



MODELLING BUS DELAY AT BUS STOP

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Abstract. A bus may be blocked from entering and exiting a stop by other buses and traffic lights. The objective of this paper is to model each type of delay under these phenomena and the overall delay a bus experiences at a stop. Occupy-based delay, transfer block-based delay and block-based delay are defined and modelled. Bus delay at stop is just the sum of these three types of delay. Bus arrival rate, bus service rate, berth number and traffic lights are taken into consideration when modelling delay. Occupy-based delay is modelled with mean waiting time in Queueing theory. Transfer block-based delay and block-based delay are modelled based on standard deviation of waiting time and the probabilities of their occurrences. Two stops in Vancouver, Canada are selected for parameter estimation and model validation. The unknown parameter is estimated as 0.4230 using Ordinary Least Squares (OLS), which indicates that 42.3% of waiting time variation can be attributed to buses being blocked by the buses in front and red light for the selected stops. Model validation shows the average accuracy rate of the proposed model is 75.07% for the selected stops. Bus delay at stop evidently increases when arrival rate is more than 85 buses per hour for the given service time (50 s), ratio of red time to cycle length (0.65) and berth number (2). We also figure out that bus delay at stop evidently increases when service time is more than 60 s for the given arrival rate (54 buses per hour), ratio of red time to cycle length (0.65) and berth number (2). The proposed model can provide a tool for bus stop design and offer the foundation for service quality evaluation of transit.

Keywords: bus delay; bus stop; queueing theory; variation of waiting time; probability of block.

Introduction

A bus may be temporarily blocked from entering and exiting a stop by other buses and traffic lights. The objective of this paper is to model each type of delay under these phenomena and the overall delay a bus experiences on average. The proposed model can provide a tool for bus stop design and the foundation for service quality evaluation of public transit (Furth, Rahbee 2000).

Bus stops are important parts in public bus network, whose operational situations influence transportation efficiency and service quality of the whole network to a great extent. Bus delay at stop affects running speed of buses, capacity of bus network and operation cost of bus companies (TRB 2003; Koshy, Arasan 2005; Nagatani 2001). It also affects bus stop design such as stop size and route number.

However, there is no general agreement for the definition of bus delay at stop. It is defined as the dif-

ference between the time a bus actually spends at bus stop and the time a bus is expected to spend there. The expected time is the sum of the typical bus deceleration and acceleration time and the typical time it takes for passengers to get on and off the bus (Xu *et al.* 2009, Zhou *et al.* 2008; Han *et al.* 2004). A similar definition states that bus delay at stop includes several components that can be summarized as time lost while a bus stops (for doors to open and close, for passengers to board and alight) and time lost during deceleration and acceleration (Furth, SanClemente 2006). These two definitions both compare the actual condition with the expected condition. Buses that travel on busy routes can be temporarily blocked from entering or exiting a stop by other buses that are serving passengers there. Thus, bus delay at stop is defined as the sum of two average delays in entry queue and in the berth due to the block from other buses (Gu *et al.* 2014). The limitation of this definition is that it does not include the impact of traffic lights.



Categories of bus delay at stop are also diverse. Researchers often categorize it according to the definitions they give. In general, bus delay at stop is classified as:

- service and non-service delay;
- single, twice and triple delay;
- fixed and non-fixed delay;
- delay inside and outside stop.

Bus delay at stop is affected by stop characteristics, bus characteristics, traffic conditions, passenger volumes and signalization parameters. Stop characteristics include berth number, stop space, stop location and road geometric condition. Bus characteristics include bus size, door number, accelerating and decelerating ability. Traffic conditions include bus arrival rate, service time, and stop capacity (Chen 1999; Xu 2001; Kuo 1999; Zhou, Chen 2002; Ma *et al.* 2006). The effects of some factors are overlapped such as service time and passenger volume. Researchers often consider some but not all of these factors when studying bus delay at stop.

Xu *et al.* (2009) propose a delay model taking bus arrival occupancy rate as an independent variable using regression analysis. Bus arrival occupancy rate is the ratio of bus arrival rate to berth number. When the bus arrival occupancy rate is less than 20 per hour, delay is quite small. However, when it increases to more than 30 per hour, delay increases significantly. Shi *et al.* (2011) analyse the effects of bus arrival rate and berth number on bus delay at stop using cellular automation model. They find out that increasing berth number cannot evidently improve operation situation at stop, but instead, it will increase delay inside the stop. Furth and SanClemente (2006) examine the impacts of grade profiles and signalization parameters on bus delay at stop using kinematic models of vehicle movements. The marginal impact of grade on bus delay at stop ranges from -4 to 11 s depending on grade size. Far-side stop is always a safe option carrying essentially zero net delay. Near-side stop can reduce delay in a few cases such as exclusive bus lanes, but more often it increases delay, sometimes considerably depending on factors such as red light ratio, ratio of volume to capacity, cycle length and stop setback. Different from Furth and SanClemente's study, Gu *et al.* (2014) isolate the impact of traffic lights and develop average bus delay model using a Markov chain embedded in the bus queueing process. This model accounts for the impacts of berth number, the coefficient of variation in service time and the ratio of bus inflow to the supremum of the bus discharge flow. This model can be used to determine the maximum bus flows while still maintaining target service levels and determine suitable berth number to achieve a specified service level. Bus delay at stop is also affected by other modes in mixed traffic flow (Lu *et al.* 2010). Authors present a model to simulate the conflicts among bus, car and bicycle in mixed traffic flow using cellular automation model. They find out that bus delay at stop increases with the increase of bicycle flow and its growth rate is determined by the car flow.

Bus arrival rate and berth number are key factors when studying bus delay at stop and they are considered in this paper. Besides, bus service rate and traffic lights are considered as well in this paper. Three types of delay are defined according to various scenarios they are generated and each type is modelled respectively. Queueing theory, some theorems for random variables and some formulas in power series are used to model delay. Parameter estimation and model validation are conducted with data collected from two stops in downtown Vancouver, Canada. The impacts of arrival rate, service time and berth number on bus delay at stop are examined using the proposed model.

1. Model Developed

1.1. The Definition and Category of Bus Delay at Stop

Near-side stop is a transit stop located on the approach side of an intersection. The buses stop there to serve passengers before crossing the intersection (Highway Capacity Manual 2000). For nearside stop, a bus may be temporarily blocked from entering and exiting a stop by other buses and traffic lights. When this happens, the corresponding stopping process is:

- decelerating;
- staying in queue to wait for entry;
- serving passengers;
- waiting for departure of the front bus and green light;
- accelerating.

Bus delay at stop is generated while waiting for entry, departure of the front bus and green light. We categorize three scenarios below to describe bus delay at stop. We assume that bus overtaking manoeuvres are prohibited.

Scenario A: When a bus arrives at a stop and intends to enter, all the berths are occupied by buses in service. Thus, it has to stay in queue and wait for entry. Waiting time is generated as a result. We define the waiting time under this scenario as occupy-based delay D_0 .

Scenario B: Buses have already finished passenger serving and intended to exit a stop. However, they are blocked by the buses in front and red light at downstream intersection so that they still occupy the berths, which leads the bus in queue to continue queuing. As a consequence, extra waiting time is imposed on the bus in queue. We define the extra waiting time under this scenario as transfer block-based delay D_t .

Scenario C: A bus has already finished passenger serving and intended to exit a stop. However, it is blocked by the front bus and red light so that it has to wait for departure of the front bus and green light. As a consequence, extra waiting time is imposed on the blocked bus itself. We define the extra waiting time under this scenario as block-based delay D_b .

Bus delay at stop D is the average waiting time a bus experiences at a stop, which is resulted from waiting for entry, departure of the front bus and green light. It is the sum of occupy-based delay, transfer block-based delay and block-based delay.

1.2. Mean Waiting Time in Queueing Theory and Occupy-Based Delay

Queueing theory is the mathematical study of queues, models in which are constructed to predict queue length and waiting time (Sundarapandian 2009). Important applications of queueing models are in production systems, transportation systems, stocking systems, communication systems and information processing systems (Adan, Resing 2002; Menasce *et al.* 2004). The basic queueing process is shown in Fig. 1. A queueing model is characterized by arrival process of customers, service time, server number, behaviour of customers, the capacity of the system and service discipline (Medhi 2002).

We use $M/M/s/\infty/\infty/FCFS$ (Fig. 2), short for $M/M/s$, to model bus queue phenomena at bus stops. For $M/M/s$ model, customers arrive according to a Poisson stream i.e. exponential inter-arrival time, the service time is exponentially distributed, there are s parallel identical servers, customers are patient and willing to wait, the limitations with respect to the number of customers are infinite and first come first served (Adan, Resing 2002). In terms of bus stop, customer refers to bus, server refers to berth and service refers to passenger serving. We assume bus arrivals follow Poisson distribution and service time follows Exponential distribution.

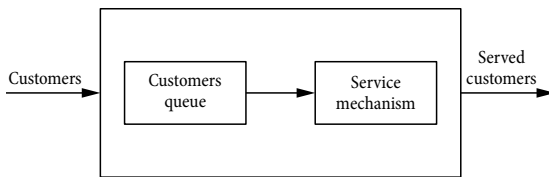


Fig. 1. The basic queueing process

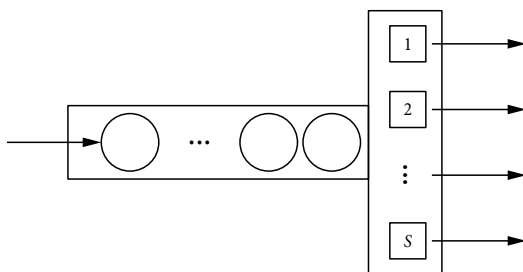


Fig. 2. $M/M/s/\infty/\infty/FCFS$ system

The notations used throughout this paper are given as follows:

- λ – denotes mean arrival rate, i.e. the mean of the number of arrival buses per unit time;
- μ – denotes mean service rate, i.e. service capacity of each berth at stop;
- ρ – denotes utilization factor, $\rho = \lambda/\mu$, ($\rho < 1$);
- n – denotes the number of buses including in service and in queue at stop;
- P_n – denotes equilibrium probabilities, i.e. the probabilities that there are n buses at stop;
- L_q – denotes the number of buses in queue, i.e. queue length;

- $E(q)$ – denotes mean queue length;
- W_q – denotes waiting time of buses in queue;
- $E(W_q)$ – denotes mean waiting time of buses in queue.

Queueing models focus on equilibrium behaviour, a kind of behaviour in which state probabilities are time-independent.

1.2.1. Mean Waiting Time for Single Berth

The flow diagram for $M/M/1$ ($s=1$) model is shown in Fig. 3. Global balance principle states that for each set of states n , the flow out of set n is equal to the flow into that set. According to this principle, we can derive expressions for P_n , $E(L_q)$ and $E(W_q)$ (Adan, Resing 2002; Wang 1990).

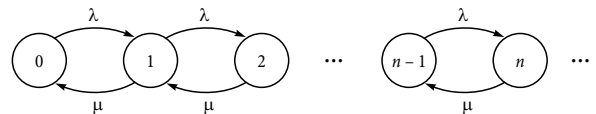


Fig. 3. The flow diagram for the $M/M/1$ model:

- State 0: $\lambda P_0 = \mu P_1$, thus, $P_1 = \rho P_0$;
- State 1: $\lambda P_1 + \mu P_1 = \lambda P_0 + \mu P_2$, thus, $P_2 = \rho^2 P_0$;
- ...
- State n : $\lambda P_n + \mu P_n = \lambda P_{n-1} + \mu P_{n+1}$, thus, $P_n = \rho^n P_0$;
- ...

The equilibrium probabilities P_n satisfy $\sum_{n=0}^{\infty} P_n = 1$ (normalization equation), thus, yield $P_0 = 1 - \rho$ and $P_n = (1 - \rho)\rho^n$.

$E(L_q)$ is the expected number of buses in queue, therefore, $E(L_q) = E(n-1) = \sum_{n=1}^{\infty} (n-1)P_n = \frac{\rho^2}{1-\rho}$.

By applying Little's law $W_q = \frac{L_q}{\lambda}$, yield:

$$E(W_q) = \frac{\rho/\mu}{1-\rho} \tag{1}$$

1.2.2. Mean Waiting Time for Multiple Berths

The flow diagram for $M/M/s$ model is shown in Fig. 4. Let $\rho_s = \lambda/(s\mu)$ ($\rho_s < 1$). Expressions for P_n , $E(L_q)$ and $E(W_q)$ can be derived according to the global balance principle (Adan, Resing 2002; Wang 1990).

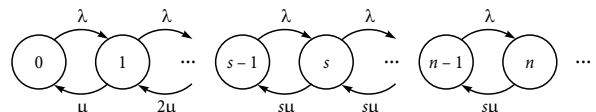


Fig. 4. The flow diagram for the $M/M/s$ model:

- State 0: $\lambda P_0 = \mu P_1$;
- State 1: $\lambda P_1 + \mu P_1 = \lambda P_0 + 2\mu P_2$;
- ...
- State s : $\lambda P_s + s\mu P_s = \lambda P_{s-1} + s\mu P_{s+1}$;
- State $s+1$: $\lambda P_{s+1} + s\mu P_{s+1} = \lambda P_s + s\mu P_{s+2}$;
- ...

Iterating gives:

$$P_n = \begin{cases} \frac{\rho^n P_0}{n!}, & n=0,1,\dots,s; \\ \frac{\rho^n P_0}{s^{n-s} s!}, & n=s+1,s+2,\dots,\infty. \end{cases} \quad (2)$$

The probability P_0 follows from normalization equation, yielding:

$$P_0 = \left(\sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s}{s!(1-\rho_s)} \right)^{-1}. \quad (3)$$

From the equilibrium probabilities in Equation (2), we obtain:

$$E(L_q) = E(n-s) = \sum_{n=s}^{\infty} (n-s)P_n = \frac{P_0 \rho^s \rho_s}{s!(1-\rho_s)^2}. \quad (4)$$

By applying Little's law, we obtain:

$$E(W_q) = \frac{P_0 \rho^s \rho_s}{\lambda s!(1-\rho_s)^2}. \quad (5)$$

1.2.3. Occupy-Based Delay

For $M/M/s$ model in Queueing theory, customers need to stay in queue if all the servers are occupied when they arrive at the system and customers can immediately depart when they finish service. Queue only occurs when all the servers are occupied (clearly by customers in service), which fits *Scenario A* (a bus needs to stay in queue because all the berths are occupied by buses in service). Therefore, we can estimate occupy-based delay with mean waiting time of $M/M/s$ model in Queueing theory. With Equation (1) and Equation (5), occupy-based delay is expressed as:

$$D_0 = \begin{cases} \frac{P_0 \rho^s \rho_s}{\lambda s!(1-\rho_s)^2} & \text{for } M/M/s; \\ \frac{\rho/\mu}{1-\rho} & \text{for } M/M/1. \end{cases} \quad (6)$$

1.3. Variation of Waiting Time and Transfer Block-Based Delay

In order to model transfer block-based delay, let's discuss standard deviation of waiting time in Queueing theory. Queue length L_q and waiting time W_q are random variables in Queueing theory. Let $\sigma(L_q)$ and $\sigma(W_q)$ denote standard deviations of queue length and waiting time respectively. The variations of queue length and waiting time partly result from the variation of service time. Buses have already finished passenger serving, but are blocked from exiting a stop by the buses in front and red light so that they still occupy berths. This phenomenon increases the service time of these occupied berths. In other words, it induces the variation of service time and further gives rise to the variations of queue length and

waiting time. This phenomenon is one of the reasons causing the variations of queue length and waiting time.

Let θ denote that the fraction of waiting time variation resulting from the phenomenon (buses are blocked from exiting a stop by the buses in front and red light). θ is an unknown parameter and needs to be estimated with field data. Then, the extra waiting time imposed on the bus in queue under *Scenario B* is equal to $\theta\sigma(W_q)$ if *Scenario B* occurs. Therefore, transfer block-based delay is equal to the product of $\theta\sigma(W_q)$ and the probability of *Scenario B* occurrence P_b .

1.3.1. Standard Deviation of Waiting Time

We deduce standard deviation of waiting time for multiple berths as below. According to the definition of variance of random variable, variance of queue length is (Jin 2000):

$$\text{var}(L_q) = E(L_q^2) - E(L_q)^2. \quad (7)$$

For a discrete random variable X , $P\{X=x_i\}=P_i$,

$i=1,2,\dots,\infty$ and $Y=f(X)$, then $E(Y)=\sum_{i=1}^{\infty} f(x_i)P_i$ (Jin 2000). Apply this theorem to L_q , yielding

$$E(L_q^2) = \sum_{n=s+1}^{\infty} (n-s)^2 P_n.$$

With Equation (2), we have:

$$\begin{aligned} E(L_q^2) &= \sum_{n=s+1}^{\infty} (n-s)^2 \frac{\rho^n P_0}{s^{n-s} s!} = \\ &= \frac{P_0}{s!} \sum_{n=s+1}^{\infty} (n-s)^2 \frac{\rho^{n-s} \rho^s}{s^{n-s}} = \\ &= \frac{P_0 \rho^s}{s!} \sum_{n=s+1}^{\infty} (n-s)^2 \rho_s^{n-s} = \\ &= \frac{P_0 \rho^s}{s!} \sum_{n=1}^{\infty} n^2 \rho_s^n = \\ &= \frac{P_0 \rho^s \rho_s}{s!} \sum_{n=1}^{\infty} n^2 \rho_s^{n-1}. \end{aligned} \quad (8)$$

Clearly:

$$\begin{aligned} \sum_{n=1}^{\infty} n^2 \rho_s^{n-1} &= \\ \sum_{n=1}^{\infty} n(n+1) \rho_s^{n-1} - \sum_{n=1}^{\infty} n \rho_s^{n-1}. \end{aligned} \quad (9)$$

According to the formulas in Power series, $\sum_{n=1}^{\infty} n(n+1) \rho_s^{n-1}$ and $\sum_{n=1}^{\infty} n \rho_s^{n-1}$ are expressed as (Xu 2004):

$$\begin{aligned} \sum_{n=1}^{\infty} n(n+1) \rho_s^{n-1} &= \sum_{n=1}^{\infty} (\rho_s^{n+1})'' = \\ \left(\frac{\rho_s^2}{1-\rho_s} \right)'' &= \frac{2}{(1-\rho_s)^3}; \end{aligned} \quad (10)$$

$$\sum_{n=1}^{\infty} n \rho_s^{n-1} = \sum_{n=1}^{\infty} (\rho_s^n)' = \left(\frac{\rho_s}{1-\rho_s} \right)' = \frac{1}{(1-\rho_s)^2}. \quad (11)$$

With Equation (9), Equation (10) and Equation (11), we have:

$$\sum_{n=1}^{\infty} n^2 \rho_s^{n-1} = \frac{1+\rho_s}{(1-\rho_s)^3}. \quad (12)$$

With Equation (8) and Equation (12), $E(L_q^2)$ is expressed as:

$$E(L_q^2) = \frac{P_0 \rho^s \rho_s (1+\rho_s)}{s!(1-\rho_s)^3}. \quad (13)$$

With Equation (4), Equation (7) and Equation (13), variance of queue length $\text{var}(L_q)$ is obtained and it is:

$$\text{var}(L_q) = \frac{P_0 \rho^s \rho_s (1+\rho_s)}{s!(1-\rho_s)^3} - \left(\frac{P_0 \rho^s \rho_s}{s!(1-\rho_s)^2} \right)^2.$$

Thus, standard deviation of queue length $\sigma(L_q)$ is:

$$\sigma(L_q) = \sqrt{\frac{P_0 \rho^s \rho_s (1+\rho_s)}{s!(1-\rho_s)^3} - \left(\frac{P_0 \rho^s \rho_s}{s!(1-\rho_s)^2} \right)^2}.$$

Little's law can be extended as:

$$\sigma(W_q) = \frac{\sigma(L_q)}{\lambda}.$$

Using the extending form of Little's law, we can gain standard deviation of waiting time $\sigma(W_q)$ for multiple berths:

$$\sigma(W_q) = \frac{1}{\lambda} \sqrt{\frac{P_0 \rho^s \rho_s (1+\rho_s)}{s!(1-\rho_s)^3} - \left(\frac{P_0 \rho^s \rho_s}{s!(1-\rho_s)^2} \right)^2}.$$

We derive standard deviation of waiting time for single berth following the same way as multiple berths, the result is:

$$\sigma(W_q) = \frac{1}{\lambda} \sqrt{\frac{\rho^2 - 2\rho^4 + \rho^5}{(1-\rho)^3}}.$$

Therefore, standard deviation of waiting time is expressed as:

$$\sigma(W_q) = \begin{cases} \frac{1}{\lambda} \sqrt{\frac{P_0 \rho^s \rho_s (1+\rho_s)}{s!(1-\rho_s)^3} - \left(\frac{P_0 \rho^s \rho_s}{s!(1-\rho_s)^2} \right)^2} & \text{for } M/M/s; \\ \frac{1}{\lambda} \sqrt{\frac{\rho^2 - 2\rho^4 + \rho^5}{(1-\rho)^3}} & \text{for } M/M/1. \end{cases} \quad (14)$$

1.3.2. Probability of Scenario B Occurrence

The process of passenger serving at bus stop is shown in Fig. 5. Assume the probability that the bus on each berth ($i = 1, 2, \dots, s$) first finishes passenger serving is identical when bus number at stop is more than s . This probability is equal to $1/s$.

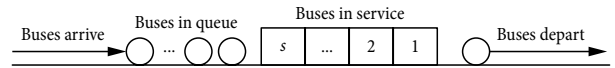


Fig. 5. The process of passenger serving at bus stop

If the bus occupying the i th ($i = 2, 3, \dots, s$) berth first finishes passenger serving, it would be blocked by the bus occupying the $(i - 1)$ th berth, which would lead the bus in queue to continue queuing. The probability of this phenomenon is equal to the sum of simultaneous probabilities that bus number is more than s and the bus occupying the i th ($i = 2, 3, \dots, s$) berth first finishes passenger serving, therefore it is $P_{(n>s)} \frac{(s-1)}{s}$. Note that this phenomenon only exists for multiple berths.

Buses have already finished passenger serving, but they are blocked by red light at downstream intersection and still occupy the berths, which also leads the bus in queue to continue queuing. The probability of this phenomenon is equal to $P_{(n>s)} \frac{t_r}{C}$. t_r is red time and C is cycle length.

The probability of Scenario B occurrence is the sum of these two probabilities, therefore, it is:

$$P_b = \begin{cases} P_{(n>s)} \left(\frac{s-1}{s} + \frac{t_r}{C} \right) & \text{for } M/M/s; \\ P_{(n>1)} \frac{t_r}{C} & \text{for } M/M/1. \end{cases} \quad (15)$$

1.3.3. Transfer Block-Based Delay

Recall that transfer block-based delay is equal to the product of $\theta \sigma(W_q)$ and the probability of Scenario B occurrence P_b , so with Equation (14) and Equation (15), transfer block-based delay is expressed as:

$$D_t = \begin{cases} D_{t1} & \text{for } M/M/s; \\ D_{t2} & \text{for } M/M/1, \end{cases} \quad (16)$$

where:

$$D_{t1} = \frac{\theta P_{(n>s)} \left(\frac{s-1}{s} + \frac{t_r}{C} \right)}{\lambda} \times \sqrt{\frac{P_0 \rho^s \rho_s (1+\rho_s)}{s!(1-\rho_s)^3} - \left(\frac{P_0 \rho^s \rho_s}{s!(1-\rho_s)^2} \right)^2};$$

$$D_{t2} = \frac{\theta P_{(n>1)} t_r}{C \lambda} \sqrt{\frac{\rho^2 - 2\rho^4 + \rho^5}{(1-\rho)^3}}.$$

1.4. Variation of Waiting Time and Block-Based Delay

A bus may be blocked by the front bus and red light when it has already finished passenger serving and intended to exit a stop. This phenomenon imposes extra waiting time on the bus in queue and imposes extra waiting time on the blocked bus itself as well. *Scenario B* pays attention to the extra waiting time imposed on the bus in queue and *Scenario C* pays attention to the extra waiting time imposed on the blocked bus itself. Since the extra waiting time imposed on the bus in queue is equal to $\theta\sigma(W_q)$ for *Scenario B*, the extra waiting time imposed on the blocked bus itself is also equal to $\theta\sigma(W_q)$ for *Scenario C*. Therefore, block-based delay is equal to the product of $\theta\sigma(W_q)$ and the probability of *Scenario C* occurrence P_c .

A bus may be blocked by the front bus and wait for its departure when exiting a stop. This phenomenon does not happen if and only if the buses in service finished passenger serving in sequence i.e. first arrives first finishes. The probability that first arrives first finishes is $\frac{1}{n!}$ (n is between 2 and s), and is $\frac{1}{s!}$ ($n > s$). Therefore, the probability of this phenomenon is equal to:

$$P_2 \left(1 - \frac{1}{2!} \right) + P_3 \left(1 - \frac{1}{3!} \right) + \dots + P_s \left(1 - \frac{1}{s!} \right) + P_{(n>s)} \left(1 - \frac{1}{s!} \right),$$

i.e. $\sum_{n=2}^s \frac{P_n (n!-1)}{n!} + \frac{P_{(n>s)} (s!-1)}{s!}$.

A bus may also be blocked by red light at downstream intersection and wait for green light when exiting a stop. The probability of this phenomenon is:

$$P_{(n>0)} \frac{t_r}{C}, \text{ i.e. } \frac{(1-P_0)t_r}{C}.$$

The probability of *Scenario C* occurrence is the sum of these two probabilities, therefore, it is:

$$P_c = \begin{cases} \sum_{n=2}^s \frac{P_n (n!-1)}{n!} + \frac{P_{(n>s)} (s!-1)}{s!} + \frac{(1-P_0)t_r}{C} & \text{for } M/M/s; \\ \frac{(1-P_0)t_r}{C} & \text{for } M/M/1. \end{cases} \quad (17)$$

With Equation (14) and Equation (17), block-based delay is expressed as:

$$D_b = \begin{cases} D_{b1} & \text{for } M/M/s; \\ D_{b2} & \text{for } M/M/1, \end{cases} \quad (18)$$

where:

$$D_{b1} = \frac{\theta \left(\sum_{n=2}^s \frac{P_n (n!-1)}{n!} + \frac{P_{(n>s)} (s!-1)}{s!} + \frac{(1-P_0)t_r}{C} \right)}{\lambda} \times \sqrt{\frac{P_0 \rho^s \rho_s (1+\rho_s)}{s!(1-\rho_s)^3} - \left(\frac{P_0 \rho^s \rho_s}{s!(1-\rho_s)^2} \right)^2};$$

$$D_{b2} = \frac{\theta(1-P_0)t_r}{C\lambda} \sqrt{\frac{\rho^2 - 2\rho^4 + \rho^5}{(1-\rho)^3}}.$$

1.5. Bus Delay at Stop

Recall that bus delay at stop is the average waiting time a bus experiences at a stop, which is resulted from waiting for entry, departure of the front bus and green light. It is the sum of occupy-based delay, transfer block-based delay and block-based delay. Therefore, bus delay at stop is obtained with Equation (6), Equation (16) and Equation (18) and is expressed as:

$$D = \begin{cases} D_1 & \text{for } M/M/s; \\ D_2 & \text{for } M/M/1, \end{cases} \quad (19)$$

where:

$$D_1 = \frac{P_0 \rho^s \rho_s}{\lambda s! (1-\rho_s)^2} + \frac{\theta \left(P_{(n>s)} \left(\frac{s-1}{s} + \frac{t_r}{C} \right) + \sum_{n=2}^s \frac{P_n (n!-1)}{n!} + \frac{P_{(n>s)} (s!-1)}{s!} + \frac{(1-P_0)t_r}{C} \right)}{\lambda} \times \sqrt{\frac{P_0 \rho^s \rho_s (1+\rho_s)}{s!(1-\rho_s)^3} - \left(\frac{P_0 \rho^s \rho_s}{s!(1-\rho_s)^2} \right)^2};$$

$$D_2 = \frac{\rho/\mu}{1-\rho} + \frac{\theta (P_{(n>1)} + 1 - P_0) t_r}{\lambda C} \sqrt{\frac{\rho^2 - 2\rho^4 + \rho^5}{(1-\rho)^3}},$$

where: θ is an unknown parameter and indicates that the fraction of waiting time variation resulting from the phenomenon (buses are blocked from exiting a stop by the buses in front and red light); λ is mean arrival rate; μ is mean service rate; s is berth number; t_r is red time; C is cycle length;

$$\rho = \lambda/\mu;$$

$$\rho_s = \lambda/(s\mu);$$

$$P_0 = \begin{cases} \left(\sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s}{s!(1-\rho_s)} \right)^{-1} & \text{for } M/M/s; \\ 1-\rho & \text{for } M/M/1; \end{cases}$$

$$P_{(n>s)} = 1 - \sum_{n=0}^s \frac{\rho^n P_0}{n!}.$$

2. Parameter Estimation and Model Validation

In the proposed model of bus delay at stop, the unknown parameter θ needs to be estimated with field data. This model also needs to be validated with field data. Two curb-side stops, located close to Granville & Georgia intersection and Granville & Pender intersection, in downtown Vancouver, Canada are selected for data collection. The stop located close to Granville & Georgia intersection is called Granville & Georgia stop

in this paper. The stop located close to Granville & Pender intersection is called Granville & Pender stop in this paper. Data collected from Granville & Georgia stop are divided into two parts, among which one part is used for parameter estimation and the other part is used for model validation. Data collected from Granville & Pender stop are only used for parameter estimation. The two selected stops are both near-side stops and the distances to stop lines of downstream intersections are about 7 m. Red time is 42 s and cycle length is 65 s at downstream intersection of Granville & Georgia stop. They are 30 s and 63 s for Granville & Pender stop. There are nine routes at Granville & Georgia stop and six at Granville & Pender stop. In addition, the two selected stops both have two berths.

Field surveys are conducted from 10:00 am to 13:00 pm on 19, 20, 22, 23 October 2012. All the buses arriving at the selected stops are recorded during the surveys. For each bus, the time when it stops outside berth t_1 (only for buses which are blocked from entering), the time when it enters berth t_2 , the time when it finishes passenger serving t_3 , and the time when it exits the stop t_4 are recorded. Average deceleration time, average door opening and closure time, and average acceleration time are observed as well.

Mean arrival rate, mean service rate and bus delay at stop are derived based on the field data described above. *Delay for each bus* [s] = $(t_2 - t_1) + (t_4 - t_3 - \text{average door closure time} - \text{average acceleration time})$. Service time is the time that a bus spends at a stop. Therefore, it is equal to the difference between the time it exits a stop and the time it begins to decelerate. For buses which are blocked from entering, *service time* [s] = $t_4 - t_1 + \text{average deceleration time}$. For buses which directly enter berths,

service time [s] = $t_4 - t_2 + \text{average deceleration time}$. Ten minutes is regarded as one calculation unit in this paper. Mean arrival rate is equal to the number of arrival buses in each unit divided by 600. Average service time is equal to the mean of service time of all the arrival buses in each unit. Mean service rate is the reciprocal of average service time. Bus delay at stop is equal to the mean of delays of all the arrival buses in each unit.

Ordinary Least Squares (OLS) method is used for parameter estimation. OLS method is programmed using *Matlab* software (<http://se.mathworks.com>). 21 data sets from Granville & Georgia stop and Granville & Pender stop are used. And each data set includes mean arrival rate, mean service rate, berth number, red time, cycle length and the corresponding bus delay at stop. Output result of the unknown parameter is 0.4230, which means that the value of 0.4230 yields the least error for the given data sets. It also means that 42.3% of waiting time variation can be attributed to buses being blocked from exiting the survey stops by the buses in front and red light. This matches the actual situations. The ratio of red time to cycle length is 0.65 at downstream intersection of Granville & Georgia stop. Being blocked by red light further induces the block by other buses and causes the bus in queue to continue queuing. Being blocked by red light is the source of bus delay at these two stops.

29 data sets from Granville & Georgia stop are used for model validation. Predicted delay is obtained with Equation (19) taking θ as 0.4230. Mean arrival rate, mean service rate, actual delay, predicted delay, absolute deviation and absolute deviation rate are shown in Table 1. Absolute deviation is equal to absolute value of actual delay minus predicted delay. Absolute deviation rate is equal to absolute deviation divided by actual de-

Table 1. Mean arrival rate, mean service rate, actual delay, predicted delay, absolute deviation and absolute deviation rate for model validation (berth number = 2, red time = 42 s, cycle length = 65 s)

Mean arrival rate [bus/s]	0.0133	0.0117	0.0150	0.0167	0.0133	0.0117	0.0150	0.0117	0.0167	0.0133	0.0150
Mean service rate [bus/s]	0.0167	0.0143	0.0167	0.0152	0.0237	0.0167	0.0166	0.0155	0.0216	0.0165	0.0150
Actual delay [s/bus]	25.50	34.86	23.78	48.34	12.87	26.43	22.89	23.43	15.20	23.00	28.22
Predicted delay [s/bus]	22.19	27.37	29.56	55.14	7.19	16.08	29.75	20.60	15.62	22.98	42.92
Absolute deviation [s/bus]	3.31	7.48	5.78	6.80	5.68	10.35	6.86	2.82	0.42	0.02	14.70
Absolute deviation rate [%]	12.99	21.47	24.32	14.06	44.12	39.17	29.99	12.06	2.77	0.10	52.08
Mean arrival rate [bus/s]	0.0150	0.0167	0.0200	0.0150	0.0183	0.0150	0.0150	0.0150	0.0117	0.0167	0.0167
Mean service rate [bus/s]	0.0186	0.0183	0.0249	0.0210	0.0270	0.0213	0.0221	0.0277	0.0102	0.0206	0.0177
Actual delay [s/bus]	21.22	22.10	13.33	19.56	13.27	18.44	16.56	9.22	63.00	24.10	25.00
Predicted delay [s/bus]	20.14	27.84	14.91	13.50	9.39	13.07	11.47	5.67	92.34	18.51	31.04
Absolute deviation [s/bus]	1.08	5.74	1.57	6.06	3.88	5.38	5.08	3.55	29.34	5.59	6.04
Absolute deviation rate [%]	5.09	26.54	11.80	30.97	29.27	29.15	30.69	38.55	46.57	23.20	24.16
Mean arrival rate [bus/s]	0.0133	0.0133	0.0117	0.0133	0.0167	0.0183	0.0150				
Mean service rate [bus/s]	0.0254	0.0163	0.0174	0.0167	0.0165	0.0217	0.0164				
Actual delay [s/bus]	10.63	22.75	19.33	23.63	25.80	21.36	34.25				
Predicted delay [s/bus]	5.79	23.95	14.10	22.03	40.08	17.66	28.53				
Absolute deviation [s/bus]	4.83	1.20	5.23	1.59	14.28	3.71	5.72	Average 6.00			
Absolute deviation rate [%]	45.46	5.29	27.05	6.74	55.34	17.35	16.69	Average 24.93			

lay. The average absolute deviation is 6.00 s for each bus and average absolute deviation rate is 24.93%. In other words, the average accuracy rate of the proposed model is 75.07% for the given data sets. Field surveys indicate that buses may be blocked by cars when they enter or exit stops, and buses in queue may be blocked from entering spare berths due to the block between buses. These phenomena can cause certain bus delay, but they are not considered in this paper, which leads to errors of the proposed model to some extent.

3. Applications of the Proposed Model

Sometimes when a bus arrives at a stop, all the berths are occupied by buses in service. Sometimes a bus needs to continue queuing because buses which have finished passenger serving are blocked by the buses in front and traffic lights. Sometimes a bus is blocked by the front bus and traffic lights when it exits a stop. The proposed model in this paper can estimate each type of delay under the phenomena described above and the overall delay. When users apply this model, they first need to estimate the unknown parameter based on local data, and then apply it to predict delay.

The proposed model also has other potential applications. The operational situations of bus stops influence service quality of the whole bus network, while this model is the fundamental work for bus service quality evaluation. This model also provides a tool for bus stop design such as determining reasonable numbers of route and berth.

The model is proposed for near-side stop. For far-side stop and midblock stop, the impact of traffic lights on delay can be ignored. Therefore, the model can be applied to far-side and midblock stop after removing the items which include $\frac{t_r}{C}$.

Models in Queueing theory are built for equilibrium behaviour of systems. Therefore, the proposed model is suitable for bus stops where mean arrival rate is less than mean service rate, which is always met.

4. Impact Analysis Using the Proposed Model

Impacts of arrival rate, service time and berth number on bus delay at stop are analysed using the proposed model, as shown in Figs 6, 7 and Table 2.

In Fig. 6, arrival rate varies from 20 buses to 120 buses per hour. Service time, red time, cycle length and berth number are taken as fixed values and they

are 50 s, 42 s, 65 s and 2 s respectively. Occupy-based delay, transfer block-based delay, block-based delay and bus delay at stop rise with the increase of arrival rate. Transfer block-based delay has the lowest value among them. When arrival rate is more than 70 buses per hour, occupy-based delay is higher than block-based delay. However, their difference is negligible when arrival rate is less than 70 buses per hour. When arrival rate is more than 85 buses per hour (shown as the arrow in Fig. 6), bus delay at stop evidently increases.

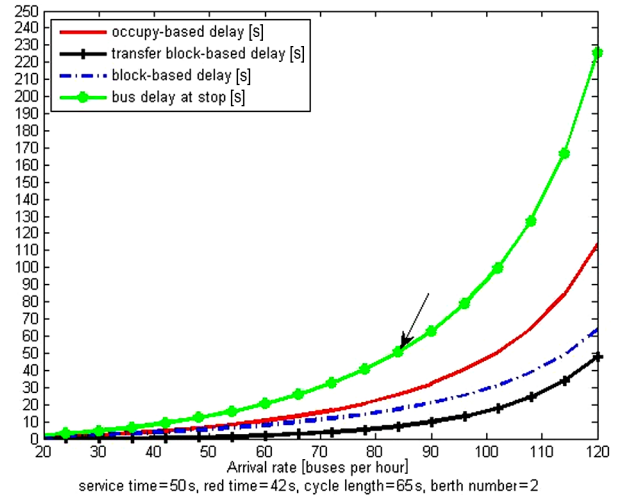


Fig. 6. The impact of arrival rate on bus delay

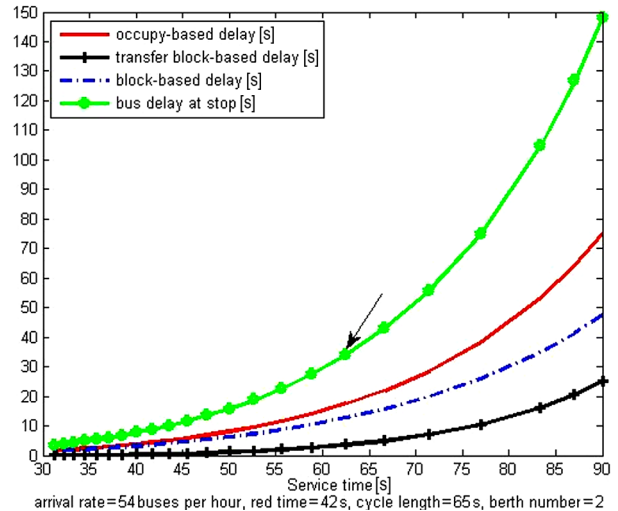


Fig. 7. The impact of service time on bus delay

Table 2. The impact of berth number on bus delay (arrival rate = 54 buses per hour, service time = 50 s, red time = 42 s, cycle length = 65 s)

Berth number	Occupy-based delay	Transfer block-based delay	Block-based delay	Bus delay at stop
1	150	33.51	44.68	228.19
2	8.18	1.25	6.47	15.9
3	0.98	0.06	1.96	3
4	0.12	0	0.63	0.75
5	0.02	0	0.2	0.22

In Fig. 7, service time varies from 30 s to 90 s. Occupy-based delay, transfer block-based delay, block-based delay and bus delay at stop rise with the increase of service time. When service time is more than 55 s, occupy-based delay is higher than block-based delay. However, their difference is negligible when service time is less than 55 s. When service time is more than 60 s (shown as the arrow in Fig. 7), bus delay at stop evidently increases.

Table 2 shows the impact of berth number on bus delay taking other variables as fixed values. Occupy-based delay, transfer block-based delay, block-based delay and bus delay at stop for multiple berths sharply decrease compared with one berth. Occupy-based delay is higher than block-based delay for one or two berths, but block-based delay becomes higher for three berths or more.

Conclusions

1. We propose a model to estimate bus delay at stop. Three types of delay can be estimated using the proposed model:
 - occupy-based delay, which is generated from a bus being blocked from entering by buses in service, can be estimated;
 - transfer block-based delay, which is generated from a bus being blocked from entering by buses finishing passenger serving, can be estimated.
 - block-based delay, which is generated from a bus being blocked from exiting by the front bus and traffic lights, can be estimated (bus delay at stop is the sum of them).
2. The proposed model has an unknown parameter, users need to first estimate the unknown parameter based on local data, and then apply this model to predict delay.
3. The unknown parameter is estimated as 0.4230 using OLS and the average accuracy rate of the proposed model is 75.07% for the survey stops in Vancouver.
4. Using the proposed model, we find out that bus delay at stop markedly rises when arrival rate exceeds 85 buses per hour and service time exceeds 60 s for the given ratio of red time to cycle length (0.65) and berth number (2). Bus delay at stop for multiple berths sharply decreases compared with one berth.
5. The proposed model is derived from theoretical analysis. Hence has some limitations. We assume bus overtaking manoeuvres are prohibited and the probability that the bus on each berth first finishes passenger serving is identical when all the berths are occupied. We do not consider the impact of other modes and the phenomenon that buses in queue cannot enter spare berths. These aspects should be further studied in the future.

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