JOURNAL OF CIVIL ENGINEERING AND MANAGEMENT
ISSN 1392-3730 / eISSN 1822-3605
2017 Volume 23(8): 985–1001
https://doi.org/10.3846/13923730.2017.1348982

WEIGHT MINIMIZATION OF TRUSS STRUCTURES WITH SIZING AND LAYOUT VARIABLES USING INTEGRATED PARTICLE SWARM OPTIMIZER

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Received 09 Mar 2017; accepted 23 Jun 2017

Abstract. This study investigates the performances of the integrated particle swarm optimizer (iPSO) algorithm in the layout and sizing optimization of truss structures. The iPSO enhances the standard PSO algorithm employing both the concept of weighted particle and the improved fly-back method to handle optimization constraints. The performance of the recent algorithm is tested on a series of well-known truss structures weight minimization problems including mixed design search spaces (i.e. with both discrete and continuous variables) over various types of constraints (i.e. nodal displacements, element stresses and buckling criterion). The results demonstrate the validity of the proposed approach in dealing with combined layout and size optimization problems.

Keywords: particle swarm optimizer, combined sizing/layout optimization, trusses, structural optimization.

Introduction

In the last decade several new metaheuristic techniques have been developed to solve the structural optimization problems. Genetic algorithms (GA), particle swarm optimization (PSO), harmony search (HS), teaching and learning based optimization (TLBO), firefly algorithm (FA) and flower pollination algorithm (FPA) are a variety of well-established methods for the optimal design of structures. Depending on the optimization purpose, cross-sectional areas of the members and/or nodal coordinates separately or simultaneously can be included as the design variables of the problem. Phan *et al.* (2013), Gholizadeh and Fattahi (2014), Gholizadeh and Poorhoseini (2015), Kaveh and Shokohi (2015), Gholizadeh (2015) and Artar (2016) applied some metaheuristic algorithms for the design optimization problems including the sizing variable only. Although it is possible to obtain better results taking into account both sizing and layout variables, in such a case the optimization problem becomes more complex due to rising number of variables (Hasançebi *et al.* 2009; Tang *et al.* 2005; Miguel *et al.* 2013; Silih *et al.* 2010; Deb, Gulati 2001; Dede, Ayvaz 2015; Bekdaş *et al.* 2015; Aydın, Çakır 2015). On the other hand, to acquire practical solutions cross-sectional areas usually should be selected from a predefined discrete list of available structural profiles, while nodal coordinates generally are chosen from a continuous search space. Hence, formulating a mixed variable optimization problem may be unavoidable and employed optimization method should be able to handle both continuous and discrete variables at the same time (Achtziger 2007; Torii *et al.* 2011).

Dede and Ayvaz (2015) performed sizing and layout optimization of truss structures using teaching learning based optimizer (TLBO). Miguel, L. F. F. and Miguel, L. F. F. (2012) optimized size and layout of truss structures subjected to dynamic constraints using Harmony Search (HS) and Firefly Algorithms (FA). Miguel *et al.* (2013) represented the efficiency of Firefly Algorithm to optimize size, layout and topology of truss structures. Tang *et al.* (2005) and Deb and Gulati (2001) carried out sizing and topology optimization of the truss structures using Genetic Algorithm (GA) and its variants.

In this study, an integrated Particle Swarm Optimizer (iPSO) proposed by Mortazavi *et al.* (2017) is employed for optimal design of 2D and 3D truss structures including both sizing and layout variables to demonstrate its performance on those type problems. The iPSO applies concept of the weighted particle to enhance the search capacity of the standard particle swarm optimizer (PSO). In order to handle the optimization constraints, iPSO implements the improved Fly-back technique. This technique aims to make the role of the weighted particle more prominent and thereby improves the characteristic search aspects (i.e. exploration and exploitation) of the proposed algorithm.

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In order to evaluate the performance of iPSO, several well-known truss structures optimization problems which previously solved in the literature adopting different metaheuristic algorithms, are considered. These problems include various constraints on displacements, stresses and critical buckling loads. Results indicate that iPSO has a good search capability at relatively low computation time to solve combined sizing and layout optimization problems of the truss structures.

The article is structured as follows. The general formulation of truss weight minimization problem is recalled in Section 1. Section 2 briefly describes standard PSO and then presents the proposed iPSO algorithm in detail. Section 3 compares the optimization results with those available in the literature. Section 4 is devoted to discuss about the affirmative features of applying improved Fly-back method on the search capability of the iPSO. Conclusions discuss overall performance and strength-points of the iPSO while summarizing the main achievements of this study.

1. Formulation of truss weight minimization problems

Since sizing and layout structural optimization problems involve element cross-sectional areas and nodal coordinates as design variables, corresponding constraints and objective function can be expressed as follows:

find $\mathbf{X}$

Such that:

$$\min.\quad W(\mathbf{X}) = \sum_{e=1}^{m} L_e\,\rho_e\,A_e$$

Subjected to

$$g_1(\mathbf{X}) = d_k(\mathbf{X}) \le d_{\max,k} \qquad (1)$$
$$g_2(\mathbf{X}) = \sigma_e(\mathbf{X}) \le \sigma_{a,e} \text{ (for tension and compression)}$$
$$g_3(\mathbf{X}) = x_{\min,e} \le x_e \le x_{\max e}$$
$$g_4(\mathbf{X}) = cor_{\min,p} \le cor_p \le cor_{\max,p},$$

in which $W(.)$ is the weight of structure and d_k and $d_{\max,k}$ are the existing and allowable displacement for node k, respectively. The length, material density and cross-sectional area of the e^{th} element are respectively L_e, ρ_e, and A_e while m is the total number of elements in the structure. $g_k(\mathbf{X})$ is the k^{th} constraint function, x_i is the i^{th} design variable. Also, σ_e is the available stress in the e^{th} element and $\sigma_{a,e}$ is the allowable tension/compression stress for the same element. While, $x_{\min,e}$ and $x_{\max,e}$, are respectively indicate the lower and upper bounds for the cross-sectional area of the eth element; Finally, $cor_{\min,p}$ indicates the lower bounds and $cor_{\max,p}$ indicates the upper bounds for coordinates of p^{th} node.

It is notable that, in the structural optimization to achieve a feasible solution the stability of the structure should be sustained during the optimization process. In this respect, several methods based on graph theory and algebraic approaches were provided to check this criterion (Kaveh 2004, 2006). Based on the essence of the problem (e.g complexity level) different approaches individually or in combination can be used (Mortazavi, Toğan 2016).

In the current study to check the stability of the structure evaluation of the condition number of the stiffness matrix is employed. Such that, if the condition number of the stiffness matrix is greater than a predefined large number, system is determined as unstable. This predefined large number (e.g. 1E15) is specified depending on precision defined for complier/interpreter environment that is employed by user to solve optimization problem.

2. Integrated particle swarm optimization (iPSO)

Standard Particle Swarm Optimizer (PSO) mimics the behavior of animals (e.g. the colony of fish or swarm of birds) to find food sources or to escape from predators (Kennedy, Eberhart 2001). In the first design cycle of this method, a swarm including several particles is randomly generated while each particle can be a potential solution of the problem. The swarm iteratively flies over the domain of the problem for a unit of time. At the end of the each iteration, each particle finds its own new position. The qualities of these recent positions are evaluated via calculating a proper objective function. In each iteration the best particle in the swarm (global best) is stored in the $\mathbf{X}^G$ vector while the best positions previously gained by each particle are stored in the $\mathbf{X}^P$ matrix.

Although PSO is applied in the various engineering fields, it is reported that the standard PSO has some drawbacks (e.g. staggering from the convergence in further steps of the process), so different modifications have been applied on the PSO in order to overcome these limitations (Van den Bergh, Engelbrecht 2003; He *et al.* 2004; Li *et al.* 2009, 2014).

The iPSO algorithm (Mortazavi, Toğan 2016; Mortazavi *et al.* 2017) attempts enhancing standard PSO by implementing an improved fly-back method and the concept of weighted particle. The iPSO formulation and its relative terms are shortly described in the following subsections (see more Mortazavi, Toğan 2016; Mortazavi *et al.* 2017).

2.1. Weighted particle

To improve the standard PSO method a unique particle named weighted particle ($\mathbf{X}^W$) was proposed by Li *et al.* (2014). Weighted particle is defined as weighted average of all available particles in the swarm. Unlike Li *et al.* (2014), the effect of it's on the solution process of PSO is examined on the structural optimization problems. The objective value of each particle is taken as weight factor to calculate $\mathbf{X}^W$. The weighted particle can hence be defined as:

$$\mathbf{X}^W = \sum_{i=1}^{m} \bar{c}_i^W \mathbf{X}_i^P, \qquad (2)$$

where $\bar{c}_i^W = \hat{c}_i^W \Big/ \sum_{i=1}^{m} \bar{c}_i^W$

and $$\hat{c}_i^W = \frac{\max_{1\le k\le m}\left(f(\mathbf{X}_k^P)\right) - f(\mathbf{X}_i^P) + \varepsilon}{\max_{1\le k\le m}\left(f(\mathbf{X}_k^P)\right) - \min_{1\le k\le m}\left(f(\mathbf{X}_k^P)\right) + \varepsilon}. \quad (3)$$

In Eqns (2)–(3), m is the number of particles, $\mathbf{X}^W$ is the position vector of the weighted particle, $\mathbf{X}_i^P$ is i^{th} particle stored in the $\mathbf{X}^P$, $f(.)$ returns the corresponding value of the objective function of the optimization problem, also max(.) and min(.), respectively, are the worst and best objective function values of the particles available in the $\mathbf{X}^P$ matrix. Also ε is a small positive value to prevent division by zero condition which in this study is taken as 0.001.

2.2. Improved fly-back mechanism

To handle the constraints of the optimization problems, the proposed fly-back mechanism (He *et al.* 2004) is improved to increase the convergence rate of the algorithm and the guidance role of the weighted particle. And it is donated as improved fly-back technique. The following scheme is employed to use the improved fly-back technique. First, problem constraints are divided in two groups: (i) numerical constraints, those do not entail structural analysis to evaluate constraint(s) violation, e.g. side constraints on cross-sectional areas or nodal coordinates; (ii) Characteristic constraints, those who entail structural analysis to determine their violation, e.g. existing stresses and displacements. Next, if a particle violates the numerical constraint(s), the corresponding violated components in the particle are replaced with the same components stored in the weighted particle. The new particle is evaluated and if it gives a better objective value it replaces the old one and if not, it is reset to its previous best position stored in the $\mathbf{X}^P$.

2.3. Formulation of the iPSO algorithm

iPSO utilizes the weighted particle ($\mathbf{X}^W$) not only to overcome reducing or vanishing the guidance effect of particle own prior best position X_i^P and position of the best particle in the swarm ($\mathbf{X}^G$) when a particle is located very close to one or even both of these landmark points but also to involve all particles' experience through weighted particle in the position updating of each particle. Consequently, iPSO is formulated as below:

$$^{t+1}\mathbf{V}_i = \varphi_{4i}\left({}^t\mathbf{X}^W - {}^t\mathbf{X}_i\right) \quad i \ge 1, \quad \varphi_{4i} = C_4 \times \text{rand}_{4i} \quad \text{if rand}_{0i} \le \alpha; \quad (4)$$

$$^{t+1}\mathbf{V}_i = w_i \times {}^t\mathbf{V}_i + \left(\varphi_{1i} + \varphi_{2i} + \varphi_{3i}\right)\left({}^t\mathbf{X}_j^P - {}^t\mathbf{X}_i\right) + \varphi_{2i}\left({}^t\mathbf{X}^G - {}^t\mathbf{X}_j^P\right) + \varphi_{3i}\left({}^t\mathbf{X}^W - {}^t\mathbf{X}_j^P\right) \quad \text{if rand}_{0i} > \alpha; \quad (5)$$

$$^{t+1}\mathbf{X}_i = {}^t\mathbf{X}_i + {}^{t+1}\mathbf{V}_i \quad i \ge 1, \quad j \le m, \quad \varphi_{1i} = C_1 \times \text{rand}_{1i} \quad \varphi_{2i} = C_2 \times \text{rand}_{2i}, \quad \varphi_{3i} = C_3 \times \text{rand}_{3i}. \quad (6)$$

In Eqns (4)–(6), superscripts of "t" and "t + 1" denote current step and next step, respectively. So, $^{t+1}\mathbf{V}_i$ is the updated velocity, w_i is the inertia term of current velocity, and $^t\mathbf{V}_i$ is the current velocity of i^{th} particle. C_1, C_2, C_3, and C_4 are acceleration factors, where respectively can be selected as $-\left(\varphi_{2i} + \varphi_{3i}\right)$, 2, 1 and 2. rand_{ki} is the random number selected from the [0, 1] interval, in which $k \in \{0, 1, 2, 3, 4\}$. Also, $^t\mathbf{X}_j^P$ indicates the randomly selected particle from the current matrix of $\mathbf{X}^P$. Moreover, $^t\mathbf{X}^G$ is the global best particle up to current step while $^{t+1}\mathbf{X}_i$ and $^t\mathbf{X}_i$, respectively, represent the updated position and current position of the i^{th} particle. The weighted particle calculated for the current step is shown by $^t\mathbf{X}^W$. In each iteration, w_i is randomly selected from [0.5, 0.55] and α = 0.4. Li *et al.* (2014) established these parameters adjustments for scalar functions with continuous search spaces.

Since discrete variables are also adopted in this study, Eqns (2)–(5) are modified as follows:

$$\mathbf{X}^W = INT\left(\sum_{i=1}^{m} c_i \mathbf{X}_i^P\right); \quad (7)$$

$$^{t+1}\mathbf{X}_i = INT\left({}^t\mathbf{X}_i + \varphi_{4i}\left({}^t\mathbf{X}^W - {}^t\mathbf{X}_i\right)\right) \text{ if rand}_{0i} \le \alpha \quad (8)$$

if $\text{rand}_{0i} > \alpha$

$$^{t+1}\mathbf{X}_i = INT\begin{pmatrix} {}^t\mathbf{X}_i + w_i \times {}^t\mathbf{V}_i + \left(\varphi_{1i} + \varphi_{2i} + \varphi_{3i}\right)\left({}^t\mathbf{X}_j^P - {}^t\mathbf{X}_i\right) + \\ \varphi_{2i}\left({}^t\mathbf{X}^G - {}^t\mathbf{X}_j^P\right) + \varphi_{3i}\left({}^t\mathbf{X}^W - {}^t\mathbf{X}_j^P\right) \end{pmatrix} \quad (9)$$

The operator INT (.) in above formulations takes the integer part of any scalar variable. The rest of the definitions are the same as for Eqns (2)–(5). As can be seen from given formulation the $\mathbf{X}^W$ plays significant role in the guidance of swarm. In the proposed method the weighted particle accompanied with $\mathbf{X}^W$ form the particular motion orientation for other particles. So, excepted particles own experiences stored in $\mathbf{X}^P$ each particle has two reasonable guidance points (i.e. $\mathbf{X}^W$ and $\mathbf{X}^G$), in comparison to standard PSO where $\mathbf{X}^G$ takes over this role individually.

It is remarkable that, the weighted particle includes the data stored in all particles while associates them related with their objective values (i.e. for a minimization problem, particle with lower objective value has a higher impact). Neglecting particles inertia, in each iteration $\mathbf{X}^W$ spots a location in search domain based on combination of detected locations by swarm up to current step. So, this particle (as gravity center of colony) lies closer to better particles. Subsequently, since it cannot stand so far away from swarm it generally provides a local search, and moving toward this particle increases the exploitation ability of the algorithm. On the other hand, similar to the standard PSO, holding the effect of particles' inertia, moving toward the $\mathbf{X}_i^P$ and $\mathbf{X}^G$ (for $\text{rand}_{0i} > \alpha$)

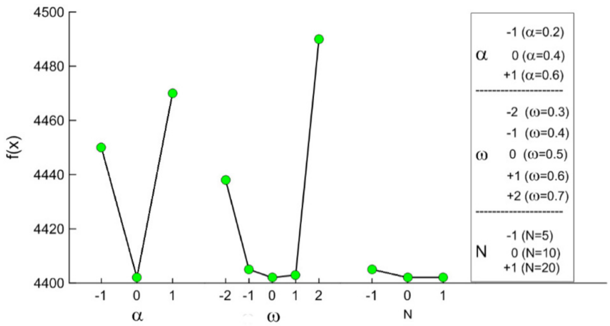
provides exploration ability of the algorithm. However, to reach the global optimum through the search space, a balance between exploration and exploitation is required (Gholizadeh 2013). In iPSO, this balance is provided via adjusting attraction factor (α).

2.4. Parameters of iPSO

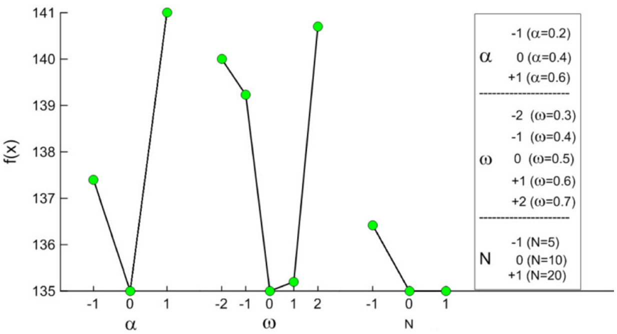
To exhibit the effect of variation of attraction factor (α) and initial velocity inertia term (w) on iPSO algorithm performance, sensitivity analysis based on the graphical sensitivity method described in Lee *et al*. (2013) was carried out. These analyses provide illustrative data about the effect of adjustable parameters, i.e. changing a parameter either has a positive or negative effect on the performance of the algorithm. The corresponding mapped ranges for the attraction factor (α), initial velocity inertia term (w), and number of population (N) are illustrated in Figure 1. Results of the sensitivity analysis performed for 18-bar and 39-bar trusses verify the results obtained by Li *et al.* (2014) and Mortazavi and Toğan (2017). According to these results, the best combination of internal parameters (α and w) are w = [0.5, 0.55] and α = 0.4, respectively. As a consequence, considering the numerical experiments conducted in the current study, it can be worth to say that the appropriate values for α and w are 0.4 and 0.5 for the examples to be investigated in the present study.

3. Design examples

Five benchmark truss optimization problems are solved to evaluate the performance of the iPSO. Due to the stochastic nature of the optimization technique, 30 independent runs are carried out for solving each test problem. For more clarity the statistical information and number of objective function evaluation (OFEs) about each solution also are provided. It is notable that some statistical data and OFEs of referred studies are not reported in the relative literature.



a. 18-bar truss



b. 39-bar truss

Fig. 1. Illustrative results of the sensitivity analysis for the internal parameters of iPSO algorithm

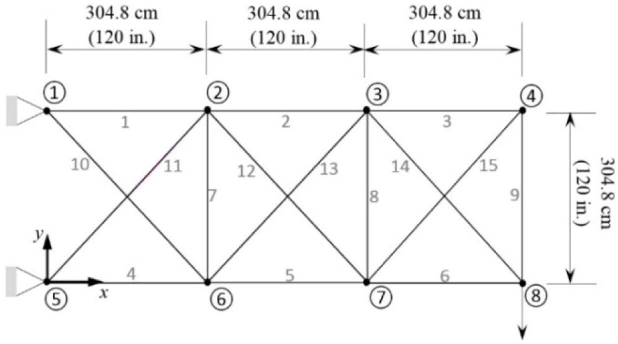


Fig. 2. Schematic of the planar 15-bar truss structure

3.1. 15-bar planar truss structure

As shown in Figure 2, the 2D 15-bar truss structure is taken as the first test case. The design parameters adopted in this example are presented in Table 1. Population size and allowable number of iterations are respectively set as 20 and 300 for this example.

Table 2 compares the optimum design obtained by the iPSO with those using other techniques available in relative studies. In addition Table 2 presents the statistical information about this problem, as well. According to data tabulated in Table 2, iPSO found the lightest solution among the reported results by Wu and Chow (1995) using GA, Tang *et al.* (2005) using improved GA, Hwang and He (2006) applying ARSAGA, Rahami *et al.* (2008) using Force method and GA, Dede and Ayvaz (2015) using TLBO, and Miguel *et al.* (2013) using FA. However, the lightest design for this example was reported by Gholizadeh (2013) using SCPSO. This result, 72.5413 lb, is only %0.3 lb smaller than the design obtained by using iPSO. The acquired optimized structure is shown in Figure 3. The best weight is achieved at 249th iteration (i.e. after 4980 structural analyses) and it remains unchanged until the last iteration is reached. The standard deviation for the 30 independent runs is 2.023 lb and it indicates that dispersion of the outcomes is in the acceptable range from mean value of the solutions. Based on the comparisons made in Table 2, it can be expressed that iPSO has adequate capability to handle the layout and sizing

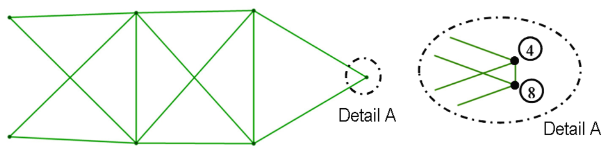


Fig. 3. Optimal layout for the planar 15-bar truss problem

Table 1. Input data for the planar 15-bar truss problem

	Material properties	
Young's modulus	E = 68947.5728 MPa (10 Msi)	
Density	ρ = 27.1447 kN/m^3 (0.1 lb/in^3)	
Allowable stress	±172.36898 MPa (±25 ksi)	
	Design variables	
Sizing variables	A_i =1, 2, . . ., 15	
Discrete area set, S	S = {0.111, 0.141, 0.174, 0.220, 0.270, 0.287, 0.347, 0.440, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.800, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.300, 10.850, 13.330, 14.290, 17.170, 19.180} in^2	
	Layout variables	
	254 cm (100 in) ≤ $X_2 = X_6$ ≤ 355.6 cm (140 in)	
	558.8 cm (220 in) ≤ $X_3 = X_7$ ≤ 660.4 cm (260 in)	
	254 cm (100 in) ≤ Y_2, Y_3 ≤ 355.6 cm (140 in)	
	127 cm (50 in) ≤ Y_4 ≤ 228.6 cm (90 in)	
	–50.8 cm (–20 in) ≤ Y_6, Y_7 ≤ 50.8 cm (20 in)	
	50.8 cm (20 in)≤ Y_8 ≤ 152.4 cm (60 in)	
	Nodal loads	
Node number	x	y
8	0	–44.537 kN (–10 kips)
	Constraints	
Displacement	–	
Euler buckling	–	
Stress	σ_e ≤ allowable stress	

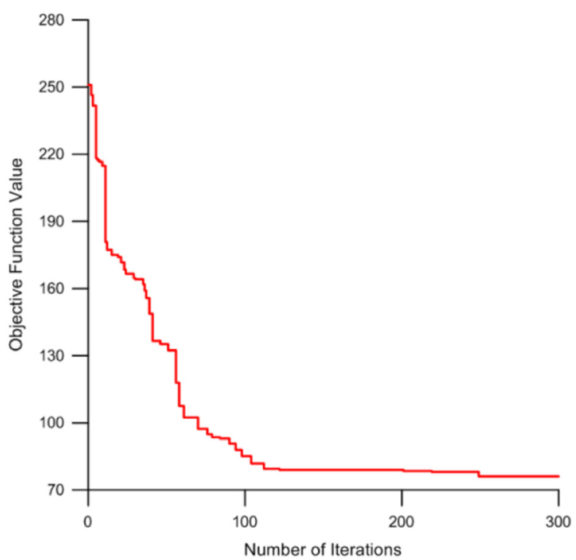


Fig. 4. Convergence history for the planar 15-bar truss using iPSO

optimization problem. Figure 4 shows the convergence history of the 15-bar truss structure for iPSO.

3.2. 18-bar planar truss structure

The 18 bar truss shown in Figure 5 is selected as the second example of the combined sizing/layout optimization. Input data required for this test case are listed in Table 3. The population size and allowable number of iterations are respectively set as 10 and 500 for this example.

The optimized layout of the structure is shown in Figure 6. The optimum design found by the iPSO is compared with those reported in the relative literature in Table 4. According to the provided information in Table 4, iPSO finds the lightest solution in comparison with the results found by Hasançebi and Erbatur (2002) using SA, Kaveh and Kalatjari (2004) using Force method and GA, Rahami *et al.* (2008) using Force method and GA, Lee and Geem (2005) using HS, and also Dede and Ayvaz (2015) using TLBO. However, comparing the designs presented in Table 4 it is notable that the better result among them is the one obtained by Gholizadeh (2013) employing SCPSO. The optimum design is reached at 445th iteration (i.e. after 4450 structural analyses) and any improvement in the solution is not obtained until reaching to the maximum allowable number of iterations. The statistical data for current example is also provided in Table 4. The data shows that the value of the standard deviation is equal to 14.899 lb for the set of obtained solutions through 30 independent runs. This value indicates that distribution of the obtained results over their mean value is statistically admissible. The corresponding convergence history of the 18-bar truss structure for iPSO is demonstrated in Figure 7.

Table 2. Comparison of the iPSO optimization results with the literature for the 15-bar truss problem

Design variables	Wu and Chow (1995) GA	Tang *et al.* (2005) iGA	Hwang and He (2006) ARSAGA	Rahami *et al.* (2008) GA	Miguel *et al.* (2013) FA	Gholizadeh (2013) SCPSO	Dede and Ayvaz (2015) TLBO	This study iPSO
Sizing variables (in^2)								
A_1	1.174	1.081	0.954	1.081	0.954	0.954	1.081	1.081
A_2	0.954	0.539	1.081	0.539	0.539	0.539	0.954	0.539
A_3	0.44	0.287	0.44	0.287	0.22	0.270	0.141	0.27
A_4	1.333	0.954	1.174	0.954	0.954	0.954	1.081	0.954
A_5	0.954	0.954	1.488	0.539	0.539	0.539	0.539	0.539
A_6	0.174	0.22	0.27	0.141	0.22	0.174	0.347	0.141
A_7	0.44	0.111	0.27	0.111	0.111	0.111	0.111	0.111
A_8	0.44	0.111	0.347	0.111	0.111	0.111	0.174	0.111
A_9	1.081	0.287	0.22	0.539	0.287	0.287	0.141	0.27
A_{10}	1.333	0.22	0.44	0.44	0.44	0.347	0.27	0.287
A_{11}	0.174	0.44	0.22	0.539	0.44	0.347	0.22	0.44
A_{12}	0.174	0.44	0.44	0.27	0.22	0.220	0.141	0.27
A_{13}	0.347	0.111	0.347	0.22	0.22	0.220	0.44	0.287
A_{14}	0.347	0.22	0.27	0.141	0.27	0.174	0.347	0.174
A_{15}	0.44	0.347	0.22	0.287	0.22	0.270	0.141	0.27
Layout variables (in)								
X_2	123.189	133.612	118.346	101.5775	114.967	137.2216	100.0042	132.2415
X_3	231.595	234.752	225.209	227.9112	247.04	259.9093	241.0473	257.4379
Y_2	107.189	100.449	119.046	134.7986	125.919	123.5006	118.8228	128.3136
Y_3	119.175	104.738	105.086	128.2206	111.067	110.0020	100.0829	111.2506
Y_4	60.462	73.762	63.375	54.863	58.298	59.9356	50	59.9894
Y_6	16.728	–10.067	–20	–16.4484	–17.564	–5.1799	3.1411	–10.5543
Y_7	15.565	–1.339	–20	–16.4484	–5.821	4.2193	–9.6997	10.7686
Y_8	36.645	50.402	57.722	54.8572	31.465	57.8829	46.8963	60.0
Statistical results								
Best weight (lb)	120.52	79.82	104.573	76.6854	75.55	72.5143	76.6519	72.7373
Worst weight (lb)						80.156		76.522
Mean weight (lb)						76.411		74.316
Std. deviation (lb)						1.922		2.023
OFEs	6000	8000	16000	8000	8000	4500	16000	4980

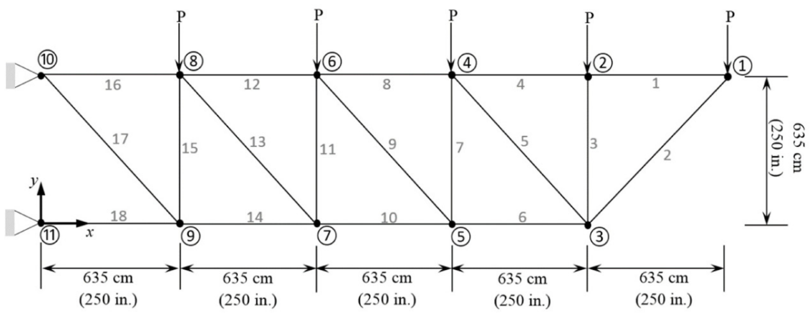


Fig. 5. Schematic of the planar 18-bar truss structure

Table 3. Input data for the planar 18-bar truss problem

	Material properties	
Young's modulus	E = 68947.5728 MPa (10 Msi)	
Density	ρ = 2768 kg/m^3 (0.1 lb/in^3)	
Allowable stress	±137.8951 MPa (±20 ksi)	
	Design variables	
Sizing variables	Members	
A_1	1, 4, 8, 12, 16	
A_2	2, 6, 10, 14, 18	
A_3	3, 7, 11, 15	
A_4	5, 9, 13, 17	
Discrete area set	{2.0, 2.25, . . ., 21.5, 21.75} in^2	
	Layout variables	
	–571.5 cm (–225 in) $\leq Y_3, Y_5, Y_7, Y_9 \leq$ 622.3 cm (245 in)	
	1968.5 cm (775 in) $\leq X_3 \leq$ 3111.5 cm (1225 in)	
	1333.5 cm (525 in) $\leq X_5 \leq$ 2349.5 cm (925 in)	
	698.5 cm (275 in) $\leq X_7 \leq$ 1841.5 cm (725 in)	
	63.5 cm (25 in) $\leq X_9 \leq$ 1206.5 cm (475 in)	
	Loading conditions	
Node number	x	y
1	0	–44.537 kN (–10 kips)
2	0	–44.537 kN (–10 kips)
4	0	–44.537 kN (–10 kips)
6	0	–44.537 kN (–10 kips)
8	0	–44.537 kN (–10 kips)
	Constraints	
Displacement	–	
Euler buckling	$\sigma_{eb} \leq \dfrac{\alpha E A_e}{L_e^2}$	
Stress	$\sigma_e \leq$ allowable stress	

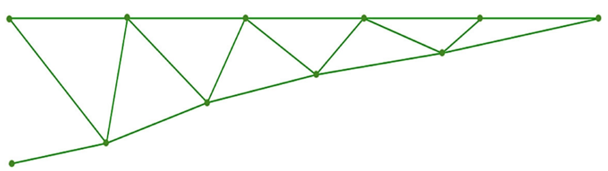


Fig. 6. Optimal layout for the planar 18-bar truss problem

3.3. 25-bar space truss tower

The third example deals with the combined sizing/layout optimization of the 3D 25-bar truss tower shown in Figure 8. Problem specifications are listed in Table 5. Initial values of nodal coordinates and members grouping are presented in Table 6. The symmetry of structure is maintained during the optimization process. The population size and allowable number of iterations are set as 10 and 500, respectively.

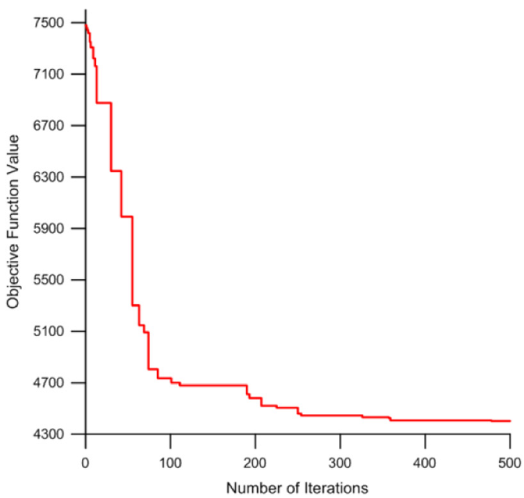


Fig. 7. Convergence history for the planar 18-bar truss using iPSO

Table 4. Comparison of the iPSO optimization results with the literature for the planar 18-bar truss problem

Design variables	Hasançebi and Erbatur (2002) SA	Kaveh and Kalatjari (2004) GA	Rahami *et al.* (2008) GA	Lee and Geem (2005)	Gholizadeh (2013) SCPSO	Dede and Ayvaz (2015) TLBO	This study iPSO
Sizing variables (in^2)							
A_1	12.25	12.25	12.75	12.62	12.50	12.25	14.25
A_2	17.5	18	18.5	17.22	17.50	17.5	11.75
A_3	5.75	5.25	4.75	6.17	5.75	5.75	6.00
A_4	4.25	4.25	3.25	3.55	3.75	4.25	8.00
Layout variables (in)							
X_3	910	913	917.4475	903.1	907.2491	906.9373	916.4975
Y_3	179	186.8	193.7899	174.3	179.8671	179.8866	190.5241
X_5	638	650	654.3243	630.3	636.7873	637.0087	916.4975
Y_5	141	150.5	159.9436	136.3	141.8271	142.617	152.9217
X_7	408	418.8	424.4821	402.1	407.9442	408.6414	649.4695
Y_7	91	97.4	108.5779	90.5	94.0559	94.1563	105.425
X_9	198	204.8	208.4691	195.3	198.7897	199.6503	205.4255
Y_9	24	26.7	37.6349	30.6	29.5157	25.3657	36.4252
Statistical results							
Best weight (lb)	4533.24	4547.9	4530.7	4525.6	4512.365	4528.797	4520.99
Worst weight (lb)					4621.227		4560.27
Mean weight (lb)					4551.709		4526.585
Std. deviation (lb)					37.691		14.889
OFEs	–	5000	8000	25000	4500	16000	4450

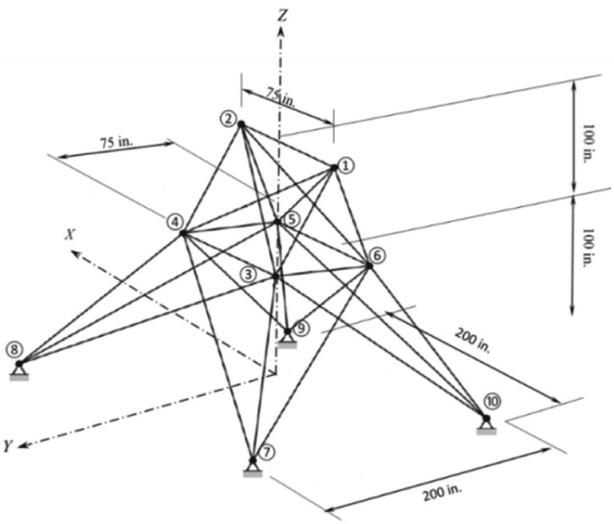


Fig. 8. Schematic of the spatial 25-bar truss tower

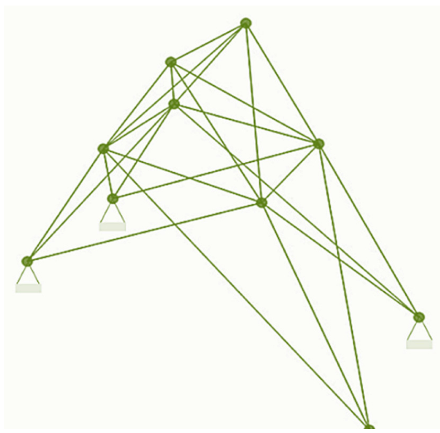


Fig. 9. Optimal layout for the spatial 25-bar truss problem

The optimum design is compared with those available in the relative literature in Table 7. The iPSO finds the lightest solution for the current structure in comparison with the reported results by Wu and Chow (1995), Tang *et al.* (2005) using different variants of GA, Rahami *et al.* (2008), Kaveh and Kalatjari (2004) using Force method and GA. The results show that iPSO and SCPSO (Gholizadeh 2013) converge to better solution in comparison with others however there is a negligible difference between these designs in the point view of engineering. The optimal layout found for this problem is shown in Figure 9. It is accomplished at 487th generation (i.e. after 4870 structural analyses) to find the optimum design reported in Table 7. Any further enhancement in the design is not attained until the maximum iteration number is reached. The graphical representation of this process is illustrated in Figure 10. The statistical data for this exam-

Table 5. Input data for the spatial 25-bar truss problem

Material properties			
Young's modulus	E = 68947.5728 MPa (10 Msi)		
Density	ρ = 2768 kg/m^3 (0.1 lb/in^3)		
Allowable stress	±275.80 MPa (±40 ksi)		
Design variables			
Sizing variables	$A_i = 1, 2, \ldots, 8$		
Nodal coordinates and symmetry conditions	$x_4 = x_5 = -x_3 = -x_6$; $x_8 = x_9 = -x_7 = -x_{10}$; $y_3 = y_4 = -y_5 = -y_6$; $y_7 = y_8 = -y_9 = -y_{10}$; $z_3 = z_4 = z_5 = z_6$		
Discrete area set	{0.1, 0.2, . . ., 2.6, 2.8, 3.0, 3.2, 3.4} in^2		
Layout variables			
50.8 cm (20 in) $\le X_4 \le$ 152.4 cm (60 in)			
101.6 cm (40 in) $\le X_8 \le$ 203.2 cm (80 in)			
101.6 cm (40 in) $\le Y_4 \le$ 203.2 cm (80 in)			
254 cm (100 in) $\le Y_8 \le$ 355.6 cm (140 in)			
228.6 cm (90 in) $\le Z_4 \le$ 330.2 cm (130 in)			
Loading conditions			
Node number	x	y	z
1	4.454 kN (–1.0 kips)	–44.537 kN (–10 kips)	–44.537 kN (–10 kips)
2	0	–44.537 kN (–10 kips)	–44.537 kN (–10 kips)
3	2.227 kN (0.5 kips)	0	0
6	2.672 kN (0.6 kips)	0	0
Constraints			
Displacement	$\lvert\Delta_i\rvert \le$ 0.89 cm (0.35 in); $i = 1, \ldots, 6$		
Euler buckling	–		
Stress	$\sigma_e \le$ allowable stress		

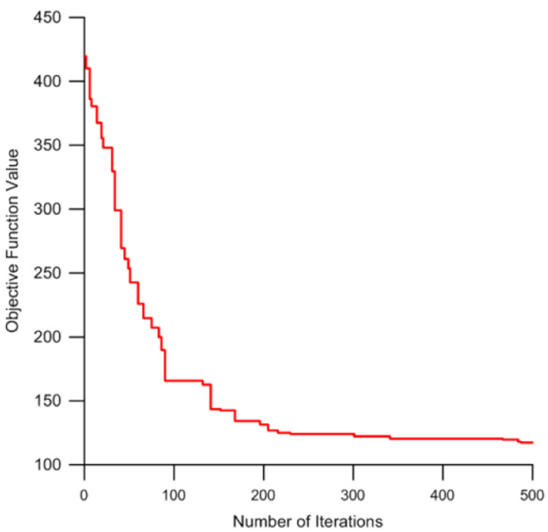


Fig. 10. Convergence history for the spatial 25-bar truss using iPSO

ple is declared in Table 7. The standard deviation value of the set of achieved solutions is 1.3908 lb. This shows that the solutions satisfactorily are spread out over the mean value of the solutions set.

3.4. 39-bar space truss tower

The fourth test problem is the combined sizing/layout optimization of the 3D 39-bar truss tower shown in Figure 11. Problem specifications are listed in Table 8. Fixed nodes' coordinates and elements' connectivity are presented in Table 9. The top and bottom nodes have fixed position while middle nodes' coordinates are taken as design variables. The symmetry of structure is maintained during the optimization process. The population size and allowable number of iterations is set to 10 and 1000, respectively.

The optimum design found by iPSO is demonstrated in Figure 12 while in Table 10 it is compared with those obtained from other methods. The solution obtained by iPSO is lighter than the cited references Wang *et al.* (2002) and Dede and Ayvaz (2015). This optimum design is attained at 725th generation (i.e. after 7250 structural analyses). No further improvement is achieved through the remaining iterations. Table 10 provides the statistical information about the 30 independent runs of iPSO for this example.

The standard deviation 1.831 lb indicates an acceptable distribution of the outcomes over the mean value of all independent runs. Since the buckling criterion is added as an extra constraint, the boundary condition of the prob-

Table 6. Initial layout and member grouping for the spatial 25-bar truss problem

Joint number	x (in)	y (in)	z (in)	Group	Member number (end joints number)
1	–95.25 cm (–37.5)	0.0	508 cm (200)	A_1	1(1,2)
2	95.25 cm (37.5)	0.0	508 cm (200)	A_2	2(1,4), 3(2,3), 4(1,5), 5(2,6)
3	–95.25 cm (–37.5)	95.25 cm (37.5)	254 cm (100)	A_3	6(2,5), 7(2,4), 8(1,3), 9(1,6)
4	95.25 cm (37.5)	95.25 cm (37.5)	254 cm (100)	A_4	10(3,6), 11(4,5)
5	95.25 cm (37.5)	–95.25 cm (–37.5)	254 cm (100)	A_5	12(3,4), 13(5,6)
6	–95.25 cm (–37.5)	–95.25 cm (–37.5)	254 cm (100)	A_6	14(3,10), 15(6,7), 16(4,9), 17(5,8)
7	–254 cm (–100)	254 cm (100)	0.0	A_7	18(3,8), 19(4,7), 20(6,9), 21(5,10)
8	254 cm (100)	254 cm (100)	0.0	A_8	22(3,7), 23(4,8), 24(5,9), 25(6,10)
9	254 cm (100)	–254 cm (–100)	0.0		
10	–254 cm (–100)	–254 cm (–100)	0.0		

Table 7. Comparison of the iPSO optimization results with the literature for the spatial 25-bar truss problem

Design variables	Wu and Chow (1995) GA	Tang *et al.* (2005) iGA	Kaveh and Kalatjari (2004) GA	Rahami *et al.* (2008) GA	Gholizadeh (2013) SCPSO	This study iPSO
			Sizing variables (in^2)			
A_1	0.1	0.1	0.1	0.1	0.1	0.1
A_2	0.2	0.1	0.1	0.1	0.1	0.1
A_3	1.1	1.1	1.1	1.1	1.0	1.0
A_4	0.2	0.1	0.1	0.1	0.1	0.1
A_5	0.3	0.1	0.1	0.1	0.1	0.1
A_6	0.1	0.2	0.1	0.1	0.1	0.1
A_7	0.2	0.2	0.1	0.2	0.1	0.1
A_8	0.9	0.7	1	0.8	0.9	0.9
			Layout variables (in)			
X_4	41.07	35.47	36.23	33.0487	36.9520	37.6
Y_4	53.47	60.37	58.56	53.5663	54.5786	54.46
Z_4	124.6	129.07	115.59	129.9092	129.9758	130
X_8	50.8	45.06	46.46	43.7826	51.7317	51.89
Y_8	131.48	137.04	127.95	136.8381	139.5316	139.55
			Statistical results			
Best weight (lb)	136.2	124.94	124.0	120.1149	117.227	117.255
Worst weight (lb)					132.672	121.969
Mean weight (lb)					122.876	119.57
Std. deviation (lb)					3.671	1.3908
OFEs	–	6000	–	10000	4500	4870

Table 8. Input data for the spatial 39-bar truss problem

		Material properties	
Young's modulus		E = 210 GPa (30457.9249233 ksi)	
Density		ρ = 7800.2 kg/m^3 (0.2818 lb/in^3)	
Allowable stress		±240 MPa (±34.809048 ksi)	
		Design variables	
Sizing variables		A_i = 1, 2, 3, 4, 5	
		Layout variable	
		$Y_{12}, Z_{12}, Y_{13}, Z_{13}, Y_{14}, Z_{14}$	
		Loading conditions	
Node number	x	y	z
5		10 kN (2.2481 kips)	
10		10 kN (2.2481 kips)	
15		10 kN (2.2481 kips)	
		Constraints	
Displacement	X	Y	Z
Node 13		0.4 cm (0.1574 in)	
Euler buckling		$\sigma_{eb} \le \frac{-K_e E A_e}{L_e^2}$	
Stress		$\sigma_e \le$ allowable stress	

Table 9. Initial layout and member grouping for the spatial 39-bar truss problem

Joint number	x (m)	y (m)	z (m)	Group	(end joints number)
1		–0.5	0	A_1	(1,2)(6,7)(11,12)
5		–0.14	4	A_2	(2,3)(7,8)(12,13)
6		–0.5	0	A_3	(3,4)(8,9)(13,14)
10		–0.14	4	A_4	(4,5)(9,10)(14,15)
11	0	1	0	A_5	Rest of the elements
15	0	0.28	4		

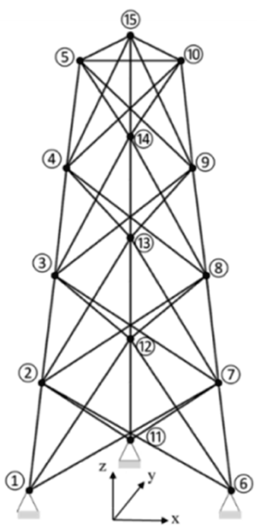


Fig. 11. Schematic of the spatial 39-bar truss tower

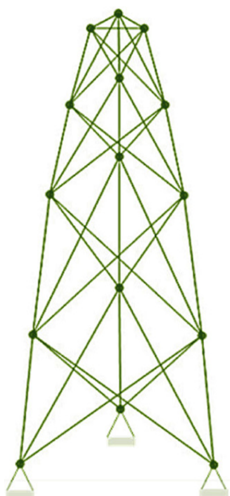


Fig. 12. Optimal layout for the spatial 39-bar truss problem

Table 10. Comparison of the iPSO optimization results with the literature for the spatial 39-bar truss problem

Design variables	Wang *et al.* (2002) FSD	Dede and Ayvaz (2015) TLBO	This study iPSO
Sizing variables (cm^2)			
A_1	11.01	11.9650	12.08
A_2	8.63	11.1457	10.1
A_3	6.69	7.8762	6.64
A_4	4.11	2.7013	3.18
A_5	4.37	2.4058	1.72
Layout variables (m)			
Y_{12}	0.805	0.8996	0.872
Z_{12}	1.186	1.3507	1.198
Y_{13}	0.654	0.6917	0.693
Z_{13}	2.204	2.3122	2.496
Y_{14}	0.466	0.4825	0.490
Z_{14}	3.092	3.3031	3.309
Statistical results			
Best weight (kg)	203.18	154.13	135.0552
Worst weight (kg)			140.865
Mean weight (kg)			138.58
Std. deviation (kg)			1.831
OFEs	–	7560	7250

Table 11. Input data for the planar 47-bar truss problem

Material properties				
Young's modulus	E = 30 Msi			
Density	ρ = 0.30 lb/in^3			
Allowable stress	+20 ksi and –15 ksi for members in tension and compression, respectively.			
Design variables				
Sizing variables	A_i=1, 2, …, 46, 47			
Layout variables	$X_2=-X_1$, $X_4=-X_3$, $Y_4=Y_3$, $X_6=-X_5$, $Y_6=Y_5$, $X_8=-X_7$, $Y_8=Y_7$, $X_{10}=-X_9$, $Y_{10}=Y_9$, $X_{12}=-X_{11}$, $Y_{12}=Y_{11}$, $X_{14}=-X_{13}$, $Y_{14}=Y_{13}$, $X_{20}=-X_{19}$, $Y_{20}=Y_{19}$, $X_{21}=-X_{18}$, $Y_{21}=Y_{18}$			
Discrete area set	{0.1, 0.2, 0.3, . . ., 4.8, 4.9, 5.0} in^2			
Loading conditions				
Load case	Node numbers	x	y	z
1	17 and 22	6.0 kips	–14.0 kips	0
2	17	6.0 kips	–14.0 kips	0
3	22	6.0 kips	–14.0 kips	0
Constraints				
Displacement				
Euler buckling	$3.96EA/L^2$			
Stress	$\sigma_e \leq$ allowable stress			

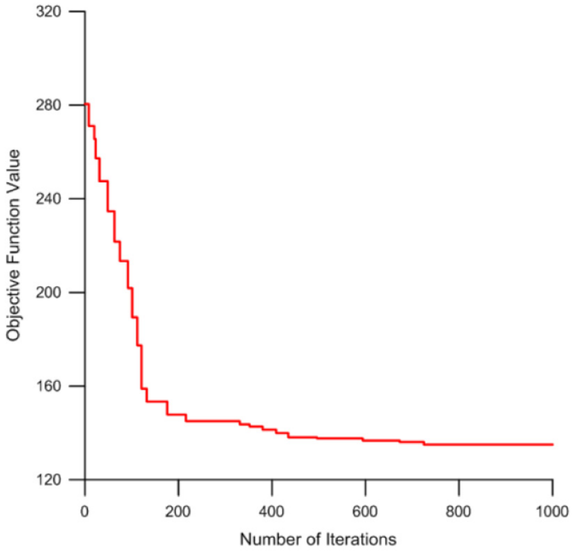


Fig. 13. Convergence history for the spatial 39-bar truss using iPSO

lem becomes more complex, thereupon the probability of existing more local optimum points in the search space is raised, and consequently the standard division of current example is higher than prior example. Figure 13 shows the convergence history of the 39-bar truss structure for iPSO.

3.5. 47-bar planar truss structure

The last test problem is devoted to investigate the sizing and layout optimization of the planar 47-bar truss structure shown in Figure 14. The necessary data to model this problem are specified in Table 11. The population size and limit number of iterations are set as 40 and 600, respectively. Considering the symmetric about the y-axis, members of the 47-bar truss are collected into 27 independent groups as sizing variables in Table 12. Layout variables corresponding to the nodal coordinates are also presented in Table 11. Therefore, totally 44 design variables are taken into consideration during the optimization process of this test example. Furthermore, it can be seen from Table 11 that 47-bar truss structure is subject to the three independent loading conditions.

The optimum design found by iPSO is illustrated in Table 13 while for the sake of comparison the results obtained in other studies using other methods are also included. According to this table the present approach, iPSO, is competitive with all other results presented in Table 13 in finding optimum solution. The best weight for this case is found at 501th iteration and any improvements are not occurred until the last iteration. The statistical information of 30 independent runs is also provided in Table 13. Standard deviation for Gholizadeh (2013) using sequential cellular particle swarm optimization (SCPSO) and this study using iPSO are 34.755 and 20.782 lb, respectively. The standard deviation on the optimized weight of the iPSO is comparatively lower than other cited methods, so there is

Table 12. Member grouping for the planar 47-bar truss problem

Group	Member	Group	Member
1	1, 3	16	28
2	2, 4	17	29, 30
3	5, 6	18	31, 32
4	7	19	33
5	8, 9	20	34, 35
6	10	21	36, 37
7	11, 12	22	38
8	13, 14	23	39, 40
9	15, 16	24	41, 42
10	17, 18	25	43
11	19, 20	26	44, 45
12	21, 22	27	46, 47
13	23, 24		
14	25, 26		
15	27		

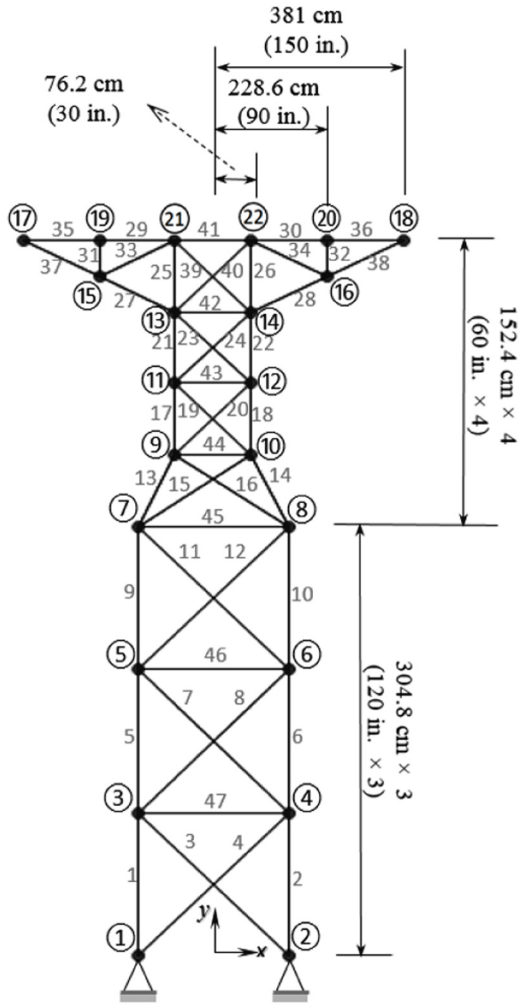


Fig. 14. Schematic of the planar 47-bar truss structure

Table 13. Comparison of the iPSO optimization results with the literature for the planar 47-bar truss problem

Design variables	Hasancebi and Erbatur (2001) iGA	Hasancebi and Erbatur (2002) SA	Gholizadeh (2013)			This study iPSO
			PSO	CPSO	SCPSO	
A_1	2.5	2.5	2.8	2.6	2.5	2.5
A_2	2.2	2.5	2.7	2.5	2.5	2.5
A_3	0.7	0.8	0.8	0.7	0.8	0.8
A_4	0.1	0.1	1.1	0.3	0.1	0.1
A_5	1.3	0.7	0.8	1.2	0.7	0.7
A_6	1.3	1.3	1.3	1.1	1.4	1.4
A_7	1.8	1.8	1.8	1.6	1.7	1.7
A_8	0.5	0.7	0.9	0.8	0.8	0.8
A_9	0.8	0.9	1.2	1.1	0.9	0.9
A_{10}	1.2	1.2	1.4	1.3	1.3	1.3
A_{11}	0.4	0.4	0.3	0.3	0.3	0.3
A_{12}	1.2	1.3	1.4	0.8	0.9	0.9
A_{13}	0.9	0.9	1.1	1.0	1.0	1.0
A_{14}	1.0	0.9	1.2	1.0	1.1	1.1
A_{15}	3.6	0.7	1.6	0.9	5.0	0.9
A_{16}	0.1	0.1	1.0	0.1	0.1	0.1
A_{17}	2.4	2.5	2.8	2.7	2.5	2.5
A_{18}	1.1	1.0	0.8	0.9	1.0	1.0
A_{19}	0.1	0.1	0.1	0.1	0.1	0.1
A_{20}	2.7	2.9	3.0	3.0	2.8	2.8
A_{21}	0.8	0.8	0.9	1.0	0.9	0.9
A_{22}	0.1	0.1	0.1	0.2	0.1	0.1
A_{23}	2.8	3.0	3.3	3.3	3.0	3.0
A_{24}	1.3	1.2	0.9	0.9	1.0	1.0
A_{25}	0.2	0.1	0.1	0.1	0.1	0.1
A_{26}	3.0	3.2	3.3	3.3	3.2	3.2
A_{27}	1.2	1.1	1.2	1.1	1.2	1.2
Layout variables (in)						
X_2	114.0	104.0	98.8628	99.363	101.3393	101.2077
X_4	97.0	87.0	78.6595	83.4439	85.9111	85.8555
Y_4	125.0	128.0	146.7331	126.3845	135.9645	135.9679
X_6	76.0	70.0	66.5231	69.5148	74.7969	74.9087
Y_6	261.0	259.0	239.0901	218.2013	237.7447	238.0442
X_8	69.0	62.0	55.6936	58.0004	64.3115	64.1206
Y_8	316.0	326.0	327.7882	322.2272	321.3416	321.5037
X_{10}	56.0	53.0	48.8641	51.4015	53.3345	53.3481
Y_{10}	414.0	412.0	398.6775	401.5626	414.3025	413.7265
X_{12}	50.0	47.0	43.14	46.8605	46.0277	46.2881
Y_{12}	463.0	486.0	464.7831	458.3021	489.9216	487.9695
X_{14}	54.0	45.0	37.8993	46.8885	41.8353	41.8603
Y_{14}	524.0	504.0	511.045	527.8575	522.4161	522.8897
X_{20}	1.0	2.0	18.2341	16.2354	1.0005	0.9892
Y_{20}	587.0	584.0	594.071	610.8496	598.3905	598.3959
X_{21}	99.0	89.0	90.9369	98.3239	97.8696	97.8656
Y_{21}	631.0	637.0	621.3943	624.958	624.055	624.0605
Statistical results						
Best weight (lb)	1925.79	1871.70	1975.84	1908.83	1864.10	1861.429
Worst weight (lb)					2007.563	1908.991
Mean weight (lb)					1894.056	1873.011
Std. deviation (lb)					34.755	20.782
OFEs	100000	N/A	25000	25000	25000	20040

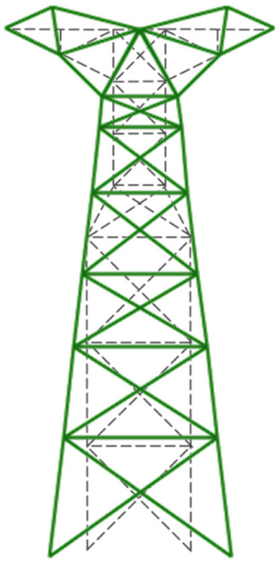


Fig. 15. Optimal layout for the planar 47-bar truss problem

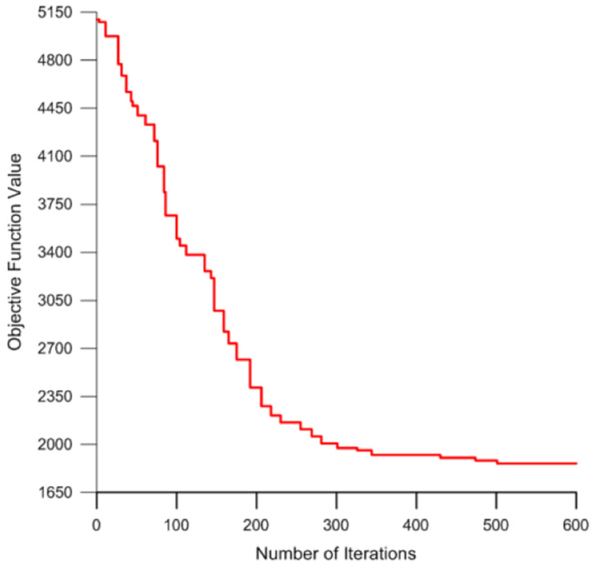


Fig. 16. Convergence history for the planar 47-bar truss using iPSO

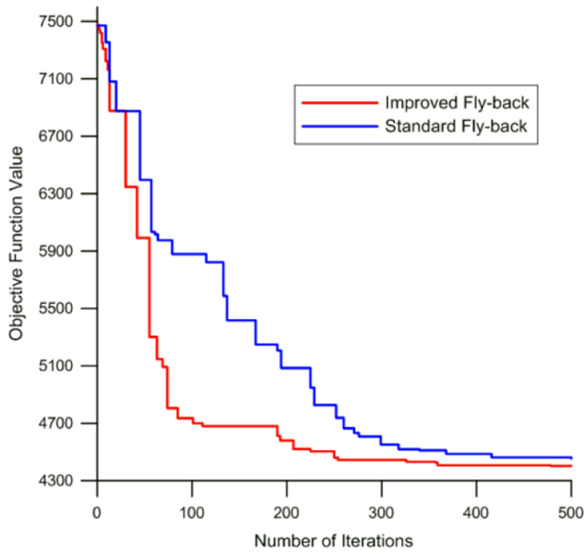


Fig. 17. Comparison between the standard and improved fly-back mechanism on 18-bar truss problem

relatively less dispersion in the obtained solutions. Considering the previous problems, this problem is a largescale optimization problem, and based on the numerical results reported in Table 13, it can be stated that the proposed iPSO algorithm can find the best optimal solution among the cited methods for the this problem. Figure 15 shows the optimal design found for this problem. Besides, Figure 16 demonstrates the convergence history of the 47-bar truss structure using iPSO.

Discussion

In this study the concept of weighted particle is utilized for both position updating and constraints handling features of the proposed algorithm. The new approach is called improved Fly-back technique. In this recent approach the search space is divided into two different categories and so the problem domain is examined more accurately. Furthermore, by incorporating the weighted particle into constraints handling process the affirmative data stored in this particle is used to handle the constraints. Since the weighted particle is the weighted average of all particles on the swarm, sharing its components with the violated particles can improve the exploitation feature of the algorithm. Especially, weighted particle lies closer, but not at same position, to the $\mathbf{X}^G$. On the other hand the weighted particle can share experience of all existing particles with the particle that violates the numeric boundary condition of the search space. This means all particles can interact with each other through the weighted particle.

Improved Fly-back mechanism plays two important roles: first, it emphasizes the role of the weighted particle as the guidance point for other particles (i.e. enhancing the exploitation) and second, it gives an opportunity to access the components stored in the weighted particle (i.e. enhancing the exploration). In this respect, Figure 17 compares the performance of the standard and improved Fly-back methods on the optimization of 18-bar truss structure. As can be seen from this figure, the convergence rate and final outcome of improved Fly-back technique are good than its old form. It is notable that, since the condition is qualitatively identical for all other examples, the diagram is plotted just for 18-bar truss problem to prevent any unnecessary congestion.

Conclusions

Integrated particle swarm optimization (iPSO) improves the standard PSO's search capability by implementing the concept of weighted particle and improved Fly-back approach. The enhancement is mostly in order to avoiding the algorithm from premature convergence into local optima(s). Furthermore, unlike the classical penalty function approach, the improved Fly-back strategy enables iPSO to consistently acquire feasible solutions without any violations in the boundary conditions. The proposed algorithm was tested on five well known weight minimization problems of truss structures. All of them include sizing and layout variables and the last one is the slightly higher scale problem consisting of totally 44 sizing and layout variables. The statistical data for each example is provided and based on acquired outcomes the iPSO shows adequate robustness and the iPSO can be considered as an efficient algorithm for solving combined sizing/layout optimization problems of truss structures.

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