# A Modified Reverse Engineering Approach Using Bezier Curve Approximation 

Mustafa Saad Ayoob Al-Khazraji<br>Al-Nahrain University, College of Engineering, Iraq<br>mustafasaadayaob@gmail.com


#### Abstract

Reverse Engineering is a process of reproducing existing parts by obtaining digital models using a special data taken from the original parts using specific techniques. It can be used to redesign existing parts either due to lost data or the parts are no longer available. In this paper, surface modelling technique using special data taken from CMM (Coordinate Measuring Machine) was employed to redesign a candle holder. Specific MATLAB code was generated to model the data taken from the surface of a candle holder made of glass. Bezier curve technique was implemented in this research to model the curve of the outer surface of the candle holder. Various orders of Bezier curves were discussed and used to give better approximation of the original data curve with error percentage monitoring each time. The thickness of the candle holder was reduced from 5 mm to 3 mm and the volume reduction was calculated. The amount of reduction in the glass volume when reducing the thickness was found to be $210 \mathrm{~mm}^{3}$. In addition, the amount of increase in the area of glass section was calculated to be $138.5 \mathrm{~mm}^{2}$. This reduction gives a better vision of the amount of glass saved using this procedure. Two different shapes were found and plotted by varying the control points coordinates.


Keywords: Reverse Engineering, Bezier Curve, Surface Modelling and Redesign procedure.

## Introduction:

Reverse engineering can be defined as the field of engineering concerned with generating three dimensional (3D) models of an existing component. This process can be performed, mainly, with the aid of computer programs and a set of data taken using various techniques from the original component. [1]

The aim from reverse engineering an object is to apply improvements or gain economic benefits from the generated object. Some incomes of these improvements could be changing the shape, dimension and function of the original object. Economically, reducing the amount of material used or the complexity of the original part will lead to minimizing the cost of producing such products. [2]

One of the most important aims of applying reverse engineering to certain parts is to reconstruct them. Some parts are very old and rare, finding these parts is very difficult in the market either because the company produces these parts has stopped or moderated these parts in the current production [3]. Therefore, it is necessary to reconstruct these parts to replace the old, broken ones. Utilizing reverse engineering, these old parts could be reproduced using a set of data taken from the original part. [4]
Au \& Yuen, (1999) concluded that applying reverse engineering techniques should not be in conflict with the quality of the object. As mentioned earlier, by reverse engineering, various improvements can be applied to an object. However, these improvements should not alter the function, strength and durability of the object. For example, reducing the thickness of an object will lead to minimizing the amount of material used in fabricating that object. However, this thickness will reduce the strength of the object to a certain extent. Therefore, some limitations should be applied and noticed when performing reverse engineering procedure to an object.

Various techniques can be applied to obtain 3D models in reverse engineering. One of these techniques was implemented by Motavalli, (1998) who utilized a geometrical set of data obtained from the original part and, using various modelling software, these data are converted to a valid geometry which is very close to the original part. Several software can be used for this purpose including CATIA, SOLIDWORKS and others. The choice of the software used is a matter of availability and the level of complexity of the obtained geometry. Geometric constraints and decomposing complex surface problems was investigated by Lin, et al., (2005) and Kovács, et al., (2015) to resolve some problems when modeling complex parts.

Fan, (1998) described the Coordinate Measuring Machine or simply (CMM) as the most common machine which is used to obtain the required data for generating the 3D model of the original part. This machine constructs, simply, from a moving probe, table, arm and a computer. The machine is capable of taking 2D and 3D measurements of the original parts by passing the probe over the surface of the part. These measurements are in the form of a table containing either X and Y coordinates (for 2D
measurements) or $\mathrm{X}, \mathrm{Y}$ and Z coordinates (for 3D measurements). In this paper, 2D measurements are sufficient because the shape of the part is not very complex.

In more complex shapes, Fisher, (2004) conducted a research to solve some of the problems arising when dealing with such complex geometries. Applying specific knowledge of surfaces including standardization and interference boundary conditions could solve some of the errors which occur when dealing with a high-detail geometry. However, these techniques require more details of the original part as well as employing extra facilities such as high definition cameras.

A very similar reverse engineering approach was adopted by Lin, et al., (2005) to reconstruct an artificial joint. The approach included scanning of a total knee arthroplasty (TKA) using a CMM machine and modelling the data using a specific 3D imaging software (SURFACER). The result of this study was implemented in modifying the design of the TKA joint according to patient comfort.

Reverse engineering approach was used to model and reconstruct various parts. One of these approaches was discussed by Barbero, (2009), who combined common knowledge with some design approximations to model complex parts such as cams. Another approach was followed by Wang, et al., (2012) who utilized surface meshes to construct the geometry of the required part using certain procedures such as partitioning, trimming, special algorithms and surface smoothing procedures.

Tai \& Huang, (2000) and Thompson, et al., (1999) built various algorithms to resolve the complexity and connectivity issues when processing the set of data. These algorithms are based on visual knowledge of parts, surface smoothing and design intent functions. However, most of these algorithms are subject-specific and are capable of solving problems with the parts for which, these algorithms were built.

## Bezier Curve Technique:

Bezier curve is a powerful tool to deal with complex curves which cannot be modelled using conventional mathematics techniques such as equation curves. Bezier curve technique deals with complex curves to obtain an approximate shape to these curves using a set of control points. The level of complexity of the curve determines the number of the control points required. As a result of that, the higher the number of the control points, the more accurate and smooth curves obtained with complex shapes. However, the level of complexity of the equations increases as the number of the control points increases due to the higher number of constants to be obtained. [5]

The basic construction of Bezier curves depends mainly on a certain polynomial called Bernstein Polynomial. This polynomial comprises of a number of constants called Control Points and a specific variable (u) which is having the value between 0 and 1 . Drawing a straight line between these points will lead to a polygon shape called Control Polygon (figure 1).

Bezier curves depend mainly on the number of control points used. The basic Bezier curve consists of two control points which is called a Single Degree Bezier Curve. Generally, the number of the control points is higher than the degree of the Bezier curve by 1. Several degrees of Bezier curves can be obtained depending on the number of control points such as quadratic, cubic and quartic Bezier curves. Figure (1) shows a sample Cubic Bezier curve with the corresponding control points ( $\mathrm{P}_{0}, \mathrm{P}_{1}, \mathrm{P}_{2}$ and $\mathrm{P}_{3}$ ).


Figure (1): Simple Cubic Bezier curve polygon with its control points. [6]

Bezier curves are based on dividing the original curve into several ( $n-1$ ) lines depending on the degree of the Bezier curve and the number of the data points obtained from CMM. The length (L) of the first line can be calculated using the formula:

$$
L_{m}=\sqrt{\left(a_{m}-a_{m-1}\right)^{2}+\left(b_{m}-b_{m-1}\right)^{2}}
$$

(1)

Where $a_{m-1}$ and $b_{m-1}$ represent the X and Y coordinates of the first data point, $a_{m}$ and $b_{m}$ represent the X and Y coordinates of the second point while $m$ gives the order of the data point.
It can be understood from the above formula that dividing the original curve into small lines follows the application of Pythagoras' theorem by finding the length of the hypotenuse (the line opposite to the right angle) of a right angle triangle. In other words, the first term of the formula ( $a_{m}-a_{m-1}$ ) represents the length of one line of the triangle while the second term ( $b_{m}-$ $b_{m-1}$ ) represents the length of the other side respectively.

As mentioned earlier, the specific variable (u) has a value of 0 to 1 depending on the proximity of the point to the starting point (0). In other words, the first point of the first line $\left(L_{1}\right)$ should have the value of (0) and the second point of the
same line should have a value less than (1) according to the following formula:
$U_{m}=U_{m-1}+\left(\frac{L_{m-1}}{\Sigma L}\right)$
(2)

Where $U_{m}$ and $U_{m-1}$ are the value of the specific variable (u) of the second and the first points respectively.

After finding the values of the specific variable (u) for every data point, Bezier polynomial can be applied to generate the set of equations that contains the control points of the curve. The Bezier polynomial $(B(n, i)$ ) is given by the formula:
$B(n, i)=\sum_{i=0}^{n}\binom{n}{i} \cdot(1-u)^{n-i} \cdot u^{i}$
Where $n$ is the order of the Bezier curve, $i$ is a reverse counter which have the same value of $n$. For example, a cubic Bezier curve should have the following polynomial:

$$
\begin{align*}
& B=(1-u)^{3} P_{1}+3 u(1-u)^{2} P_{2}+3 u^{2}(1-u) P_{3}+ \\
& u^{3} P_{4} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . . \tag{4}
\end{align*}
$$

Where $P_{1}, P_{2}, P_{3}$ and $P_{4}$ are the values of the control points for a specific data point. The values of these control points could be found by solving a series of simultaneous equations. Each data point should have an equation containing four control points as shown above. It should be noted that each control point is a set of X and Y coordinates and will be found separately.

## Data Acquisition:

A candle holder made of glass was mounted on the CMM and an axial probe was passed over the outer surface of the holder from the bottom base to the upper edge of the holder. Due to the symmetry of the candle holder, 2D data is sufficient for building the geometry of the holder. Therefore, a set of X and Y coordinates was recorded as the probe moves along the surface of the holder from bottom to top. Ninety-six data points was recorded and the acquired data was arranged in an (XLSX) sheet to be imported to the next stage.

## Data Processing using MATLAB:

## 1. Plotting the data:

As mentioned earlier, 96 data points were taken from the outer surface of the original candle holder using the CMM, and stored in an XLSX sheet. These data points were plotted in figure (2).

Because the aim of this research is to redesign the candle holder, the base and the lower neck of the holder will be omitted as they are not concerned with changing the thickness or the size of the holder. Thus, only 82 data points will be considered in this analysis.

Using a special MATLAB code, the 82 data points were imported from the XLSX file into

MATLAB and the outer surface of the candle holder were plotted and shown in figure (3)


Figure (2): 3D modelling of the original candle holder.


Figure (3): Original data points plot before equating the axis.


Figure (4): Original data points plot after equating the axis.

By equating the axis of the curve in figure (3), a new curve was plotted which represents the actual shape of the outer surface of the holder. The actual shape is shown in figure (4) after equating the axis.

## 2. Dividing the Curve and Finding the values of $L$ and $U$ :

After plotting the original curve using MATLAB, the curve was divided into 81 segments (which is consistent with $n-1$ by setting n to 82). After dividing the curve into 81
segments, equations (1) and (2) were applied to find the values of (L) and (U) for each segment.

Again, using a special MATLAB code and a pair of FOR loops to find the values of ( L ) and (U). The result of this is a set of 81 values of 81 segments (which represents (L)) and 82 values of (U). A sample of the values of ( U ) are listed in table (1)for the first 32 points. The first and the last points are 0 and 1 respectively.

Table (1): U-coordinates of the first 32 points.

| Point No. | U-Coordinate | Point No. | U-Coordinate |
| :---: | :---: | :---: | :---: |
| 1 | 0 | 17 | 0.1212 |
| 2 | 0.0035 | 18 | 0.1281 |
| 3 | 0.0069 | 19 | 0.1356 |
| 4 | 0.0111 | 20 | 0.1457 |
| 5 | 0.0202 | 21 | 0.1564 |
| 6 | 0.0209 | 22 | 0.1686 |
| 7 | 0.0321 | 23 | 0.1789 |
| 8 | 0.0402 | 24 | 0.194 |
| 9 | 0.0510 | 25 | 0.2074 |
| 10 | 0.0625 | 26 | 0.2199 |
| 11 | 0.0740 | 27 | 0.2313 |
| 12 | 0.0857 | 28 | 0.2417 |
| 13 | 0.0862 | 29 | 0.2547 |
| 14 | 0.0932 | 30 | 0.2666 |
| 15 | 0.1038 | 31 | 0.2836 |
| 16 | 0.1125 | 32 | 0.2940 |

## 3. Cubic Bezier curve:

Modelling the outer surface of the holder will start with the first approximation. This approximation is the use of four control points (Cubic Bezier Curve).

After finding the values of the u-coordinates of the curve, Bezier polynomial was applied for each value of $U$. Using equation (3), each value of $U$ was applied and a series of 82 equations was generated. Each equation contains four unknown control points. It should be noted that the above procedure was applied to find the X and Y coordinates of each control point which means that the procedure should be applied twice.

The analysis of the Bezier curve was first implemented using Cubic Bezier curve. Utilizing equation (4), Cubic Bezier approximation was applied to all of the values of $U$, and using a special MATLAB code, four control point coordinates were found and listed in table (2).

Table (2): X-coordinates and corresponding Y coordinates of the cubic Bezier curve.

| x-coordinates for the <br> Cubic curve control <br> points (mm) | Corresponding $\mathbf{y}-$ <br> coordinates for the Cubic <br> curve control points (mm) |
| :---: | :---: |
| 20.1969 | -0.2236 |
| 30.8854 | 26.6741 |
| 20.9926 | 52.9726 |
| 3.7266 | 73.2733 |

After finding the coordinates of all control points, the Cubic curve was plotted in (fig (4)) with the original curve (from the given data points). As can be seen from figure (5), an error
percentage exist between the original and Bezier curves due to the degree of the Bezier approximation used (Cubic order in this case). This percentage of error was calculated for 11 average points, equally spaced along the curve. The error was found to be $50.1 \%$ which is a very high percentage and should be reduced by increasing the degree of the Bezier curve in the next section using the same procedure.


Figure (5): Cubic Bezier Curve vs. the original curve.

## 4. Quartic Bezier curve:

After the high error percentage when using Cubic Bezier curve, the degree of the Bezier curve was increased to have five control points (Quartic Bezier curve). The same procedure of the Cubic Bezier curve was applied in this case and the coordinates of the five control points were calculated and a new curve approximation was obtained and plotted along with the original curve and shown in figure (6)

As can be seen from figure (6), the Quartic Bezier curve gives an approximation which is closer to the original curve as compared with the Cubic Bezier approximation. Again the error of the Quartic curve was calculated in the same way as the Cubic curve (for the same 11 points) and it was found to be $7.59 \%$ which is a great reduction in the error percentage which means a better approximation. However, a higher approximation will be applied to further decrease the error percentage and get a better approximation.


Figure (6): Quartic Bezier Curve vs. the original curve.

## 5. Quantic Bezier curve:

Another approximation of the Bezier curve was implemented by increasing the order of the curve to have six control points (Quantic Bezier curve). Again the same procedure of the Cubic and Quartic Bezier curves was applied to the Quantic curve. The coordinates of the six control points were calculated by solving a series of simultaneous equations and are listed in table (3)

Table (3): X-coordinates and their corresponding Y coordinates of the Quantic Bezier curve.

| x-coordinates for the <br> Quantic curve control <br> points (mm) | Corresponding y- <br> coordinates for the <br> Quantic curve control <br> points (mm) |
| :---: | :---: |
| 20.9891 | -0.0713 |
| 24.5733 | 16.1089 |
| 23.9837 | 28.7252 |
| 33.3234 | 53.3938 |
| 3.9868 | 56.3081 |
| 6.0875 | 74.3171 |

The Quantic Bezier approximation was plotted along with the original curve and shown in figure (7). As can be seen from figure (7), the Quantic Bezier curve is very close to the original curve with a very narrow gap between the two curves. The error of the Quantic curve was calculated in the same way as for the Cubic and Quartic curves and it was found to be $1 \%$. This error percentage is very small and is almost acceptable in this paper. Thus, the Quantic curve is a very good representative of the original curve and could be used in the next analysis where the shape of the candle holder will be modified.


Figure (7):Quantic Bezier Curve vs. the original curve.

## Redesigning the Candle Holder:

The aim of this paper is to redesign the candle holder by reducing the thickness of glass from 5 mm to 3 mm using the same material and height and calculating the amount of increase in the volume as a result of that. In order to perform this, the wall thickness (inside surface) of the holder should be plotted.

The method used to draw the wall thickness is to find the tangent vector at each point of the curve by taking the $1^{\text {st }}$ derivative of the curve
function and then obtaining the normal vector to that tangent. This can be performed by switching the (i) and ( $j$ ) coordinates of the tangent vector and inverting their sign.


Figure (8): Inner and outer curves of the candle holder.
After finding the normal vector at each point on the curve, each point was moved along this vector with a value of 5 mm to find a new x and y coordinates of the points that will form the thickness curve. Figure (8) shows the inner and outer curves obtained for 5 mm thickness.

In order to redesign the candle holder to the new thickness (3mm), the volumes of the original and new shapes of glass were calculated. To perform this, the area of the original candle holder glass was calculated by dividing the wall thickness to small segments and finding the area of each segment which is assumed to be rectangular by multiplying the thickness by the distance between every two adjacent points. The volume of glass is then calculated by sweeping each segment (multiply by $2 \pi r$ ). The reason behind using this approximation was to reduce the complexity of using the integration method to find the volume under high order Bezier curve (5th order in this case) which will lead to more than 70 parameters!

The area of glass was calculated using the above procedure to be ( $392.6990 \mathrm{~mm}^{2}$ ) and the volume to be (4.0896* $10^{4} \mathrm{~mm}^{3}$ ) which should be attained when changing the thickness of the candle holder.

Getting the first new shape with (3mm) thickness was made by changing the coordinates of the control points (from table 3) until the same volume achieved. This was done by an iteration process and applying the same procedure for obtaining the thickness used with the 5 mm thickness (discussed earlier) followed by finding the area and the volume using the same approximation method. The new control points used to obtain the new shape are listed in table 4.The new figure obtained is shown in figure (9)
and the area of glass of the new shape is 254.2395 $\mathrm{mm}^{2}$. However, the volume is $4.0686 * 10^{4} \mathrm{~mm}^{3}$.

Table (4): Coordinates of the control points of the first shape obtained.

| x-coordinates for the <br> Quantic curve control <br> points (mm) | Corresponding $\mathbf{y -}$ <br> coordinates for the <br> Quantic curve control <br> points (mm) |
| :---: | :---: |
| 35 | 0 |
| 35 | 10 |
| 25 | 37 |
| 52 | 58 |
| 6 | 60 |
| 6 | 75 |

It can be seen from the calculations of the reduced thickness that the area of the glass is less than the original one while the volume is, almost, the same. The reason behind that is the expansion of the curved surface in all directions when stretching it leaving the same material with different thickness.

Another point of interest is that the movement of the control points was made to all points except the upper one because we can move the free end of the holder but not the end that connected to the stem (Upper end in figure 9). Furthermore, the y coordinates of the top upper and bottom lower control points were kept without change because we want to keep the same height of the candle holder.


Figure (9): First obtained shape of the candle holder after changing the volume.

Using the same procedure for obtaining the first shape, the second shape can be obtained by further changing the coordinates of the control points as listed in table 5 giving the volume of $4.0844 * 10^{4} \mathrm{~mm}^{3}$. Figure (10) shows the second shape.

Different shapes can be obtained by changing the position of control points and keeping the same material volume.
The difference between the first and second new shapes is that when changing the $y$ coordinates of the control points, the resulting curve will compressed or stretched vertically and we can
increase or decrease the radius further in order to compensate the reduction in the material volume and vice versa.

Table (5): Coordinates of the second shape control

| points. |  |
| :---: | :---: |
| x-coordinates for the <br> Quantic curve control <br> points (mm) | Corresponding y- <br> coordinates for the <br> Quantic curve control <br> points (mm) |
| 35 | 0 |
| 32 | 15 |
| 30 | 37 |
| 51.5 | 53 |
| 6 | 60 |
| 6 | 75 |



Figure (10): The second shape of the candle holder after changing the volume.

The amount of increase in the containing volume of the candle holder has been calculated using the approximation method (same as calculating the volume of glass). The older volume of the candle holder is $5.8233 * 10^{4} \mathrm{~mm}^{3}$ while the new volume for both new shapes is approximately $1.7621^{*} 10^{5} \mathrm{~mm}^{3}$ which is as twice as the original volume.

## Discussion and Conclusion:

The reverse engineering procedure was followed in order to redesign the candle holder by reducing the wall thickness from 5 mm to 3 mm for a given data points from CMM. The redesign process started by finding the appropriate curve function for the given points using Bezier curves. The order of the Bezier curve has been changed each time in order to reduce the deviation of the new curve from the original one.

It has been observed through the research that the effect of the degree of Bezier curve on the curve increases the proximity of the obtained curve to the original one. The higher the degree, the more accurate the curve. However, increasing the degree of Bezier curve is accompanied with growing number of parameters and equations which will require tedious math to solve it. Thus,

## NJES Vol. 20 No.5, 2017

## Al-Khazraji, pp.1097-1104

increasing the degree of the curve will make it more precise and close to the original curve but attention should be drawn to the complexity of calculations.

The approximation is the best solution for complex math. In order to change the thickness of the candle holder, the volume of glass was calculated as discussed earlier. Without this approximation, other ways of finding the volume of Bezier curve are very complex and complicated because it requires derivation and integration of the function. Any integration or deriving of the function will produce a huge number of parameters which might approach 100! Thus, although the approximation approach is not an accurate one, it can be used to reduce the complicated math operations.

Reverse engineering is a powerful tool. Many components can be reverse engineered using the same procedure and, in turn, the design can be improved by changing the thickness or dimensions or may be obtaining new shapes. For instance, for the new shape obtained, the amount of increase in the volume is as twice as the original one. This should be a great advantage in increasing the containing volume of the original component using the same material.

One of the limitations of this research is the symmetry of the shape of the candle holder. This symmetry makes the curve of the holder easier to model. However, complex shapes may require complicated calculations which will produce a series of complex equations with higher order Bezier curves.

Various shapes can be obtained using this approach. Changing the position of the control points of Bezier curve will produce a new shape. The new shape should follow some restrictions in terms of using the same material and with the same height. Following these rules along with changing the coordinates of the control points in an iteration method will result in various shapes with the same volume.

## References

[1] T. Varady, R. R. Martin and J. Cox, "Reverse engineering of geometric models an introduction," Computer-Aided Design, vol. 29, no. 4, pp. 255-268, 1997.
[2] R. B. Fisher, "Applying knowledge to reverse engineering problems," ComputerAided Design, vol. 36, p. 501-510, 2004.
[3] Q. Peng and M. Loftus, "A new approach to reverse engineering based on vision information," International Journal of Machine Tools \& Manufacture, vol. 38, p. 881-899, 1998.
[4] E. Bagci, "Reverse engineering applications
for recovery of broken or worn parts and remanufacturing: Three case studies," Advances in Engineering Software, vol. 40, p. 407-418, 2009.
[5] G. Farin and D. Hansford, "B'ezier Curves: Cubic and Beyond," in The Essentials of CAGD, CRC Press, 2000, pp. 1-33.
[6] R. B. Agarwal, CURVES: ME 165 Lecture Notes, Computer Aided Design in Mechanical Engineering.
[7] V. Raja and K. J. Fernandes, Reverse Engineering : An Industrial Perspective, USA: Springer, 2008.
[8] A. Karniel, Y. Belsky and Y. Reich, "Decomposing the problem of constrained surface fitting in reverse engineering," Computer-Aided Design, vol. 37, p. 399417, 2005.
[9] Y. P. Lin, C. T. Wang and K. R. Dai, "Reverse engineering in CAD model reconstruction of customized artificial joint," Medical Engineering \& Physics, vol. 27, p. 189-193, 2005.
[10] B. R. Barbero, "The recovery of design intent in reverse engineering problems," Computers \& Industrial Engineering, vol. 56, p. 1265-1275, 2009.
[11] J. Wang, D. Gu, Z. Yu, C. Tan and L. Zhou, "A framework for 3D model reconstruction in reverse engineering," Computers \& Industrial Engineering, vol. 63, p. 11891200, 2012.
[12] I. Kovács, T. Várady and P. Salvi, "Applying geometric constraints for perfecting CAD models in reverse engineering," Graphical Models, vol. 82, pp. 44-57, 2015.
[13] B. Sarkar and C.-H. Menq, "Smooth-surface approximation and reverse engineering," computer-aided design, vol. 23, no. 9, pp. 623-628, 1991.
[14] S. Motavalli, "Review of Reverse Engineering Approaches," Computers ind. Engng, vol. 35, no. 1, 2, pp. 25-28, 1998.
[15] K.-C. Fan, "A non-contact automatic measurement for free-form surface profiles," Computer integrated Manufacturing Systems, vol. 10, no. 4, pp. 277-285, 1998.
[16] C. Au and M. Yuen, "Feature-based reverse engineering of mannequin for garment design," Computer-Aided Design, vol. 31, p. 751-759, 1999.
[17] C.-C. Tai and M.-C. Huang, "The processing of data points basing on design intent in reverse engineering," International Journal of Machine Tools \& Manufacture, vol. 40, p. 1913-1927, 2000.
[18] I. Kovács, T. Várady and P. Salvi, "Applying geometric constraints for perfecting CAD models in reverse engineering," Graphical Models, vol. 82, pp. 44-57, 2015.
[19] V. Jain, S. Jain and K. Yada, "Functional Reverse Engineering for Mechanical Components," International Journal of Advanced Mechanical Engineering., vol. 4, no. 2, pp. 139-144, 2014.
[20] N. Singh and J. Singh, "REVERSE ENGINEERING OF BRAKE ROD OF BAJAJ PULSAR 150CC MOTOR BIKE USING SOLIDWORKS AND AUTODESK INVENTOR," Journal of Engineering Research and Studies, vol. 3, no. 1, pp. 4048, 2012.
[21] V. Raja and K. J. Fernandes, "Reverse Engineering : An Industrial Perspective," in Springer, Springer, 2008, pp. 242-251.
[22] I. R. M. Junior, A. Ogliari and N. Back, "Guidelines for Reverse Engineering Process Modeling of Technical Systems," Complex

Systems Conccurent Engineering, vol. 28, pp. 831-840, 2007.
[23] A. KUMAR, P. K. JAIN and P. M. PATHAK, "REVERSE ENGINEERING IN PRODUCT MANUFACTURING: AN OVERVIEW," in INTERNATIONAL SCIENTIFIC BOOK, Vienna, Austria, DAAAM International, 2013, pp. 665-678.
[24] W. B. Thompson, J. C. Owen, H. J. de St. Germain, S. R. Stark and T. C. Henderson, "Feature-Based Reverse Engineering of Mechanical Parts," IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION, vol. 15, no. 1, pp. 57-66, 1999.


الخلاصة
الهندسة العكسية هي عملية اعادة تصنيع اجزاء مصنعة مسبقا عن طريق بناء نموذج الكتروني للجزء الاصلي وفقا لحسـابات ميينـة. هذه
 طريقة الهندسة العكسية لعمل نماذج مقاربة جدا للثكل الاصلي وذلك بسبب ضياع المخططات الاصلية التي تم تصنيع الجزء على اسناسـها او
 مصنوع من الزجاج باستخدام معلومات ماخوذة من ماكنة خاصة تسمى (CMM) تعطي احداثيات للسطح الخارجي لحامل الشمع. استخدم في هذا البحث برنامج ماتلاب لبناء كود خاص لمعالجة احداثيات السطح الخـارجي للحامل. تـم اعتمـاد طريقة منحني (بزير) عدة مرات (بتغيير درجة المنحني) لغرض ايجاد شكل تقريبي للسطح الخارجي للحامل مـع حسـاب نسبة الخطا في كل مرة. تم تغيير سمك الجدار للحامل من
 138.5ملم². هذه النتائج تعطي تصور واضح عن فائدة الهندسة العكسية في تقليل كمية الزجاج المستخدم فَّي صناعة حامل الشمع. كمـا و تم ايجاد اشكال مختلفة اضافية للحامل بتغيير مواقع النقاط في منحني بزير.

