2007), a limit particle size can be determined for every solid material. Solid – liquid mixtures made by particles smaller than the limit can be transported as fine suspension flow in pipelines. For example, the limit particle size is  $160 \mu\text{m}$  for fly ashes and  $50 \mu\text{m}$  for sands. Such small particles are able to penetrate into the boundary layer near the pipe wall. The integral of the shear stress on the particle surface is a take-off the wall force and it is negligible for small particles; therefore the particle is able to alter the flow behavior of the boundary layer and indirectly the rheology of the fine suspension also changes. The fine suspension flow can be considered as single phase liquid flow with its own density and rheology. Flow behavior of such mixtures can be measured with viscometers, and on the basis of the measured rheological parameters the pressure loss can be calculated for any flow rate. The described Tarján–Faitli fine suspension flow model is consistent with the Sengun and Probstein (Sengun, Probstein 1989a, 1989b) bimodal model. Based on the bimodal model the fine fraction can be subdivided into a submicron – colloidal – and a micron range fraction. The colloidal size range is responsible for non-Newtonian behaviour and the relatively coarse – some 10 microns-size particles contribute to the viscosity rise through hydrodynamic dissipation.

If the transported particles are coarser than the particle size limit, particles do not fit into the boundary layer, the surface integral of the shear stress results in a considerable take-off force for higher flow rates. This model explains the often measured fact, that at high flow rates the pressure drop of coarse sand – water mixtures flow is almost the same as just the water flow alone, but at lower flow rates coarse particles are sliding along the pipe wall, and the pressure drop is high, – much higher than the water's – because of the mechanical, not flow friction between the particles and the pipe wall. In this coarse mixture flow case, the fluid is the only phase which flows and the particle movement and particle friction on the pipe wall are purely the consequence of mechanical forces. The force of mechanical friction does not depend on the velocity of the moving bodies, but on the normal force and the coefficient of friction. Contrary to mechanical friction, flow frictional loss highly depends on the flow velocity of the fluid. According to the model, the rheology of the coarse mixture cannot be interpreted, only the rheology of the liquid. In extreme conditions – when a river is rolling huge rocks – it is expedient that only the fluid is flowing while the movement of the solid rocks is the result of various mechanical forces due to the flow. This paper does not deal with this coarse mixture flow.

The most important equipment for measuring the rheology of fine suspensions is the tube viscometer. The principal of the measurement is that in straight

horizontal pipes, stable laminar flow has to be established and the pressure loss as a function of the flow rate has to be measured. The tube viscometer developed by our Institute (Fig. 1) has three pipe sections, each six meters long, while their diameters vary at 16; 21 and 27 mm. The mixed suspension (120 liter) is circulated by a rigid characteristic screw pump. Another device for measuring the rheology of fine suspensions is the rotational viscometer. There is an Anton-Paar rotational viscometer at our institute (cylinder-cylinder and cone-plate test systems, 50 cm<sup>3</sup> sample volume, 0.007–7 Pas viscosity and 40–1200 1/s shear rate measuring range). Measuring the rheology of fine suspensions, the cylinder – cylinder geometry is used with a 0.5 mm gap in the Couette ring. The tube viscometer is a large laboratory instrument, but at the same time it is a small pilot scale industrial hydraulic transport system and therefore, its test data is reliable for industrial size design. Contrary, test data of rotational viscometer measurements can be used for verifying only; however the results of the two devices generally do correspond. Measurement temperatures were 22 °C room temperature for both viscometers. The rotational viscometer is equipped with a thermostat. The tube viscometer – however – is equipped with a casing pipe section where cooling water was circulated to stabilize the temperature. There are vertical up and down pipe sections built into the tube viscometer to mix the transported suspension. In addition there are separate sampling points in the main tube where samples can be taken from the upper half, lower half and full cross sections of the pipe. In addition, any accidental sedimentation can be monitored based on the measured pressure loss. In such case the revolution number of the screw pump must be switched into maximum. Both viscometers were calibrated by calibrating the sensors. In the case of the rotational viscometer the revolution number and the torque sensors, in the case of the tube viscometer the pressure sensors were calibrated. The flow rate measuring technique – filling a tank and measuring the time – is absolute technique, no additional calibration was necessary.

In the following the results of some rheological tests with different materials are presented. Based on the

Tarján – Faitli fine suspension – coarse mixture flow model (Tarján, Faitli 1998; Böhm *et al.* 2007) – the rheological properties of only the fine fraction were measured in the tube viscometer. The full granular materials were tested in a hydraulic test loop – 400 liter sample. There are many working coal power plant fly-ash deposition system installations worldwide (Jacksonville Power Plant – USA, Craiova 2, Isalnita, Rovinari, Turceni Power Plants – Romania, Mátra Power Plant – Hungary, etc...) which were designed on the basis of the model and measurements presented.

## 1. Materials and methodology

### 1.1. Flow behavior of fine suspensions, measurement results

Before starting a rheological test the physical properties of the materials had to be measured. The particle densities of the solid materials were measured by laboratory picnometers. Particle size distributions of all the tested bulks had been determined by laboratory sieving using 200 mm laboratory sieve and micro sieve series. The measured particle size distribution of most of the tested materials can be seen on Figure 2. The rheological tests

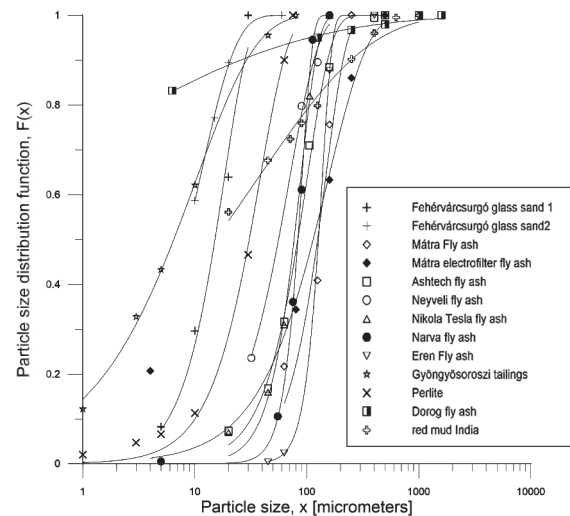


Fig. 2. Measured particle size distributions of most of the tested bulks

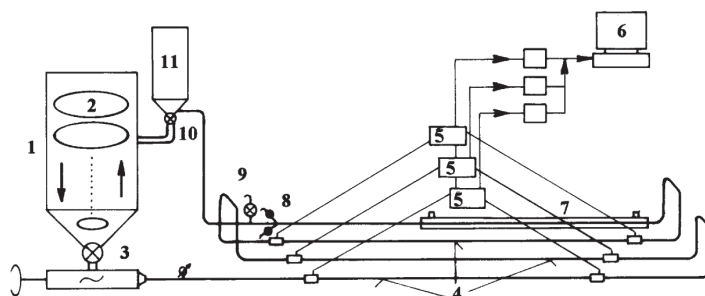


Fig. 1. Tube viscometer with three measuring pipes

1. Mixing tank
2. Mixing hole discs
3. Screw pump
4. Measuring pipes
5. Differential pressure transducers
6. Data acquisition system
7. Casing pipe for cooling water
8. Sampling points
9. Sampling point
10. Valve
11. Flow rate measuring tank

presented here were carried out mainly with fine suspensions. Coarse fractions were removed by sieving at the limit particle size before the tests.

The tube viscometer measurements were started using pure water firstly, and then the required masses of the solid for different concentrations were added into the system step by step. By this method discrete concentration suspensions

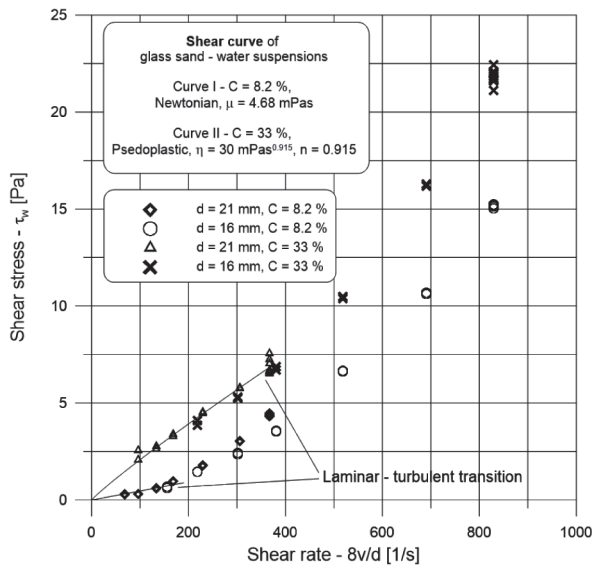


Fig. 3. Flow curves of sand glass from Fehérvárcsurgó

Table 2. Flow properties of sand glass from Fehérvárcsurgó (5 ... 30 μm, ρ<sub>s</sub> = 2644 kg/m<sup>3</sup>)

Volumetric concentration	μ absolute viscosity [mPas]	Flow behavior	
0.1	2.99	Newtonian	
0.2	4.63	Newtonian	
0.3	8.47	Newtonian	
	K	m flow behavior index	
0.35	19.3 mPas <sup>0.916</sup>	0.916	pseudoplastic
0.4	75.5 mPas <sup>0.713</sup>	0.713	pseudoplastic

Table 3. Flow properties of sand glass from Fehérvárcsurgó (x<sub>90</sub> = 20.4 μm, ρ<sub>s</sub> = 2734 kg/m<sup>3</sup>)

C Volumetric concentration	μ absolute viscosity [mPas]	Flow behavior	
0.082	4.68	Newtonian	
0.196	5	Newtonian	
0.26	8	Newtonian	
	K	m flow behavior index	
0.33	30 mPas <sup>0.915</sup>	0.915	pseudoplastic

were tested gradually. The device is equipped by a propeller mixer. After the feeding of a solid portion some time was waited while the suspension was continuously circulated at high speed by the pump until pressure loss equilibrium.

This paper reports about many rheological tests, it summarizes the results of almost a hundred shear curves. It is not possible to show them here, just two shear curves of tube viscometer tests are shown in Figure 3 as an example.

Sand glass is mined at Fehérvárcsurgó as a basic raw material for glass production. During mineral processing, the raw material is also classified, thus fine, narrow particle size classes can be achieved. These narrow particle size fractions are really good test materials for testing flow behavior. Sand glass with a particle size less than 50 μm can be hydraulically transported as a fine suspension flow. Fine sand glass has been frequently tested (Tables 2 and 3) and it behaves as a Newtonian fluid below 30% volumetric concentration, and generally becomes pseudoplastic at higher concentrations.

The most important industrial application of hydraulic transport in Hungary, as previously mentioned, is slag and fly ash transport from coal fired power plants to tailings areas. Fly ash particles with a size less than 160 μm can be transported as fine suspensions in normal size pipelines. During the last decade flow properties of several fly ash samples were analyzed from all over the world. These tests were carried out with particles finer than the limit particle size of fly ashes – 160 μm, and they were sieved if it was necessary. It was determined that all the high concentration fly ash fine suspension samples behaved as a Bingham – plastic fluid (Tables 4–8).

Some of the tested fly ash samples (Tables 4–8) had moderate friction resistance with increasing concentration and they became Bingham plastic over 20% volumetric concentration and the coefficient of rigidity remains under 100 mPas up to 30% volumetric concentration.

Table 4. Flow properties of Mátra Power Plant, Fly Ash: (x<sub>95</sub> = 160 μm, ρ<sub>s</sub> = 1900 kg/m<sup>3</sup>)

C Volumetric concentration	μ absolute viscosity [mPas]	Flow behavior	
0.13	5	Newtonian	
0.16	5.6	Newtonian	
0.2	7	Newtonian	
	η coefficient of rigidity [ mPas ]	τ <sub>0</sub> Yield Stress [ Pa ]	
0.23	9	0.8	Bingham plastic
0.27	17	1.1	Bingham plastic
0.33	51	1.5	Bingham plastic
0.35	102	2.3	Bingham plastic



Table 5. Flow properties of Mátra Power Plant, Fly Ash from electrofilter: ( $x_{90} = 200 \mu\text{m}$ ,  $\rho_s = 1984 \text{ kg/m}^3$ )

C Volumetric concentration [-]	$\eta$ coefficient of rigidity [mPas]	$\tau_0$ Yield Stress [Pa]	Flow behavior
0.16	3.4	1.24	Bingham plastic
0.25	11.3	1.89	Bingham plastic
0.31	60	3.63	Bingham plastic

Table 6. Flow properties of Ashtech – India, Fly Ash ( $x_{80} = 110 \mu\text{m}$ ,  $\rho_s = 1974 \text{ kg/m}^3$ )

C Volumetric concentration	$\mu$ absolute viscosity [mPas]	Flow behavior
0.1	6	Newtonian
0.19	8	Newtonian
0.26	10	Newtonian
0.3	13	Newtonian
0.36	16	Newtonian

C Volumetric concentration	$\eta$ coefficient of rigidity [mPas]	$\tau_0$ Yield Stress [Pa]	Flow behavior
0.4	42	1	Bingham plastic
0.5	80	4	Bingham plastic

Table 7. Flow properties of Neyveli – India, Fly Ash: ( $x_{95} = 160 \mu\text{m}$ ,  $\rho_s = 2492 \text{ kg/m}^3$ )

C Volumetric concentration	$\mu$ absolute viscosity [mPas]	Flow behavior
0.1	9	Newtonian

C Volumetric concentration	$\eta$ coefficient of rigidity [mPas]	$\tau_0$ Yield Stress [Pa]	Flow behavior
0.25	12	0.75	Bingham plastic
0.3	15	1.05	Bingham plastic
0.36	22	4.5	Bingham plastic
0.38	86	15	Bingham plastic
0.43	140	29	Bingham plastic

Table 8. Flow properties of Nikola Tesla B Power Plant – Serbia, Fly Ash: ( $x_{95} = 160 \mu\text{m}$ ,  $\rho_s = 1630 \text{ kg/m}^3$ )

C Volumetric concentration	$\mu$ absolute viscosity [mPas]	Flow behavior
0.1	11	Newtonian

C Volumetric concentration	$\eta$ coefficient of rigidity [mPas]	$\tau_0$ Yield Stress [Pa]	Flow behavior
0.195	15	0.86	Bingham plastic
0.336	24	1.9	Bingham plastic
0.448	99	6	Bingham plastic

The fly ash samples (Tables 9 and 10) were originally generated by another type of combustion method. The water mixing of these fly ashes was much more difficult and they had higher friction resistance. The measuring range of our tube viscometer is 1 to 250 mPas; this is determined by the differential pressure transducers and the drive system of the screw pump. These types of fly ashes reached a level of approximately 250 mPas, around 35% volumetric concentration; therefore, there is no measured point indicated in the tables over this concentration.

The base metal mine near Gyöngyösorszi, Hungary was decommissioned a long time ago; however final closure work is still continuing. One of the problems was to handle the former froth flotation tailings deposited on the tailing facility. The planned method of handling was to utilize these tailings for abandoned underground mine backfilling. This tailing material has a very fine particle size range and contains a high amount of clay minerals. These material features are good to make paste state mixtures. Using this tailing for tests, it was determined that it becomes Bingham plastic over 18% of volumetric concentration with extreme high coefficient of rigidity and yield stress corresponding to increasing concentration (Table 11). For testing this material, a pilot scale hydraulic transport test loop was built applying a high capacity

Table 9. Flow properties of Narva Pow. Plant N°8. Balti CFB Furnace, Fly Ash: ( $x_{95} = 160 \mu\text{m}$ ,  $\rho_s = 2701 \text{ kg/m}^3$ )

C Volumetric concentration	$\eta$ coefficient of rigidity [mPas]	$\tau_0$ Yield Stress [Pa]	Flow behavior
0.1	9	0.4	Bingham plastic
0.22	12	0.6	Bingham plastic
0.3	44	7.5	Bingham plastic
0.34	190	21	Bingham plastic
0.37	230	31	Bingham plastic

Table 10. Flow properties of Eren Pow. Plant – Turkey, Fly Ash: ( $x_{95} = 160 \mu\text{m}$ ,  $\rho_s = 2148 \text{ kg/m}^3$ )

C Volumetric concentration	$\mu$ absolute viscosity [mPas]	Flow behavior
0.1	8.5	Newtonian
0.15	22	Newtonian
0.2	20	Newtonian
0.22	26	Newtonian

C Volumetric concentration	$\eta$ coefficient of rigidity [mPas]	$\tau_0$ Yield Stress [Pa]	Flow behavior
0.28	70	6	Bingham plastic
0.3	130	10	Bingham plastic
0.34	125	16	Bingham plastic
0.35	220	18	Bingham plastic

Mono type screw pump able to produce a high pressure head and able to transport paste like suspensions over the test loop. Pressure transducers were changed as well to be able to measure extreme pressures in the system. The test material was considered to be paste by a standard Slump Cone Test (Phasias *et al.* 1996; Gombkötő, Faitli 2008), over 48% of volumetric concentration.

Tests were carried out for perlite – water mixtures as well (Table 12). It was determined that perlite suspensions remain Newtonian fluids up to 40% of volumetric concentration.

The flow behavior of all of the previously presented granular materials was time independent. This means that during the tube viscometer tests, the pressure loss of a given concentration fine suspension flow, at a set constant flow rate, remained the same for over an hour, while the suspension was circulated in the closed loop.

The following two samples showed rheopectic behavior. These tests were carried out in the rotational viscometer, where the rheology of the suspensions was determined during the first minute of the test by changing the shear rate. The initial results of the rheology tests are presented. The time dependency of resistance against shear was analyzed using the same revolution for the rotor of the viscometer over a long time interval. The torque of the rotor became two – three times higher than the initial value over a few minutes, and thus the apparent viscosity become higher as well. Fly ash originating from the hazardous waste treatment facility in Dorog (Table 13) behaved as Bingham plastic over 10% of volumetric concentration and it was a time dependent rheopectic fluid.

Table 11. Flow properties of base metal tailing, GyöngyöSOROSZI: ( $x_{95} = 50 \mu\text{m}$ ,  $\rho_s = 2701 \text{ kg/m}^3$ )

C Volumetric concentration	$\eta$ coefficient of rigidity [mPas]	$\tau_0$ Yield Stress [Pa]	Flow behavior
0.18	30	26	Bingham plastic
0.26	60	32	Bingham plastic
0.4	95	122	Bingham plastic
0.485	285	308	Bingham plastic
0.525	490	638	Bingham plastic

Table 12. Flow properties of perlite – water mixtures

C Volumetric concentration	$\mu$ absolute viscosity [mPas]	Flow behaviour
0.1	8	Newtonian
0.2	15	Newtonian
0.3	30	Newtonian
0.4	59	Newtonian

The other sample that had rheopectic behavior was red mud from an Indian bauxite operation (Table 14). Even though the Dorog fly ash was Bingham plastic, Indian red mud was determined to be Newtonian over a short time period; however its shear resistance was increased over time as well.

Table 13. Flow properties of fly ash, Dorog hazardous waste treatment facility ( $x_{80} = 60 \mu\text{m}$ ,  $\rho_s = 2429 \text{ kg/m}^3$ ), 1<sup>st</sup> minute

C Volumetric concentration	$\eta$ coefficient of rigidity [mPas]	$\tau_0$ Yield Stress [Pa]	Flow behaviour
0.1	3.8	0.21	Bingham plastic
0.2	6.5	0.46	Bingham plastic
0.3	150	7.5	Bingham plastic

Table 14. Flow properties of red Mud, India: ( $x_{80} = 1250 \mu\text{m}$ ,  $\rho_s = 3680 \text{ kg/m}^3$ ), 1<sup>st</sup> minute

C Volumetric concentration	$\mu$ absolute viscosity [mPa]	Flow behaviour
0.35	5	Newtonian
0.4	13	Newtonian
0.45	40	Newtonian
0.5	63	Newtonian
0.55	145	Newtonian

## 2. Results and discussion

It can be seen that in the cases of the presented solid – liquid fine suspensions, viscosity is slightly increasing in a nearly linear way over concentrations from  $C = 0$  (clear water) of  $\mu_0 = 1 \text{ mPas}$ . Generally, these suspensions become non-Newtonian fluids over 20% of volumetric concentration and the viscosity like parameter (absolute viscosity – Newtonian, coefficient of rigidity – Bingham plastic, consistency index – Pseudoplastic) gets exponentially higher. The highest tested concentration was 55% by volume. Parameters of rheology can be calculated by function fitting over the measured domain. Viscous flow of suspensions is possible up to the theoretical maximum concentration  $C_{\text{max}}$  – solid particles are placed with the greatest amount of space filled in. Experimental determination of  $C_{\text{max}}$  is difficult. In the case of fly ash tests, samples were put into settling columns – Batch settling test – in order to determine the final concentration the settled solid can reach over time. It was experienced, that higher final concentration was achieved using higher initial concentration.

The following well known relation between concentration and viscosity is taken from the literature:

$$\mu = \mu_0 \left[ 1 + \frac{2.5C}{2 \left( 1 - \frac{C}{C_{\text{max}}} \right)} \right]. \quad (1)$$

The Einstein equation can be recognized in it,  $\mu = \mu_0(1+2.5C)$  which is valid only at low concentrations ( $C < 2\%$  by vol.), and viscosity becomes infinite at  $C_{max}$ . However, this equation does not describe our experiences! At low concentrations viscosity slightly increases almost linearly, but at higher concentrations it starts to increase exponentially. To better describe the phenomenon, the following equation was introduced. It consists of a linear and an exponential part.

$$\eta = \mu_0(1 + K_1C + K_2C^{K_3}), \quad (2)$$

where  $\mu_0$  – viscosity of the carrier liquid.

Viscosity should be written on the left side of the equation in the case of perlite – water mixtures, because they are Newtonian fluids at even high concentrations too. The coefficient of rigidity ( $\eta$ ) and the consistency index ( $K$ ) should be involved in case of Bingham plastic – fly ashes – and pseudoplastic – sands – fluids.

Most of the tested materials behaved as Bingham – plastic fluids. For yield stress the following equation can be used:

$$\tau_0 [Pa] = K_4 e^{K_5 C}. \quad (3)$$

Equation 3 is not a physical relation. If the concentration is substituted as a non-dimensional value, the yield stress can be calculated in Pa units. The rheological measurements should be carried out at some discrete concentrations of a given material, as was shown earlier. Then, the  $K_1$ – $K_5$  constants in Equations 2 and 3 can be determined by curve fitting, and rheological parameters can be calculated for any concentration – within the measured range. Figure 4 summarizes the measured rheological parameters and the fitted curves (Eqs 2 and 3) of all the presented tests.

The constants for each tested material can be seen in Table 15. Using these constants and equations 2 and 3,

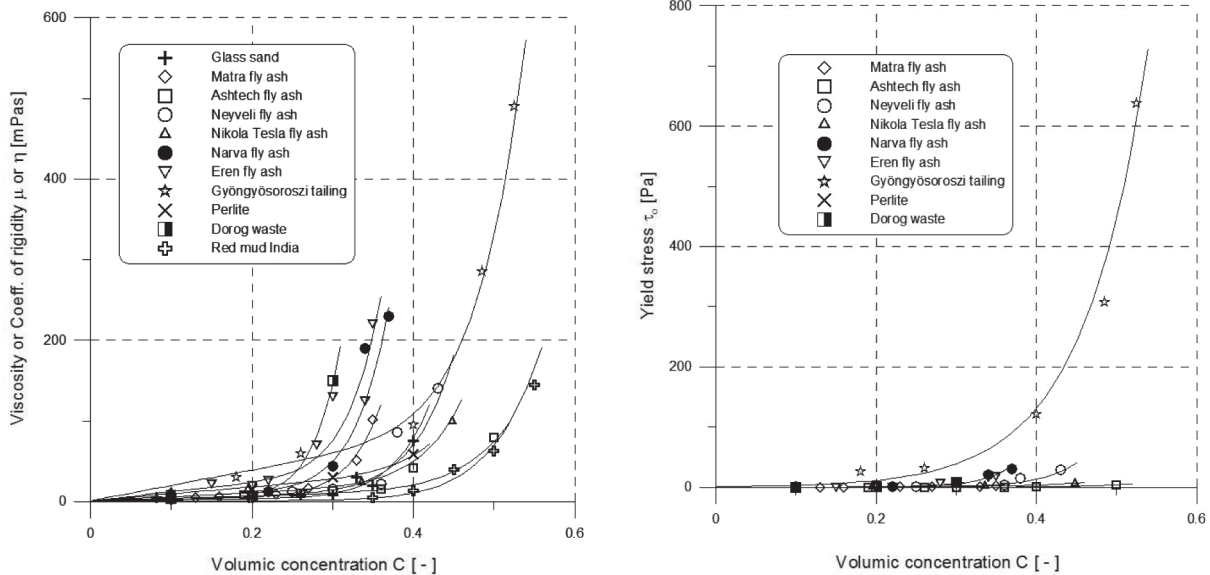


Fig. 4. Corresponding measurement results (symbols) and calculated functions (continuous lines) according to Eqs 2 and 3

Table 15. Collected constants comparison of all the tested material

Material	Measured max. Conc.	$K_1$	$K_2$	$K_3$	R-squared	$K_4$	$K_5$	R-squared
Fehérvárcsurgó, sand glass	0.4	12	280,000	9	0.988	–	–	–
Máttra, fly ash	0.35	20	1,100,000	9	0.986	0.002	21	0.866
Ashtech, fly ash	0.5	30	30,000	9	0.930	0.0002	19.8	0.980
Neyveli, fly ash	0.43	30	220,000	9	0.917	0.002	22	0.955
Nikola, fly ash	0.45	50	110,000	9	0.999	0.04	11.4	0.991
Narva, fly ash	0.37	50	1,700,000	9	0.903	0.002	26.5	0.951
Eren, fly ash	0.35	110	2,100,000	9	0.875	0.002	26.5	0.925
Gyöngyösoroszi, tailing	0.53	190	120,000	9	0.998	1	12.2	0.979
Perlite	0.4	85	85,000	9	0.995	–	–	–
Dorog, fly ash (1st minute)	0.3	20	7,000,000	9	0.999	0.009	22	0.996
Red mud (1st minute)	0.55	1	35,000	9	0.989	–	–	–

rheological parameters can be calculated for any concentration. This is a simple engineering method, but it is also a very powerful method to design industrial sized applications. In Table 15 the coefficient of determination – R-squared – values of curve fittings are also shown, and they indicate good fit. The underground backfilling possibilities of the abandoned base metal mine near Gyöngyöösorosi were also investigated. For this purpose mixed solids were chosen to be used. The tested mixtures were made of Máttra fly ash ( $x_{57} = 160 \mu\text{m}$ ,  $\rho_s = 2355 \text{ kg/m}^3$ ), sand ( $x_{57} = 160 \mu\text{m}$ ,  $\rho_s = 2720 \text{ kg/m}^3$ ), and Mátraszele bentonite ( $x_{41} = 160 \mu\text{m}$ ,  $\rho_s = 2574 \text{ kg/m}^3$ ). For these materials, detailed flow parameters are not presented, but the  $K_1 - K_5$  parameters can be found in Table 16. The fine suspensions of these mixed materials (bentonite was added to fly ash and sand to alter the hydraulic conductivity of the backfill material) were Bingham plastic fluids, even sand – bentonite mixtures too.

Table 16.  $K_1 - K_5$  Constants of Máttra fly ash ( $x_{57} = 160 \mu\text{m}$ ,  $\rho_s = 2355 \text{ kg/m}^3$ ), sand: ( $x_{57} = 160 \mu\text{m}$ ,  $\rho_s = 2720 \text{ kg/m}^3$ ), and bentonite (Mátraszele): ( $x_{41} = 160 \mu\text{m}$ ,  $\rho_s = 2574 \text{ kg/m}^3$ )

Material	Measured					
	max. Conc.	$K_1$	$K_2$	$K_3$	$K_4$	$K_5$
fly ash – bentonite (5%) – water	0.35	30	19,500	5	0.01	26
fly ash – bentonite (10%) – water	0.35	30	39,000	5	0.01	26
sand – bentonite (5%) – water	0.35	50	14,000	5	0.01	26
sand – bentonite (10%) – water	0.35	50	17,000	5	0.01	26

## Conclusions

Systematic slurry testing has been done in the last decades on the basis of the Tarján–Faitli fine suspension – coarse mixture flows model. This paper summarizes the results of the rheological testing of the separated fine fractions.

The empirical equations of Equation 2 and 3 were introduced based on the presented results of many different particulate materials from many places of the world. These equations better characterize the presented test results than the well-known Equation 1.

The rheological parameters of any feasible concentration and any kind of solid/liquid mixtures can be determined by using these equations. However, these equations are empirical ones and the  $K_1 - K_5$  parameters in them should be determined by curve fitting. They are really powerful for the engineering design of slurry handling, and hydraulic transport systems. This knowledge and practice are also important for fields where environmental awareness is required.

The described method, using the Tarján–Faitli model to determine pressure loss of hydraulic transport pipelines using measured rheology of given concentration fine suspensions is a practical approach. The application of this method necessitates a concentration – viscosity function. The most important advantage of the introduced method against theoretical concentration – viscosity equations in the literature is that equation 2 aptly describes the first, almost linear, part (0 – ~20%) and also the second exponential part (~20% –  $C_{\text{max}}$ ).

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