International Journal of Operations Research

International Journal of Operations Research Vol. 14, No. 4, 177−185 (2017)

Aggregation Weights for Linguistic Hybrid Geometric Averaging Operator

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Abstract: This paper tries to point out that the aggregation weights in linguistic hybrid geometric averaging operator will dominate the final result of the ranking for alternatives. We examined the linguistic hybrid geometric averaging operator that was proposed by previous studies and found it contained several questionable results. The major defect of the previous approach was that it failed to demonstrate two core factors: accuracy and speed, both of which have been explicitly uncovered and discussed in the study. With previous work the pivotal and dominant element, distribution of weights, in finding subjectively by decision maker of linguistic hybrid geometric averaging operators for group decision-making problems, lacks solid foundation and is unjustified. Here we provide the mathematical rationale and reliable advices, to point out that deficiency. In addition, we have detected and rectified some redundancies of operational laws in the procedure of previous study due to the improper utilization of negative operators. It certainly should be noted that the careless applications of those highly dependant operators may significantly diminish the efficiency and performance of entire mechanism for decision making under fuzzy environment. We develop an easy aggregation approach based on the arithmetic mean to solve the most favorable alternative problem. A comprehensive numerical examination of 1296 tests supports our result.
Keyword — Decision-making, Linguistic preference relation.

1. INTRODUCTION

Accuracy and speed have grabbed the attention of enterprisers and academicians for years owing to the former, i.e. accuracy, being viewed as effectiveness and the latter being viewed as efficiency which are the two major pillars of performance and profitability. However some studies have been ambiguous and deviated away from this pivotal rationale.

We discovered that, after reviewing a great deal of relevant papers, the approach of preference relation aggregations has been frequently employed in finding optimal solutions of group decision-making problems with diverse criteria, decision makers (DMs), and preference weights. Some respectable contributions include development of ordered weighted averaging (OWA) operator accomplished by Yager (1988), introduction of new families of OWA operators by Xu and Da (2003), output of linguistic representation model carried out by Herrera and Martinez (2000), and other significantly wide range of applications by Yager and Kacprzyk (1997) and Xu (2002). Yet among these papers, the most relevant factor, distribution of weights, which dominantly influence the objectivity, reliability, and usability of outcomes seems to be either deliberately ignored, recklessly processed, or over-simplified.

On the other hand, some researchers have tried to revise those improvements. For example, Chu and Liu (2002) illustrated problems in Apostolou and Hassell (1993). Yang et al. (2004) demonstrated that the method of Bernhard and Canada (1990) was incomplete and then modified it. Chao et al. (2004) explained that the diagonal procedure of Finan and Hurley (1996) did not pass the consistency test of Saaty (1980). Lin et al. (2008) pointed out that the proof in Xu (2000) contains questionable results. Following this trend, we will study the aggregation weights for linguistic hybrid geometric averaging operator.

In the study, the biased shortcomings generated by one of the most productive contributors in related fields, Xu (2004), is pinpointed. The given values of weights in his work, the pivotal factors, are groundless and unjustified when applied to a host of operators, such as weighted geometric averaging (WGA) operator, ordered weighted geometric averaging (OWGA) operator, linguistic geometric averaging (LGA) operator, linguistic weighted geometric averaging (LWGA) operator, linguistic ordered weighted geometric averaging (LOWGA) operator, and linguistic hybrid geometric averaging (LHGA) operator. Furthermore, Xu improperly utilized the negative operator in the procedure he created which derived some redundancies of operational laws. This paper thereby tries to rectify his negligence in a more rigorous manner as a constructive patchwork since the unreliable outcomes, as applied in subsequent studies,

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may take place by inadequately distributing numbers to exponential weight vectors.

In fact Xu has quoted Xu (2004) at least twenty times since its publication. Meanwhile some other papers, studied in turn by Buyukozkan and Feyzioglu (2005), Cheng and Chang (2006), Nakamori and Ryoke (2006), Pankowska and Wygralak (2006), Wang and Parkan (2006), Wu (2006), Xia et al. (2006), Yucheng and Yinfeng (2006), Martinez et al. (2007), and Wu et al. (2007), have referred to the same outcomes in their references. In recent years, Wei (2009), who took Xu's study (2006) as a reference, proposed an uncertain linguistic hybrid geometric mean (ULHGM) operator. Wei and Yi (2010) who constructed a new aggregation operator: induced trapezoidal fuzzy ordered weighted harmonic averaging (ITFOWHA) operator also referred to the result of Xu. Based on the Dempster-Shafer theory and Xu's operators, Merigo et al. (2010) developed a new linguistic aggregation operator for decision makings. However, none of those papers have discovered that the results may be questionable. Yet, because of the high citation rate, it may be worthwhile to go for a deeper examination of it and point out those problems.

In the meantime we suggest that some developed and well-applicable operators have covered inherent properties and provided optimal alternative combinations that Xu's generalized but not-well-theoretically-grounded LHGA operator intended to tackle. The two possible substitute directions will be mentioned below. Those operators are mainly Weighted Ordered Weighted Averaging (WOWA) derived by Torra (1997) and Induced Ordered Weighted Averaging (IOWA) derived by Yager and Filev (1999) that can be enhanced to be applied to linguistic terms under diverse expected or planned circumstances.

2. REVIEW OF XU'S RESULTS AND OUR REVISIONS

From Aczel and Saaty (1983), and Xu and Da (2003), Xu assumed the WGA operator as

$$WGA_{\omega}\left(a_1, a_2, \cdots, a_n\right) = \prod_{j=1}^{n} a_j^{\omega_j} \tag{1}$$

where $\omega = \left(\omega_1, \omega_2, \cdots, \omega_n\right)$ is the exponential weighting vector of the a_j, for $j = 1,2,\ldots,n$, and $\omega_j \in \left[0,1\right]$,

$$\sum_{j=1}^{n} \omega_j = 1.$$

According to Herrera et al. (2001), and Xu and Da (2002, 2003), Xu defined the OWGA operator as

$$OWGA_{w}\left(a_1, a_2, \cdots, a_n\right) = \prod_{j=1}^{n} b_j^{w_j} \tag{2}$$

where b_j is the *j*th largest of a_i and $w = \left(w_1, w_2, \cdots, w_n\right)$ is the exponential weighting vector of the b_j, for $j = 1,2,\ldots,n$, with $w_j \in \left[0,1\right]$, and $\sum_{j=1}^{n} w_j = 1$.

Refer to Herrera et al. (1996), Herrera and Martinez (2000), and Yager and Kacprzyk (1997), Xu considered that $S = \left\{s_i : 1 \le i \le t, \text{ where } i \text{ is a natural number}\right\}$ is a finite and totally descending ordered set where s_i represents a possible value for a linguistic variable such that $s_i \ge s_j$ as $i \ge j$ and there is the negative operator: $neg\left(s_i\right) = s_j$ where $i + j = t + 1$.

To preserve all the given information, Xu (2004) generated a continuous linguistic term set $\overline{S} = \left\{s_\alpha : 1 \le \alpha \le t, \text{ where } \alpha \text{ is a real number}\right\}$ by extending the discrete term set S such that the following operational laws are satisfied:

(a) $\left(s_\alpha\right)^\mu = s_{\alpha^\mu}$; (b) $\left(s_\alpha\right)^{\mu_1} \otimes \left(s_\alpha\right)^{\mu_2} = \left(s_\alpha\right)^{\mu_1+\mu_2}$; (c) $\left(s_\alpha \otimes s_\beta\right)^\mu = \left(s_\alpha\right)^\mu \otimes \left(s_\beta\right)^\mu$; and (d) $s_\alpha \otimes s_\beta = s_\beta \otimes s_\alpha = s_{\alpha\beta}$,

where μ and μ_i are exponential weights.

We found that among those four operational laws (b) and (c) are redundant because they can be derived from the other two: (a) and (d). Moreover Xu (2004) didn't define the order relation for the continuous set $\overline{S}$. Hence we have provided a patchwork such that the relation:

$$s_\alpha \ge s_\beta \text{ holds if } \alpha \ge \beta \tag{3}$$

where α and β are two real numbers.

Meanwhile Xu defined the LWGA operator

$$LWGA_{\omega}\left(s_{\alpha_1},\cdots,s_{\alpha_n}\right)=\left(s_{\alpha_1}\right)^{\omega_1}\otimes\cdots\otimes\left(s_{\alpha_n}\right)^{\omega_n} \tag{4}$$

where $\omega=\left(\omega_1,\omega_2,\cdots,\omega_n\right)$ is the exponential weighting vector of the s_{α_j}, and $\omega_j\in\left[0,1\right]$, $\sum_{j=1}^{n}\omega_j=1$. In particular when $\omega=\left(1/n,\cdots,1/n\right)$, LWGA is called LGA operator.

Xu (2004) further assumed the LOWGA operator as

$$LOWGA_{w}\left(s_{\alpha_1},\cdots,s_{\alpha_n}\right)=\left(s_{\beta_1}\right)^{w_1}\otimes\cdots\otimes\left(s_{\beta_n}\right)^{w_n} \tag{5}$$

where s_{β_j} is the *j*th largest of s_{α_j} and $w=\left(w_1,w_2,\cdots,w_n\right)$ is the exponential weighting vector, with $w_j\in\left[0,1\right]$, $\sum_{j=1}^{n}w_j=1$.

We may at this point pose a question:

$$\text{what is the difference of } \omega \text{ and } w\,? \tag{6}$$

Through the comparison of Equations (1) and (4), we learned that ω is used for operators WGA, LWGA and LGA such that each ω_i is associated with corresponding a_i in WGA and s_{α_j} in LWGA and LGA and its order of appearance is not arranged by any particular rule. On the other hand, it is found that w is used for the new orders of a_i in OWGA and s_{α_j} in LOWGA.

In the numerical example 3 of Xu (2004), he assumed that

$$\omega=\left(0.3,0.1,0.4,0.2\right) \text{ and } w=\left(0.3,0.1,0.4,0.2\right) \tag{7}$$

which probably indicated that Xu himself couldn't provide different vectors for ω and w, where both ω and w are the exponential weighting vectors.

Xu (2004) tried to develop a new operation that generates LWGA and LOWGA to create the following LHGA operator

$$LHGA_{\omega,w}\left(s_{\alpha_1},\cdots,s_{\alpha_n}\right)=\left(s_{\beta_1}\right)^{w_1}\otimes\cdots\otimes\left(s_{\beta_n}\right)^{w_n} \tag{8}$$

where s_{β_j} is the *j*th largest of the linguistic weighted argument $\left(s_{\alpha_i}\right)^{n\omega_i}$ and $w=\left(w_1,w_2,\cdots,w_n\right)$ and $\omega=\left(\omega_1,\omega_2,\cdots,\omega_n\right)$ are the exponential weighting vector, with $w_j\in\left[0,1\right]$, $\sum_{j=1}^{n}w_j=1$, and $\omega_j\in\left[0,1\right]$, $\sum_{j=1}^{n}\omega_j=1$.

In Theorem 7 of Xu (2004), he showed that LWGA is a special case of LHGA and then in Theorem 8, he further proved that LOWGA is a special case of LHGA. He developed the following five-step approach for group decision-making with linguistic preference relations:

Step 1. Let $X=\left\{x_1,\cdots,x_n\right\}$ be a set of alternatives, $D=\left\{d_1.\cdots,d_m\right\}$ be a set of decision-makers and $\lambda=\left\{\lambda_1,\cdots,\lambda_m\right\}$ be assigned weighted vector of decision-makers, with $\lambda_k\geq 0$ and $\sum_{k=1}^{m}\lambda_k=1$. DMs compare these alternatives with respect to a single criterion by the linguistic terms in the set $S=\left\{s_1,\ldots,s_t\right\}$ and constructs the linguistic preference relation matrix $R_k=\left(r_{ij}^{(k)}\right)_{n\times n}$, where the diagonal elements in R_k are expressed as "-" which mean "undefined", and $r_{ij}^{(k)}\oplus r_{ji}^{(k)}=s_t$ for $i\neq j$ where $i,j=1,\ldots,n$. Xu (2004) mistyped "t" for "$t+1$". The correct expression should be $r_{ij}^{(k)}\oplus r_{ji}^{(k)}=s_{t+1}$ which can be understood from numerical examples.

Step 2. Utilize the LGA operator

$$z_i^{(k)} = LGA\left(r_{i1}^{(k)},\ldots,r_{in}^{(k)}\right) = \left(r_{i1}^{(k)} \otimes \cdots \otimes r_{in}^{(k)}\right)^{1/(n-1)} \tag{9}$$

for $i = 1,\ldots,n$ and $k = 1,\ldots,m$ to aggregate the preference information $r_{ij}^{(k)}$ in the *i*th row of R_k, and obtain the preference degree $z_i^{(k)}$ of the *i*th alternative over all the other alternatives.

Step 3. Utilize the LHGA operator

$$z_i = LHGA_{\lambda,w}\left(z_i^{(1)}, z_i^{(2)}, \cdots, z_i^{(m)}\right) \tag{10}$$

to aggregate $z_i^{(k)}$ $(k = 1,2,\cdots,m)$ corresponding to the alternative x_i and obtain the preference degree z_i of the *i*th alternative over all the other alternatives, where $\lambda = \left(\lambda_1,\cdots,\lambda_m\right)$ is the weight vector of DMs, with $\lambda_k \geq 0$ and $\sum_{k=1}^{m}\lambda_k = 1$ and $w = \left(w_1,\cdots,w_m\right)$ is the exponential weighting vector of the LHGA where $0 \leq w_k$ and $\sum_{k=1}^{m} w_k = 1$.

Step 4. Rank all the alternatives and select the optimal one(s) in accordance with the preference degrees z_i for $i = 1,\ldots,n$.

Step 5. End.

Xu (2004) introduced a complex two-stage aggregation method to synthesize the results of three decision-makers. Xu (2004) only provided his new aggregation method without a reasonable explanation what motivates his new method. We will apply a well know approach, the arithmetic mean, that is the AS operator. Our approach is easier to execute.

3. THE INHERENT PROBLEM IN PREVIOUS APPROACH

Xu (2004) created a new operator, LHGA, to generalize LWGA and LOWGA based on which the above five-step approach was developed in order to synthesize group opinion for *m* decision-makers. However, there are two blurry and dubious points. First, he did not explain how to derive the exponential weighting vectors, ω and w in Equation (8). Then second, why or under what situations did he replaced ω and w with λ and w for operator LHGA in Step 3?

We have learned up to this point that the exponential weighting vectors are the key of influencing which alternative is the optimal solution. In the next section, we shall demonstrate through quoting the same numerical example provided by Xu (2004) that the exponential weighting vectors play the pivotal role of determining the best alternative. And based on which we set forth the inquiry that gives rise to the inherent problem in Xu's paper:

How did he derive ω, w and λ and what were their relations with each other? (11)

In the Section 4, we will demonstrate that with different pairs of λ and w, the ordering of five alternatives will change to reveal that the intimate relation between the ordering of alternatives and the pairs of λ and w. In Section 5, we will run a comprehensive numerical examination to test all possible 1296 combinations of λ and w to show that there are eight different possible ordering for these five alternatives.

4. NUMERICAL EXAMPLES TO DEMONSTRATE THE INHERENT PROBLEM

Let's employ the numerical example from Xu (2004). A group decision-making problem involves the evaluation of five schools x_i $(i = 1,\cdots,5)$ of a university. There are three DMs: d_k (*k*=1, 2, 3) who provide the weight vector:

$\lambda = (0.3, 0.4, 0.3)$. The DMs compared these five schools by means of one major criterion and using the linguistic terms in the set $S = \{s_i : i = 1, 2, \cdots, 9\}$ to construct the linguistic preference relations R_k (k=1, 2, 3) shown in the following Tables 1-3, respectively. To save space and make the whole undergoing process more efficient, we use Step 2 of Xu's procedure to calculate each $z_i^{(k)}$ listed as well in Tables 1-3, respectively.

Xu made the assumption that the weight vector $w = (0.3, 0.5, 0.2)$. However, he did not state what rule or principle he chose to derive λ and w and why λ and w were different. In the following operations, we quote Tables 1, 2 and 3 from his previous results.

Table 1: Reproduction of the linguistic preference relations R_1 and $z_i^{(1)}$

	x_1	x_2	x_3	x_4	x_5	$z_i^{(1)}$
x_1	−	s_2	s_4	s_3	s_7	$z_1^{(1)} = s_{3.6002}$
x_2	s_8	−	s_5	s_4	s_6	$z_2^{(1)} = s_{5.5663}$
x_3	s_6	s_5	−	s_2	s_4	$z_3^{(1)} = s_{3.9360}$
x_4	s_7	s_6	s_8	−	s_3	$z_4^{(1)} = s_{5.6346}$
x_5	s_3	s_4	s_6	s_7	−	$z_5^{(1)} = s_{4.7381}$

Table 2: Reproduction of the linguistic preference relations R_2 and $z_i^{(2)}$

	x_1	x_2	x_3	x_4	x_5	$z_i^{(2)}$
x_1	−	s_3	s_4	s_6	s_5	$z_1^{(2)} = s_{4.3559}$
x_2	s_7	−	s_7	s_4	s_5	$z_2^{(2)} = s_{5.5951}$
x_3	s_6	s_3	−	s_4	s_6	$z_3^{(2)} = s_{4.5590}$
x_4	s_4	s_6	s_6	−	s_4	$z_4^{(2)} = s_{4.8990}$
x_5	s_5	s_5	s_4	s_6	−	$z_5^{(2)} = s_{4.9492}$

Table 3: Reproduction of the linguistic preference relations R_3 and $z_i^{(3)}$

	x_1	x_2	x_3	x_4	x_5	$z_i^{(3)}$
x_1	−	s_2	s_6	s_4	s_7	$z_1^{(3)} = s_{4.2814}$
x_2	s_8	−	s_4	s_3	s_4	$z_2^{(3)} = s_{4.4267}$
x_3	s_4	s_6	−	s_5	s_7	$z_3^{(3)} = s_{5.3836}$
x_4	s_6	s_7	s_5	−	s_3	$z_4^{(3)} = s_{5.0100}$
x_5	s_3	s_6	s_3	s_7	−	$z_5^{(3)} = s_{4.4093}$

From the numerical example, with $\lambda = (0.3, 0.4, 0.3)$ and $w = (0.3, 0.5, 0.2)$, for each decision-maker, $r_{ij}^{(k)} \oplus r_{ji}^{(k)} = s_{t+1}$ for $i, j = 1, \ldots, n$ and $i \neq j$. It implies that $neg\left(r_{ij}^{(k)}\right) = r_{ji}^{(k)}$. By Xu's approach, the following outcomes were obtained:

$$z_1 = s_{4.1155},\ z_2 = s_{5.2603},\ z_3 = s_{4.7129},\ z_4 = s_{5.1558}, \text{ and } z_5 = s_{4.6778}. \tag{12}$$

Based on which the schools ranking are:

$$x_2 \succ x_4 \succ x_3 \succ x_5 \succ x_1 \tag{13}$$

and the best school is x_2.

However if we assume that another setting of λ and ω as $\lambda = (0.1, 0.1, 0.8)$ and $w = (0.8, 0.1, 0.1)$, then yield

$$z_1 = s_{17.7217},\ z_2 = s_{19.2877},\ z_3 = s_{27.6231},\ z_4 = s_{24.3732}, \text{ and } z_5 = s_{18.9802}. \tag{14}$$

and the relations:

$$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1 \tag{15}$$

are concluded such that x_3 is the best alternative.

Further, by assuming another setting of λ and ω as $w = \lambda = (0.9, 0.05, 0.05)$, the outcome:

$$z_1 = s_{22.9829},\ z_2 = s_{66.4024},\ z_3 = s_{28.6024},\ z_4 = s_{68.3954}, \text{ and } z_5 = s_{44.8506} \tag{16}$$

can be calculated and the relations:

$$x_4 \succ x_2 \succ x_5 \succ x_3 \succ x_1 \tag{17}$$

are concluded such that x_4 is the best alternative.

From above discussion, the decisive factor of finding best alternative is the collection of exponential weighting vectors ω and w of LHGA (or λ and w in Xu's approach). Yet the mechanism of attaining ω and w or whether or not $\omega = w$ or $\lambda =$ w in Xu's study is still outstanding and in question.

From Equations (13), (15) and (17), it is obvious that different derivations can be selected as the best alternative in accordance with different exponential weighting vectors λ and w as defined in Xu's approach. This conclusion indicates that Xu's (2004) study and outcomes face the shortcoming of insufficiency for the evaluation of ω and w of LHGA which are significantly vital on the whole. As a matter of fact, the determinations of λ and w should be based on the rationality of logic as well as the meaning of practicality and management.

Traditionally, there are two main approaches in linguistic decision analysis. The first approach is to use the concept of fuzzy sets to model linguistic terms and then use fuzzy set based techniques for developing solution methods (mainly making use of the extension principle and the process of linguistic approximation). The second one is to develop methods which directly perform computations on the set of linguistic values in which only a totally ordered structure is assumed. Note that the use of a linguistic approach is only necessary when the information in decision situations cannot be assessed precisely in a quantitative form by numerical values. Then, Xu's approach which reduces the computation on linguistic values, i.e., indices or numbers, seems to be paradoxical. In fact, by reducing linguistic values to single numbers by means of their indices, we are losing much of the information we have purposely been keeping throughout the structural stage of linguistic decision problems.

5. OUR APPROACH

In this section, we develop our proposed approach. We directly take the row sum. Moreover, to use the expression of s_2 is unnecessary. Instead, we will directly use the suffix "2" to simplify the expression.
On the other hand, Xu (2004) used a complicated OWGA operator (ordered weighted geometric average) that is a two-stage weighted geometric mean.
The OWGA operator is an interesting approach. To the best of our knowledge, we can not find any reasonable meaning of OWGA operator. There is no practical example to demonstrate that the OWGA operator is more reasonable than the weighted geometric average (WGA operator).
We will apply the arithmetic sum (AS operator), owing to the computation for the AS operator is easier than that of WGA operator by ordinary practitioners. Hence, our operation is easier to execute for practitioners.
We will apply the AS operator that is familiar to practitioners and researchers.

For example, for the decision maker d_1 and alternative x_1, we compute the row sum of suffix as $2+4+3+7=16$. For example, for the decision maker d_2 and alternative x_1, we compute the row sum of suffix as $3+4+6+5=18$. For example, for the decision maker d_3 and alternative x_1, we compute the row sum of suffix as $2+6+4+7=19$. Hence, for alternative x_1, we find that $16+18+19=53$. We list the computation results in the next table.

Table 4: The results for AS operator

	x_1	x_2	x_3	x_4	x_5
d_1	16	23	17	24	20
d_2	18	23	19	20	20
d_3	19	19	22	21	19
total	53	65	58	65	59

We derive the ordering for alternatives as

$$x_2 = x_4 \succ x_5 \succ x_3 \succ x_1 .$$

To decide the most favorable alternative, we directly compare x_2 with x_4 to find that x_2/x_4 as s_4, s_4 and s_3 from the three decision-makers such that sum of suffix $=11$. On the other hand, we obtain that x_4/x_2 as s_6, s_6 and s_7 such that sum of suffix $=19$.
This computation result is supported by $11+19=3(t+1)=30$.
From $19>11$, we derive that $x_4 \succ x_2$ so the most favorable alternative by our approach is x_4.

We assume that $\lambda=(\lambda_1,\lambda_2,\lambda_3)$ and $w=(w_1,w_2,w_3)$, with $\sum_{i=1}^{3}\lambda_i=1$ and $\sum_{i=1}^{3}w_i=1$, where λ_j and $w_k\in\{0.1,0.2,...,0.8\}$, for $j,k=1,2,3$. There are 36 different combinations for $(\lambda_1,\lambda_2,\lambda_3)$. Similarly, there are 36 different combinations for (w_1,w_2,w_3). Hence, For the exhaust examination, there are 1296 different combination for $\lambda=(\lambda_1,\lambda_2,\lambda_3)$ and $w=(w_1,w_2,w_3)$. With the help of Professor Tung, we know the outcomes for those 1296 examples. There are 120 possible orders among x_1, x_2, x_3, x_4 and x_5. We compute the frequency for 120 possible orders. Most orders have zero frequency. There are only eight of them with nonzero frequency and then we list them in the following table.

Table 5: List of 8 orders with nonzero frequency

$x_4 \succ x_3 \succ x_2 \succ x_5 \succ x_1$	103

$x_4 \succ x_2 \succ x_5 \succ x_3 \succ x_1$	163
$x_4 \succ x_2 \succ x_3 \succ x_5 \succ x_1$	303
$x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$	120
$x_2 \succ x_4 \succ x_3 \succ x_5 \succ x_1$	72
$x_2 \succ x_4 \succ x_5 \succ x_3 \succ x_1$	525
$x_2 \succ x_3 \succ x_4 \succ x_5 \succ x_1$	5
$x_2 \succ x_5 \succ x_4 \succ x_3 \succ x_1$	5

From the above table, the first four orders satisfy $x_4 \succ x_2$ to imply that it happens 689 times.

On the other hand, the rest four orders satisfy $x_2 \succ x_4$ to yield that it occurs 607 times.

From the above discussion, we derivation of $x_4 \succ x_2$ is supported by the exhaust examination.

6. CONCLUSION

Xu (2004) derived a new operator, the LHGA operator, to generalize (a) the LWGA operator and (b) the LOWGA operator. However, he did not explain how those two crucial exponential weighting vectors were decided. By the same token, we have shown that the outcomes are quite volatile and unreliable under the same approach and examples but with different exponential weight vectors. This indicates that the LHGA operator Xu developed in connection with the five-step approach is not well-developed and faces significant deficiency. Therefore, we advise researchers not to apply this new algorithm to avoid unsound results.

Acknowledgment

The authors greatly appreciate the partially financial support of MOST 105-2410-H-015-006. We express our sincere gratitude to Professor Tung (tung0827@gmail.com) for his help to provide a computer program to compute synthesized weight of 1296 examples and frequency for 120 possible orders.

REFERENCES

1. Aczel. J., and Saaty, T. L. (1983). Procedures for synthesizing ratio judgments. Journal of Mathematical Psychology, Vol. 27: 93-102.
2. Apostolou, B., and Hassell, J. M., (1993). An empirical examination of the sensitivity of the analytic hierarchy process to departures from recommended consistency ratios. Mathematical and Computer Modelling, Vol. 17: 163-170.
3. Bernhard, R. H., and Canada, J. R. (1990). Some problems in using benefit/cost ratios with the analytic hierarchy process. The Engineering Economist, Vol. 36: 56-65.
4. Buyukozkan, G., and Feyzioglu, O. (2005). Group decision making to better respond customer needs in software development. Computers and Industrial Engineering, Vol. 48, No. 2: 427-441.
5. Chao, H. C. J., Yang K. L., and Chu, P. (2004). Note on diagonal procedure in analytic hierarchy process. Mathematical and Computer Modelling, Vol. 40: 1089-1092.
6. Cheng, C., and Chang, J. (2006). MCDM aggregation model using situational ME-OWA and ME-OWGA operators. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, Vol. 14, No. 4: 421-443.
7. Chu. P., and Liu, J. K. H. (2002). Note on consistence ratio. Mathematical and Computer Modelling, Vol. 35: 1077-1080.
8. Finan, J. S., and Hurley, W. J. (1996). A note on a method ensure rank-order consistency in the analytic hierarchy process. International Transactions of Operational Research, Vol. 3: 99-103.
9. Herrera, F., Herrera-Viedma, E., and Chiclana, F. (2001). Multiperson decision-making based on multiplicative

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preference relations. European Journal of Operational Research, Vol. 129: 372-385.
10. Herrera, F., and Martinez, L. (2000). A 2-tuple fuzzy linguistic representation model for computing with words. IEEE Transactions on Fuzzy Systems, Vol. 8: 746-752.
11. Herrera, F., Herrera-Viedma, E., and Verdegay, J. L. (1996). A model of consensus in group decision making under linguistic assessments. Fuzzy Sets and Systems, Vol. 78: 73-87.
12. Lin, R., Lin, J. S. J., Chang, J., Tang, D., Chao, H., and Julian, J. C. (2008). Note on group consistency in analytic hierarchy process. European Journal of Operational Research, Vol. 190: 672-678.
13. Martinez, L., Liu, J., Ruan, D., and Yang, J., (2007). Dealing with heterogeneous information in engineering evaluation processes. Information Sciences, Vol. 177, No. 7: 1533-1542.
14. Merigo, J. M., Casanovas, M., and Martinez, L., (2010). Linguistic aggregation operators for linguistic decision making based on the Dempster-Shafer theory of evidence. International Journal of Uncertainty, Fuzziness, and Knowledge-Based Systems, Vol. 16, No. 3: 287-304.
15. Nakamori, Y., and Ryoke, M., (2006). Treating fuzziness in subjective evaluation data. Information Sciences, Vol. 176, No. 24: 3610-3644.
16. Pankowska, A., and Wygralak, M. (2006). General IF-sets with triangular norms and their applications to group decision making. Information Sciences, Vol. 176, No. 18: 2713-2754.
17. Saaty, T. L. (1980). The Analytic Hierarchy Process, McGraw-Hill, New York.
18. Torra, V. (1997). The Weighted OWA Operator. International Journal of Intelligent Systems, Vol. 12: 153-166.
19. Wang, Y., and Parkan, C. (2006). Two new approaches for assessing the weights of fuzzy opinions in group decision analysis. Information Sciences, Vol. 176, No. 23: 3538-3555.
20. Wei, G. W. (2009). Uncertain linguistic hybrid geometric mean operator and its application to group decision making under uncertain linguistic environment. Fuzziness and Knowledge-Based Systems, Vol. 17, No. 2: 251-267.
21. Wei, G. W., and Yi, W. (2010). Induced Trapezoidal Fuzzy Ordered Weighted Harmonic Averaging Operator. Journal of Information & Computational Science, Vol. 7, No. 3: 625-630.
22. Wu, D. (2006). Fuzzy group decision making based on grey relative analysis. International Journal of Management and Decision Making, Vol. 7, No. 4: 454-472.
23. Wu, J., Liang, C., and Huang, Y. (2007). An argument-dependent approach to determining OWA operator weights based on the rule of maximum entropy. International Journal of Intelligent Systems, Vol. 22, No. 2: 209-221.
24. Xia, H., Li, D., Zhou, J., and Wang, J. (2006). Fuzzy LINMAP method for multiattribute decision making under fuzzy environments. Journal of Computer and System Sciences, Vol. 72, No. 4: 741-759.
25. Xu, Z. (2000). On consistency of the weighted geometric mean complex judgment matrix in AHP. European Journal of Operational Research, Vol. 126: 683–687.
26. Xu, Z. S. (2004). A method based on linguistic aggregation operators for group decision making with linguistic preference relations. Information Science, Vol. 166: 19-30.
27. Xu, Z. S. (2002). Study on methods for multiple attribute decision making under some situations, Ph.D. Thesis, Southeast University, Nanjing, China.
28. Xu, Z. S., and Da, Q. L. (2002). The ordered weighted geometric averaging operators. International Journal of Intelligent Systems, Vol. 17: 709-716.
29. Xu, Z. S., and Da, Q. L. (2003). An overview of operators for aggregating information. International Journal of Intelligent Systems, Vol. 18: 953-969.
30. Xu, Z. S. (2006). An approach based on the uncertain LOWG and induced uncertain LOWG operators to group decision making with uncertain multiplicative linguistic preference relations. Decision Support Systems, Vol. 41: 488-499.
31. Yager, R. R. (1988). On ordered weighted averaging aggregation operators in multicriteria decision making. IEEE Transactions on Systems, Man, and Cybernetics, Vol. 18: 183-190.
32. Yager, R. R., and Filev, D. P. (1999). Induced Ordered Weighted Averaging Operators. IEEE Transactions on Systems, MAN, and Cybernetics —Part B: Cybernetics, Vol. 29, No. 2: 141-150.
33. Yager, R. R., and Kacprzyk, J. (1997). The Ordered Weighted Averaging Operators, Theory and Applications, Kluwer, Norwell, MA.
34. Yang, K. L., Chu, P., and Chouhuang, W. T. (2004). Note on incremental benefit/cost ratios in analytic hierarchy process. Mathematical and Computer Modelling, Vol. 39: 279-286.
35. Yucheng, D., and Yinfeng, X. (2006). Consistency measures of linguistic preference relations and its properties in group decision making. Lecture notes in computer Science, 4223 LNAI: 501-511.