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# RELIABILITY ANALYSIS OF THE LATERAL TORSIONAL BUCKLING RESISTANCE AND THE ULTIMATE LIMIT STATE OF STEEL BEAMS WITH RANDOM IMPERFECTIONS

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**Abstract.** The paper deals with the analysis of reliability of a hot-rolled steel IPE-beam designed according to Eurocodes. A beam at its ultimate limit state is considered. The load acting on the beam consists of permanent and long-term single variation actions. The beam is loaded with end bending moments about the major principal axis. The beam is susceptible to lateral torsional buckling between the end supports. Reliability of the beam is assessed using probabilistic analysis based on the Monte Carlo method. Failure probability is a function of the random variability of the load-carrying capacity and the random variability of load effects. The variability of the load-carrying capacity is influenced by the variability of initial imperfections. Imperfections are considered according to experimental research. Numerical studies showed that the failure probability is significantly misaligned. High values of failure probability were obtained for slender beams, for beams loaded only by permanent load action, and for beams loaded only by long-term single variation load. In further studies the values of partial safety factors of load and resistance were calibrated so that the failure probability had a target value of 7.2E–5. Relatively high values of partial safety factors were obtained especially for beams with high slenderness.

**Keywords:** steel, beam, reliability, safety, probability, failure, bending, buckling, imperfections, Eurocode.

## Introduction

Present structural design codes such as the Eurocodes provide a simple, economic and safe way for the design of civil engineering structures (JCSS 2001). Eurocode 3 (2005) provides the basic rule for the design of reliable steel structures. The design of building structures should be in accordance with the general rules given in EN 1990 (2002). The guidelines for structural reliability are based on the concept of limit states used in conjunction with the partial safety factor method.

Reliability of steel structures is traditionally achieved by deterministic methods using partial safety factors calculated generally under conservative estimators of influent parameters. Characteristic values or nominal values of material properties must be used as is indicated in Eurocode 3 (2005). The characteristic value can be fixed on statistical basis, often as quantiles of corresponding random variables. The nominal value is a value fixed on non-statical basis, for example on acquired experience or on physical conditions. Geometrical data for cross sections and systems may be taken as nominal values from product standards. In Eurocodes, numerical values of the partial safety factors and other reliability parameters are recommended as basic values that provide an acceptable level of reliability.

The degree of reliability can be identified according to the rules of standard EN 1990 (2002). The basic reliability targets for design values for the ultimate limit state recommended in EN 1990 (2002) are based on a semi-probabilistic approach, with the target value of reliability index $\beta_d = 3.80$ for a 50 years reference period (Sedlacek, Müller 2006). The target reliability level and the reference period are considered as parameters and can be adjusted, so the framework remains valid under alternative economic considerations or requirements for human safety (Caspeele *et al.* 2013). Reliability is usually expressed in probabilistic terms, although it is not the only possible mathematical approach to describe uncertainty, see for e.g. fuzzy sets (Yazdani-Chamzini 2014), non-probability convex models (Bai *et al.* 2014) or Gray systems (Zavadskas *et al.* 2010). Methods for the analysis of reliability are an important part of optimization procedures (Kalanta *et al.* 2012a, b) and of methods for the analysis of risk (Fouladgar *et al.* 2012).

One of the basic indicators of reliability of steel structures is the probability of failure. The reliability in-

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dex $\beta_d = 3.80$ corresponds to a target failure probability 7.2E–5, see EN 1990 (2002). If the evaluated failure probability is greater than the pre-set target value, then the structure is considered to be unsafe.

All random phenomena, which have an effect on the reliability of the structure must be taken into account during the evaluation of the failure probability. The probability of failure is usually a function of random material and geometrical characteristics of the structure and the random load effect. With regards to mass produced hot-rolled steel members statistical data of material and geometric properties are relatively well known from experimental research (Melcher *et al.* 2004; Kala *et al.* 2009). Other usable materials for the determination of input random variables of computational models include the tolerance standards EN 1090-2:2008+A1:2011 (2011), EN 10034:1993 (1993), which list maximum permissible deviations in the geometry of the structure or elements. The maximum allowable geometric deviations specified in the Eurocodes can never be met with 100% probability during production of steel structures; usually 5% non-compliance of the tolerance limits is considered (Kala 2011, 2012).

The most important characteristic for the analysis of reliability in the case of the ultimate limit state is the static load-carrying capacity. The load-carrying capacity is the maximum static load that a structure can safely carry. The load-carrying capacity can be determined experimentally or can be the output variable of a mathematical computational model developed on the basis of the principles of the theory of elasticity and plasticity. As the material and geometric characteristics of the structure are random variables, the load-carrying capacity is also a random variable.

A basic idea of the statistical analysis of the load-carrying capacity is provided by physical experiment, which however, is usually limited by the possibilities of the laboratory, particularly the size and shape of the examined specimen and the number of repetitions of the experiment. Standard EN 1990 (2002) considers a high number of tests such as $n \geq 100$, which is a lot in terms of the economy of real experiments, but little for the theoretical evaluation of small values of failure probability. If the failure probability is small then it is customary to have a large number of realizations of the experiment, which can be performed using virtual computer simulation based on methods of the Monte Carlo type.

The goal of the presented article is the analysis of the failure probability of a laterally unrestrained steel beam IPE200 subjected to static bending around the major axis, see Figure 1. The end-fork boundary condition is modelled as hinged ends of the beam for bending and warping torsion.

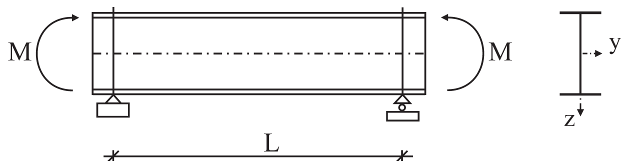


Fig. 1. End-fork IPE-beam with equal end moments

The beam has initial imperfections and is susceptible to lateral torsional buckling between the end supports. Beams IPE200 of varying lengths (varying slenderness) designed according to standards Eurocode 3 (2005) and EN 1990 (2002) for maximum utilization, i.e. the load is equal to the load-carrying capacity of the beam, were considered. The reliability of design of the steel beam designed according to standards Eurocode 3 (2005) and EN 1990 (2002) was evaluated using probabilistic analysis elaborated on the basis of methodology (Kala 2007; Lukoseviciene, Daniunas 2013). The Monte Carlo method was used for the evaluation of the probability of failure. Loading of the beam consists of permanent and long-term single variation actions. Different values of the ratio of permanent and long-term single variation action were considered in the probabilistic calculations. The partial safety factors in the calibration studies were calculated in such a manner that the probability of failure was fixed at 7.2E–5. Results of the probabilistic analyses show the limits of the applicability of the method of the partial coefficients of the Eurocode 3 (2005) standard for securing balanced reliability of steel elements.

## 1. Probability based reliability analysis

The standard EN 1990 (2002) permits the use of probabilistic methods which take into account the variability of input parameters and loading states. The probabilistic calculation is formulated as an alternative, which is to be verified with standard conventional methods using partial safety factors. Probabilistic methods can be used to determine the probability of failure and reliability index, the minimum values of which are recommended in EN 1990 (2002) for the ultimate limit state and individual classes of reliability. The results of probabilistic reliability analyses can be useful not only for the assessment of whether or not a structure fulfils requirements, but also for the verification of the reliability of design or for the calibration of partial safety factors of Eurocodes.

It is assumed for the reliability condition (1) that a steel structure is reliable, when and if the load action $S$ is lower than the structure load-carrying capacity $R$:

$$\Gamma = R - S \geq 0, \tag{1}$$

where $R$ is the load-carrying capacity and $S$ is the load action. $R$ and $S$ are generally random variables.

### 1.1. Random effect of load action *S*

The random load action $S$ was introduced as the sum $G+Q$ of random permanent load $G$ and random long-term single variation action $Q$, see Table 1. For random permanent loading $G$, the characteristic value $G_k$ can be considered as the mean value of a Gauss probability density function (pdf), variation coefficient of 0.1 (Honfi

Table 1. Input random load actions

| Symbol | Mean | St. deviation | Skewness | Kurtosis |
|---|---|---|---|---|
| $G$* | $G_k$ | $0.1 \cdot G_k$ | 0 | 3 |
| $Q$** | $0.6 \cdot Q_k$ | $0.21 \cdot Q_k$ | 1.14 | 5.4 |

* Gauss pdf, ** Gumbel-max.

*et al.* 2012). Gumbel-max pdf with mean value 0.6 $Q_k$ and standard deviation 0.21 $Q_k$ was considered for the determination of the random load action $Q$ (Kala 2007). The value $Q_k$ is approximately 95% fractile of $Q$.

Mean values and standard deviations of random variables $G$ and $Q$ are functions of characteristic values $G_k$, $Q_k$, which will be evaluated from Eqn (7) and are derived in the following manner. Let us consider value δ according to the equation:

$$\delta = \frac{Q_k}{G_k + Q_k}. \quad (2)$$

It may be noted that Eqn (2) has also been used for example in Kala (2007) and Honfi *et al.* (2012). Design according to Eurocodes requires that the design resistance $R_d$ is greater than or equal to the design load action $S_d$.

Let us assume that the steel beam IPE200 is designed according to Eurocode 3 (2005) for maximum utilization, i.e.:

$$S_d = R_d, \quad (3)$$

where $R_d$ is a function of the characteristic value of resistance $R_k$ and global partial safety factor $\gamma_M$:

$$R_d = \frac{R_k}{\gamma_M} = \chi_{LT} \cdot W_{pl,y} \cdot \frac{f_{y,n}}{\gamma_M}, \quad (4)$$

where $\chi_{LT}$ is the reduction factor for the relevant buckling mode, $W_{pl,y}$ is the plastic section modulus and $f_{y,n}$ is the nominal value of yield strength. For example: beam IPE200 under major axis bending with unbraced length $L = 3.02$ m, $\gamma_M = 1.0$, $f_{y,n} = 235$ MPa have $W_{pl,y} = 2.206\text{E}-4$ m$^3$, $\bar{\lambda}_{LT} = 1.0 \rightarrow \chi_{LT} = 0.666$ (buckling mode $a$) and $R_d = 34.506$ kNm. The value of $\chi_{LT}$ is a function of non-dimensional slendernes defined in Eurocode 3 (2005) as:

$$\bar{\lambda}_{LT} = \sqrt{\frac{M_{n(L=0)}}{M_{cr,n}}} = \sqrt{\frac{W_{pl,y} f_{y,n}}{M_{cr,n}}}, \quad (5)$$

where $M_{n,(L=0)}$ is the cross-section based resistance corresponding to a theoretical "zero-length" beam and $M_{cr,n}$ is the critical buckling moment corresponding to bifurcation from in-plane bending response to out-of-plane lateral bending and twisting, (see, e.g. Trahair 1977; Kala 2013). $M_{cr,n}$ is calculated from Eqn (9) using nominal values of geometric and material characteristics. The design load action can be expressed according to Eurocode 3 (2005) as:

$$S_d = \gamma_G \cdot G_k + \gamma_Q \cdot Q_k. \quad (6)$$

To meet the requirement that $R_d$ be greater than or equal to $S_d$, Eurocodes prescribe that, for structural design, loads are increased by partial safety factors. The partial safety factors applied in the calculations are the ones given by Eurocode: $\gamma_G = 1.35$, $\gamma_Q = 1.5$, $\gamma_M = 1.0$, see also (Honfi *et al.* 2012). Given the above assumptions, $G_k$, $Q_k$ are determined from the equation:

$$1.35 \cdot G_k + 1.5 \cdot Q_k = \frac{R_k}{1.0}, \quad (7)$$

where $\bar{\lambda}_{LT}$ and δ are deterministic (non-random) computational parameters. The goal of the presented article is to investigate the effect of $\bar{\lambda}_{LT}$ and δ on the failure probability.

### 1.2. Random load-carrying capacity *R*

The elastic load-carrying capacity $M_R$ of the beam IPE shown in Figure 1 can be derived (Trahair 1977). Lateral deflection and twisting of the ends of the beam are prevented. The initial geometrical imperfections were assumed to follow the shape of the first eigenmode pertaining to lateral torsional buckling. The elastic load-carrying capacity $M_R$ is the maximum elastic load. It was assumed that failure of the imperfect beam occurs when the maximum longitudinal stress is equal to the yield strength $f_y$. The formula for the calculation of $M_R$ was derived in Kala (2013) as:

$$M_R = -\frac{\sqrt{4D_1^2 + (D_4 + D_5)^2 + 4D_1(D_4 - 2M_{cr}D_3)}}{4M_{cr}W_z} + \frac{2D_1 + D_4 + D_5}{4M_{cr}W_z}, \quad (8)$$

where:

$$D_1 = f_y M_{cr} W_y W_z;$$
$$D_2 = M_{cr} W_z + P_z |a_{v0}| W_y;$$
$$D_3 = M_{cr} W_z - P_z |a_{v0}| W_y;$$
$$D_4 = 2P_z^2 I_y |a_{v0}|;$$
$$D_5 = 2M_{cr} D_2;$$
$$P_z = \pi^2 \frac{EI_z}{L^2},$$

and where $E$ is Young's modulus, $W_y$ is section modulus about axis $y$, $W_z$ is section modulus about axis $z$, $I_y$ is second moment of area about axis $y$, $I_z$ is second moment of area about axis $z$, $f_y$ is the yield strength and $a_{v0}$ is the amplitude of a half-wave of the sine function of the initial lateral imperfection of the beam axis. $M_{cr}$ is expressed as:

$$M_{cr} = \frac{\pi}{L} \sqrt{\frac{E^2 I_z I_t}{2(1+\mu)}} \sqrt{1 + 2\frac{\pi^2 I_\omega (1+\mu)}{I_t L^2}}, \quad (9)$$

where $\mu$ is Poisson's ratio, $I_t$ is torsion constant, $I_\omega$ is warping constant. The basic form of the empirical equation for the evaluation of the plastic load-carrying capacity $M_{pl,R}$ can be derived in accordance with (Galambos 1998) as:

$$M_P = M_R \frac{W_{pl,y}}{W_y} \alpha + M_R (1 - \alpha), \quad (10)$$

where:

$$\alpha = \left( \frac{1}{1 + \overline{\lambda}_{LT}^{-4}} \right)^4 . \quad (11)$$

### 1.3. Input random imperfections

The following basic random parameters were considered for the computational model (10) of the steel beam: geometry of the section (tolerances of plate elements and section shape), geometry of the beam (initial curvature and twist), yield strength $f_y$, modulus of elasticity $E$ and Poisson's ratio $\mu$. Random geometric and material imperfections of the profile IPE200 were considered according to Kala *et al.* (2009) and Soares (1988). Profile IPE was considered in accordance with Kala (2005, 2009, 2011) as perfectly biaxially symmetrical (without imperfections distorting the symmetry), see Figure 2. Statistical characteristics of yield strength of steel grade S235 were considered according to Melcher *et al.* (2004). The effects of residual stresses were neglected.

If the beam is curved according to the first eigenmode, it is then valid that:

$$a_{v0} = \frac{e_0}{1 + \frac{h}{2} \frac{P_z}{M_{cr}}}; \quad (12)$$

$$a_{\phi_0} = a_{v_0} \frac{P_z}{M_{cr}}, \quad (13)$$

where $a_{v0}$, $e_0$ $a_{f0}$ (see Kala (2013) and Fig. 3). Input random variables are clearly listed in Table 2. All input variables are considered as statistically independent.

Hermite pdf is a Gaussian pdf multiplied by the Hermite polynom with respective skewness and kurtosis (program Statrel 3.10).

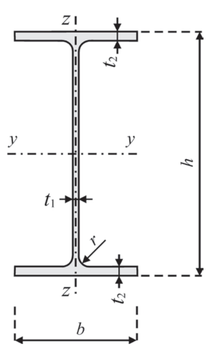


Fig. 2. Geometry of symmetric IPE cross section

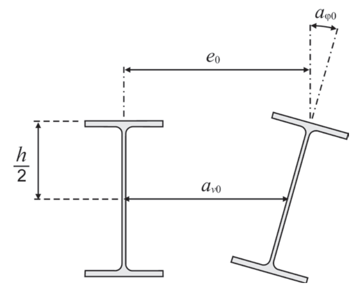


Fig. 3. Curvatures in the middle of the span

Table 2. Input random imperfections

| Symbol | Mean | St. deviation | Skewness | Kurtosis |
|---|---|---|---|---|
| $h$** | 200.18 mm | 0.8847 mm | –0.409 | 3.038 |
| $b$** | 101.39 mm | 0.9868 mm | –0.3711 | 3.730 |
| $t_1$** | 5.902 mm | 0.2187 mm | 0.5306 | 4.9671 |
| $t_2$** | 8.438 mm | 0.3898 mm | –0.1039 | 2.7782 |
| $r$* | 12 mm | 0.552 mm | 0 | 3 |
| $e_0$* | 0 m | $L$/1960 | 0 | 3 |
| $E$* | 210 GPa | 10 GPa | 0 | 3 |
| $\mu$* | 0.3 | 0.009 | 0 | 3 |
| $f_y$** | 297.3 MPa | 16.8 MPa | 0.3246 | 2.5415 |

* Gauss pdf, ** Hermite.

### 1.4. Failure probability

Failure occurs in the event that inequality (1) is not fulfilled. The failure probability $P_f$ (14) was calculated using the Monte Carlo method for 10 million simulation runs. The program Statrel 3.1 was used:

$$P_f = P(\Gamma < 0). \quad (14)$$

Input random variables needed for the evaluation of $P_f$ in (14) are listed in Table 1 and Table 2.

It is necessary to specify $L$, $G_k$, $Q_k$ in Table 1 and Table 2. Practically, the procedure is as follows: the pair of values $\overline{\lambda}_{LT}$ and $\delta$ is selected. $L$ is evaluated from Eqn (5) using the value of $\overline{\lambda}_{LT}$. $G_k$, $Q_k$ are evaluated from Eqn (7) using the value of $\delta$.

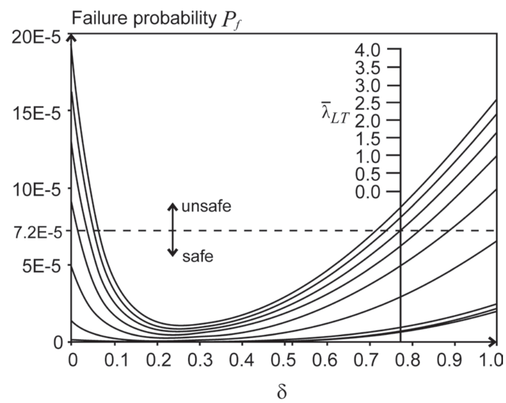


Fig. 4. Misalignment of failure probability $P_f$

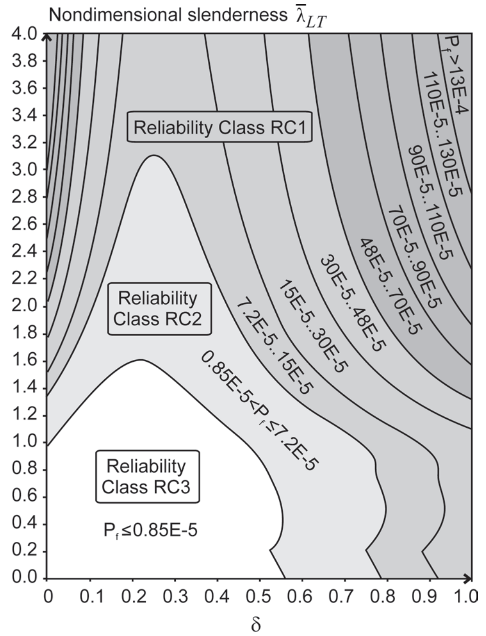


Fig. 5. 2-D graph of $P_f$ for $\gamma_G$ = 1.35, $\gamma_Q$ = 1.5, $\gamma_M$ = 1.0

## 2. Reliability verification of design according to standard

The curves of the misalignments of failure probability $P_f$ for a set of fixed values of $\bar{\lambda}_{LT}$ are shown in Figure 4. Parameter δ was considered with a step of 0.02. A convex curve $P_f$ is obtained for each fixed (constant) $\bar{\lambda}_{LT}$. If $\bar{\lambda}_{LT} \leq 3.1$, the maximum values of $P_f$ are obtained for δ = 1.0; otherwise, the maximum $P_f$ is for δ = 0.

More general results are shown in Figure 5. Figure 5 shows a 2-D top view of the 3-D surface chart, in which the outlines (similarly as isolines) depict curves connecting points with the same (predetermined) value of $P_f$. Parameter δ (horizontal axis) was considered with a step of 0.02, parameter $\bar{\lambda}_{LT}$ (vertical axis) was considered with a step of 0.05. Values of $P_f$ for plotting the curves were selected so that Figure 5 was as illustrative as possible.

The shaded regions between the lines depict areas according to values of $P_f$. Three reliability classes RC1, RC2 and RC3 are differentiated. Reliability classes RC are defined in standard EN 1990 (2002) for reference periods of 1 and 50 years according to the β reliability index concept. The reliability index β is related to $P_f$ by:

$$P_f = \Phi(-\beta), \qquad (15)$$

where Φ is the cumulative distribution function of the standardised Gauss distribution. Values in Table 3 are taken from Table B2 in EN 1990 (2002). Table 3 lists the minimum values of the reliability index β for ultimate limit state and 50 years reference period.

Table 3. Recommended minimum values of β and related $P_f$

| Reliability class | β | $P_f$ |
|---|---|---|
| RC3 | 4.3 | 8.5E–6 |
| RC2 | 3.8 | 7.2E–5 |
| RC1 | 3.3 | 4.8E–4 |

Three reliability classes RC1, RC2 and RC3 may be associated with the three consequence classes CC1, CC2 and CC3 listed in Table B1 of standard EN 1990 (2002), see Table 4.

Table 4. Consequence classes according to EN 1990 (2002)

| Consequences class | Description | Examples of building and civil engineering works |
|---|---|---|
| CC3 | **High** consequences for loss of human life, or economic, social or environmental consequences **very great** | Grandstands, public buildings where consequences of failure are high (e.g. a concert hall) |
| CC2 | **Medium** consequences for loss of human life, economic, social or environmental consequences **considerable** | Residential and office buildings, public buildings where consequences of failure are medium (e.g. an office building) |
| CC1 | **Low** consequences for loss of human life, and economic, social or environmental consequences **small or negligible** | Agricultural buildings where people do normally enter (e.g. storage buildings), greenhousses |

Reliability index β has a target value of 3.8 provided that we consider the ultimate limit state for common design situations within the reference period of 50 years, see Table C2 in EN 1990 (2002).

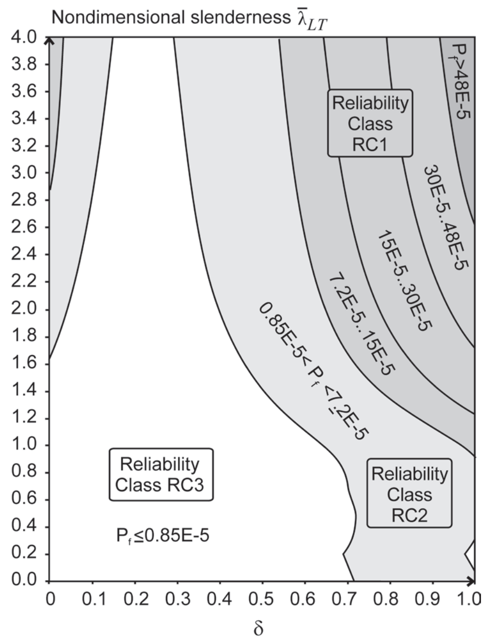


Fig. 6. 2-D graph of $P_f$ for $\gamma_G$ = 1.35, $\gamma_Q$ = 1.5, $\gamma_M$ = 1.1

The graphs for other selected $\gamma_G$, $\gamma_Q$, $\gamma_M$, see Figures 6, 7 and 8, are plotted similarly as was the graph in Figure 5. It may be noted that the case in Figure 8 corresponds essentially to the criteria of allowable stress design, which is the design approach used in the USA.

It is apparent from the graphs in Figure 5 to Figure 8 that the failure probability $P_f$ is significantly misaligned. High values of $P_f$ were obtained for $\delta \to 0$ and for $\delta \to 1.0$.

High values of $P_f$ were also obtained for slender beams with $\bar{\lambda}_{LT} > 1.5$. For $\gamma_G = 1.35$, $\gamma_Q = 1.5$, it can be recommended that if $\bar{\lambda}_{LT} \leq 1.5$, then $\gamma_M = 1.0$, otherwise $\gamma_M = 1.1$, see Figures 5 and 6.

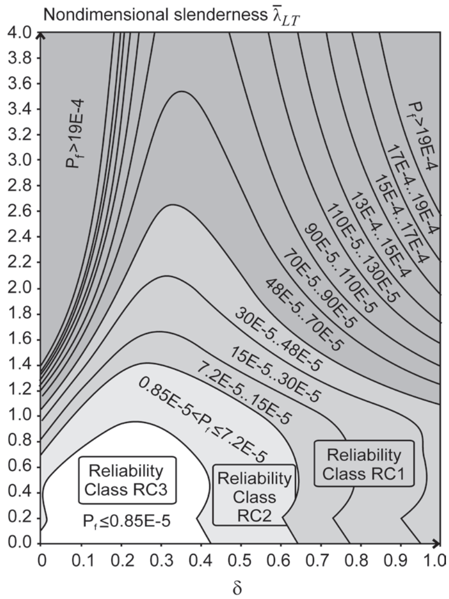


Fig. 7. 2-D graph of $P_f$ for $\gamma_G = 1.1$, $\gamma_Q = 1.3$, $\gamma_M = 1.1$

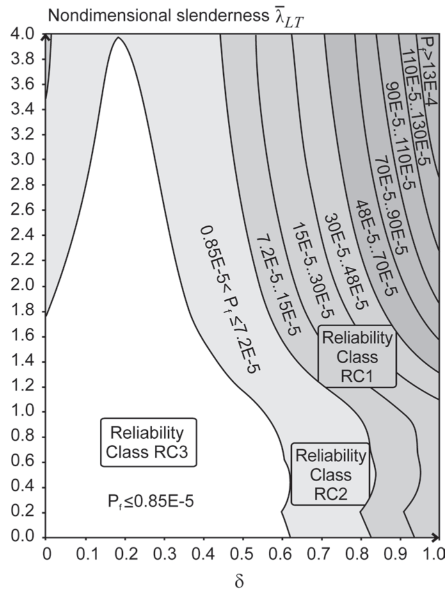


Fig. 8. 2-D graph of $P_f$ for $\gamma_G = 1.0$, $\gamma_Q = 1.0$, $\gamma_M = 1.5$

Comparing the graphs in Figures 5 and 8 it can be noted that the values of $P_f$ are the same for $\delta = 1.0$. This is because $\gamma_Q = \gamma_M = 1.5$.

## 3. Partial safety factor calibration

From the previous chapter, it can be generalized that $P_f$ decreases if there is an increase in $\gamma_M$, $\gamma_G$, or $\gamma_Q$. Let us now ask ourselves what values of $\gamma_G$, $\gamma_Q$, $\gamma_M$ would have to be implemented in the probabilistic study (14) in order to attain the target value $P_f = 7.2E–5$.

Let us assume that $\bar{\lambda}_{LT}$ (and thus also $L$) and $\gamma_M$ are known. Let us select $\delta$ and seek the pair $\gamma_G$, $\gamma_Q$ such that $P_f = 7.2E–5$. Figure 9 illustrates the pair $\gamma_G$, $\gamma_Q$ evaluated for $\bar{\lambda}_{LT} = 0$ ($L \to 0$), $\gamma_M = 1.0$ and $\delta = 0, 0.1, ..., 1.0$. Practically, the procedure is as follows: Variables $\bar{\lambda}_{LT}$, $\gamma_M$, $\delta$, $\gamma_G$ are fixed as constants and the value of $\gamma_Q$ is evaluated using the bisection method with the starting interval $\gamma_Q \in \langle 0; 3 \rangle$ so that $P_f \to 7.2E–5$. It is apparent from Figure 9 that for constant $\delta$ the pairs $\gamma_G$, $\gamma_Q$ lie on a line. This is an important finding, which was not evident in advance. Each line in Figure 9 was plotted from ten pairs $\gamma_G$, $\gamma_Q$ (from ten points).

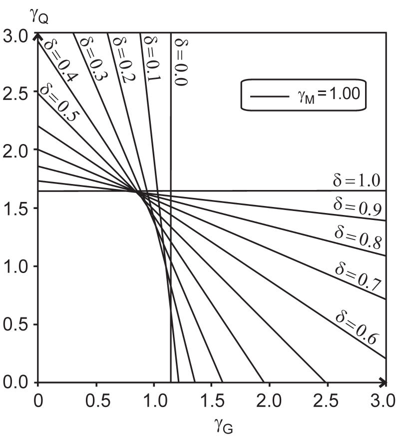


Fig. 9. Partial safety factors for $\bar{\lambda}_{LT} = 0$ and $P_f = 7.2E–5$

The procedure was similar for the calculation of $\gamma_M$, $\gamma_G$, $\gamma_Q$ plotted in Figures 10–15.

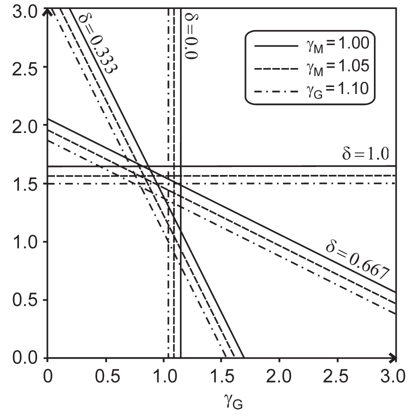


Fig. 10. Partial safety factors for $\bar{\lambda}_{LT} = 0.0$, $P_f = 7.2E–5$

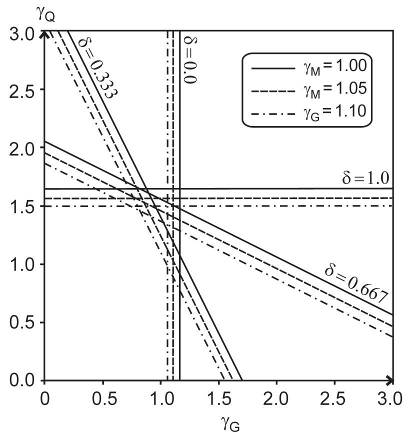


Fig. 11. Partial safety factors for $\overline{\lambda}_{LT} = 0.5$, $P_f = 7.2E{-}5$

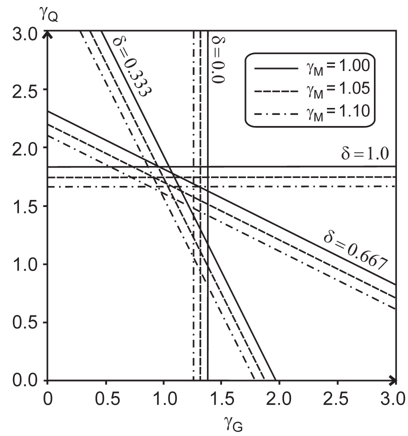


Fig. 13. Partial safety factors for $\overline{\lambda}_{LT} = 1.5$, $P_f = 7.2E{-}5$

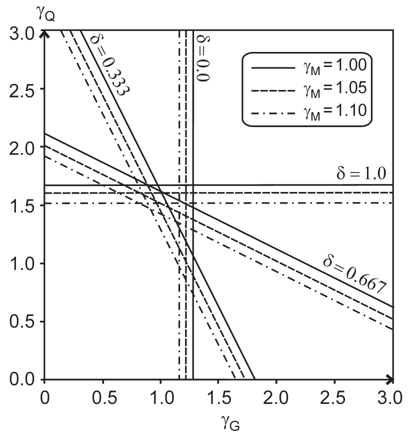


Fig. 12. Partial safety factors for $\overline{\lambda}_{LT} = 1.0$, $P_f = 7.2E{-}5$

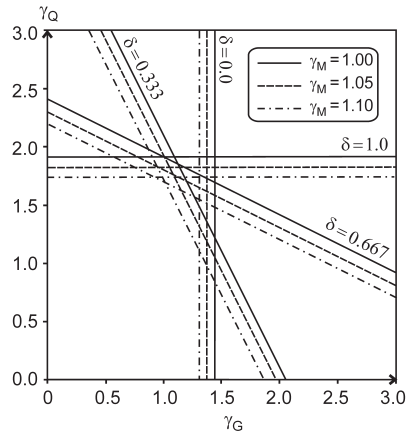


Fig. 14. Partial safety factors for $\overline{\lambda}_{LT} = 2.0$, $P_f = 7.2E{-}5$

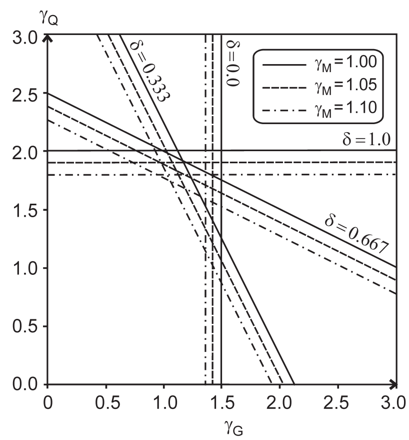


Fig. 15. Partial safety factors for $\overline{\lambda}_{LT} = 3.0$, $P_f = 7.2E{-}5$

## 4. Reliability of structures

The presented study showed one of the strategies in the verification of reliability of design of steel elements according to the Eurocodes and the complexity of the calibration of partial safety factors. Results of the numerical studies have shown that the probability of failure is significantly misaligned. With regard to the variety of structural elements and building materials used in construction attainment of optimal reliability of building structures based on the system of Eurocode standards can be very complicated. Mechanical parameters of building materials and foundation soil obtained within the framework of experimental research often have significant variance. In this context, mention can be made of concrete and reinforced concrete (J. Králik, J. Jr. Králik 2009; Kala *et al.* 2010), brickware and masonry (Li *et al.* 2014), wood (Sousa *et al.* 2013), metals (Strauss *et al.* 2006), glass (Badalassi *et al.* 2014) and soil (Amšiejus *et al.* 2014). Brandl (2004) found that according to European statistics 80–85% of all failures and damages in buildings are due to problems inherent in the subsoil. The properties of the subsoil, however, compared with other types of building materials, are very variable, generally of significantly lower quality, and are very difficult to determine (Kelevišius *et al.* 2014; Martinkus *et al.* 2014). The approaches for the study of reliability are not even unified (Möller, Reuter 2007; Wang *et al.* 2012). The greater the epistemic (fuzzy) uncertainty, the less suitable is the use of probabilistic methods for the study of reliability.

The general principles for the reliability of different structures are given by the international standard ISO 2394 (1998). Generally, different levels of reliability can be accepted for safety, serviceability and durability of structures, however, the current requirement of the Eurocode standards for simplicity of calculation procedures eliminates the advantages of the partial safety factors method and may often lead to uneconomic design. However, one thing seems apparent today. Further improvement of current regulations will be based on calibration methods, optimization techniques and other reliability approaches of the theory of reliability, including the application of methods of the theory of probability and mathematical statistics.

## Conclusions

The reliability analysis of the steel member loaded under bending and solved with regard to the influence of lateral torsional buckling showed that the probability of failure is highly misaligned. Beams designed according to the Eurocode 3 (2005) standard for maximum utilization (with zero resistance reserve) were studied.

The results presented in this article show that the design of very slender beams may be risky. High values of failure probability were obtained for slender members (approx. for $\bar{\lambda}_{LT} > 1.5$) and for the beam loaded solely by permanent load action ($\delta = 0$) as well as for the beam loaded only by long–term single variation action ($\delta = 1.0$).

Eurocode 3 (2005) prescribes for $\gamma_M$ a constant value of $\gamma_M = 1.0$. Results of the probabilistic studies presented in the article show that a more balanced failure probability would be obtained if higher values of $\gamma_M$ (for e.g. $\gamma_M = 1.1$) are considered for higher values of $\bar{\lambda}_{LT}$. Step change of $\gamma_M$, for e.g., from 1.0 to 1.1, however, cannot be recommended for practical design of steel structures according to the Eurocodes. The main reason is human factor. Engineers under economic pressure are forced to design structures with the lowest production costs leading to structures of small masses. In practical design, where the influence of many load case combinations (in the thousands) is considered, the buckling length is often chosen subjectively by an expert estimate as the same for all load cases. Small inaccuracies in the calculation of buckling lengths, however, must not lead to large differences in the calculation of standardized design load-carrying capacities of steel members.

Optimization of reliability of slender beams can be relatively feasibly calibrated using the coefficient $\chi_{LT}$. European buckling curves are based on the Ayrton-Perry equation. Buckling curves are smooth curves for $\bar{\lambda}_{LT} > 0.2$, see Eurocode 3 (2005). If we decrease $\chi_{LT}$ in (4), then $P_f$ is decreased in (14). Calibration of $\chi_{LT}$ can be used to ensure that with constant values of $\gamma_M$, $\gamma_G$, $\gamma_Q$ more optimized reliability is achieved for all considered slenderness. Practically, the values of $\chi_{LT}$ could be calibrated so that the graphical outputs in Figures 10–15 were as similar as possible.

Misalignment of the probability of failure due to changes in parameter $\delta$ can be optimized by calibrating the values of $\gamma_G$, $\gamma_Q$. It is apparent that the values of $\gamma_G$ can be increased for small values of $\delta$, and similarly $\gamma_Q$ can be increased for large values of $\delta$. However, it could be argued that extreme values $\delta = 0$ or $\delta = 1.0$ cannot realistically occur and the complex calculation of $\gamma_G$, $\gamma_Q$ may be less feasible for practical design according to the Eurocodes.

In further reliability studies aimed at stability problems it will be necessary to focus attention also on the serviceability limit state. The serviceability limit state identifies civil engineering structures, which fail to meet technical requirements for use, despite being strong enough to remain erect. Serviceability limit state is the deciding state for design of a number of structures (see for e.g. Kamiński, Pawlak 2011; Juozapaitis *et al.* 2013; Majcher *et al.* 2014). Bearing in mind stability problems of steel structures, the buckling of slender members can reach high values, which may adversely affect the use of the structure. Nevertheless Eurocode 3 (2005) does not limit the maximum values of buckling of slender members. It is advisable to specify limit values of maximum deflections according to the type of structure depending on the method of use, the expected life span of the structure and projected cost of repairs. In the case of technologically important structures, for e.g. power plants, specification and quantification of the risks that society is still willing to accept is also important.

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