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*Article*

# Delayed control of ecological and biological networks

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## Abstract

Evolutionary Network Control (ENC) was introduced in 2011 to permit the control of any kind of ecological and biological networks, with an arbitrary number of nodes and links. To date, ENC has been applied with the idea to control biological and ecological networks since the beginning of their system dynamics. This approach has shown to be effective in the control of both continuous-time and discrete-time networks. However a delayed control, where network dynamics are controlled only from a certain point on, could be more economic from a computational viewpoint, and also more feasible from an applicative perspective. For this reason, ENC is further upgraded here to realize the delayed control of ecological and biological nets.

**Keywords** delayed network control; dynamical networks; genetic algorithms; Euclidean distance; network optimization; system dynamics.

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## 1 Introduction

Evolutionary Network Control (ENC; Ferrarini, 2011) has been conceived to enable the control of any kind of ecological and biological networks, with an arbitrary number of nodes and links, from inside (Ferrarini, 2013) and from outside (Ferrarini, 2013b). The endogenous control requires that the network is optimized at the beginning of its dynamics so that it will inertially go to the desired state. Instead, the exogenous control requires that one or more exogenous controllers act upon the network at each time step.

ENC makes use of an integrated solution (system dynamics - genetic optimization - stochastic simulations) to compute uncertainty about network control (Ferrarini, 2013c) and to compute control success and feasibility (Ferrarini, 2013d). ENC opposes the common idea in the scientific literature that controllability of networks should be based on the identification of the set of driver nodes that can guide the system's dynamics (Ferrarini, 2011), in other words on the choice of a subset of nodes that should be selected to be permanently controlled. ENC has been applied to both discrete-time (systems of difference equations) and continuous-time (systems of differential equations) networks.

ENC employs intermediate control functions to locally (step-by-step) drive ecological and biological networks, so that also intermediate steps (not only the final state) are under its strict control (Ferrarini, 2014).

ENC can also globally subdue nonlinear networks (Ferrarini, 2015), impose early or late stability to any kind of ecological and biological network (Ferrarini 2015b) and locally control nonlinear networks (Ferrarini 2016).

ENC has been also expanded to incorporate the multipurpose control of any kind of ecological and biological network (Ferrarini, 2016b). The rationale is that, not one, but at least two, or even more than two, variables can be contemporaneously driven towards the desired equilibrium values. It is useful whenever ecological and biological networks present several taxonomic resolutions that are worthy to be controlled simultaneously.

A decentralized variant of ENC (Ferrarini, 2016c), where only one node and the correspondent input/output links are controlled, has been also introduced as it could be more economic from a computational viewpoint, in particular when the network is very large (i.e. big data).

A further ENC variant, based on the inhibition of one or several nodes and/or edges, permits to more easily and parsimoniously subdue biological and ecological networks (Ferrarini, 2016d). Another task of ENC is the control of network flows at equilibrium (Ferrarini, 2017).

To date, ENC has been applied with the idea in mind to control biological and ecological networks since the beginning of their system dynamics. However a delayed control, where network dynamics are controlled only from a certain point on, could be more economic from a computational viewpoint, and also more feasible from an applicative perspective. For this reason, ENC has been further upgraded here to realize the delayed control of ecological and biological nets.

**Table 1** Evolutionary Network Control (ENC) and its applications.

| Reference | Goal |
|---|---|
| Ferrarini 2011 | Theoretical bases of Evolutionary Network Control |
| Ferrarini 2013 | Endogenous control of linear ecological and biological networks |
| Ferrarini 2013b | Exogenous control of linear ecological and biological networks |
| Ferrarini 2013c | Computing the uncertainty associated with network control |
| Ferrarini 2013d | Computing the degree of success and feasibility of network control |
| Ferrarini 2014 | Local control of linear ecological and biological networks |
| Ferrarini 2015 | Global control of nonlinear ecological and biological networks |
| Ferrarini 2015b | Imposing early/late stability to linear and nonlinear networks |
| Ferrarini 2016 | Local control of nonlinear ecological and biological networks |
| Ferrarini 2016b | Multipurpose control of ecological and biological networks |
| Ferrarini 2016c | Decentralized control of ecological and biological networks |
| Ferrarini 2016d | Structural control of ecological and biological networks |
| Ferrarini 2017 | Control of network flows at equilibrium |
| This work | Delayed control of ecological and biological networks |

## 2 Delayed Evolutionary Network Control: Mathematical Formulation

An ecological (or biological) dynamical system of *n* interacting taxonomic resolutions (species, genera, family, etc.) or aggregated assemblages of taxa (e.g., phytoplankton) is as follows

$$\frac{d\mathbf{S}}{dt} = \gamma(\mathbf{S}(t)) \tag{1}$$

where $S_i \in \mathbf{S}$ is the number of individuals (or the total biomass) of the generic *i-th* taxonomic resolution (species, genera, family, or aggregated assemblages of taxa). If we also consider inputs (e.g. immigration) and outputs (e.g. emigration) from-to outside, we must write:

$$\frac{d\mathbf{S}}{dt} = \gamma(\mathbf{S}(t)) + \mathbf{I}(t) + \mathbf{O}(t) \tag{2}$$

At the beginning of its dynamics, the network values are

$$\mathbf{S}_0 = <S_1(0), S_2(0) \ldots S_n(0)> \qquad (3).$$

while at the generic time *t*

$$\mathbf{S}_t = <S_1(t), S_2(t) \ldots S_n(t)> \tag{4}$$

Now let's introduce a desired solution at equilibrium for the dynamics of the studied network

$$\mathbf{S}_d = <S_1(d), S_2(d) \ldots S_n(d)> \tag{5}$$

At each time step, the distance between $\mathbf{S}_t$ and $\mathbf{S}_d$ can be computed as Euclidean distance in the *n*-dimension space of the *n*-variable network dynamics:

$$Dist\ (\mathbf{S}_t, \mathbf{S}_d) = \sqrt{(S_{1t} - S_{1d})^2 + (S_{2t} - S_{2d})^2 + \ldots + (S_{nt} - S_{nd})^2} \tag{6}$$

Now, let's introduce a desired threshold distance $T_d$ which triggers the activation of the evolutionary network control (ENC) as follows:

$$\begin{cases} IF & Dist\ (\mathbf{S}_t, \mathbf{S}_d) > T_d \\ THEN & \text{ENC is off} \\ ELSE & \text{ENC is on} \end{cases} \tag{7}$$

In other words, ENC is activated if, and only if, the ecological and biological net under study is sufficiently near to the desired solution at equilibrium.

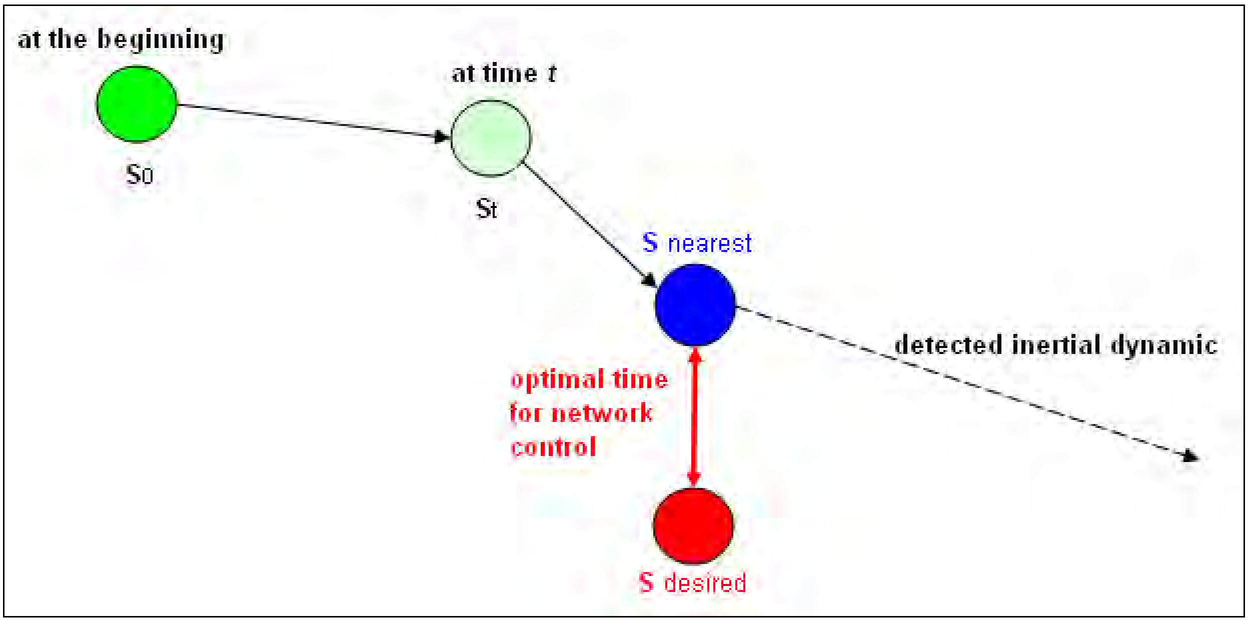

**Fig. 1** The idea behind Delayed Evolutionary Network Control (D-ENC). Rather than controlling network dynamics since their beginning, D-ENC seeks the optimal time ($T_{opt}$) to activate network control. The optimal time happens when network dynamics are closer to the desired solution than a pre-defined threshold value. At that point in time, ENC activates to drive the ecological/biological network towards the desired solution.

Once that ENC is on, it solves the control of Eq. 2 using the following approach (Ferrarini, 2013)

$$\begin{cases} \dfrac{dS_1}{dt} = a_{11*}S_1^{\ *} + \ldots + a_{1n*}S_n^{\ *} + I_{1*} + O_{1*} \\ \ldots \\ \dfrac{dS_n}{dt} = a_{n1*}S_1^{\ *} + \ldots + a_{nn*}S_n^{\ *} + I_{n*} + O_{n*} \end{cases} \tag{8}$$

where any component (variable, parameter or coefficient) of Eq. 8 can be tamed, as denoted by the asterisk, using genetic optimization (Holland, 1975; Goldberg, 1989) to drive the network towards the desired state.

ENC can also use an exogenous network control using an external controller $C_1$ (Ferrarini, 2013b)

$$\begin{cases} \dfrac{dS_1}{dt} = a_{11}S_1 + \ldots + a_{1n}S_n + I_1 + O_1 + c_{11*}C_{1*} \\ \ldots \\ \dfrac{dS_n}{dt} = a_{n1}S_1 + \ldots + a_{nn}S_n + I_n + O_n + c_{n1*}C_{1*} \\ \dfrac{dC_1}{dt} = f_1S_1 + \ldots + f_nS_n \end{cases} \tag{9}$$

where asterisks stand for the genetic optimization of exogenous node's edges (i.e., coefficients of interaction with the inner system) and exogenous node's stock. The controller $C_1$ that can also receive feedbacks from the network, which could be subject to control by subduing $<f_1 \ldots f_n>$. In case 1 controller is not enough, the model in (9) must be expanded to the following *k*-external-controllers model (Ferrarini, 2013b):

$$\begin{cases} \dfrac{dS_1}{dt} = a_{11}S_1 + \ldots + a_{1n}S_n + I_1 + O_1 + c_{11*}C_{1*} + \ldots + c_{1k*}C_{k*} \\ \ldots \\ \dfrac{dS_n}{dt} = a_{n1}S_1 + \ldots + a_{nn}S_n + I_n + O_n + c_{n1*}C_{1*} + \ldots + c_{nk*}C_{k*} \\ \dfrac{dC_1}{dt} = f_{11}S_1 + \ldots + f_{1n}S_n \\ \ldots \\ \dfrac{dC_k}{dt} = f_{k1}S_1 + \ldots + f_{kn}S_n \end{cases} \tag{10}$$

Many ecological (or biological) dynamical systems can be more properly described using difference (recurrent) equations rather than differential ones. This is true for many systems where dynamics happen on discrete, rather than continuous, time. In this case, the ruling equation becomes

$$\begin{cases} (S_1)_{t+1} = a_{11}(S_1)_t + \ldots + a_{1n}(S_n)_t + (I_1)_t + (O_1)_t \\ \ldots \\ (S_n)_{t+1} = a_{n1}(S_1)_t + \ldots + a_{nn}(S_n)_t + (I_n)_t + (O_n)_t \end{cases} \tag{11}$$

ENC solves the control of Eq. 11 using the following approach

$$\begin{cases} \left(S_1\right)_{t+1} = a_{11*}\left(S_1\right)_t + \ldots + a_{1n*}\left(S_n\right)_t + \left(I_1\right)_{t*} + \left(O_1\right)_{t*} \\ \ldots \\ \left(S_n\right)_{t+1} = a_{n1*}\left(S_1\right)_t + \ldots + a_{nn*}\left(S_n\right)_t + \left(I_n\right)_{t*} + \left(O_n\right)_{t*} \end{cases} \tag{12}$$

Delayed ENC can be applied using the software Control-Lab 8 (Ferrarini, 2017b).

## 3 Conclusions

Evolutionary network control (ENC) has been introduced as a methodology where an arbitrary number of network nodes and links can be subdued to drive the network dynamics towards the desired outputs. In previous studies, ENC has shown to be very effective in the control of ecological and biological networks. A delayed control could be more economic from a computational viewpoint, in particular when the network is very large. In this sense, delayed ENC results very promising when applied to big data, the new frontier of network dynamics and control.

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