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Statistical analysis of wind speed and wind power potential of Port Elizabeth using Weibull parameters

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Abstract

This paper analyses wind speed characteristics and wind power potential of Port Elizabeth using statistical Weibull parameters. A measured 5–minute time series average wind speed over a period of 5 years (2005 - 2009) was obtained from the South African Weather Service (SAWS). The results show that the shape parameter (k) ranges from 1.319 in April 2006 to 2.107 in November 2009, while the scale parameter (c) varies from 3.983m/s in May 2008 to 7.390 in November 2009.The average wind power density is highest during Spring (September–October), 256.505W/m^2 and lowest during Autumn (April-May), 152.381W/m^2. This paper is relevant to a decision-making process on significant investment in a wind power project.

Keywords: statistical analysis, wind power density, wind speed, Weibull parameters, Port Elizabeth

1. Introduction

Detailed knowledge of wind speed characteristics and distribution of a particular location is important to the wind industry. This basic information enables the turbine designers to optimise the output of their turbines at the lowest generating cost. Wind power investors use it to estimate possible income from their investment, and it also serves as a control tool for utility operators charged with generation dispatch to reduce threats to the security of the power system as a result of variation in wind speed.

In South Africa, the major indigenous energy resource for electricity generation is coal, which constitutes 88% of the country's primary energy mix. This makes South Africa the 14th highest emitter of greenhouse gas (GHG) in the world (Africa.info, 2010). However, as a signatory to the UN Framework Convention on Climate Change and the Kyoto Protocol, the country is committed to reducing GHG emissions. A step towards honouring this commitment occurred in 2008 when the first phase of the 5.2MW Darling wind farm comprising 4 wind turbines (1.3 MW Fuhrlaender Gmbh-Germany each) was commissioned. Another two, 30 MW in the Eastern Cape and 50 MW at St Helena Bay, are being planned. An estimated 25-40 GW new generating capacity is anticipated by the year 2025 to meet the increasing demand in South Africa (Refocus, 2002), with 20% of this forecast expected to come from wind power (SAWEA, 2010). In view of this development, there is a need to have a clear appreciation of the characteristics and distribution of wind speed data in the country. This is an important aspect of decision-making process on significant investment in a wind power project.

Wind speed variability is often described by the Weibull two-parameter distribution function. It is considered a standard approach for evaluating local wind load probabilities, because it has been found to fit a wide collection of wind data (Gupta and Biswas, 2010, Lun and Lam, 2000, Seguro and Lambert, 2000, Ulgen and Hepbasli, 2002, Weisser, 2003).

Many authors point out that the Weibull distribution gives a better fit for wind speed data when compared to other families of distribution. The probability of observing various wind speeds was compared for both Weibull and Rayleigh's distribution in Celik (2004), Gupta and Biswas (2010) and Ulgen and Hepbasli (2002). It was concluded that the Weibull distribution gives a better fit than Rayleigh's distribution. The lognormal and the Weibull distributions were compared in (Zaharim et al., 2009). The Weibull distribution was also reported to give a superior fit compared to lognormal distribution. Several methods to calculate the Weibull parameters have been proposed and are well documented in the literature. These include the maximum likelihood method, least square techniques and method of moment (Harter and Moore, 1965, Seguro and Lambert, 2000). Various studies recommend the maximum likelihood method when wind data are in time–series format, while the modified likelihood method is preferable for wind data in frequency distribution format (Seguro and Lambert, 2000).

A number of studies have been conducted on wind power potential around the world in the past few years (Gupta and Biswas, 2010, Ulgen and Hepbasli, 2002, Weisser, 2003) with a few conducted in South Africa.

Port Elizabeth that is situated on the coast of South Africa has four distinct seasons: Spring (September–October), Summer (November–March), Autumn (April–May) and Winter (June–August). By virtue of its location it is deemed to have abundant wind resources for grid integration of wind power. However, to date no detailed statistical analyses of wind speed characteristics and wind power potential of this area for wind power generation have been done.

The objective of this paper is to assess statistically the wind speed distribution and to determine the wind power density in Port Elizabeth for grid integration of wind power using Weibull parameters.

2. Wind speed data

The wind speed data for the study in 5-minute time series format were measured from 2005–2009 at station 0035209B1 located in Port Elizabeth, South Africa. The data was obtained from the South African Weather Services (SAWS) which has been in existence for over 150 years. Wind speeds were recorded at an anemometer height of 10m. A total of 525 871 wind speed observations were analysed with 4171 missing data. This is probably due to a breakdown of the anemometer at such times. The missing data which constitutes less than 1% of the total data was removed, the 99% of the remaining data is considered to be valid.

3. The Weibull distribution and wind turbine characteristics

3.1 Statistical estimation of Weibull parameters

The Weibull probability density function for representing the Weibull distribution can be expressed as (1) (Bilgili et al., 2004, Ulgen and Hepbasli, 2002):

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1}\exp\left[-\left(\frac{v}{c}\right)^{k}\right] \quad (1)$$

where $f(v)$ is the probability of observing wind speed, v.Wind speed is a stochastic process that its descriptive parameters (mean, variance, standard deviation) can be obtained. The Weibull shape and scale parameter are denoted by k and c, respectively. k is dimensionless and it indicates how peak the site under consideration is, while c has a unit of wind speed (m/s) and it shows how windy the site is. The cumulative distribution function is given by:

$$F(v) = 1 - \exp\left[-\left(\frac{v}{c}\right)^{k}\right] \quad (2)$$

The double logarithm of equation (2) yields:

$$1n[-1n(1 - F(v))] = k\ln(v) - k\ln(c) \quad (3)$$

A plot of $1n[-1n(1 - F(v))$ against $1n(v)$ results in a straight line. The gradient of the line give,and the intercept on the $1n[-1n(1 - F(v))]$ axis will yield $-k\ln(c)$. Hence, k and c can be graphically determined.

The Weibull parameters can be estimated once the average, v_m, and the variance, σ^2,of the wind speed are known. The Weibull parameters and average wind speed are related by equations (4) and (5) (Gupta and Biswas, 2010, Lu et al., 2002):

$$k = \left(\frac{\sigma}{v_m}\right)^{-1.086} \quad (1 \le k \le 10) \quad (4)$$

$$c = \frac{v_m}{\Gamma\left(1 + \frac{1}{k}\right)} \quad (5)$$

The average wind speed and the variance are calculated by (6) and (7):

$$v_m = \frac{1}{n}\left[\sum_{i=1}^{n} v_i\right] \quad (6)$$

$$\sigma^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n}(v_i - v_m)^2\right] \quad (7)$$

$\Gamma(.)$ is a gamma function. It has the properties of (8):

3.2 Probability assessment of wind power density

The average available power in the flow of wind can be written as (9) (Lu et al., 2002):

$$P = \frac{1}{2}\rho A\int_0^{\infty} v^3 f(v)\,dv \qquad (9)$$

A is the swept area in m^2 normal to the flow and ρ is the air density in kg/m^3, it is assumed to be $1.225 kg/m^3$ in this article. Substituting (1) in (9) yields (10):

$$P = \frac{1}{2}\rho A v_m^3 \left| \frac{\Gamma(1+3/k)}{\Gamma^3(1+1/k)} \right| \qquad (10)$$

Substituting (5) in (10) results in (11):

$$P_d = \frac{1}{2}\rho c^3 \Gamma(1+3/k) \qquad (11)$$

Pd = P/A is the power density which can be evaluated once k and c are known.

The most probable wind speed, vmp (m/s), and the wind speed carrying the maximum energy, v_{max} (m/s), can be determined using the Weibull parameters k and c (Akpinar and Akpinar, 2005). The most probable wind speed is the most frequent wind speed for a given wind probability distribution and is given by equation (12) (Akpinar and Akpinar, 2005):

$$v_{mp} = c\left(\frac{k-1}{k}\right)^{\frac{1}{k}} \qquad (12)$$

The wind speed that carries the maximum amount of wind energy is expressed as (13):

$$v_{max} = c\left(\frac{k+2}{k}\right)^{\frac{1}{k}} \qquad (13)$$

3.3 Wind turbine selection

For any given site, the wind turbine choice is made on the basis of the wind profile of the site. Once the most probable and the maximum wind speeds are known, the wind turbine operating range can be estimated and is given by (14) (Nigim and Parker, 2007):

$$\begin{aligned} & v_{max} \le v_{co} \le (2 \text{ to } 4) v_{max} \\ & (1.5 \text{ to } 3) v_{mp} \le v_{rated} \le (2 \text{ to } 4) v_{ci} \\ & 0.3 v_{mp} \le v_{ci} \le 0.8 v_{mp} \end{aligned} \qquad (14)$$

where v_{co} is the wind speed at which the wind turbine shuts down (cut–out wind speed), vci is the wind speed at which the wind turbine starts to produce power known as cut–in wind speed and v_{rated} is the wind speed at which the wind turbine operates at full rating.

The power output performance curve of a wind turbine varies from one wind turbine to another. The equation for simulating the power output of a wind turbine can be expressed as (15) (Lu et al., 2002):

$$P_G(v) = \begin{cases} P_{rated}\dfrac{v^k - v_{ci}^k}{v_{rated}^k - v_{ci}^k} & (v_{ci} < v < v_{rated}) \\ P_{rated} & (v_{rated} \le v \le v_{co}) \\ 0 & v < v_{ci} \text{ and } v > v_{co} \end{cases} \qquad (15)$$

3.4 Wind speed variation with height

Wind speed changes with height, and most wind speeds are observed at a height less than the hub height. It is therefore necessary to re–define the wind speed from the observed height to the hub height using the expression in (16) (Di Piazza et al., 2010):

$$v_2 = v_1\left(\frac{H_2}{H_1}\right)^m \qquad (16)$$

where v_2 is the wind speed at the hub height H_2 and v1is the wind speed at the observed height H_1. The exponential is the factor that depends on surface roughness and atmospheric stability. It is usually in the range of 0.05-0.5, the most frequently adopted value is 0.14 (Akpinar and Akpinar, 2005). This is widely applicable to a low surface, well exposed site and it is adopted in this paper.

4. Result and analysis

4.1 The Weibull parameters and probability distribution

The monthly and annual Weibull shape and the scale parameters are presented in Table 1. Calculations for these parameters were based on the mathematical model in section 3 and the data obtained from the SAWS. On the table, the shape and the scale parameters show a wide monthly and annual variation. In 2009, the highest scale parameter is 7.390m/s in November while the lowest scale parameter is 5.038m/s in July. The annual shape and scale parameters range from 1.545 in 2005 to 1.753 in 2009, and from 5.204m /s in 2006 to 6.368m /s in 2009.

The probability distribution showing the monthly variation in wind speed in 2009 is shown in Figure 1 and it indicates that probability of observing higher wind speeds in the months of October and November is greater than in other months. The monthly cumulative distributions of the wind speeds in 2009 are depicted in Figure 2. The probability distributions and the cumulative distributions showing the annual wind speed distribution are illustrated in Figures 3 and 4. These figures show an increase in the probability of observing higher wind speeds from 2006 to 2009.This trend shows that higher wind speeds have been experienced every year for the past 4 years.

Seasonal variations in the shape and scale parameters are presented in Table 2. The scale parameter is highest during Spring of each year, indicating the most windy season of the year. Autumn presents the lowest scale parameter showing that it is the least windy season. The probability densities, showing the seasonal variations in wind speed, are shown in Figures 5 and 6. These figures indicate that there is a higher probability of observing high wind speeds in Spring and Summer when compared to Winter and Autumn. Figure 7 depicts the distributions of wind speed in the same season (Summer) in different years and it shows that there are wide variations in the distribution.

4.2 The wind speed characteristics and power densities in Port Elizabeth

The annual and the seasonal wind speed characteristics are presented in Tables 3 and 4. The mean monthly variations in wind speed are plotted in Figure 8, which shows that lower wind speeds are experienced between April and August in all the years. Figure 9 shows the prospect of more wind power generation from September to March. Low average power density is experienced from April to July. The monthly wind average power densities for

Table 1: Monthly shape parameters (k) and scale parameters, c (m/s) (2005 - 2009)

Month	Parameters	2005	2006	2007	2008	2009	Whole year
January	k	1.996	1.887	1.893	1.934	2.067	1.912
		5.951	5.494	6.146	5.580	7.308	6.095
February	k	1.753	1.785	1.784	1.881	1.798	1.765
	c	5.452	5.371	6.939	5.932	6.390	6.015
March	k	1.500	1.612	1.684	1.787	1.986	1.675
	k	4.601	5.508	5.974	5.651	6.998	5.743
April	k	1.626	1.319	1.582	1.559	1.576	1.504
	c	4.448	4.505	5.349	5.670	5.522	5.094
May	k	1532	1.469	1.388	1.567	1.600	1.478
	c	4.932	4.888	4.785	3.983	5.721	4.850
June	k	1.384	1.321	1.554	1.604	1.441	1.430
	c	4.648	4.097	4.548	5.133	5.720	4.825
July	k	1.359	1.604	1.349	1.509	1.749	1.486
	c	4.396	4.499	5.037	4.863	5.038	4.770
August	k	1.438	1.509	1.674	1.710	1.652	1.585
	c	5.459	5.820	5.600	6.385	5.852	5.825
September	k	1.852	1.624	1.870	1.626	1.768	1.668
	c	6.412	5.771	5.311	6.999	6.698	6.130
October	k	1.640	1.791	1.883	1.785	2.071	1.760
	c	7.053	5.412	6.735	6.692	7.273	6.625
November	k	1.620	2.134	1.882	1.886	2.107	1.838
	c	5.738	5.349	5.808	7.251	7.390	6.306
December	k	1.6326	1.647	2.074	1.601	1.904	1.745
	c	5.887	5.644	6.112	6.243	6.466	6.077
Yearly	k	1.545	1.594	1.669	1.633	1.753	1.629
	c	5.366	5.204	5.700	5.848	6.368	5.696

Table 2: Seasonal shape parameters (k) and scale parameters, c (m/s) (2005–2009)

Season	Parameters	2005	2006	2007	2008	2009	Whole year
Summer	k	1.670	1.778	1.834	1.766	1.962	1.786
	c	5.528	5.482	6.185	6.124	6.919	6.029
Winter	k	1.382	1.430	1496	1.580	1.574	1.487
	c	4.832	4.801	5.072	5.456	5.528	5.138
Spring	k	1.639	1.693	1.821	1.705	1.838	1.712
	c	6.481	5.588	6.027	6.847	6.985	6.381
Autumn	k	1.563	1.391	1.478	1.482	1.588	1.489
	c	4.690	4.700	5.064	4.784	5.622	4.970

all the years are compared in Figure 10. The seasonal variations in the average power densities are illustrated in Figure 11. There is a steady increase in seasonal power densities from 2006 to 2009 with Spring having the highest wind power density.

Wind speed and power density are calculated for different heights using (16), assuming a surface factor of 0.14, and this is presented in Table 5. The variations in wind speed and power density at different heights are illustrated in Figure 12. The shape parameter does not change with height while the scale parameter increases as the height increases.

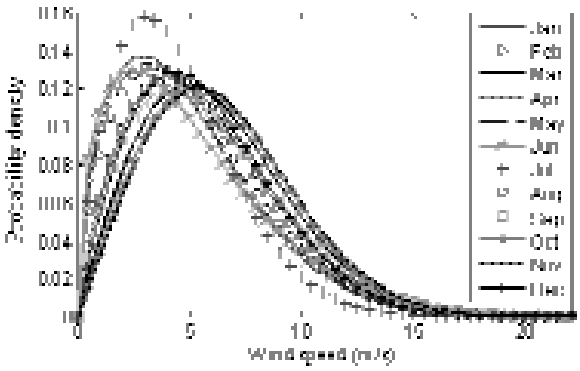

Figure 1: Probability density function showing the monthly variation in wind speed (Jan–Dec, 2009)

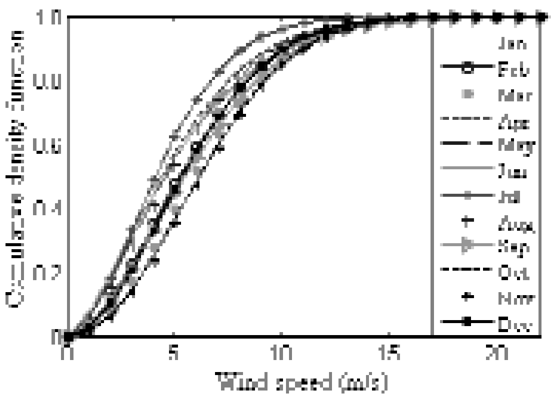

Figure 2: Monthly wind speed cumulative probaility distribution (Jan–Dec, 2009)

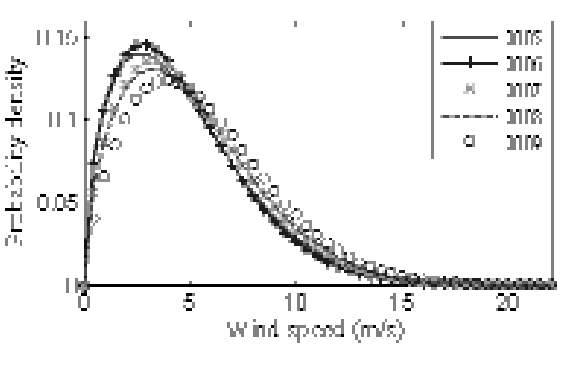

Figure: 3 Probability density function showing the annual variations in wind speed (2005–2009)

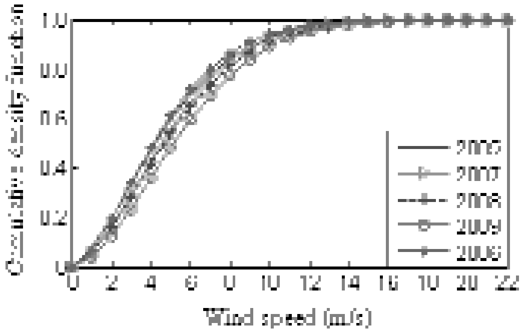

Figure 4: Yearly wind speed cumulative probability distribution for each of the years (2005–2009)

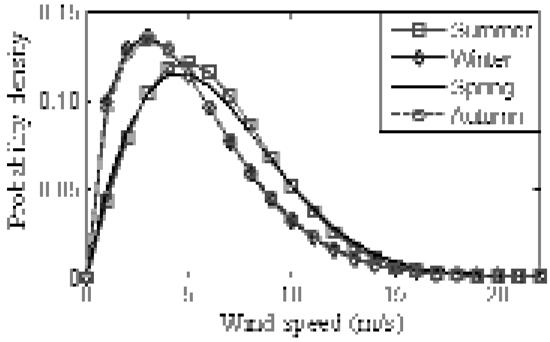

Figure 5: Probability density function showing the seasonal variations in wind speed in 2009

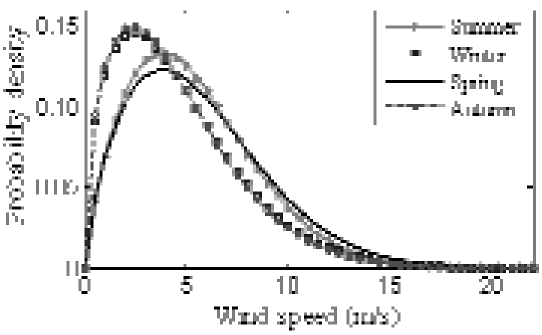

Figure 6: Probability density function showing seasonal variations in wind speed considering the five years period at a time (whole year)

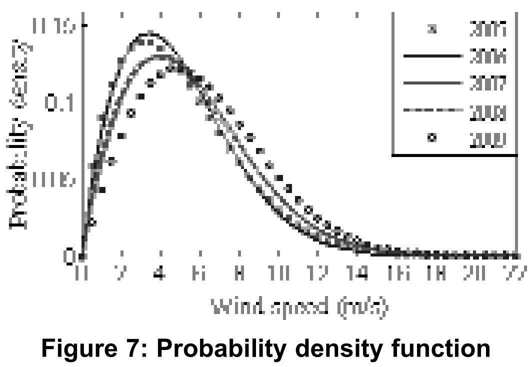

Figure 7: Probability density function comparing variations in the same season (Summer) from 2005–2009

Table 3: Annual wind speed characteristics in Port Elizabeth

Year	*vm (m/s)*	*σ2(m/s)*	*k*	*c(m/s)*	v_{mp} *(m/s)*	v_{max} *(m/s)*	P_d *(W/m²)*
2005	4.828	10.464	1.545	5.366	2.734	9.187	179.539
2006	4.667	9.228	1.594	5.204	2.802	8.665	155.224
2007	5.092	10.092	1.669	5.700	3.297	9.136	189.683
2008	5.234	11.099	1.633	5.848	3.274	9.542	211.895
2009	5.671	11.434	1.753	6.368	3.933	9.830	246.499
Whole year	5.099	10.585	1.629	5.696	3.176	9.314	196.625

Table 4: Seasonal wind speed characteristics for the whole year

	v_m *(m/s)*	σ^2*(m/s)*	*k*	*c(m/s)*	v_{mp} *(m/s)*	v_{max} *(m/s)*	P_d *(W/m²)*
Summer	5.363	9.881	1.786	6.029	3.808	9.180	203.926
Winter	4.643	10.379	1.487	5.138	2.426	9.112	168.806
Spring	5.691	12.036	1.712	6.381	3.821	10.029	256.505
Autumn	4.490	9.679	1.489	4.970	2.353	8.803	152.381

Table 5: Variation of the wind characteristic with height considering the whole year

Height (m)	v_m (m/s)	σ^2(m/s)	k	c(m/s)	v_{mp} (m/s)	v_{max} (m/s)	P_d (W/m²)
10	5.099	10.585	1.629	5.696	3.176	9.314	196.625
20	5.619	12.853	1.629	6.277	3.500	10.163	263.073
50	6.388	16.612	1.629	7.136	3.979	11.668	386.568
80	6.822	18.948	1.629	7.621	4.250	12.462	470.921
100	7.039	20.170	1.629	7.863	4.384	12.857	517.181

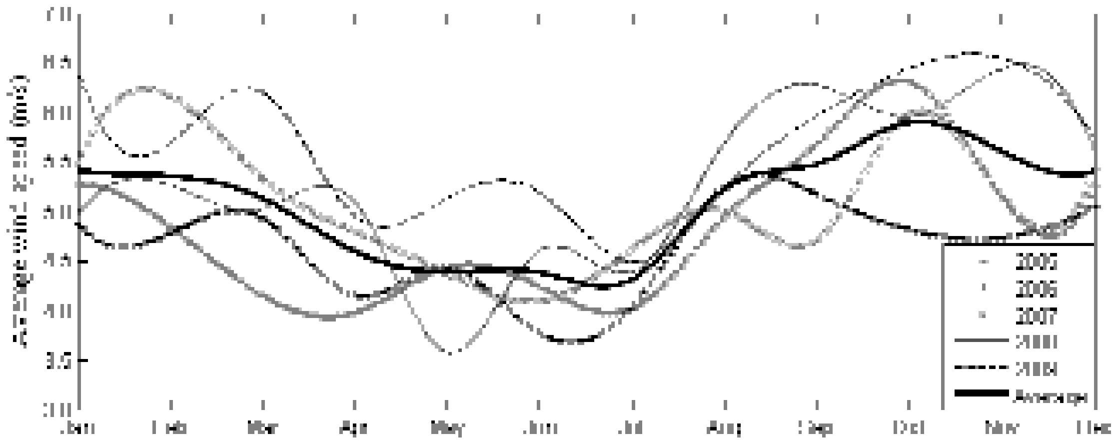

Figure 8: Average monthly variations of wind speeds

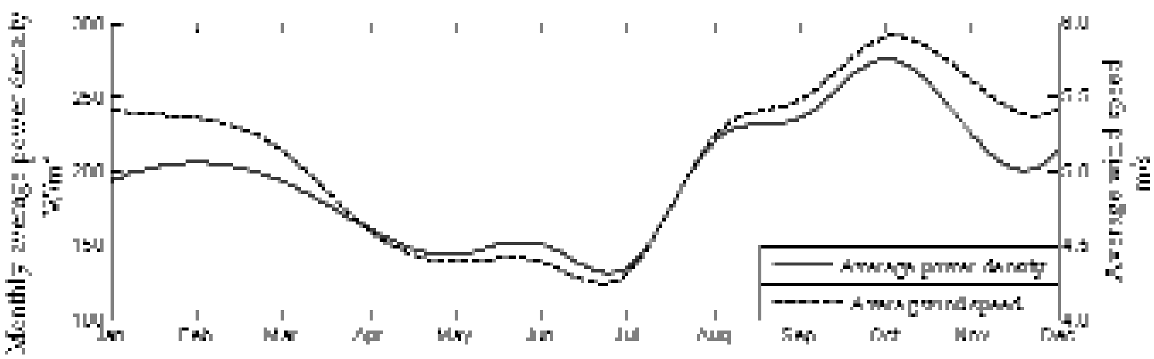

Figure 9: Variations in monthly average power densities of wind speed for the whole year

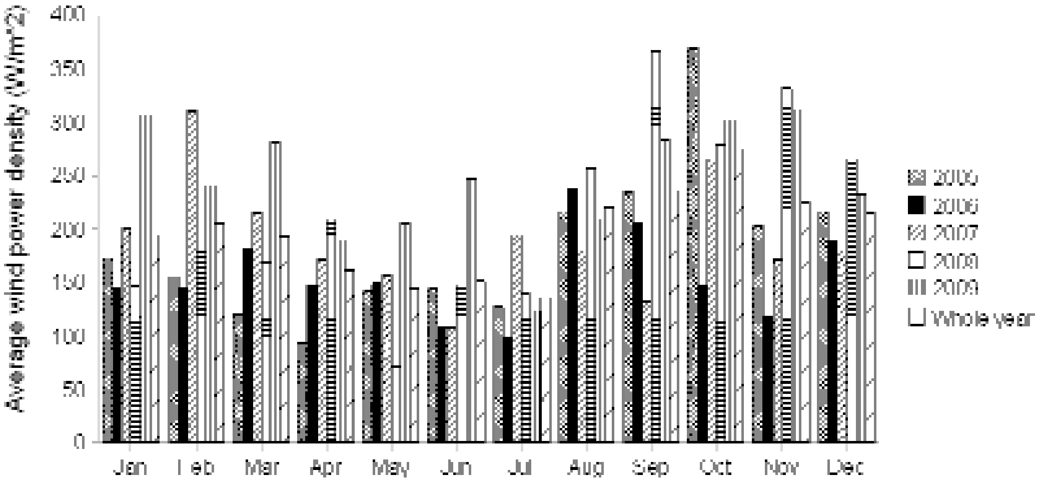

Figure 10: Plot of monthly variations in wind power densities (2005–2009)

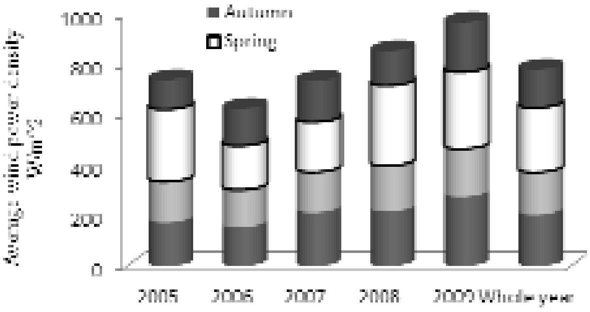

Figure 11: Seasonal variation in average wind power density (2005–2009)

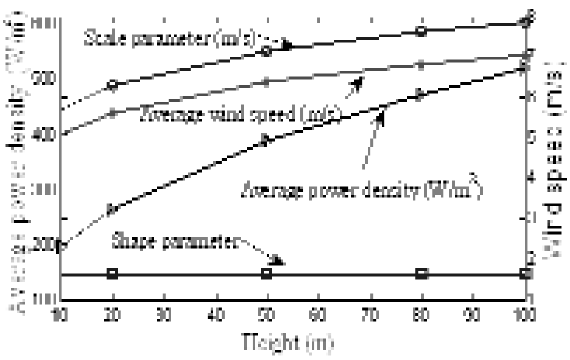

Figure 12: Variations in average power density, wind speed, shape parameter (k) and scale parameter (c) with increase in height

4.3 Wind turbine characteristics and selection

The probability densities and the characteristics of the wind speed at the hub height of 50m for the five year period (whole year) are depicted in Figure 13. The cut-in, cut-out and the rated wind speed operating region for an enhanced power production of a wind turbine is determined as shown in Figure 14 for a 1.3MW wind turbine.

Typical characteristic of wind turbine used in the Darling wind farm in South Africa (1.3MW Fuhrlaender, Gmbh-Germany, 50Hz) is given in Table 6. The probability of distribution of the output power of this wind turbine if it were to be installed on the site under consideration based on the wind characteristics of the site are depicted in Figure 15.

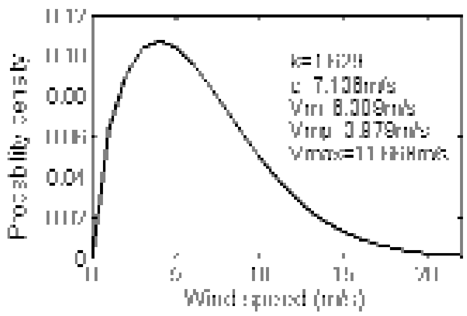

Figure 13: Probability density at 50m hub height

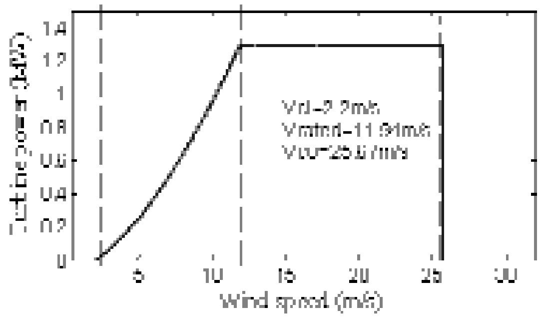

Figure 14: Wind turbine power operating range base on estimated v_{mp} and v_{max} in Figure 13

The probability of the turbine being in operation during the 5 year period is 82.83%, of which the probability of operating at rated power is 4.18%. If this turbine is to be operated, based on the determined power curve in Figure 14, then the probability of the turbine operating at the rated power advances to 13.18%, as depicted in Figure 16.

Table 6: Typical wind turbine characteristics

Source: www.darlingwindfarm.co.za/projectfactsheet.htm

Description	*Values*
Rated power	1.3 MW
Cut-in wind speed	8 km/hr (2.22m/s)
Rated wind speed	54km/hr (15m/s)
Cut-out wind speed	97km/hr(26.94m/s)
Rotor diameter	62m
Hub height	50m

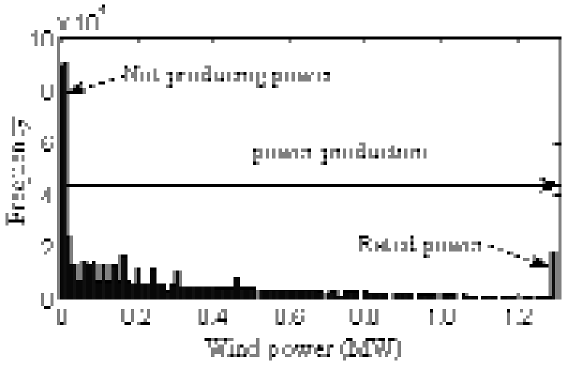

Figure 15: Probability of operating at rated power with the wind turbine operating range in Table 6

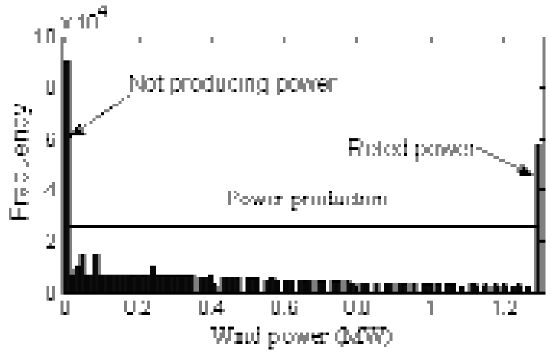

Figure 16: Probability of operating at rated power with the new determined operating range in Figure 14

5 Conclusion

The wind speed characteristics and the wind power densities of Port Elizabeth, South Africa have been analysed using a mathematical model that is found applicable for assessing the wind distribution of a location. The results show wide monthly, seasonal and annual variations in the distribution of wind speed. The power density is higher from September to March in all the years. At 10 m height, the average power density in Summer is 203.926 W/m^2; Spring is 256.505 W/m^2; Winter is 168.806 W/m^2 and Autumn is 152.381 W/m^2. At the hub height of 50 m, the estimated average wind speed is determined to be 6.389m/s, the most probable wind speed is 3.979m/s and the wind speed that carries the maximum amount of wind power is calculated to be 11.668m/s. The wind speeds (cut-in, cut-out and rated wind speeds) that will enhance the capacity factor of the site compared to wind turbine in table 6 are estimated as 2.2m/s, 25.67m/s and 11.94m/s, respectively. However, the wind speed at hub height is based on the surface factor assumption, actual measurement at the hub height may be necessary.

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