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AN EXTENSION OF THE RATIO SYSTEM APPROACH OF MOORA METHOD FOR GROUP DECISION-MAKING BASED ON INTERVAL-VALUED TRIANGULAR FUZZY NUMBERS

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Abstract. Decision-making in fuzzy environment is often a very complex, especially when related to predictions and assessments. The Ratio system approach of the MOORA method and Interval-valued fuzzy numbers have already proved themselves as the effective tools for solving complex decision-making problems. Therefore, in this paper an extension of the Ratio system approach of the MOORA method, which allows a group decision-making as well as the use of interval-valued triangular fuzzy numbers, is proposed. Interval-fuzzy numbers are rather complex, and therefore, they are not practical for direct assigning performance ratings. For this reason, in this paper it has also been suggested the approach which allows the expression of individual performance ratings using crisp, interval or fuzzy numbers, and their further transformation into the group performance ratings, expressed in the form of interval-valued triangular fuzzy numbers, which provide greater flexibility and reality compared to the use of linguistic variables. Finally, in this paper the weighted averaging operator was proposed for defuzzification of interval-valued triangular fuzzy numbers.

Keywords: MCDM, MOORA, ratio system approach, fuzzy set theory, interval-valued triangular fuzzy numbers, weighted Hamming distance.

JEL Classification: D81, C61, C44.

Introduction

Decision-making is often associated with the process of selecting the best alternative from the set of available alternatives. In many cases when selecting the best alternative, it is necessary to take into account the impact of multiple criteria.

Since the 1970s, the multiple criteria decision-making (MCDM) approach has been developed rapidly and very soon it has become a main area of research for dealing with complex decision-making problems, which require the consideration of multiple objectives or criteria (Lertprapai 2013).

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As a result of that development, many MCDM methods are proposed, out of which only the most prominent are mentioned, such as: SAW (MacCrimmon 1968), Compromise Programming (Zeleny 1973; Yu 1973), AHP (Saaty 1980), TOPSIS (Hwang, Yoon 1981), PROMETHEE (Brans, Vickine 1985), ELECTRE (Roy 1991) and VIKOR (Opricovic 1998).

These methods were successfully used to solve many decision-making problems. However, these methods, the so-called ordinary MCDM methods, were based on the use of crisp numbers, and because of that, they were not fully adequate for solving many real-world problems. Therefore, many MCDM methods were extended to enable the use of fuzzy numbers or interval-valued fuzzy numbers.

Many real-world decision-making problems also require the participation of multiple decision makers and/or experts in the decision-making processes. Therefore, many MCDM methods wereextended also to provide a group decision-making approach.

The Multi-Objective Optimization on the basis of Ratio Analysis (MOORA) method was proposed by Brauers and Zavadskas (2006). The MOORA method consists of two components, that is: the Ratio system approach and the Reference point approach. Brauers and Zavadskas (2010a) also proposed the MULTIMOORA method, as an extension of the MOORA method in which there has been added a third component: Full Multiplicative Form.

Similarly to other MCDM methods, for the MOORA and the MULTIMOORA method, some extensions have been proposed. Brauers *et al.* (2011) proposed the first fuzzy extension of the MOORA method, or more precisely the MULTIMOORA method. In that extension, the MULTIMOORA method has been updated by the fuzzy sets theory, and all three components of the MULTIMOORA method: the Ratio system approach, the Reference point approach and the Full multiplicative form have been modified to enable the usage of triangular fuzzy numbers. Balezentis *et al.* (2012a, 2012b) further modified the fuzzy MULTIMOORA, and proposed the fuzzy extension, named MULTIMOORA-FG, which includes the use of linguistic variables and enables the group decision-making approach. Balezentis and Zeng (2013) also proposed an extension of MULTIMOORA based on interval-valued fuzzy numbers.

The MOORA, MULTIMOORA and their extensions were used for solving numerous problems, such as the regional development (Brauers, Zavadskas 2010b, 2011a; Brauers, Ginevicius 2010), the choice of bank loan (Brauers, Zavadskas 2011b), the personnel selection (Balezentis *et. al* 2012a, 2012b), forming a multiple criteria decision-making framework for the energy crops prioritization (Balezentiene *et al.* 2013), the selection of building elements for the renovation of energy efficient buildings (Kracka *et al.* 2013), and so on.

In the literature, it can also be identified a characteristic approach, initiated by Kalibatas and Turskis (2008), which is based on the use of the Ratio system approach of the MOORA method.

Karande and Chakraborty (2012), and Dey *et al.* (2012) proposed the fuzzy extensions of the Ratio system approach of the MOORA method. Both of these extensions enable the use of fuzzy triangular numbers. The extension proposed by Dey *et al.* (2012) also included the group decision-making approach, but in this extension, the decision matrix has been defuzzified in the initial stage, and then the crisp MOORA has been further employed.

Stanujkic *et al.* (2012a, 2012b) proposed the extensions of the Ratio system approach of the MOORA method, based on the use of interval fuzzy numbers.

The interval-valued fuzzy numbers provide an opportunity for a much more adequate modelling and solving complex real-world problems, but they are not suitable for direct assigning the values by decision makers. Therefore, they are often used in a combination with various linguistic scales.

In this paper, an extension of the Ratio system approach of the MOORA method, where the performance ratings of alternatives are expressed in interval-valued triangular fuzzy numbers, is proposed. The proposed extension also includes the group decision-making approach, where decision makers can express their individual performance ratings using crisp, interval or triangular fuzzy numbers. So obtained individual performance ratings are further transformed into the group, interval-valued triangular fuzzy performance ratings. This approach should provide a greater flexibility and more adequate determination of the group performance ratings compared to the use of linguistic scales.

Because of all above mentioned reasons, the rest of this paper is organized as follows: In Section 1, some basic definitions and notations are given. In Section 2, some procedures for determining the weight of criteria and performance ratings, based on the group decision-making approach are presented, and in Section 3 the interval-valued fuzzy extension of the Ratio system approach of the MOORA method, is proposed. In Section 4, an example is considered with the aim to explain in details the proposed methodology. Finally, the conclusions are given.

1. Preliminaries

In this section some basic definitions and notations, relevant for forming the extension of the Ratio system approach of the MOORA method for the multiple criteria group decision-making based on interval-valued triangular fuzzy numbers, proposed in Section 3, are given.

1.1. Interval-valued fuzzy sets

In the classical set theory, an element can belong or does not belong to a set. Let X be a classical set of objects, called the *universe*, whose generic elements are denoted by x. The membership, in a classical subset A of X, is often represented by a degree of the membership, such that:

$$\mu_A(x) = \begin{cases} 1 & x \in A; \\ 0 & x \notin A, \end{cases} \tag{1}$$

where: $\mu_A(x)$ denotes the membership function, and $\mu_A(x) \in \{0, 1\}$.

Unfortunately, many real-world decision-making problems are often related to the influence of uncertainty, which can not be easily expressed using the classical sets.

Zadeh (1965) introduced the Fuzzy sets theory, which allows a partial membership in a set. The fuzzy set A, illustrated in Figure 1, is completely defined by the set of pairs, such that

$$A = \left\{ \langle x, \mu_A(x) \rangle \middle| x \in X \right\}, \tag{2}$$

where $\mu_A(x) \in [0, 1]$.

The fuzzy sets theory was later extended. The concept of the interval-valued fuzzy set was proposed by Turksen (1986, 1996) and Gorzalczany (1987). The interval-valued fuzzy set A, illustrated in Figure 2, is given by:

$$A = \left\{ \left\langle x, \left[\mu_A^l(x), \mu_A^u(x) \right] \right\rangle \middle| x \in X \right\}, \tag{3}$$

where: $\mu_A^l(x)$ is the lower limit of the degree of the membership, $\mu_A^l(x) \in [0, 1]$; $\mu_A^u(x)$ denotes the upper limit of the degree of the membership, $\mu_A^u(x) \in [0, 1]$; and $\mu_A^l(x) \le \mu_A^u(x)$.

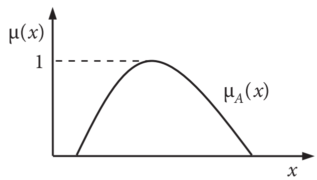

Fig. 1. Fuzzy set

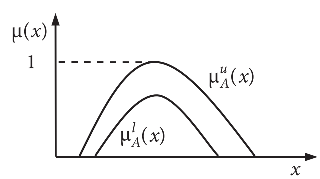

Fig. 2. The Interval-valued fuzzy set

1.2. Some typical types of numbers

As previously stated, the classical sets theory is based on the use of crisp numbers, and these numbers are not suitable for solving complex real-world decision-making problems.

The partial membership in a set, enabled in the fuzzy sets theory, introduced some new types of numbers such as interval, triangular, trapezoidal, and bell-shaped fuzzy numbers. The interval-valued fuzzy sets also introduced some new types of numbers such as interval-valued triangular fuzzy numbers and interval-valued trapezoidal fuzzy numbers.

Some of the commonly used numbers are listed below.

Crisp number. A crisp number, shown in Figure 3, is fully determined by its value m.

The membership function of the crisp number A is defined as follows:

$$\mu_A(x) = \begin{cases} 1 & x = m; \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Fuzzy number. A real fuzzy number A is described as a fuzzy subset of the real line $\Re$ with the membership function μ_A that represents uncertainty, where the membership function is defined from the universe of discourse to [0, 1].

Interval fuzzy number. An interval fuzzy number is fully determined by a pair of real numbers (l, u), where l and u are the lower bound and the upper bound, respectively; and (l, $u \in \Re$; $l < u$). The interval fuzzy number $\bar{A}$, is shown in Figure 4.

The membership function of interval fuzzy number $\bar{A}$ is defined as follows:

$$\mu_{\bar{A}}(x) = \begin{cases} 1 & x \in [l, u]; \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

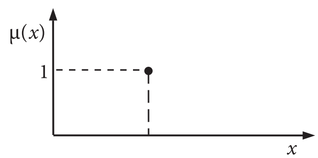

Fig. 3. A crisp number

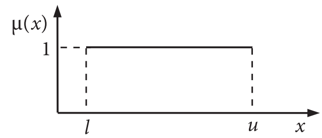

Fig. 4. An interval fuzzy number

Triangular fuzzy number. A triangular fuzzy number, shown in Figure 5, is fully characterized by a triplet of real numbers (l,m,u), where parameters *l*, *m* and *u*, denote the smallest possible value, the most promising value, and the largest possible value that describes a fuzzy event (Dubois, Prade 1980; Ertugrul, Karakasoglu 2009).

The membership function of the triangular fuzzy number is defined as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases}(x-l)/(m-l) & l\le x\le m;\\ (u-x)/(u-m), & m\le x\le u;\\ 0 & \text{otherwise.}\end{cases} \tag{6}$$

Interval-valued triangular fuzzy number. According to Yao and Lin (2002), the interval-valued triangular fuzzy number, shown in Figure 6, can be presented as follows:

$$\tilde{A}=\left[\tilde{A}^l,\tilde{A}^u\right]=\left[(a_l',a_m',a_u';\omega_A'),(a_l,a_m,a_u;\omega_A)\right], \tag{7}$$

where: $\tilde{A}^L$ and $\tilde{A}^U$ denote the lower and upper triangular fuzzy numbers, $\tilde{A}^L\subset\tilde{A}^U$, respectively; l', m' *and* u', denote the smallest possible value, the most promising value, and the largest possible value that describes a fuzzy event $\tilde{A}^L$; *l*, *m and u*, denote the smallest possible value, the most promising value, and the largest possible value that describes a fuzzy event $\tilde{A}^U$; ω_A' and ω_A denote the maximum values of the lower $\mu_{\tilde{A}^L}(x)$ and upper $\mu_{\tilde{A}^U}(x)$ membership functions, respectively.

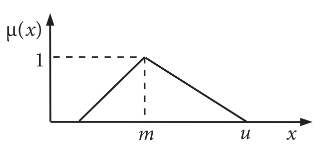

Fig. 5. A triangular fuzzy number

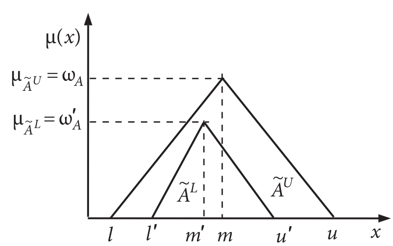

Fig. 6. The interval-valued triangular fuzzy number

Normalized interval-valued triangular fuzzy number. The particular case of the interval-valued triangular fuzzy numbers are the normalized interval-valued triangular fuzzy numbers, $\omega_A'=\omega_A=1$, with the same mode. The normalized interval-valued triangular fuzzy number, shown in Figure 7, can be presented as follows:

$$\tilde{A}=[\tilde{A}^l,\tilde{A}^u]=\left[(l,l_l'),m,(u',u)\right]. \tag{8}$$

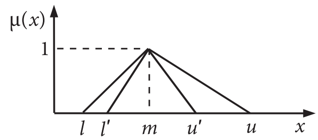

Fig. 7. The normalized interval-valued triangular fuzzy number with the same mode

1.3. The basic operations of interval-valued triangular fuzzy numbers

Suppose that $\tilde{A}=[(a_l,a_l'),a_m,(a_u',a_u)]$ and $\tilde{B}=[(b_l,b_l'),b_m,(b_u',b_u)]$ are two normalized interval-valued triangular fuzzy numbers. Then, the basic operations on these fuzzy numbers (Chen 1997; Chen, S. J., Chen, S. M. 2008) are defined as follows:

$$\tilde{A}+\tilde{B}=[(a_l+b_l,a_l'+b_l'),a_m+b_m,(a_u'+b_u',a_u+b_u)]; \quad (9)$$

$$\tilde{A}-\tilde{B}=[(a_l-b_u,a_l'-b_u'),a_m-b_m,(a_u'-b_l',a_u-b_l)]; \quad (10)$$

$$\tilde{A}\times\tilde{B}=[(a_l\times b_l,a_l'\times b_l'),a_m\times b_m,(a_u'\times b_u',a_u\times b_u)]; \quad (11)$$

$$\tilde{A}\div\tilde{B}=[(a_l\div b_u,a_l'\div b_u'),a_m\div b_m,(a_u'\div b_l',a_u\div b_l)]; \quad (12)$$

$$\frac{1}{k}\times\tilde{A}=[(\frac{1}{k}\times a_l,\frac{1}{k}\times a_l'),\frac{1}{k}\times a_m,(\frac{1}{k}\times a_u',\frac{1}{k}\times a_u)]. \quad (13)$$

1.4. Hamming distance

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy sets. The Hamming distance $d(\tilde{A},\tilde{B})$ is as follows:

$$d(\tilde{A},\tilde{B})=\sum_{i=1}^{n}|\mu_A(x_i)-\mu_B(x_i)|. \quad (14)$$

Suppose that $\tilde{A}=[(a_1,a_2),a_3,(a_4,a_5)]$ and $\tilde{B}=[(b_1,b_2),b_3,(b_4,b_5)]$ are two normalized interval-valued triangular fuzzy numbers. Then, the Hamming, normalized Hamming and weighted normalized Hamming distances are as follows:

$$d(\tilde{A},\tilde{B})=\sum_{i=1}^{n}|a_i-b_i|; \quad (15)$$

$$d(\tilde{A},\tilde{B})=\frac{1}{5}\sum_{i=1}^{n}|a_i-b_i|; \quad (16)$$

$$d_w(\tilde{A},\tilde{B})=\frac{1}{5}\sum_{i=1}^{n}w_i|a_i-b_i|, \quad (17)$$

where w_i denotes the weight (significance) of i-th parameter of an interval-valued triangular fuzzy number, $\sum_{i=1}^{n}w_i=1$ and $w_j\in[0,1]$.

1.5. Defuzzification of interval-valued triangular fuzzy numbers

As the time goes by, a number of defuzzification methods were proposed. However, these methods were mainly intended for the defuzzification of trapezoidal and triangular fuzzy numbers, such as, the methods proposed by Liou and Wang (1992) and Opricovic and Tzeng (2003).

The defuzzification method proposed by Liou and Wang (1992) can be shown as:

$$mf(\tilde{A}) = \frac{1}{2}\,[(1-\lambda)\,l + m + \lambda\,u], \tag{18}$$

where $mf(\tilde{A})$ denotes a mapping function that transforms the fuzzy numbers into the crisp ones, λ denotes an index of optimism, and $\lambda \in [0,\,1]$.

By assigning different values to the coefficient λ, decision makers can express their opinions more realistically, i.e. levels of their optimism or pessimism about something.

Opricovic and Tzeng (2003) also proposed a simple to use defuzzification method, which can be shown as:

$$mf(\tilde{A}) = \frac{l+m+u}{3}. \tag{19}$$

On the basis of Eqs (18) and (19), in this paper, the following formula was proposed for the defuzzification of normalized interval-valued triangular fuzzy numbers:

$$fm(\tilde{A}) = \frac{1}{5}\Big[(1-\alpha)l + (1-\beta)l' + \gamma m + \beta u' + \alpha u\Big], \tag{20}$$

where α, β, and γ are coefficients, $\alpha, \beta \in [0,1]$, and $\gamma \geq 0$.

By giving different values to coefficients α, β, and γ in the Eq. (20), the decision makers have the opportunity to express their opinions more accurately. In addition, the Eq. (20) has some special forms such as:

– when $\alpha = \beta$

$$fm(\tilde{A}) = \frac{1}{5}\Big[(1-\alpha)(l+l') + \gamma m + \alpha(u'+u)\Big]. \tag{21}$$

– when $\alpha = \beta = 0.5$, $\gamma = 1$, $l = l'$ and $u = u'$

$$mf(\tilde{A}) = \frac{l+m+u}{5}. \tag{22}$$

As it can be seen, the Eq. (22) is very similar to the Eq. (19) and what is more, it takes into account the specificities of normalized interval-valued triangular fuzzy numbers.

2. Multiple criteria group decision making

A typical MCDM problem can be more concisely presented in the following form:

$$D = [x_{ij}]_{m \times n}; \tag{23}$$

$$W = [w_j]_n, \tag{24}$$

where: D denotes the decision matrix, x_{ij} is the performance rating of alternative A_i with respect to criterion C_j, W denotes the weight vector, w_j is the weight of criterion C_j, $i = 1,2, \ldots m$; m is the number of compared alternatives, $j = 1,2, \ldots, n$; n is the number of the criteria.

In Eqs (23) and (24) the values of x_{ij} and w_j are crisp numbers. Many previously published studies have indicated that crisp numbers are not fully adequate for solving complex real-world decision-making problems. These studies have also pointed out the importance of the group-decision making approaches.

The interval-valued fuzzy multiple criteria group decision-making can be presented in the following form:

$$\tilde{D}^k = [\tilde{x}_{ij}^k]_{m\times n}; \tag{25}$$

$$\tilde{W}^k = [\tilde{w}_j^k]_n, \tag{26}$$

where: $\tilde{D}^k$ denotes a fuzzy decision matrix formed by the decision maker/expert k, $\tilde{x}_{ij}^k$ is the interval-valued fuzzy performance rating of alternative A_i with respect to criterion C_j obtained from decision maker/expert k, $\tilde{W}^k$ denotes the fuzzy weight vector obtained from the decision maker/expert k, and k = 1,2, …, K; K denotes a number of decision makers or experts.

In the multiple criteria group decision-making, it is very important how to aggregate individual criteria weights and individual performance ratings into the group (aggregated) criteria weights and performance ratings.

2.1. Determining the weights of criteria

Many published papers have also indicated that the use of the group decision-making approaches and pairwise comparison procedure provides an efficient approach for precisely determining the relative importance of criteria i.e. weights of criteria.

Pairwise comparison. The pairwise comparison procedure is quite simple and understandable, even for decision makers who are not familiar with the MCDM. For a decision making problem that contains n criteria, the process of determining weights of criteria begins by forming reciprocal square matrix:

$$A = [a_{ij}]_{n\times n}, \tag{27}$$

where: A denotes a pairwise comparison matrix, a_{ij} is the relative importance of criterion C_i in relation to criterion C_j, $i = 1,2,\ldots,n$, $j = 1,2,\ldots,n$, and n is the number of criteria. In the matrix A, $a_{ij} = 1$ when $i = j$ and $a_{ji} = 1/a_{ij}$.

The nine-point scale, shown in Table 1, proposed by Saaty (1980), is used to assign a relative importance of criteria.

Table 1. The scale of relative importance for pairwise comparison

Intensity of Importance	Definition
1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong importance
9	Extreme importance
2, 4, 6, 8	For interpolation between the above values

After forming the matrix A, by using one of several available procedures, weights of criteria can be calculated. Using the Normalization of the Geometric Mean of the Rows procedure, the weights of criteria are calculated as follows:

$$w_i = \left(\prod_{j=1}^{n} a_{ij} \right)^{1/n} \Bigg/ \sum_{i=1}^{n} \left(\prod_{j=1}^{n} a_{ij} \right)^{1/n} . \tag{28}$$

While forming the matrix A, it is very important that each decision maker should perform its comparisons consistently. The decision about the consistency of performed comparisons and their acceptability, are made on the basis of the Consistency Ratio. If the consistency ratio is higher than 0.1, then the pairwise comparison matrix A is inconsistent, and therefore the comparisons should be reviewed and improved.

The Consistency Ratio is calculated as follows:

$$CR = CI / RI , \tag{29}$$

where: CR denotes the consistency ratio of the pairwise comparison matrix A, CI is the Consistency Index and RI is the Random Consistency Index.

The values of CI can be calculated as follows:

$$CI = (\lambda_{\max} - n)/(n-1) , \tag{30}$$

where $\lambda_{\max}$ is the maximum eigenvalue of the pairwise comparison matrix and it can be calculated as follows:

$$\lambda_{\max} = \sum_{j=1}^{n} \left\{ \left(\sum_{i=1}^{n} a_{ij} \right) w_j \right\} , \tag{31}$$

where w_j is the weight of criterion C_j and n is the number of criteria.

The values for RI are determined based on matrix size n. Table 2 shows the value of the Random Consistency Index RI for different matrix sizes (Saaty 1980).

Table 2. The Random Consistency Index for different matrix sizes

Matrix size (n)	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.58	0.9	1.12	1.24	1.32	1.41	1.46	1.49

Thanks to this controlling mechanism the above mentioned procedure for calculation of criteria weights has become very popular and frequently used.

Group decision-making approach to determine criteria weights. In many published papers, the use of the different group decision-making approaches to determine the group criteria weights, were considered. In this approach, the simplest and the most efficient one is accepted and used.

For a group that contains K decision makers, the group weight of each criterion w_j is calculated using the geometric mean, as follows:

$$w_j = \left(\prod_{k=1}^{K} w_j^k \right)^{1/K} , \tag{32}$$

where: w_j^k is the weight of criterion C_j, obtained on the basis of pairwise comparisons performed by decision maker k.

2.2. Determining the interval-valued fuzzy performance ratings

The interval-valued fuzzy numbers provide much greater opportunities than the ordinary fuzzy numbers, especially in the case of solving complex real-world decision-making problems.

However, these numbers are not suitable for assigning values directly by decision makers, and therefore the interval-valued fuzzy numbers are often used in combination with linguistic scales. In such approaches, in order to express their attitudes, the decision maker uses linguistic variables, whose values are defined in advance, which in some way can limit the precise expression of their attitudes.

In order to allow the decision makers to express more accurately their attitudes about the performance ratings of alternatives, instead of linguistic variables, in this approach, they have an opportunity to express their individual opinions using crisp, interval or triangular fuzzy numbers.

After that, the transformation of individual performance ratings into the corresponding group, the interval-valued triangular fuzzy, performance ratings can be performed using the following procedure:

Let x_{ij}^k denote performance ratings of the alternative A_i with respect to criterion C_j obtained from the decision maker/expert k, and x_{ij}^k can be expressed using crisp, $x_{ij}^k = m_{ij}^k$, interval, $x_{ij}^k = [l_{ij}^k, u_{ij}^k]$, or triangular fuzzy numbers $x_{ij}^k = (l_{ij}^k, m_{ij}^k, u_{ij}^k)$. Then, the individual performance ratings are transformed into a group, the interval-valued triangular $\tilde{x}_{ij} = [(l_{ij}, l'_{ij}), m_{ij}, (u'_{ij}, u_{ij})]$, performance ratings using the following equations:

$$l_{ij} = \min_k (l_{ij}^k); \tag{33}$$

$$l'_{ij} = \left(\prod_{k=1}^{K} l_{ij}^k \right)^{1/K}; \tag{34}$$

$$m_{ij} = \left(\prod_{k=1}^{K} m_{ij}^k \right)^{1/K}; \tag{35}$$

$$u_{ij}^k = \left(\prod_{k=1}^{K} u_{ij}^k \right)^{1/K}; \tag{36}$$

$$u_{ij} = \max_k (u_{ij}^k). \tag{37}$$

The proposed equations unfortunately contain some limitations, i.e., they are not sufficient for determining the group interval-valued triangular fuzzy performance ratings when all individual performances are presented using: i) only interval fuzzy numbers or ii) only crisp numbers. Therefore, when individual performance ratings of a certain criterion are expressed:

- using only the interval fuzzy numbers, the mode m_{ij}^k must be determined first, as follows:

$$m_{ij}^k = 0.5\ (l_{ij}^k + u_{ij}^k), \tag{38}$$

after which the Eq. (25) to (29) should be used.

– using only crisp numbers, the following equations should be used:

$$l_{ij} = \min_k (m_{ij}^k); \tag{39}$$

$$l'_{ij} = \min_k (m_{ij}^k); \tag{40}$$

$$m_{ij} = \left(\prod_{k=1}^{K} m_{ij}^k \right)^{1/K}; \tag{41}$$

$$u_{ij}^k = \max_k (m_{ij}^k); \tag{42}$$

$$u_{ij} = \max_k (m_{ij}^k). \tag{43}$$

3. The Interval-valued fuzzy extension of the Ratio system approach of the MOORA method

When compared to other MCDM methods, the Ratio system approach of the MOORA method is based on a specific idea that the overall performance rating of an alternative can be determined as a difference between its sum of weighted normalized ratings of benefit criteria[1] and the sum of weighted normalized ratings of cost criteria[2], as follows:

$$S_i = \sum_{j \in \Omega_{\max}} w_j r_{ij} - \sum_{j \in \Omega_{\min}} w_j r_{ij}, \tag{44}$$

where: S_i denotes the overall performance rating of alternative A_i, r_{ij} is the normalized performance rating of alternative A_i with respect to criterion C_j, w_j is the weight of criterion C_j, $\Omega_{\max}$ and $\Omega_{\min}$ denotes the set of benefit criteria and cost criteria, respectively.

After that, the considered alternatives are ranked on the basis of their S_i in ascending order, and the alternative with the highest value of S_i is the best ranked.

For normalization, the authors of the MOORA method (Brauers, Zavadskas 2006) proposed the use of vector normalization procedure, as follows:

$$r_{ij} = \frac{x_{ij}}{\left(\sum_{i=1}^{n} x_{ij} \right)^{1/2}}. \tag{45}$$

The above discussed computational procedure is based on the use of crisp numbers and attitudes of the single decision maker. To enable the use of interval-valued fuzzy triangular numbers for expressing the performance ratings and th use of the group decision-making approach, some changes have to be made in it.

The detailed step-by-step computational procedure of the extended Interval-valued Fuzzy Ratio system approach of the MOORA method, which enables the use of the interval-valued triangular fuzzy numbers and the group decision-making approach, can be precisely expressed using the following steps:

[1] Criteria to be maximized, i.e. the larger the better type.

[2] Criteria to be minimized, i.e. the smaller the better type.

Step 1. Identify available alternatives and select the evaluation criteria. In this step, the team of decision makers/experts identifies the available alternatives and chooses the criteria for their evaluation.

Step 2. Determine the relative importance of the evaluation criteria. In this step, the team of decision makers/experts determines weights of evaluation criteria using the procedure, proposed in subsection 2.1.

Step 3. Construct the interval-valued fuzzy decision matrix. In this step, using the procedure proposed in the subsection 2.2, the interval-valued fuzzy decision matrix is formed.

Step 4. Construct the normalized interval-valued fuzzy decision matrix. Similarly to the TOPSIS method, the MOORA method is based on the use of the vector normalization procedure. In many fuzzy extensions of the TOPSIS method, the vector normalization procedure has been replaced by the less complex normalization procedures, usually with the linear scale transformation - max method (Saremi *et al.* 2009; Mahdavi *et al.* 2008; Wang, Elhag 2006), and sometimes by some other normalization as the linear scale transformation – max-min method (Yang, Hung 2007).

For this reason, in order to determine the normalized interval-valued triangular fuzzy performance ratings $\tilde{r}_{ij}$, the following equation is proposed:

$$\tilde{r}_{ij} = \left[\left(\frac{l_{ij}}{x_j^+}, \frac{l'_{ij}}{x_j^+} \right), \frac{m}{x_j^+}, \left(\frac{u'_{ij}}{x_j^+}, \frac{u_{ij}}{x_j^+} \right) \right], \tag{46}$$

where

$$x_j^+ = \max_i x_{ij}. \tag{47}$$

Step 5. Calculate the overall interval-valued fuzzy performance ratings based on the Ratio system approach of the MOORA method, for each alternative. The overall interval-valued fuzzy performance ratings, based on the Ratio system approach of the MOORA method, can be determined using the following equation:

$$\tilde{S}_i = \tilde{S}_j^+ - \tilde{S}_j^-, \tag{48}$$

where:

$$S_i^+ = \sum_{j \in \Omega_{\max}} w_j \, \tilde{r}_{ij}; \tag{49}$$

$$S_i^- = \sum_{j \in \Omega_{\min}} w_j \, \tilde{r}_{ij}. \tag{50}$$

Step 6. Rank the alternatives and select the best one. The results obtained by using the Eq. (41) are in fact the interval-valued triangular fuzzy numbers. Therefore, in order to rank the available alternatives, the proposed extension should have the possibility of ranking interval-valued triangular fuzzy numbers or it should include a defuzzification method which must be used before their ranking.

Therefore, in this extension of the Ratio system approach of the MOORA method the use of the two approaches were considered, namely:

– the distance based approach, and

- the approach based on the defuzzification of the interval-valued triangular fuzzy numbers.

The distance based approach. The most widely used distances for fuzzy sets are the Hamming distance and the Euclidean distance. In this approach, the use of the Hamming distance is proposed, or more precisely the Hamming distance between the overall interval-valued performance ratings of alternatives and the anti-ideal reference point $\tilde{A}^-$.

For simplicity, in this approach as the anti-ideal reference point is choosen $\tilde{A}^- = [(0,\ 0),\ 0,(0,\ 0)]$, and therefore the Eqs (16) and (17) can be written as follows:

$$d(\tilde{A},\ \tilde{A}^-) = \frac{1}{5}\sum_{i=1}^{n} a_i\ ; \tag{51}$$

$$d_w(\tilde{A},\ \tilde{A}^-) = \frac{1}{5}\sum_{i=1}^{n} w_i a_i\ . \tag{52}$$

Depending on the fact whether the decision maker wants or does not want to give more importance to some of the parameters of the interval-valued triangular fuzzy number (*l*, *l*', *m*, *u*' and *u*) relating to the alternatives ranking, there can be used equations (52) or (51).

The approach based on the defuzzification. As previously mentioned, for the defuzzification of the ordinary fuzzy numbers a number of procedures were proposed, such as the procedures proposed by Liou and Wang (1992), Chiu and Park (1994), Opricovic and Tzeng (2003), and so on. However, these procedures can not be so easily applied to the defuzzification of interval-valued fuzzy numbers.

In the subsection 1.5, an approach, which was specially adapted for the defuzzification of the normalized interval-valued triangular fuzzy numbers was proposed.

By giving different values to the coefficients α, β, and γ in the Eq. (20), the decision makers have the opportunity to express their opinions more accurately.

4. Numerical example

To demonstrate the applicability and efficiency of the proposed approach, in this section is shown its use to solve a particular problem, on an example adopted from Stanujkic *et al.* (2014). In order to demonstrate briefly the advantages of the proposed methodology, this example is slightly modified.

The mining company XYZ is planning to start the exploitation of a new mine with surface mining. The geographical location of the new mine, i.e. the distance of the new mine to the existing flotation does not provide a cost-effective transportation of the excavated ore. Therefore, the team of three experts was formed with the aim to evaluate some comminution circuit designs and propose the most appropriate one.

In the preliminary selection, the team formed of three experts considers three typical comminution circuit designs:

- A_1, comminution circuit designs based on the combined use of rod mills and ball mills;

– A_2, comminution circuit designs based on the use of ball mills; and

– A_3, comminution circuit designs based on the use of semi-autogenous mills.

To evaluate the aforementioned comminution circuit designs and select the most appropriate one, the team of experts has proposed the following criteria:

– C_1, Grinding efficiency;

– C_2, Economic efficiency;

– C_3, Capital investment; and

– C_4, Environmental impact.

At the beginning of the evaluation, each expert, using the procedure previously proposed in the subsection 3.1, has performed an assignment of the criteria weights. The results of pairwise comparisons, obtained from experts, are shown in Tables 3, 4 and 5.

After that, the team of experts has estimated the performance ratings of the three specified comminution circuit designs in relation to the selected evaluation criteria, using the procedure described in the subsection 3.2. They have evaluated the specified designs in relation to the characteristics of the ore which will be excavated from the ore body Cerovo.

Table 3. The pairwise comparisons matrix and the relative criteria weights obtained from the first decision maker

Criteria		C_1	C_2	C_3	C_4	w_i
Grinding efficiency	C_1	1	2	3	5	0.44
Economic efficiency	C_2	1/2	1	3	7	0.34
Capital investment costs	C_3	1/3	1/3	1	5	0.16
Environmental impact	C_4	1/5	1/7	1/5	1	0.05
				$CR = 0.078$		

Table 4. The pairwise comparisons matrix and the relative criteria weights obtained from the second decision maker

Criteria		C_1	C_2	C_3	C_4	w_i
Grinding efficiency	C_1	1	3	4	5	0.52
Economic efficiency	C_2	1/3	1	3	5	0.28
Capital investment costs	C_3	1/4	1/3	1	3	0.13
Environmental impact	C_4	1/5	1/5	1/3	1	0.06
				$CR = 0.070$		

Table 5. The pairwise comparisons matrix and the relative criteria weights obtained from the third decision maker

Criteria		C_1	C_2	C_3	C_4	w_i
Grinding efficiency	C_1	1	4	3	2	0.49
Economic efficiency	C_2	1/4	1	3	1	0.20
Capital investment costs	C_3	1/3	1/3	1	1	0.13
Environmental impact	C_4	1/2	1/1	1/1	1	0.18
				$CR = 0.089$		

In order to simplify the procedure of determining the performance ratings of the considered alternatives in relation to the selected criteria, the experts have used the following approach:

- Commonly used comminution circuit design, based on the use of the rod mills and ball mills, i.e. alternative A_1, was chosen as a standard solution, which is why the performance ratings of alternative A_1, in relation to the selected criteria, have set to the value 1.
- The performance ratings of the remaining alternatives were assigned in relation to the standard alternative A_1, where decision makers have had the opportunity to express their attitudes using crisp, interval fuzzy or triangular fuzzy numbers.

The results of evaluation are shown in Table 6.

The group performance ratings, obtained using the procedure described in the subsection 2.2, are shown in Table 7.

Table 6. The performance ratings of alternatives obtained from experts

		C_1	C_2	C_3	C_4
	A_1	1.00	1.00	1.00	1.00
Expert 1	A_2	[0.90, 0.95]	[1.05, 1.07]	[0.90, 0.95]	(0.90, 1.02, 1.03)
	A_3	(1.05, 1.07, 1.10)	[1.05, 1.10]	(1.00, 1.10, 1.20)	[0.90, 0.95]
Expert 2	A_2	[1.02, 1.05]	[1.08, 1.10]	[0.90, 1.05]	1.00
	A_3	[1.00, 1.05]	1.00	1.10	1.10
Expert 3	A_2	[0.90, 0.95]	(1.04, 1.05, 1.06)	(0.80, 0.90, 0.95)	(0.85, 0.90, 1.00)
	A_3	(0.80, 1.05, 1.07)	(0.90, 1.02, 1.06)	[0.90, 1.20]	[0.90, 1.00]

Table 7. The group performance ratings

	A_1	A_2	A_3
C_1	[(1.00, 1.00), 1.00, (1.00, 1.00)]	[(0.90, 0.94), 0.96, (0.98, 1.05)]	[(0.80, 0.94), 1.06, (1.07, 1.10)]
C_2	[(1.00, 1.00), 1.00, (1.00, 1.00)]	[(1.04, 1.06), 1.05, (1.08, 1.10)]	[(0.90, 0.97), 1.01, (1.08, 1.10)]
C_3	[(1.00, 1.00), 1.00, (1.00, 1.00)]	[(0.80, 0.87), 0.90, (0.98, 1.05)]	[(0.90, 0.95), 1.10, (1.20, 1.20)]
C_4	[(1.00, 1.00), 1.00, (1.00, 1.00)]	[(0.85, 0.87), 0.97, (1.01, 1.03)]	[(0.90, 0.90), 1.10, (0.97, 1.00)]

The process of transforming the individual performance ratings into the group performance ratings can seem complex. Therefore, it is given an example of the transformation performed for the criterion C_2 of the alternative A_2, because it involves the use of crisp, interval and triangular fuzzy numbers.

As it can be seen from Table 7, the performance ratings obtained from the three decision makers are as follows: $x_{32}^1 = [1.05,\ 1.10]$, $x_{32}^2 = 1.00$, and $x_{32}^3 = (0.90,\ 1.02,\ 1.06)$.

The corresponding group performance rating for the alternative A2 in Table 8, expressed in the form of the interval-valued triangular fuzzy number, $x_{32} = [(0.90,\ 0.97),\ 1.01,\ (1.08,\ 1.10)]$, is obtained as follows:

- the parameter l, i.e. number 0.90, was determined by using the Eq. (33), i.e. as a minimum of numbers 1.05 and 0.90;

- the parameter l', i.e. 0.97, was determined by using the Eq. (34), i.e. as a geometric mean of numbers 1.05 and 0.90;
- the parameter m, i.e. 1.01, was determined by using the Eq. (35), i.e. as a geometric mean of numbers 1.00 and 1.02;
- the parameter u', i.e. 1.08, was determined by using the Eq. (36), i.e. as a geometric mean of numbers 1.10 and 1.06; and
- the parameter u, i.e. 1.10, was determined by using the Eq. (37), i.e. as a minimum of numbers 1.10 and 1.06.

The group weights of criteria, obtained by using the Eq. (32), and the normalized group performance ratings, obtained by using the Eq. (46), are shown in Table 8.

The overall interval-valued fuzzy performance ratings based on the cost and benefit criteria, obtained by using Eq. (49) and Eq. (50), and overall interval-valued fuzzy performance ratings of alternatives, obtained by using Eq. (48), are shown in Table 9.

Table 8. The group criteria weights and the normalized performance ratings

	w_j		A_1	A_2	A_3
C_1	0.48	max	[(0.91, 0.91), 0.91, (0.91, 0.91)]	[(0.82, 0.85), 0.87, (0.89, 0.95)]	[(0.73, 0.86), 0.96, (0.98, 1.00)]
C_2	0.28	max	[(0.91, 0.91), 0.91, (0.91, 0.91)]	[(0.95, 0.96), 0.95, (0.98, 1.00)]	[(0.82, 0.88), 0.92, (0.98, 1.00)]
C_3	0.15	max	[(0.83, 0.83), 0.83, (0.83, 0.83)]	[(0.67, 0.72), 0.75, (0.82, 0.88)]	[(0.75, 0.79), 0.92, (1.00, 1.00)]
C_4	0.09	min	[(0.97, 0.97), 0.97, (0.97, 0.97)]	[(0.83, 0.85), 0.94, (0.99, 1.00)]	[(0.87, 0.87), 1.07, (0.95, 0.97)]

Table 9. The results of the comminution circuit design evaluation

	S_i^+	S_i^-	S_i
A_1	[(0.82, 0.82), 0.82, (0.82, 0.82)]	[(0.09, 0.09), 0.09, (0.09, 0.09)]	[(0.73, 0.73), 0.73, (0.73, 0.73)]
A_2	[(0.75, 0.79), 0.80, (0.82, 0.86)]	[(0.07, 0.08), 0.08, (0.09, 0.09)]	[(0.68, 0.71), 0.72, (0.73, 0.77)]
A_3	[(0.69, 0.78), 0.86, (0.89, 0.91)]	[(0.08, 0.08), 0.10, (0.09, 0.09)]	[(0.61, 0.70), 0.76, (0.80, 0.82)]

The overall interval-valued fuzzy performances, obtained by using the proposed extension of the Ratio system approach of the MOORA method, are also interval-valued triangular fuzzy numbers. To select the most appropriate alternative(s), or rank the alternatives, the interval-valued triangular fuzzy numbers should be transformed into the corresponding crisp numbers.

The ranking results and the ranking order of the alternatives obtained on the basis of distance based approach, i.e. by using Eq. (16), are shown in Table 10.

Table 10. Ranking order of the alternatives

	S_i	*Rank*
A_1	0.73	2
A_2	0.72	3
A_3	0.74	1

In this case, all distances from the anti-ideal point had the same significance. When necessary, by using the formula (17), the decision makers can give more importance to some of the distances.

The ranking results and the ranking order of the alternatives obtained on the basis of the distance based approach are show in Table 11. In this case, for the sake of transformation of the interval-valued triangular fuzzy numbers into the crisp ones, the Eq. (21), with α = 0.5 and γ = 1, was used.

Table 11. Ranking order of the alternatives

	S_i	*Rank*
A_1	0.44	2
A_2	0.43	3
A_3	0.45	1

By using Eqs (20) or (21), with different values of coefficients α, β and γ , the decision makers can carry out the necessary analysis and make a selection of the most suitable alternative(s).

Conclusions

Compared to crisp and ordinary fuzzy numbers, the interval-valued fuzzy numbers provide much greater opportunities for solving real-world problems, especially the problems which are placed in a fuzzy environment or problems which include some forms of predictions or forecasting. Therefore, in this paper an extension of the Ratio system approach of the MOORA method that enables the use of interval-valued triangular fuzzy numbers is proposed.

However, direct assign of values to the performance ratings of alternatives in relation to the evaluation criteria, when these are expressed using interval-valued triangular fuzzy numbers, is not so common and easy for the decision makers and/or experts. Because of that, the proposed extension enables the use of crisp, interval and triangular fuzzy numbers for expressing the individual performance ratings of alternatives and their transformation into the interval-valued group performance ratings. This approach should ensure a greater flexibility in relation to the use of linguistic variables as well as the possibility of the more precise expressing performance ratings in a fuzzy environment.

Defuzzification of interval-valued triangular fuzzy numbers is also more complex than the defuzzification of ordinary fuzzy numbers. Therefore, in this paper, the weighted averaging operator was proposed for the defuzzification of interval-valued triangular fuzzy numbers. In the literature, the use of numerous averaging operators has been discussed, but in this approach the weighted averaging operator was chosen because of its simplicity and efficiency.

The usability and efficiency of the proposed extension is presented on the example of the comminution circuit design selection. Characteristics of ore in ore deposits can vary within certain limits, and these characteristics may also be related to the depth of the ore body. This makes the selection of comminution circuit design a complex problem. Using the proposed extension of the Ratio system approach of the MOORA method decision, the makers can analyze different scenarios and make the most appropriate choice.

Certainly, the proposed approach is not limited to the selection of comminution circuit design. Its application is much broader, and it can be applied to solve a number of other complex problems.

Finally, it should be noted that in addition to the proposed extension of the Ratio system approach, in a similar way, also should be proposed an extension of the Reference point approach, or even more an extension of the Full multiplicative form. With these extensions, a new extension of the MULTIMOORA method can be obtained.

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