An Inventory Model under Space Constraint in Neutrosophic Environment: A Neutrosophic Geometric Programming Approach

Chaitali Kar[1], Bappa Mondal [2], Tapan Kumar Roy [3]

[1]Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah-711 103, West Bengal, India. E-mail: chaitalikar12@gmail.com

[2]Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah-711 103, West Bengal, India. E-mail: bappa802@gmail.com

[3]Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah-711 103, West Bengal, India. E-mail: roy_t_k@yahoo.co.in

Abstract: In this paper, an inventory model is developed without shortages where the production cost is inversely related to the set up cost and production quantity. In addition, the holding cost is considered time dependent. Here impreciseness is introduced in the storage area. The objective and constraint functions are defined by the truth (membership) degree, indeterminacy (hesitation) degree and falsity (non-membership) degree. Likewise, a non-linear programming problem with a constraint is also considered. Then these are solved by Neutrosophic Geometric Programming Technique for linear membership, hesitation and non-membership functions. Also the solution procedure for Neutrosophic Non-linear Programming Problem is proposed by using additive operator and Geometric Programming method. Numerical examples are presented to illustrate the models using the proposed procedure and the results are compared with the results obtained by other optimization techniques.

Keywords: Neutrosophic Sets, Non-linear Programming, Inventory, Additive Operator, Geometric Programming, Neutrosophic Optimization.

1 Introduction

In general, most of the classical inventory models assume that the unit production cost and the holding cost of an item are constant and independent in nature. But these assumptions may not be true in real life. In practical situations, unit production cost may depend on the production quantity. Also the unit holding cost may depend on the amount produced. Cheng [1, 2] used these ideas to formulate inventory models and solved them by Geometric Programming (GP) method and obtained closed form optimal solutions. Later on, Jung and Klein [3] developed three cost minimization inventory models: Model 1 considered demand dependent unit cost, Model 2 assumed order quantity dependent unit cost and both of demand and order quantity dependent unit cost is considered in Model 3. All these models are then solved by GP method.

In general, GP is an effective method to solve a class of non-linear problem in comparison with other non-linear methods. The main advantage of GP method is that in this method a complicated problem with non-linear and inequality constraints (primal problem) is converted into an equivalent problem with linear and equality constraints (dual problem). Therefore the dual problem is easier to solve than the primal problem. GP method was first introduced by Zener [4]. Later on, Duffin et al. [5] developed GP method for optimization problems. Kotchenberger [6] was the first Scientist who tackled the inventory problem by GP method. After that, Lee [7] presented a profit maximizing selling price and order quantity problem where the demand is taken as non-linear function of price with a constant elasticity and solved by GP approach. After that, Hariri and Ata [8] presented GP approach for solving a multi-item production lot size inventory model with varying order cost. Later on, Jung and Klein [9] discussed a comparative analysis between the total cost minimization model and the profit maximization model via GP. Then a constrained inventory model of deteriorated items was built-up with and without trancation on the deterioration term and solved using GP method by Mandal et al. [10]. Leung [11] proposed an EPQ model with a flexible and imperfect production process by GP approach and also established more general results using the arithmetic-geometric mean inequality. In the recent era, Wakeel et al. [12] discussed multi-product, multi-vendors inventory models with different cases of rational function under linear and non-linear constraints via GP method.

In many inventory models the objective and constraint goals are assumed to be known. The optimum cost in an inventory model is affected by the restrictions on the storage area, number of orders and production cost. But, in real life, it is not always possible to predict the total cost and resources precisely. So these may be assumed to be fuzzy in nature. In this case, the inventory problem along with the constraints may be realistically represented formulating the model under fuzzy environment and the fuzzy model can be solved by different fuzzy programming methods.

In 1965, Zadeh [13] first introduced the concept of fuzzy set theory. Later on, Bellman and Zadeh [14] introduced fuzzy decision making process. Then Zimmermann [15] solved multi objective linear programming problem based on fuzzy decision making process. Many researchers used fuzzy set theory in inventory control system. Sommer [16] applied fuzzy concept to inventory model. After that, Roy and Maiti [17] studied and solved a fuzzy EOQ model with demand dependent unit cost and limited storage capacity by GP and non-linear programming method. Mandal et al. [18] applied GP method to solve a multi-item inventory problem with three constraints under fuzzy environment. Again, Islam and Roy [19] proposed and solved a fuzzy production inventory model considering fuzziness in objective function, constraint goals and coefficients of the objective function and the constraint. Later on, Sadjadi et al. [20] suggested a pricing and marketing planning model where demand and cost function depend on price and marketing expenditure in imprecise environment and solved the problem by GP method.

In the case, where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set, the concept of an Intuitionistic Fuzzy Set (IFS) can be viewed as an alternative approach to define a fuzzy set. The IFS may represent information more abundant and flexible than the fuzzy set when uncertainty such as hesitancy degree is involved and hereby seems to be suitable for dealing with natural attributes of physical phenomena in complex management situations. The IFS uses two indexes, degree of membership and degree of non-membership, to describe the fuzziness. The degree of membership and degree of non-membership can be arbitrary satisfying the condition that the sum of the both is less than one.

The concept of IFS was introduced as a successful generalization of the fuzzy set by Atanassov [21]. Atanassov also analysed open problems in IFS theory in an explicit way. After that, Atanassov and Gargov [22] discussed interval valued IFS. Then Angelov [23] presented an optimization problem in intuitionistic fuzzy environment and solved the problem by converting into a crisp one. Pramanik and Roy [24–26] applied intuitionistic fuzzy goal programming approach to solve vector optimization problem, quality control problem and multi objective transportation problem respectively. After that, Pramanik et al. [27] investigated bilevel programming in intuitionistic environment. Later on, Jana and Roy [28] suggested a new intuitionistic fuzzy optimization approach for solving a multi objective intuitionistic fuzzy linear programming problem with equality and inequality constraints with intuitionistic fuzzy goals. They also discussed the application of this approach in transportation problems. After that, Banerjee and Roy [29] considered a stochastic inventory model with fuzzy cost components and solved by fuzzy GP and intuitionistic fuzzy GP techniques. Banerjee and Roy [30] also analysed and solved a stochastic inventory model with deterministic constraint by fuzzy GP and intuitionistic fuzzy GP method. A constrained multi objective inventory model of deteriorating items was solved under intuitionistic fuzzy environment by Mahapatra [31]. In recent era, Jafarian et al. [32] proposed a process to solve multi objective non-linear programming problem under intuitionistic environment using GP method.

The IFS can only handle incomplete information. In the case of indeterminate or inconsistent information the concept of IFS becomes insufficient. So, it cannot deal with all types of uncertainties in real life problems. In that case, Neutrosophic Set (NS) was introduced as a generalization of fuzzy set and IFS. In 1995, Smarandache [33–35] introduced the term 'Neutrosophy' which means knowledge of neutral thought. Neutrosophic is the derivative of neutrosophy and it includes neutrosophic set, neutrosophic probability, neutrosophic statistics, neutrosophic logic etc. NS is defined by three independent degrees; truth (membership) degree, indeterminacy (hesitation) degree and falsity (non-membership) degree. Here all the three degrees are standard or non-standard subsets of $]0^-, 1^+[$.

Nowadays, NS is used in different fields of research work. Roy and Das [36] solved multi objective production planning problem by neutrosophic linear programming approach. Banerjee et al. [37] discussed single objective linear goal programming problem in neutrosophic number environment. In recent era, Pramanik and Banerjee [38] formulated three new neutrosophic goal programming model to solve multi objective programming problems with neutrosophic number coefficients. Basset et al. [39, 40], S. Pramanik [41] analysed neutrosophic goal programming problem under neutrosophic sets environment. Again, a multi objective neutrosophic optimization technique is investigated and its application to structural design is developed by Sarker et al. [42]. Then Basset et al. [43] introduced and solved a neutrosophic linear programming model where the parameters are considered as trapezoidal neutrosophic numbers. Again, Basset et al. [44] represented a framework to estimate different cloud services by providing a neutrosophic multi-criteria decision analysis approach and devolved a model depending on neutrosophic Analytic Hierarchy Process (AHP) using triangular neutrosophic numbers and estimated the quality of cloud services.

There are several papers on decision making using NS and Single Valued Neutrosophic Sets (SVNSs) environment. Basset et al. [45] discussed AHP decision making model under neutrosophic environment. Also, Basset et al. [46] extended AHP-SWOT analysis in neutrosophic environment. After that, NS was introduced for decision making and evaluation method to determine the factors influencing the selection of SCM suppliers by Basset et al. [47]. Basset et al. [48] also introduced a new neutrosophic association rule algorithm for big data analysis and discovered all of the possible association rules and minimized the losing processes of rules. Afterwards, Mondal and Pramanik [49, 50] explained neutrosophic decision making model for school choice and clay-brick selection respectively. The research field is then extended to neutrosophic Multi-Attribute Decision Making (MADM) process by some researchers. Biswas et al. [51] discussed neutrosophic MADM with unknown weight information. Again, Pramanik et al. [52] investigated the contribution of some Indian researchers to MADM in neutrosophic environment. Later on, Mondal and Pramanik [53] applied tangent similarity measure to neutrosophic MADM process. Ye and Zhang [54] established MADM with the help of similarity measures between SVNSs. After that, J. Ye [55] presented MADM model using a proposed form of correlation coefficient of SVNSs under neutrosophic environment. Recently, Mondal et al. [56] developed MADM process for SVNSs using similarity measures based on hyperbolic sine functions. Moreover, Multi-Criteria Decision Making (MCDM) approach is presented in neutrosophic environment by Zhang and Wu [57]. Mondal and Pramanik [58] extended Multi-Criteria Group Decision Making (MCGDM) approach in neutrosophic environment. Mondal et al. [59] used hybrid binary logarithm similarity measure to solve Multi-Attribute Group Decision Making (MAGDM) problem under SVNSs environment. Also, Biswas et al. [60] discussed MADM using entropy based grey relational analysis method under SVNSs environment. In recent era,, Pramanik et al. [61] solved MAGDM problem using NS cross entropy.

Rough neutrosophic sets also have been used by several investigators to solve the decision making problems. Mondal et al. [62] discussed decision making process based on several trigonometric hamming similarity measures under rough neutrosophic environment. Recently, Pramanik et al. [63] used trigonometric hamming similarity measures to develop MADM model under rough neutrosophic environment. Also, Mondal et al. [64] presented MAGDM based on rough neutrosophic TOPSIS. Later on, the same authors extended MADM on rough neutrosophic variational coefficient similarity measure [65]. After that, Mondal and Pramanik [66] proposed tri-complex rough neutrosophic similarity measure and its applications in MADM. In recent era, Pramanik et al. [67, 68] discussed MCDM using projection and bidirectional projection measures and correlation coefficient under rough neutrosophic environment respectively. Again, Mondal and Pramanik [69] investigated decision making approach based on some similarity measure using interval rough neutrosophic sets. The same authors also discussed rough neutrosophic MADM using rough accuracy function [70]. Afterwards, Pramanik and Mondal [71] investigated rough neutrosophic similarity measures and MADM. Mondal and Pramanik [72] studied rough neutrosophic MADM based on grey relational analysis. Later on, Pramanik and Mondal [73, 74] used cotangent and cosine similarity measures under rough neutrosophic environment and its application in medical diagnosis. Later, Basset and Mohamed [75] proposed a general framework for dealing with imperfectness and incompleteness using single valued neutrosophic and rough set theories.

There are some developments on neutrosophic programming method which have been applied on some real life problems. Jiang and Ye [76] defined neutrosophic functions and numbers for optimization models. They formulated a two bar truss structure design problem and minimized its weight under stress and stability constraints using the neutrosophic number optimization method. Later, Ye [77] applied the neutrosophic number (NN) optimization method to a three bar planer truss structural design for minimum weight under stress and deflection constraints. Ye [78] and Ye et al. [79] developed neutrosophic number linear and non-linear programming methods respectively. In both cases, authors made applications under NN environment.

In spite of the above developments, there are several gaps in the literature of Neutrosophic Optimization. Till now, none has demonstrated that a non-linear Neutrosophic Optimization Problem can be reduced to a Geometric Programming Problem (GPP) with posynomial terms and solved by GP technique. Thus the motivation of the present investigation is to develop a procedure to reduce a non-linear Neutrosophic Problem to a corresponding GPP and then to solve it by the appropriate technique depending upon its degree of difficulty. Hence the main contributions of the present paper are the following.

- Representation of a non-linear Neutrosophic Programming Problem to a corresponding Geometric Programming Problem with posynomial terms.
- For illustration, a virgin non-linear inventory programming problem is formulated under neutrosophic environment.
- The said Neutrosophic Problem is reduced to a GPP with zero degree of difficulty.
- Reduced GPP is now solved by three methods-(i) Fuzzy Optimization technique, (ii) Intuitionistic Optimization method and (iii) Neutrosophic Optimization procedure.
- The superiority of Neutrosophic Optimization procedure is demonstrated with the help of some numerical data.

In this paper, we formulate an inventory model along with the space constraint. The holding cost has been taken as time dependent and the production cost has been taken as inversely related with set-up cost and production quantity. The constraint is considered here in neutrosophic environment. The inventory model is then converted into a crisp programming problem using additive operator and Neutrosophic Optimization Technique. Finally, it has been solved by GP method. Also a non-linear problem has been considered and solved proceeding the same procedure. At last, the numerical examples are considered to illustrate the problems.

This research paper is organized as follows. The introduction is described in Section 1. In Section 2, the basic definitions and operations are presented. Some notations and assumptions are made in Section 3. An inventory model is developed and solved in Section 4. The general form of Neutrosophic Non-linear Programming problem is given in Section 5. In Section 6, a solution procedure to solve Neutrosophic Non-linear Programming problem is described. Application of Neutrosophic Optimization Technique on a non-linear inventory model and a non-linear programming problem are illustrated in Section 7. In Section 8, the numerical experiments are presented. Discussion on numerical experiments is presented in Section 9. The conclusions and future research scope are described in Section 10.

2 Mathematical Preliminaries

2.1 Fuzzy Set [13]

Let X be a space of points (objects). A fuzzy set A in X is an object of the form $A = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_A : X \to [0, 1]$ is called the membership function of the fuzzy set A.

2.2 Intuitionistic Fuzzy Set [21]

Let X denotes the universal set. An intuitionistic fuzzy set A in X is an object of the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ with the condition $0 < \mu_A(x) + \nu_A(x) < 1 \ \forall\, x \in X$. Here $\mu_A, \nu_A : X \to [0, 1]$ define the membership function and the non-membership function for every element x in X respectively.

2.3 Neutrosophic Set [35]

Let the set X be a space of points (objects) and $x \in X$. A neutrosophic set A in X is defined by a truth (i.e., membership) function $\mu_A(x)$, an indeterminacy (i.e., hesitation) function $\sigma_A(x)$ and a falsity (i.e., non-membership) function $\nu_A(x)$ and having the form $A = \{(x, \mu_A(x), \sigma_A(x), \nu_A(x)) : x \in X\}$. Here $\mu_A(x)$, $\sigma_A(x)$ and $\nu_A(x)$ are real standard or real non standard subset of $]0^-, 1^+[$, that is, $\mu_A, \sigma_A, \nu_A : X \to]0^-, 1^+[$.
There is no restriction on the sum of $\mu_A(x)$, $\sigma_A(x)$ and $\nu_A(x)$, so $0^- \leq Sup\, \mu_A(x) + Sup\, \sigma_A(x) + Sup\, \nu_A(x) \leq 3^+ \ \forall \ x \in X$.

2.4 Single Valued Neutrosophic Set (SVNS) [33]

Let the set X be the universe of discourse. A single valued neutrosophic set A over X is an object having the form $A = \{(x, \mu_A(x), \sigma_A(x), \nu_A(x)) : x \in X\}$, where $\mu_A, \sigma_A, \nu_A : X \to [0, 1]$ with the condition $0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3 \ \forall\, x \in X$. $\mu_A(x)$, $\sigma_A(x)$ and $\nu_A(x)$ denote the truth degree, indeterminacy degree and falsity degree of the member x to A respectively.

2.5 Complement of SVNS [33]

The complement of a single valued neutrosophic set A is denoted by $c\,(A)$ whose truth, indeterminacy and falsity functions are respectively given by

$$\begin{aligned} \mu_{c(A)}(x) &= \nu_A(x), \\ \sigma_{c(A)}(x) &= 1 - \sigma_A(x), \\ \nu_{c(A)}(x) &= \mu_A(x) \qquad for\ \ all\ \ x \in X. \end{aligned}$$

2.6 Union of SVNS [33]

The union of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as $C = A \cup B$, whose truth, indeterminacy and falsity functions are respectively given by

$$\begin{aligned} \mu_{A\cup B}(x) &= max\left(\mu_A(x), \mu_B(x)\right), \\ \sigma_{A\cup B}(x) &= max\left(\sigma_A(x), \sigma_B(x)\right), \\ \nu_{A\cup B}(x) &= min\left(\nu_A(x), \nu_B(x)\right) \qquad for\ \ all\ \ x \in X. \end{aligned}$$

2.7 Intersection of SVNS [33]

The intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as $C = A \cap B$, whose truth, indeterminacy and falsity functions are respectively given by

$$\begin{aligned} \mu_{A\cap B}(x) &= min\left(\mu_A(x), \mu_B(x)\right), \\ \sigma_{A\cap B}(x) &= min\left(\sigma_A(x), \sigma_B(x)\right), \\ \nu_{A\cap B}(x) &= max\left(\nu_A(x), \nu_B(x)\right) \qquad for\ \ all\ \ x \in X. \end{aligned}$$

3 Mathematical Model

An Inventory model is developed under the following notations and assumptions:

3.1 Notations

The inventory model is developed under the following notations:

D: Demand per unit time (Decision variable).
f(S,Q): Total production cost per cycle.
H: Holding cost per unit item, which is time depended.
I(t): Inventory level at any time, $t \geq 0$.
Q: Production quantity per batch (Decision variable).
S: Set-up cost per unit time (Decision variable).
T: Cycle of length.
TAC(D,S,Q): Total average cost per unit time.
W: Total storage space area.
w_0: Space area per unit quantity.

3.2 Assumptions

Following assumptions have been considered in the model:
a) The inventory system involves only one item.
b) The replenishment occurs instantaneously at infinite rate.
c) The lead time is negligible.
d) Demand rate is constant.
e) The total production cost is inversely related to set up cost(S) and production quantity(Q) i.e., $f(S,Q) = bS^{-x}Q^{-y}, \quad b,x,y \in R(>0)$.
(It is a fact that modern machineries which may be costlier than the earlier ones perform better in terms of production rate, products' quality, etc. The cost of machineries are considered to a part of set up cost. As the high production rate reduces the unit price, set up cost may be considered to be inversely related to the production cost. Moreover, it is well known that when the quantities are procured in lot, the per unit cost reduces with the size of procured units, i.e., the production cost is inversely related with the procured amount.)
f) In general, holding cost is assumed to be constant. But it is more realistic if we consider the holding cost increases with time, that is, it is time depended. Assume $H = at$.

4 Model Formation

In this model the inventory level gradually decreases to meet the demand (See Fig. 1). Therefore the differential equation describing I(t) at time t over the time period (0,T) is given by

$$\frac{dI(t)}{dt} = -D, \qquad 0 \leqslant t \leqslant T \tag{1}$$

with the initial and boundary conditions $I(0) = Q$ and $I(T) = 0$.

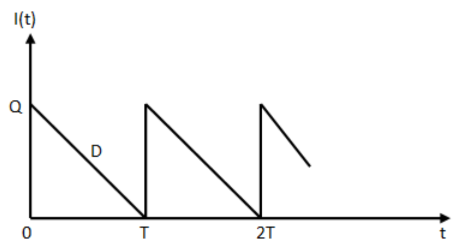

Figure 1: Crisp inventory model

The solution of the above differential equation is $I(t) = Q - Dt$.
Also we have, $T = \frac{Q}{D}$
Inventory holding cost = $\int_0^T H\ I(t)\ dt = \int_0^T at\ I(t)\ dt = \frac{aQ^3}{6D^2}$.
Total inventory related cost per cycle = set up cost + holding cost + production cost $= S + \frac{aQ^3}{6D^2} + f(S,Q)$
Total average cost per unit cycle is $TAC(D,S,Q) = \frac{SD}{Q} + \frac{aQ^2}{6D} + \frac{bD}{S^xQ^{1+y}}$
There is a limitation on the available storage space area where the items are to be stored, i.e., $w_0Q \leq W$. This restriction on available storage space in the inventory problem cannot be ignored to derive the optimal total cost.

Thus the primal problem for the inventory model can be written as:

$$\begin{aligned} &\text{Min} \quad TAC(D,S,Q) = \frac{SD}{Q} + \frac{aQ^2}{6D} + \frac{bD}{S^xQ^{1+y}} \\ &\text{subject to} \quad C(Q) \equiv w_0Q \leq W, \\ &\qquad D,S,Q > 0. \end{aligned} \tag{2}$$

The problem (2) is a constrained posynomial Primal Geometric Programming Problem (PGPP). Here Degree of Difficulty (DD) [It is defined as DD = total number of terms in objective and constraint functions - total number of decision variables - 1] for the problem = 4-3-1 =0.

Therefore the Dual Geometric Programming Problem (DGPP) of (2) is as follows:

$$Max\ \ d_c(w) = \left(\frac{1}{w_{01}}\right)^{w_{01}} \left(\frac{a}{6w_{02}}\right)^{w_{02}} \left(\frac{b}{w_{03}}\right)^{w_{03}} \left(\frac{w_0}{W}\right)^{w_{11}} \tag{3}$$

subject to the normality and orthogonality conditions

$$\begin{aligned} w_{01}+w_{02}+w_{03}&=1,\\ w_{01}-w_{02}+w_{03}&=0,\\ w_{01}-xw_{03}&=0,\\ -w_{01}+2w_{02}-(1+y)w_{03}+w_{11}&=0, \end{aligned}$$

and the positivity conditions are $w_{01}, w_{02}, w_{03}, w_{11} \geq 0$
where $w = (w_{01}, w_{02}, w_{03}, w_{11})^T$.
Solving the above equations we get the dual variables and the dual objective function as given below:

$$w^*_{01} = \frac{x}{2(1+x)},\ w^*_{02} = \frac{1}{2},\ w^*_{03} = \frac{1}{2(1+x)},\ w^*_{11} = \frac{y-x-1}{2(1+x)},\ and\ d^*_c(w^*) = \left(\frac{2a(x+1)}{3}\right)^{1/2}\left[\frac{b}{x^x}\left(\frac{w_0}{W}\right)^{y-x-1}\right]^{\frac{1}{2(1+x)}} \tag{4}$$

where $w^* = (w^*_{01}, w^*_{02}, w^*_{03}, w^*_{11})^T$.
[Noted that from positivity conditions we have, $x > 0\ and\ y > x+1$.]
Now primal-dual relations for obtaining the decision variables are

$$\frac{SD}{Q} = w^*_{01}d^*_c(w^*),\quad \frac{aQ^2}{6D} = w^*_{02}d^*_c(w^*),\quad \frac{bD}{S^xQ^{1+y}} = w^*_{03}d^*_c(w^*)\quad and\quad \frac{w_0Q}{W} = \frac{w^*_{11}}{w^*_{11}} \tag{5}$$

Solving the above equations (5), the optimum decision variables are obtained as follows:

$$D^* = \left(\frac{a}{6(x+1)}\right)^{1/2}\left[\frac{x^x}{b}\left(\frac{W}{w_0}\right)^{3x+y+3}\right]^{\frac{1}{2(1+x)}},\quad S^* = \left[bx\left(\frac{w_0}{W}\right)^y\right]^{\frac{1}{1+x}},\quad and\quad Q^* = \frac{W}{w_0}. \tag{6}$$

The corresponding optimal value of the cost function $T^*_c(D^*, S^*, Q^*)$ is obtained as

$$T^*_c(D^*, S^*, Q^*) = \left(\frac{2a(x+1)}{3}\right)^{1/2}\left[\frac{b}{x^x}\left(\frac{w_0}{W}\right)^{y-x-1}\right]^{\frac{1}{2(1+x)}} \tag{7}$$

5 Neutrosophic Non-linear Programming

A non-linear programming problem can be written in the following general form:

$$\begin{aligned} &\text{Min} && f(x)\\ &\text{subject to} && g_j(x) \leq c_j, \quad (j = 1, 2, ..., n)\\ &&& x \equiv (x_1, x_2, ..., x_m)^T \geq 0. \end{aligned} \tag{8}$$

Usually the constraint goals are taken as fixed. But in real life one can find that the constraint goals may be imprecise. So let us take the constraint goal be at least $c_j$and the maximum allowable tolerance due to impreciseness be c_{1j} for the j^{th} constraint. For this fact the constraint goals are converted into neurosophic constraint goals.

Thus the non-linear programming problem reduces to the following Neutrosophic Non-linear Programming (NNP) Problem:

$$\begin{aligned} &\text{Min} && f(x)\\ &\text{subject to} && g_j(x) \preceq c_j\ \ with\ maximum\ allowable\ tolerance\ c_{1j},\ \ (j = 1, 2, ..., n)\\ &&& x \geq 0. \end{aligned} \tag{9}$$

6 Solution Procedure

6.1 Step I:

To solve the NNP problem (9) following Werner's Approach the problem is divided into two sub-problems; one sub-problem is considered without maximum allowable tolerance and another sub-problem is considered with maximum allowable tolerance in the constraints. Therefore the two sub-problems are as follows:

Sub-problem I:

$$\begin{aligned} &\text{Min} && f(x)\\ &\text{subject to} && g_j(x) \leq c_j, \quad (j = 1, 2, ..., n)\\ &&& x \geq 0. \end{aligned} \tag{10}$$

Sub-problem II:

$$\begin{aligned} &\text{Min} && f(x)\\ &\text{subject to} && g_j(x) \leq c_j + c_{1j}, \quad (j = 1, 2, ..., n)\\ &&& x \geq 0. \end{aligned} \tag{11}$$

Let the optimum solutions for the two sub-problems (10) and (11) be $(x^{1*}, f(x^{1*}))$ and $(x^{2*}, f(x^{2*}))$ respectively.

6.2 Step II:

For Neutrosophic Optimization Problem (NOP) problem we assume that $U^{\mu}_{f(x)}, U^{\sigma}_{f(x)}, U^{\nu}_{f(x)}$ and $L^{\mu}_{f(x)}, L^{\sigma}_{f(x)}, L^{\nu}_{f(x)}$ be the upper and lower bounds of the truth, indeterminacy and falsity functions for objective respectively.

Now we define those upper and lower bounds as follows:

$$
\begin{aligned}
&U^{\mu}_{f(x)} = max\{f(x^{1*}), f(x^{2*})\}, && L^{\mu}_{f(x)} = min\{f(x^{1*}), f(x^{2*})\},\\
&U^{\sigma}_{f(x)} = L^{\mu}_{f(x)} + \delta_{\sigma}(U^{\mu}_{f(x)} - L^{\mu}_{f(x)}), && L^{\sigma}_{f(x)} = L^{\mu}_{f(x)},\\
&U^{\nu}_{f(x)} = U^{\mu}_{f(x)}, && L^{\nu}_{f(x)} = L^{\mu}_{f(x)} + \delta_{\nu}(U^{\mu}_{f(x)} - L^{\mu}_{f(x)}).
\end{aligned}
$$

where δ_{σ} and δ_{ν} are predetermined real numbers in $(0, 1)$.

Similarly, for the j^{th} constraint let $U^{\mu}_{g_j(x)}, U^{\sigma}_{g_j(x)}, U^{\nu}_{g_j(x)}$ and $L^{\mu}_{g_j(x)}, L^{\sigma}_{g_j(x)}, L^{\nu}_{g_j(x)}$ be the upper and lower bounds of the truth, indeterminacy and falsity functions respectively. Then let us define them as

$$
\begin{aligned}
&U^{\mu}_{g_j(x)} = c_j + c_{1j}, && L^{\mu}_{g_j(x)} = c_j,\\
&U^{\sigma}_{g_j(x)} = L^{\mu}_{g_j(x)} + \epsilon_{\sigma j}, && L^{\sigma}_{g_j(x)} = L^{\mu}_{g_j(x)},\\
&U^{\nu}_{g_j(x)} = U^{\mu}_{g_j(x)}, && L^{\nu}_{g_j(x)} = L^{\mu}_{g_j(x)} + \epsilon_{\nu j}.
\end{aligned}
$$

where $\epsilon_{\sigma j}$ and $\epsilon_{\nu j}$ are predetermined real numbers in with $0 \leq \epsilon_{\sigma j}, \epsilon_{\nu j} \leq c_{1j},\ \ j = 1, 2, ..., n.$

6.3 Step III:

According to the assumptions given in step II the truth, indeterminacy and falsity functions for the objective are defined as follows (See Fig. 2):

$$
\mu_{f(x)}(x) = \begin{cases} 1 & if \ \ f(x) \leq L^{\mu}_{f(x)} \\ \frac{U^{\mu}_{f(x)} - f(x)}{U^{\mu}_{f(x)} - L^{\mu}_{f(x)}} & if \ \ L^{\mu}_{f(x)} \leq f(x) \leq U^{\mu}_{f(x)} \\ 0 & if \ \ f(x) \geq U^{\mu}_{f(x)} \end{cases}
$$

and

$$
\sigma_{f(x)}(x) = \begin{cases} 1 & if \ \ f(x) \leq L^{\sigma}_{f(x)} \\ \frac{U^{\sigma}_{f(x)} - f(x)}{U^{\sigma}_{f(x)} - L^{\sigma}_{f(x)}} & if \ \ L^{\sigma}_{f(x)} \leq f(x) \leq U^{\sigma}_{f(x)} \\ 0 & if \ \ f(x) \geq U^{\sigma}_{f(x)} \end{cases}
$$

and

$$
\nu_{f(x)}(x) = \begin{cases} 0 & if \ \ f(x) \leq L^{\nu}_{f(x)} \\ \frac{f(x) - L^{\nu}_{f(x)}}{U^{\nu}_{f(x)} - L^{\nu}_{f(x)}} & if \ \ L^{\nu}_{f(x)} \leq f(x) \leq U^{\nu}_{f(x)} \\ 1 & if \ \ f(x) \geq U^{\nu}_{f(x)} \end{cases}
$$

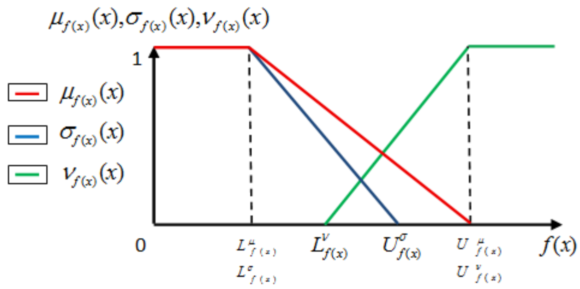

Figure 2: Rough sketch of truth, indeterminacy and falsity functions for objective function

Similarly, according to the assumptions the truth, indeterminacy and falsity functions for the j^{th} constraint are defined as follows (See Fig. 3):

$$
\mu_{g_j(x)}(x) = \begin{cases} 1 & if \ \ g_j(x) \leq L^{\mu}_{g_j(x)} \\ \frac{U^{\mu}_{g_j(x)} - g_j(x)}{U^{\mu}_{g_j(x)} - L^{\mu}_{g_j(x)}} & if \ \ L^{\mu}_{g_j(x)} \leq g_j(x) \leq U^{\mu}_{g_j(x)} \\ 0 & if \ \ g_j(x) \geq U^{\mu}_{g_j(x)} \end{cases}
$$

and

$$
\sigma_{g_j(x)}(x) = \begin{cases} 1 & if \ \ g_j(x) \leq L^{\sigma}_{g_j(x)} \\ \frac{U^{\sigma}_{g_j(x)} - g_j(x)}{U^{\sigma}_{g_j(x)} - L^{\sigma}_{g_j(x)}} & if \ \ L^{\sigma}_{g_j(x)} \leq g(x) \leq U^{\sigma}_{g_j(x)} \\ 0 & if \ \ g_j(x) \geq U^{\sigma}_{g_j(x)} \end{cases}
$$

and

$$\nu_{g_j(x)}(x) = \begin{cases} 0 & if \;\; g_j(x) \le L^{\nu}_{g_j(x)} \\ \frac{g_j(x) - L^{\nu}_{g_j(x)}}{U^{\nu}_{g_j(x)} - L^{\nu}_{g_j x)}} & if \;\; L^{\nu}_{g_j(x)} \le g_j(x) \le U^{\nu}_{g_j(x)} \\ 1 & if \;\; g_j(x) \ge U^{\nu}_{g_j(x)} \end{cases}$$

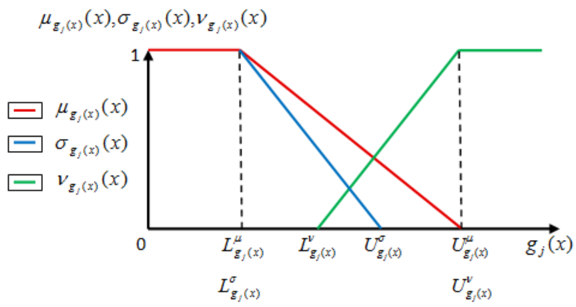

Figure 3: Rough sketch of truth, indeterminacy and falsity functions for j^{th} constraint

6.4 Step IV:

In Neutrosophic Optimization Technique the decision maker wants to maximize the degree of truth and to minimize the degree of indeterminacy and the degree of falsity of the objective and the constraints both. Therefore the NNP problem can be formulated in the following form:

$$\begin{aligned} \text{Max} \quad & \mu_{f(x)}(x), \mu_{g1(x)}(x), \mu_{g2(x)}(x), ..., \mu_{gn(x)}(x), \\ \text{min} \quad & \sigma_{f(x)}(x), \sigma_{g1(x)}(x), \sigma_{g2(x)}(x), ..., \sigma_{gn(x)}(x), \\ \text{min} \quad & \nu_{f(x)}(x), \nu_{g1(x)}(x), \nu_{g2(x)}(x), ..., \nu_{gn(x)}(x) \\ \text{subject to} \quad & \mu_{f(x)}(x) \ge \sigma_{f(x)}(x), \;\; \mu_{g_j(x)}(x) \ge \sigma_{g_j(x)}(x), \quad (j = 1, 2, ..., n) \\ & \mu_{f(x)}(x) \ge \nu_{f(x)}(x), \;\; \mu_{g_j(x)}(x) \ge \nu_{g_j(x)}(x), \quad (j = 1, 2, ..., n) \\ & \mu_{f(x)}(x), \; \sigma_{f(x)}(x), \; \nu_{f(x)}(x), \; \mu_{g_j(x)}(x), \; \sigma_{g_j(x)}(x), \; \nu_{g_j(x)}(x) \in \; [0, 1], \quad (j = 1, 2, ..., n) \\ & x \ge 0. \end{aligned} \tag{12}$$

Based on weighted sum approach with equal weights the above problem reduces to the following crisp non-linear programming problem:

$$\begin{aligned} \text{Max} \quad & VF_A(x) = \mu_{f(x)}(x) + \sum_{j=1}^{n} \mu_{g_j(x)}(x) - \sigma_{f(x)}(x) - \sum_{j=1}^{n} \sigma_{g_j(x)}(x) - \nu_{f(x)}(x) - \sum_{j=1}^{n} \nu_{g_j(x)}(x) \\ \text{subject to} \quad & \mu_{f(x)}(x) \ge \sigma_{f(x)}(x), \;\; \mu_{g_j(x)}(x) \ge \sigma_{g_j(x)}(x), \quad (j = 1, 2, ..., n) \\ & \mu_{f(x)}(x) \ge \nu_{f(x)}(x), \;\; \mu_{g_j(x)}(x) \ge \nu_{g_j(x)}(x), \quad (j = 1, 2, ..., n) \\ & \mu_{f(x)}(x), \; \sigma_{f(x)}(x), \; \nu_{f(x)}(x), \; \mu_{g_j(x)}(x), \; \sigma_{g_j(x)}(x), \; \nu_{g_j(x)}(x) \in \; [0, 1], \quad (j = 1, 2, ..., n) \\ & x \ge 0. \end{aligned} \tag{13}$$

The above problem is equivalent to

$$\begin{aligned} \text{Max} \quad & VF_A(x) = K - VF_{A1}(x) \\ \text{subject to} \quad & f(x) \in [L_f, U_f] \;\; and \;\; g_j(x) \in [L_{g_j}, U_{g_j}], (j = 1, 2, ..., n) \\ & x > 0. \end{aligned} \tag{14}$$

where

$$L_f = \frac{U^{\mu}_{f(x)} L^{\sigma}_{f(x)} - U^{\sigma}_{f(x)} L^{\mu}_{f(x)}}{U^{\mu}_{f(x)} - L^{\mu}_{f(x)} - U^{\sigma}_{f(x)} + L^{\sigma}_{f(x)}}, \qquad U_f = \frac{U^{\mu}_{f(x)} U^{\nu}_{f(x)} - L^{\mu}_{f(x)} L^{\sigma}_{f(x)}}{U^{\mu}_{f(x)} - L^{\mu}_{f(x)} + U^{\nu}_{f(x)} - L^{\nu}_{f(x)}},$$

$$L_{g_j} = \frac{U^{\mu}_{g_j(x)} L^{\sigma}_{g_j(x)} - U^{\sigma}_{g_j(x)} L^{\mu}_{g_j(x)}}{U^{\mu}_{g_j(x)} - L^{\mu}_{g_j(x)} - U^{\sigma}_{g_j(x)} + L^{\sigma}_{g_j(x)}}, \qquad U_{g_j} = \frac{U^{\mu}_{g_j(x)} U^{\nu}_{g_j(x)} - L^{\mu}_{g_j(x)} L^{\sigma}_{g_j(x)}}{U^{\mu}_{g_j(x)} - L^{\mu}_{g_j(x)} + U^{\nu}_{g_j(x)} - L^{\nu}_{g_j(x)}}, \quad (j = 1, 2, ..., n)$$

$$K = \frac{U^{\mu}_{f(x)}}{U^{\mu}_{f(x)} - L^{\mu}_{f(x)}} - \frac{U^{\sigma}_{f(x)}}{U^{\sigma}_{f(x)} - L^{\sigma}_{f(x)}} + \frac{L^{\nu}_{f(x)}}{U^{\nu}_{f(x)} - L^{\nu}_{f(x)}} + \sum_{j=1}^{n} \left[\frac{U^{\mu}_{g_j(x)}}{U^{\mu}_{g_j(x)} - L^{\mu}_{g_j(x)}} - \frac{U^{\sigma} g_j(x)}{U^{\sigma}_{g_j(x)} - L^{\sigma}_{g_j(x)}} + \frac{U^{\nu}_{g_j(x)}}{U^{\nu}_{g_j(x)} - L^{\nu}_{g_j(x)}} \right],$$

and

$$VF_{A1}(x) = \left[\frac{f(x)}{U^{\mu}_{f(x)} - L^{\mu}_{f(x)}} - \frac{f(x)}{U^{\sigma}_{f(x)} - L^{\sigma}_{f(x)}} + \frac{f(x)}{U^{\nu}_{f(x)} - L^{\nu}_{f(x)}} \right] + \sum_{j=1}^{n} \left[\frac{g_j(x)}{U^{\mu}_{g_j(x)} - L^{\mu}_{g_j(x)}} - \frac{g_j(x)}{U^{\sigma}_{g_j(x)} - L^{\sigma}_{g_j(x)}} + \frac{g_j(x)}{U^{\nu}_{g_j(x)} - L^{\nu}_{g_j(x)}} \right].$$

Now it is sufficient to solve the following crisp minimization problem

$$\text{Min} VF_{A1}(x) = \left[\frac{f(x)}{U^{\mu}_{f(x)} - L^{\mu}_{f(x)}} - \frac{f(x)}{U^{\sigma}_{f(x)} - L^{\sigma}_{f(x)}} + \frac{f(x)}{U^{\nu}_{f(x)} - L^{\nu}_{f(x)}} \right] + \sum_{j=1}^{n} \left[\frac{g_j(x)}{U^{\mu}_{g_j(x)} - L^{\mu}_{g_j(x)}} - \frac{g_j(x)}{U^{\sigma}_{g_j(x)} - L^{\sigma}_{g_j(x)}} + \frac{g_j(x)}{U^{\nu}_{g_j(x)} - L^{\nu}_{g_j(x)}} \right]$$

subject to the same restrictions as given in (14)

(15)

If $f(x) = \sum_{k=1}^{P_0} C_{0k} \prod_{r=1}^{m} x_r^{a_{0kr}}$ and $g_j(x) = \sum_{k=1+P_{j-1}}^{P_j} C_{jk} \prod_{r=1}^{m} x_r^{a_{jkr}}$
where $C_{jk} > 0$ for $k = 1, 2, ..., P_j;$ $j = 0, 1, 2, ..., n$ and a_{jkr} $(k = 1, 2, ..., 1 + P_{j-1}, ..., P_j;$ $j = 0, 1, 2, ..., n;$ $r = 1, 2, ..., m.)$ are real numbers.
Then the problem can be taken as a crisp unconstrained posynomial PGPP with $DD = \sum_{j=0}^{n} P_j - m - 1$, provided that the optimal solution of $f(x) \in [L_f, U_f]$ and that of $g_j(x) \in [L_{g_j}, U_{g_j}]$, $j = 1, 2, ..., n$.

7 Application of Neutrosophic Optimization Technique

7.1 An Inventory Model

Consider the inventory model (2) and assume that the storage area is flexible. Also assume that the maximum allowable tolerance be w_p due to impreciseness in the space constraint.
Therefore the inventory problem is converted into the NOP with flexible space constraint as given below:

$$\begin{aligned} &\text{Min} \quad TAC(D, S, Q) = \frac{SD}{Q} + \frac{aQ^2}{6D} + \frac{bD}{S^x Q^{1+y}} \\ &\text{subject to} \quad w_0 Q \preceq W \text{ with maximum allowable tolerance } w_p, \\ &\quad D, S, Q > 0. \end{aligned} \tag{16}$$

According to step I following Werner's Approach we first have to solve the following two sub-problems.
Sub-problem I:

$$\begin{aligned} &\text{Min } TAC(D, S, Q) = \frac{SD}{Q} + \frac{aQ^2}{6D} + \frac{bD}{S^x Q^{1+y}} \\ &\text{subject to} \quad w_0 Q \le W, \\ &\quad D, S, Q > 0. \end{aligned} \tag{17}$$

Sub-problem II:

$$\begin{aligned} &\text{Min } TAC(D, S, Q) = \frac{SD}{Q} + \frac{aQ^2}{6D} + \frac{bD}{S^x Q^{1+y}} \\ &\text{subject to} \quad w_0 Q \le W + w_p, \\ &\quad D, S, Q > 0. \end{aligned} \tag{18}$$

Solving (17) by GP method the optimum decision variables and the corresponding optimal objective T_1 are obtained as:

$$D^* = \left(\frac{a}{6(x+1)} \right)^{1/2} \left[\frac{x^x}{b} \left(\frac{W}{w_0} \right)^{3x+y+3} \right]^{\frac{1}{2(1+x)}}, \quad S^* = \left[bx \left(\frac{w_0}{W} \right)^y \right]^{\frac{1}{1+x}}, \quad Q^* = \frac{W}{w_0},$$

$$\text{and} \quad T_1 = \left(\frac{2a(x+1)}{3} \right)^{1/2} \left[\frac{b}{x^x} \left(\frac{w_0}{W} \right)^{y-x-1} \right]^{\frac{1}{2(1+x)}}. \tag{19}$$

Similarly, solving (18) by GP method the optimum decision variables and the corresponding optimal objective T_0 are obtained as:

$$D^* = \left(\frac{a}{6(x+1)} \right)^{1/2} \left[\frac{x^x}{b} \left(\frac{W + w_p}{w_0} \right)^{3x+y+3} \right]^{\frac{1}{2(1+x)}}, \quad S^* = \left[bx \left(\frac{w_0}{W + w_p} \right)^y \right]^{\frac{1}{1+x}}, \quad Q^* = \frac{W + w_p}{w_0},$$

$$\text{and} \quad T_0 = \left(\frac{2a(x+1)}{3} \right)^{1/2} \left[\frac{b}{x^x} \left(\frac{w_0}{W + w_p} \right)^{y-x-1} \right]^{\frac{1}{2(1+x)}}. \tag{20}$$

According to step II assume the upper and the lower bounds for the truth, indeterminacy and falsity functions for the objective respectively as given below:

$$U_T^{\mu} = max\{T_0, T_1\} = T_1, \qquad L_T^{\mu} = min\{T_0, T_1\} = T_0,$$
$$U_T^{\sigma} = T_0 + \delta_{\sigma}(T_1 - T_0), \qquad L_T^{\sigma} = T_0,$$
$$U_T^{\nu} = T_1, \qquad L_T^{\nu} = T_0 + \delta_{\nu}(T_1 - T_0).$$

where δ_{σ} and δ_{ν} are predetermined real numbers in $(0, 1)$.
Similarly, the upper and the lower bounds for the truth, indeterminacy and falsity functions for the constraint are as follows:

$$U_C^{\mu} = W + w_p, \qquad L_C^{\mu} = W,$$
$$U_C^{\sigma} = W + \epsilon_{\sigma}, \qquad L_C^{\sigma} = W,$$
$$U_C^{\nu} = W + w_p, \qquad L_C^{\nu} = W + \epsilon_{\nu}.$$

where $0 < \epsilon_{\sigma}, \epsilon_{\nu} < w_p$.
Now let us write the truth, indeterminacy and falsity functions for the objective with the help of step III (See Fig. 4).

$$\mu_T(TAC(D,S,Q)) = \begin{cases} 1 & \text{if } \ TAC(D,S,Q) \leq T_0 \\ \frac{T_1 - TAC(D,S,Q)}{T_1 - T_0} & \text{if } \ T_0 \leq TAC(D,S,Q) \leq T_1 \\ 0 & \text{if } \ TAC(D,S,Q) \geq T_1 \end{cases} \tag{21}$$

and

$$\sigma_T(TAC(D,S,Q)) = \begin{cases} 1 & \text{if } \ TAC(D,S,Q) \leq T_0 \\ \frac{T_0 + \delta_{\sigma}(T_1 - T_0) - TAC(D,S,Q)}{\delta_{\sigma}(T_1 - T_0)} & \text{if } \ T_0 \leq TAC(D,S,Q) \leq T_0 + \delta_{\sigma}(T_1 - T_0) \\ 0 & \text{if } \ TAC(D,S,) \geq T_0 + \delta_{\sigma}(T_1 - T_0) \end{cases} \tag{22}$$

and

$$\nu_T(TAC(D,S,Q)) = \begin{cases} 0 & \text{if } \ TAC(D,S,Q) \leq T_0 + \delta_{\nu}(T_1 - T_0) \\ \frac{TAC(D,S,Q) - \{T_0 + \delta_{\nu}(T_1 - T_0)\}}{(T_1 - T_0)(1 - \delta_{\nu})} & \text{if } \ T_0 + \delta_{\nu}(T_1 - T_0) \leq TAC(D,S,Q) \leq T_1 \\ 1 & \text{if } \ TAC(D,S,Q) \geq T_1 \end{cases} \tag{23}$$

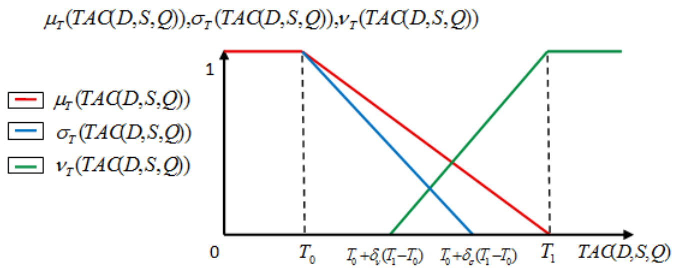

Figure 4: Rough sketch of truth, indeterminacy and falsity functions for objective function

In the same way, the truth, indeterminacy and falsity functions for the constraint are respectively as follows (See Fig. 5):

$$\mu_C(C(Q)) = \begin{cases} 1 & \text{if } \ C(Q) \leq W \\ \frac{W + w_p - C(Q)}{w_p} & \text{if } \ W \leq C(Q) \leq W + w_p \\ 0 & \text{if } \ C(Q) \geq W + w_p \end{cases} \tag{24}$$

and

$$\sigma_C(C(Q)) = \begin{cases} 1 & \text{if } \ C(Q) \leq W \\ \frac{W + \epsilon_{\sigma} - C(Q)}{\epsilon_{\sigma}} & \text{if } \ W \leq C(Q) \leq W + \epsilon_{\sigma} \\ 0 & \text{if } \ C(Q) \geq W + \epsilon_{\sigma} \end{cases} \tag{25}$$

and

$$\nu_C(C(Q)) = \begin{cases} 0 & \text{if } \ C(Q) \leq W + \epsilon_{\nu} \\ \frac{C(Q) - (W + \epsilon_{\nu})}{w_p - \epsilon_{\nu}} & \text{if } \ W + \epsilon_{\nu} \leq C(Q) \leq W + w_p \\ 1 & \text{if } \ C(Q) \geq W + w_p \end{cases} \tag{26}$$

where $0 < \epsilon_1, \epsilon_2 < w_p$.

According to step IV the NOP can be written as an equivalent crisp non-linear programming problem as follows:

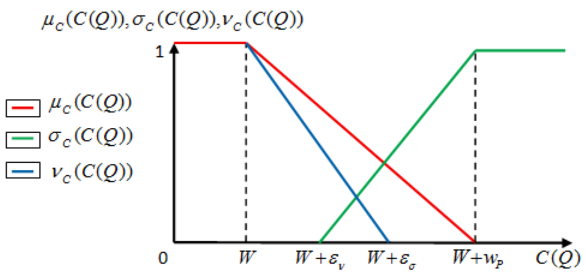

Figure 5: Rough sketch of truth, indeterminacy and falsity functions for constraint

Sub-model I:

$$\begin{aligned}
&\text{Max} \quad VF_{A2}(D,S,Q) = \mu_T(TAC(D,S,Q)) + \mu_C(C(Q) - \sigma_T(TAC(D,S,Q)) - \sigma_C(C(Q)) - \nu_T(TAC(D,S,Q)) - \nu_C(C(Q)) \\
&\text{subject to} \quad \mu_T(TAC(D,S,Q)) \geq \sigma_T(TAC(D,S,Q)), \\
&\qquad \mu_C(C(Q)) \geq \sigma_C(C(Q)), \\
&\qquad \mu_T(TAC(D,S,Q)) \geq \nu_T(TAC(D,S,Q)), \\
&\qquad \mu_C(C(Q)) \geq \nu_C(C(Q)), \\
&\qquad \mu_T(TAC(D,S,Q)), \sigma_T(TAC(D,S,Q)), \nu_T(TAC(D,S,Q)), \\
&\qquad \mu_C(C(Q)), \sigma_C(C(Q)), \nu_C(C(Q)) \in [0,1], \\
&\qquad D,S,Q > 0.
\end{aligned} \tag{27}$$

Above non-linear programming problem can be reduced into the following unconstrained non-linear programming problem.

$$\begin{aligned}
&\text{Min} \quad VF_{A3}(D,S,Q) = K_1\left(\frac{SD}{Q} + \frac{aQ^2}{6D} + \frac{bD}{S^x Q^{1+y}}\right) + K_2 Q \\
&\text{subject to} \quad D,S,Q > 0.
\end{aligned} \tag{28}$$

provided that

$$TAC(D,S,Q) \in \left[T_0, \frac{T_1 + (1-\delta_\nu)T_0}{2-\delta_\nu}\right] \quad and \quad Q \in \left[\frac{W}{w_p}, \frac{W}{w_p} + \frac{{w_p}^2}{(2w_p - \epsilon_\nu)w_0}\right]. \tag{29}$$

where $VF_{A2}(D,S,Q) = \left(\frac{1}{T_1-T_0}\right)\left(T_0 - \frac{T_0}{\delta_\sigma} + \frac{T_0+\delta_\nu(T_1-T_0)}{1-\delta_\nu}\right) + \frac{W}{w_p} - \frac{W}{\epsilon_\sigma} + \frac{W+\epsilon_\nu}{w_p-\epsilon_\nu} - VF_{A3}(D,S,Q),$
$K_1 = \frac{1}{T_1-T_0}\left[1 - \frac{1}{\delta_\sigma} + \frac{1}{1-\delta_\nu}\right] \quad and \quad K_2 = \left[\frac{w_0}{w_p} - \frac{w_0}{\epsilon_\sigma} + \frac{w_0}{w_p-\epsilon_\nu}\right].$

It is an unconstrained posynomial PGPP with DD = 0.
Therefore the DGPP of (28) is as follows:

$$Max\ d_n(w) = \left(\frac{K_1}{w_{01}}\right)^{w_{01}}\left(\frac{aK_1}{6w_{02}}\right)^{w_{02}}\left(\frac{bK_1}{w_{03}}\right)^{w_{03}}\left(\frac{K_2}{w_{04}}\right)^{w_{04}}$$

subject to the normality and orthogonality conditions,

$$\begin{aligned}
w_{01} + w_{02} + w_{03} &= 1, \\
w_{01} - w_{02} &= 0, \\
w_{01} - xw_{03} &= 0, \\
-w_{01} + 2w_{02} - yw_{03} + w_{04} &= 0,
\end{aligned}$$

and the positivity conditions are $\quad w_{01}, w_{02}, w_{03}, w_{04} \geq 0$
$where \ \ w = (w_{01}, w_{02}, w_{03}, w_{04})^T.$
Solving the above equations we get the dual variables as,

$$w_{01}^* = \frac{x}{1+x+y}, \quad w_{02}^* = \frac{1+x}{1+x+y}, \quad w_{03}^* = \frac{1}{1+x+y}, \quad and \quad w_{04}^* = \frac{y-x-1}{1+x+y}. \tag{30}$$

Using the above values we get

$$d_n^*(w^*) = (1+x+y)\left[\left(\frac{a}{6(1+x)}\right)^{1+x} K_1^{2+2x}\frac{b}{x^x}\left(\frac{K_2}{y-x-1}\right)^{y-x-1}\right]^{\frac{1}{1+x+y}} \tag{31}$$

To find the decision variables, the primal dual relations are,

$$\frac{SDK_1}{Q} = w_{01}^* d_n^*(w^*), \quad \frac{aQ^2K_1}{6D} = w_{02}^* d_n^*(w^*), \quad \frac{bDK_1}{S^x Q^{1+y}} = w_{03}^* d_n^*(w^*), \quad K_2 Q = w_{04}^* d_n^*(w^*). \tag{32}$$

By solving the above equations the optimum decision variables and the corresponding optimum objective function $T_n^*(D^*, S^*, Q^*)$ are obtained as:

$$D^* = \left[\left(\frac{a}{6(1+x)}\right)^{2+2x+y} \frac{b}{x^x} \left(\frac{K_1(y-x-1)}{K_2}\right)^{3x+y+3}\right]^{\frac{1}{1+x+y}}, \quad S^* = \left[\left(\frac{6(1+x)}{a}\right)^{y} bx^{1+y} \left(\frac{K_2}{K_1(y-x-1)}\right)^{2y}\right]^{\frac{1}{1+x+y}},$$

$$Q^* = \left[\left(\frac{a}{6(1+x)}\right)^{1+x} \frac{b}{x^x} \left(\frac{K_1(y-x-1)}{K_2}\right)^{2x+2}\right]^{\frac{1}{1+x+y}},$$

$$and \quad T_n^*(D^*, S^*, Q^*) = \left[x^{\frac{1-x+y}{1+x+y}} + (1+x) + 1\right]\left[\left(\frac{a}{6(1+x)}\right)^{1+x} \frac{b}{x^x} \left(\frac{K_1(y-x-1)}{K_2}\right)^{1+x-y}\right]^{\frac{1}{1+x+y}}. \quad (33)$$

We know the fact that in case of neutrosophic set there is no restriction on truth function, indeterminacy function and falsity function other than they are subsets of $]0^-, 1^+[$, thus; $0^- \leq Inf\,\mu + Inf\,\sigma + Inf\,\nu \leq Sup\,\mu + Sup\,\sigma + Sup\,\nu \leq 3^+$.

In the Sub-model I, the indeterminacy function is taken as monotonically non increasing function like truth function. But one can define it as monotonically non decreasing function like falsity function also. In that case, the indeterminacy function for the objective and the constraint respectively will be defined as follows (See Fig. 6 and 7):

$$\sigma_T^{'}(TAC(D,S,Q)) = \begin{cases} 0 & \text{if } TAC(D,S,Q) \leq T_0 + \delta_\sigma(T_1 - T_0) \\ \frac{TAC(D,S,Q)-(T_0+\delta_\sigma(T_1-T_0))}{(T_1-T_0)(1-\delta_\sigma)} & \text{if } T_0 + (T_1 - T_0)\delta_1 \leq TAC(D,S,Q) \leq T_1 \\ 1 & \text{if } TAC(D,S,q) \geq T_1 \end{cases} \quad (34)$$

and

$$\sigma_C^{'}(C(Q)) = \begin{cases} 0 & \text{if } C(Q) \leq W + \epsilon_\sigma \\ \frac{C(Q)-(W+\epsilon_\sigma)}{w_p-\epsilon_\sigma} & \text{if } W + \epsilon_\sigma \leq C(Q) \leq W + w_p \\ 1 & \text{if } C(Q) \geq W + w_p \end{cases} \quad (35)$$

where $\delta_\sigma \in [0,1]$ and $0 < \epsilon_\delta < w_p$.

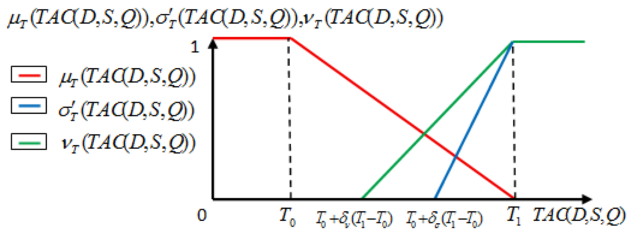

Figure 6: Rough sketch of truth, indeterminacy and falsity functions for objective in Sub-model II

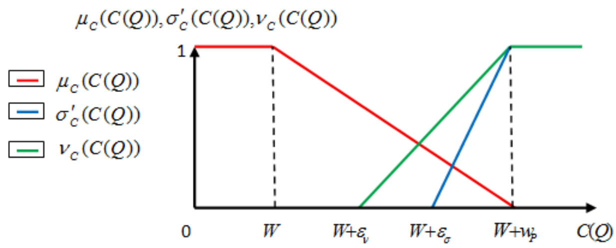

Figure 7: Rough sketch of truth, indeterminacy and falsity functions for constraint in Sub-model II

Now depending on the choice of indeterminacy function we can change the model formulation. If we take the indeterminacy functions as given in (34) and (35) and the truth and falsity functions as they are in Sub-model I, then after applying Neutrosophic Optimization technique the problem (16) reduces to the following crisp non-linear programming problem:

Sub-model II:

$$\begin{aligned}
\text{Max} \quad & VF_{AII}(D,S,Q) = \mu_T(TAC(D,S,Q)) + \mu_C(C(Q)) - \sigma'_T(TAC(D,S,Q)) - \sigma'_C(C(Q)) - \nu_T(TAC(D,S,Q)) - \nu_C(C(Q)) \\
\text{subject to} \quad & \mu_T(TAC(D,S,Q)) \geq \sigma'_T(TAC(D,S,Q)), \\
& \mu_C(C(Q)) \geq \sigma'_C(C(Q)), \\
& \mu_T(TAC(D,S,Q)) \geq \nu_T(TAC(D,S,Q)), \\
& \mu_C(C(Q)) \geq \nu_C(C(Q)), \\
& \mu_T(TAC(D,S,Q)), \sigma'_T(TAC(D,S,Q)), \nu_T(TAC(D,S,Q)), \\
& \mu_C(C(Q)), \sigma'_C(C(Q)), \nu_C(C(Q)) \in [0,1], \\
& D,S,Q > 0.
\end{aligned} \tag{36}$$

In this case also, required restrictions like (29) can be derived as before. Now this problem also can be solved by GP method as shown in Sub-model I.

In another case, one can take the indeterminacy function for the constraint as considered in (35) and consider the indeterminacy function for the objective and the truth and falsity functions for the objective and constraints both as shown in Sub-model I. In this case, we have

Sub-model III:

$$\begin{aligned}
\text{Max} \quad & VF_{AIII}(D,S,Q) = \mu_T(TAC(D,S,Q)) + \mu_C(C(Q)) - \sigma_T(TAC(D,S,Q)) - \sigma'_C(C(Q)) - \nu_T(TAC(D,S,Q)) - \nu_C(C(Q)) \\
\text{subject to} \quad & \mu_T(TAC(D,S,Q)) \geq \sigma_T(TAC(D,S,Q)), \\
& \mu_C(C(Q)) \geq \sigma'_C(C(Q)), \\
& \mu_T(TAC(D,S,Q)) \geq \nu_T(TAC(D,S,Q)), \\
& \mu_C(C(Q)) \geq \nu_C(C(Q)), \\
& \mu_T(TAC(D,S,Q)), \sigma_T(TAC(D,S,Q)), \nu_T(TAC(D,S,Q)), \\
& \mu_C(C(Q)), \sigma'_C(C(Q)), \nu_C(C(Q)) \in [0,1], \\
& D,S,Q > 0.
\end{aligned} \tag{37}$$

Similarly, one can take the indeterminacy function for the objective as given in (34). Now assume that the indeterminacy function for the constraint and the truth and falsity functions for objective and constraint both are same as in Sub-model I. In this case, the problem (16) reduces to

Sub-model IV:

$$\begin{aligned}
\text{Max} \quad & VF_{AIV}(D,S,Q) = \mu_T(TAC(D,S,Q)) + \mu_C(C(Q)) - \sigma'_T(TAC(D,S,Q)) - \sigma_C(C(Q)) - \nu_T(TAC(D,S,Q)) - \nu_C(C(Q)) \\
\text{subject to} \quad & \mu_T(TAC(D,S,Q)) \geq \sigma'_T(TAC(D,S,Q)), \\
& \mu_C(C(Q)) \geq \sigma_C(C(Q)), \\
& \mu_T(TAC(D,S,Q)) \geq \nu_T(TAC(D,S,Q)), \\
& \mu_C(C(Q)) \geq \nu_C(C(Q)), \\
& \mu_T(TAC(D,S,Q)), \sigma'_T(TAC(D,S,Q)), \nu_T(TAC(D,S,Q)), \\
& \mu_C(C(Q)), \sigma_C(C(Q)), \nu_C(C(Q)) \in [0,1], \\
& D,S,Q > 0.
\end{aligned} \tag{38}$$

In Sub-models III and IV, appropriate restrictions like (29) can be derived and they can also be solved proceeding the same solution procedure as in Sub-model I.

7.2 A Non-linear Problem

We consider the following non-linear problem with a constraint in neutrosophic environment:

$$\begin{aligned}
\text{Min} \quad & f(x,y,z) = x^{-1}y^{-2}z^{-3} \\
\text{subject to} \quad & g(x,y,z) \equiv x^3 + y^2 + z \leq 1 \ with\ maximum\ allowable\ tolerance\ 0.5, \\
& x,y,z > 0.
\end{aligned} \tag{39}$$

Now proceeding as in 7.1, we get the following four Sub-models.

Sub-model I:

$$\begin{aligned}
\text{Min} \quad & f_I(x,y,z) = K_I(x^{-1}y^{-2}z^{-3}) + K'_I(x^3 + y^2 + z) \\
\text{subject to} \quad & x,y,z > 0.
\end{aligned} \tag{40}$$

where $K_I = \frac{1}{f_1 - f_0}\left[1 - \frac{1}{\delta_\sigma} + \frac{1}{1-\delta_\nu}\right] \quad and \quad K'_I = \left[\frac{1}{0.5} - \frac{1}{\epsilon_\sigma} + \frac{1}{0.5-\epsilon_\nu}\right].$

Sub-model II:

$$\begin{aligned}
\text{Min} \quad & f_{II}(x,y,z) = K_{II}(x^{-1}y^{-2}z^{-3}) + K'_{II}(x^3 + y^2 + z) \\
\text{subject to} \quad & x,y,z > 0.
\end{aligned} \tag{41}$$

where $K_{II} = \frac{1}{f_1 - f_0}\left[1 + \frac{1}{\delta_\sigma} + \frac{1}{1-\delta_\nu}\right]$ and $K'_{II} = \left[\frac{1}{0.5} + \frac{1}{\epsilon_\sigma} + \frac{1}{0.5-\epsilon_\nu}\right]$.

Sub-model III:

$$\begin{aligned} &\text{Min} \quad f_{III}(x,y,z) = K_{III}(x^{-1}y^{-2}z^{-3}) + K'_{III}(x^3 + y^2 + z) \\ &\text{subject to} \quad x, y, z > 0. \end{aligned} \tag{42}$$

where $K_{III} = \frac{1}{f_1 - f_0}\left[1 + \frac{1}{1-\delta_\sigma} + \frac{1}{1-\delta_\nu}\right]$ and $K'_{III} = \left[\frac{1}{0.5} + \frac{1}{\epsilon_\sigma} + \frac{1}{0.5-\epsilon_\nu}\right]$.

Sub-model IV:

$$\begin{aligned} &\text{Min} \quad f_{IV}(x,y,z) = K_{IV}(x^{-1}y^{-2}z^{-3}) + K'_{IV}(x^3 + y^2 + z) \\ &\text{subject to} \quad x, y, z > 0. \end{aligned} \tag{43}$$

where $K_{IV} = \frac{1}{f_1 - f_0}\left[1 - \frac{1}{\delta_\sigma} + \frac{1}{1-\delta_\nu}\right]$ and $K'_{IV} = \left[\frac{1}{0.5} + \frac{1}{0.5-\epsilon_\sigma} + \frac{1}{0.5-\epsilon_\nu}\right]$.

8 Numerical Experiments

8.1 For Inventory Model 7.1

A manufacturing company produces machines in lots. The company has a warehouse with total floor area (W)= 1000 sq. ft. which is flexible upto (w_p)= 500 sq. ft. to store any excess spare parts, if necessary. The space area per unit quantity is (w_0)= 216 sq. ft. The production cost of the machine is related inversely with the set up (S) and the production quantity (Q). It is known from the past records that the production cost is $5S^{-1}Q^{-3}$. The holding cost per unit item is (H)= 21t.

The decision maker wants to determine the optimal values of demand (D), set-up cost (S), production quantity (Q) and the optimal total average cost $TAC(D, S, Q)$.

According to the input data, the problem (16) becomes

$$\begin{aligned} &\text{Min} \quad TAC(D,S,Q) = \frac{SD}{Q} + \frac{21Q^2}{6D} + \frac{5D}{SQ^4} \\ &\text{subject to} \quad 216Q \preceq 1000 \; with\ maximum\ allowable\ tolerance\ 500, \\ &\qquad D, S, Q > 0. \end{aligned} \tag{44}$$

The values of pre-assigned real numbers on the indeterminacy and falsity functions for objective and constraints are given in Table 1. The optimum results of the Sub-models i.e., Sub-model I, II, III and IV by Neutrosophic Optimization Technique are given in Table 2 and 3. Now we evaluate the optimum solutions of Sub-model I by different optimization techniques; Fuzzy Optimization Technique, Intuitionistic Optimization Technique [80] and Neutrosophic Optimization Technique and present the results in Table 4.

8.2 For Non-linear Problem 7.2

For this problem, the pre-assigned numbers for indeterminacy and falsity functions are assumed as shown in Table 1. The optimum solutions for all the Sub-models (Sub-model I, II, III and IV) by Neutrosophic Optimization Technique are described in Table 2 and 3. In Table 4, we express the optimum solutions of Sub-model I of problem 7.2 by different optimization techniques like Fuzzy Optimization Technique, Intuitionistic Optimization Technique [80] and Neutrosophic Optimization Technique.

Model/Problem	Indeterminacy functiuon		Falsity function	
	Objective δ_σ	Constraint ϵ_σ	Objective δ_ν	Constraint ϵ_ν
Inventory Model 7.1	0.6	250	0.4	240
Non-linear Problem 7.2	0.6	0.3	0.4	0.1

Table 1: Values of pre-assigned numbers in indeterminacy and falsity functions

Model/Problem	Sub-models	Optimum dual variables				Optimum decision variables			Optimum objective function
		w^*_{01}	w^*_{02}	w^*_{03}	w^*_{11}/w^*_{04}	S^*/x^*	D^*/y^*	Q^*/z^*	
Inventory Model 7.1	Sub-problem I	0.25	0.50	0.25	0.25	0.22	27.81	4.3	$T_1 = 5.39$
	Sub-problem II	0.25	0.50	0.25	0.25	0.11	69.26	6.94	$T_0 = 4.87$
	Sub-model I	0.20	0.40	0.20	0.20	0.15	51.22	6.07	$T^*_n(D^*, S^*, Q^*)$ =5.04
Non-linear Problem 7.2	Sub-problem I	1.00	0.33	1.00	3.00	0.46	0.55	0.60	$f_1 = 19.95$
	Sub-problem II	1.00	0.33	1.00	3.00	0.53	0.67	0.90	$f_0 = 5.16$
	Sub-model I	0.19	0.06	0.19	0.56	0.51	0.64	0.82	$f^*(x^*, y^*, z^*)$ =8.60

Table 2: Optimal solution of Sub-model I

9 Discussion

9.1 Verification of restrictions

Here we verify the restrictions given in (29) using the result in Table 2.

Here $T_0 = 4.87$, $TAC^*_n(D^*, S^*, Q^*) = 5.04$ and $\frac{T_1 + (1-\delta_\nu)T_0}{2-\delta_\nu} = 5.20$. i.e., $TAC^*_n(D^*, S^*, Q^*) = 5.04 \in [4.87, 5.20]$.

Model/Problem	Sub-models	Optimum decision variables			Optimum objective function
		S^*/x^*	D^*/y^*	Q^*/z^*	
Inventory Model 7.1	Sub-model II	0.15	48.41	5.93	5.07
	Sub-model III	1.13	2.52	1.59	7.04
	Sub-model IV	0.21	948.46	22.59	3.63
Non-linear Problem 7.2	Sub-model II	0.50	0.61	0.74	13.31
	Sub-model III	0.44	0.50	0.51	1.66
	Sub-model IV	0.58	0.77	1.20	69.91

Table 3: Optimum result of different Sub-models using Neutrosophic Optimization Technique

Model/Problem	Technique	Optimum decision variables			Optimum objective function
		S^*/x^*	D^*/y^*	Q^*/z^*	
Inventory Model 7.1	Fuzzy Optimization	0.16	44.39	5.70	5.12
	Intuitionistic Optimization	0.15	48.67	5.94	5.07
	Neutrosophic Optimization	0.15	51.22	6.07	5.04
Non-linear Problem 7.2	Fuzzy Optimization	0.49	0.60	0.72	14.72
	Intuitionistic Optimization	0.50	0.61	0.45	12.44
	Neutrosophic Optimization	0.51	0.64	0.82	8.60

Table 4: Optimal solution of Model 7.1 and Problem 7.2 using different optimization techniques

Also $\frac{W}{w_0} = 4.63,\ Q^* = 6.07 \quad and \quad \frac{W}{w_0} + \frac{w_p^2}{(2w_p - \epsilon_\nu)w_0} = 6.15$ i.e., $Q^* = 6.07 \in [4.63, 6.15]$.
Similarly for other Sub-models, this type of verifications can be performed.

9.2 Comparison by different methods

Though the fuzzy, intuitionistic fuzzy and neutrosophic fuzzy environments are different, still from the Table 4 it is concluded that Neutrosophic Optimization Technique gives better optimum solution compared with the other techniques for these models.

9.3 Model with best optimum results

From Table 2 and 3 , it is seen that, for the present model 7.1 and problem 7.2, the Sub-model I gives the best result. From this, it does not mean that always Sub-model I will give the best one. In other models, any of the four Sub-models may give the best results.

10 Conclusions

The main objective of this work is to illustrate how Neutrosophic Geometric Programming Technique can be utilized to solve a non-linear programming problem. The concept allows one to define the degree of truth, indeterminacy and falsity functions simultaneously. This research work also presents how to convert NNP into a crisp PGPP with the help of the above mentioned degrees and solve the problem.

In this paper, we have considered two examples- (i) an inventory model and (ii) a non-linear problem with space constraint under neutrosophic environment. From the numerical examples of these two problems, one can observe that Neutrosophic Optimization Technique has given better solution than Fuzzy Optimization and Intuitionistic Optimization Technique. The positive advantages of this technique is that it allows to imitate the real life situation more accurately and hence furnishes more useful solution to the management. This feature has been nicely illustrated in this paper. The limitation of the present investigation is that for illustration, we have restricted ourselves to the problem of GP type. Obviously, it limits the scope of modelling as GP problems demand posynomial expressions with zero degree of difficulty for less computation. However, the proposed method can be applied to any type of linear and non-linear optimization problems in the areas of supply chain management, portfolio management, etc. In the present problem, we have considered only one objective function. Present method can also be applied to multi-objective problems using any one of the available methods to convert multi-objective problem to a single objective one.

Thus, in future, this research work can be extended to develop Neutrosophic Geometric Programming Technique for solving several types of single objective and multi objective inventory models.

11 Acknowledgement

This research work is supported by Indian Institute of Engineering Science and Technology, Shibpur, Howrah. The first author sincerely acknowledges the contributions and is very grateful to them.

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Received: July 27, 2018. Accepted: August 20, 2018.