УДК 539.142

SEMICLASSICAL APPROACH TO THE LOW-LYING COLLECTIVE EXCITATIONS IN NUCLEI

A. M. Gzhebinsky, A. G. Magner, S. N. Fedotkin

Institute for Nuclear Research, National Academy of Sciences of Ukraine, Kyiv

For low-lying collective excitations we derived the inertia within the semiclassical Gutzwiller approach to the one-body Green's function at lowest orders in $\hbar$. The excitation energies, reduced probabilities and energy-weighted sum rules are in agreement with main features of the experimental data.

Introduction

The collective dynamics of complex nuclei at low excitation energies, such as the vibration modes, can be described within several theoretical approaches [1, 2]. One of the most powerful tools for its description is based on the response function theory [1, 3]. The collective variables are introduced explicitly as deformation parameters of a mean single-particle field. In [3], the nuclear collective excitations are parametrized in terms of the transport coefficients like the stiffness, the inertia, and the friction parameters through the adequate collective response functions.

The extended Gutzwiller path-integral approach (EGA) [4 - 6], so successful for a semiclassical description of the nuclear shell structure [5, 6], was applied to the response functions of collective dynamics in [7]. At a few lowest orders in $\hbar$ like the extended Thomas - Fermi approach (ETF), it would be worth to exploit also the Strutinsky procedure of averaging of the shell correction method (SCM) for calculations of the smooth transport coefficients for slow collective motion.

The main scope of this paper is to derive the explicit analytical expressions of a smooth inertia at leading orders in $\hbar$ for the low-lying nuclear collective excitations within the EGA by using more traditional way than in [8] in order to study also their reduced transition probabilities and contributions into the energy-weighted sum rule (EWSR) [1]. The basic key point of these derivations is to show analytically a significant enhancement of the ETF inertia with respect to that of the hydrodynamical (irrotational-flow liquid drop) model.

Nuclear response and transport coefficients

Many-body collective excitations are conveniently described in terms of the nuclear response to an external perturbation $V_{\rm ext} = \hat{\rm Q}\, q_\omega^{\rm ext}\, e^{-i\omega t}$, where $q_\omega^{\rm ext}$ is a vibration amplitude of the frequency ω, and $\hat{\rm Q}$ is one-body operator. Its quantal average variations $\delta\langle\hat{\rm Q}\rangle_t$ at time t can be calculated through the Fourier transform $\delta\langle\hat{\rm Q}\rangle_\omega$ within the linear response theory [3],

$$\delta\langle\hat{\rm Q}\rangle_\omega = -\chi_{\rm coll}(\omega)\, q_\omega^{\rm ext}, \quad \hat{\rm Q} = r^L\, Y_{L0}, \tag{1}$$

where $\chi_{\rm coll}(\omega)$ is the collective response function. For the axially symmetric multipole vibrations of the nuclear surface with the radius $R(\theta)$ near the spherical shape in the spherical coordinates r, θ, φ, one writes $R(\theta) = R[1 + q(t) Y_{L0}(\theta)]$, $q(t) = q_\omega\, e^{-i\omega t}$ is the time-dependent deformation parameter. The consistency condition,

$$\delta\langle\hat{\rm Q}\rangle_\omega = \kappa\, \delta q_\omega, \tag{2}$$

relates the variations of nuclear potential and particle density, $\delta V \propto \delta\rho$, κ is the coupling constant, δq_ω is the variation of deformation parameter [1, 3, 9].

For one dominating separate peak in the strength function, ${\rm Im}\chi_{\rm coll}(\omega)$, in the low energy region, we can approximate the collective response function by the harmonic oscillator form [3, 9, 10]:

$$\chi_{\rm coll}(\omega) \approx \frac{\kappa}{-M\omega^2 - i\gamma\omega + C}, \tag{3}$$

with transport coefficients, such as the inertia M, the friction γ, and the stiffness C. The inertia M is expressed in terms of the one-body Green function G,

$$M \propto \hbar^2 \int_0^\infty {\rm d}\varepsilon\, n(\varepsilon) \int {\rm d}\mathbf{r}_1 \int {\rm d}\mathbf{r}_2\, \hat{\rm Q}_1(\mathbf{r}_1)\, \hat{\rm Q}_2(\mathbf{r}_2)\, {\rm Im}G(\mathbf{r}_1, \mathbf{r}_2, \varepsilon)\, \frac{\partial^2}{\partial\varepsilon^2}\, {\rm Re}G(\mathbf{r}_1, \mathbf{r}_2, \varepsilon), \tag{4}$$

where $n(\varepsilon)$ is the Fermi occupation numbers at the energy ε for temperature T, $n(\varepsilon) = \{1+\exp[(\varepsilon-\lambda)/T]\}^{-1}$, λ is the chemical potential. Finally, with the strength function $S_L(\omega)$ for the first lowest peak, we may evaluate the contribution of the low-lying collective state into the sum rule

$$S_{L,\ell} = \hbar^2 \int_0^\infty \mathrm{d}\omega\, \omega^\ell\, S_L(\omega), \quad S_L(\omega) = -\frac{1}{\pi}\,\mathrm{Im}\,\chi_{\mathrm{coll}}(\omega),$$

$$\ell = 0,\, 1,\, \ldots . \qquad (5)$$

Semiclassical approach

We use now in Eq. (4) the semiclassical expansion of Green's function G derived by Gutzwiller [4, 5] from the quantum path-integral propagator,

$$G(\mathbf{r}_1,\, \mathbf{r}_2,\, \varepsilon) = \sum_\alpha G_\alpha(\mathbf{r}_1,\, \mathbf{r}_2,\, \varepsilon) =$$

$$= \sum_\alpha A_\alpha(\mathbf{r}_1,\, \mathbf{r}_2,\, \varepsilon) \exp\left[\frac{i}{\hbar} S_\alpha(\mathbf{r}_1,\, \mathbf{r}_2,\, \varepsilon) - \frac{i\pi}{2}\,\mu_\alpha\right]. \quad (6)$$

The index α covers all classical paths inside the potential well, which connect the two spatial points $\mathbf{r}_1$ and $\mathbf{r}_2$ for a given energy ε (Fig. 1). The S_α is the classical action along a trajectory α, and μ_α is related to the Maslov index of the path α. The oscillation amplitude A_α in Eq. (6) is determined by the classical trajectory stability.

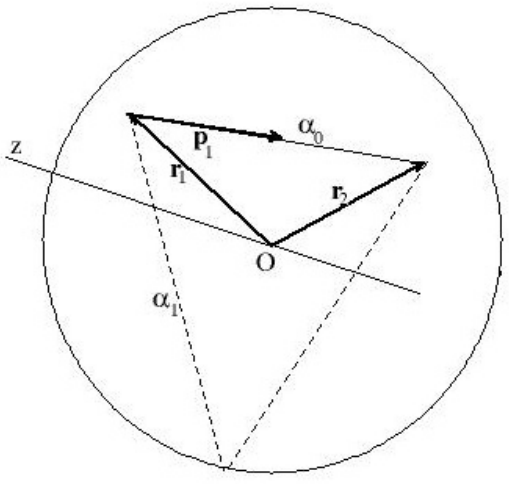

Fig. 1. The trajectory α_0 from the initial $\mathbf{r}_1$ to the final $\mathbf{r}_2$ point; the spherical coordinate system with the polar axis z and the center O is shown; dashed line denotes another trajectory α_1 with one mirror reflection from the spherical boundary.

Among all classical trajectories α, we may single out the straight line path α_0 from $\mathbf{r}_1$ to $\mathbf{r}_2$ without reflections from the potential well edge. For the Green's function G [Eq. (6)] one has then a separation, $G = G_{\alpha 0} + G_{osc}$, which leads to the corresponding splitting of the slightly averaged level density, $g(\varepsilon)$, into a smooth part of the extended Thomas - Fermi model $g_{\mathrm{ETF}}(\varepsilon)$, and its shell structure correction $g_{\mathrm{osc}}(\varepsilon)$, $g(\varepsilon) = g_{\mathrm{ETF}}(\varepsilon) + g_{\mathrm{osc}}(\varepsilon)$ [5, 6]. The ETF level density $g_{\mathrm{ETF}}(\varepsilon)$ includes the surface and curvature $\hbar$ corrections to the volume part of the Thomas - Fermi model $g_{\mathrm{TF}}(\varepsilon)$. The periodic orbit theory (POT) sum over the periodic orbits, $g_{\mathrm{osc}}(\varepsilon)$, describes the shell effects in the single-particle spectrum.

The averaging over phase-space variables of the inertia, which includes Strutinsky averaging over energy spectrum, leads to the nearly local approximation $S_\alpha(\mathbf{r}_1,\, \mathbf{r}_2,\, \varepsilon_F)/\hbar = k_F\, L_\alpha \lesssim 1$, $\varepsilon_F = \hbar^2 k_F^2/2m$, in which the only short trajectories α with small lengths L_α alive, as shown in [9, 11]. The Fermi momentum k_F in units of $\hbar$ is determined approximately by the well-known ETF particle number conservation for the edge-like potentials [6]

$$A = 4\int_0^\infty \mathrm{d}\varepsilon\, g_{\mathrm{ETF}}(\varepsilon) \approx \frac{8(k_F R)^3}{9\pi} -$$

$$-(k_F R)^2 + \frac{8 k_F R}{3\pi}\,. \qquad (7)$$

In that approach, the diffuseness parameter for mean field potential is small with respect to the nuclear radius as $A^{-1/3}$ at large enough particle numbers A in nucleus. Shell fluctuations are owing to longer trajectories (see α_1 as example in Fig. 1) of periodic orbits for smaller Gaussian averaging parameter Γ, $\Gamma \ll \hbar\Omega \sim \varepsilon_F / A^{1/3}$, in the case of non-local contributions, $k_F\, L_\alpha \gg 1$, see [9, 12].

Within the nearly local approximation, the contribution into the smooth transport coefficients is coming from the first component of Green's function $G_{\alpha 0}$, corresponding to the short trajectory α_0 (see Fig. 1) with a small relatively action $S_\alpha(\mathbf{r}_1, \mathbf{r}_2, \varepsilon_F)$, which can be reduced approximately to a simple analytical form for free particle motion,

$$G_{\alpha 0}(\mathbf{r}_1, \mathbf{r}_2, \varepsilon) \approx G_0(\mathbf{r}_1, \mathbf{r}_2, \varepsilon) = -\frac{m}{2\pi\hbar^2 s}\exp(iks),$$

$$s = |\mathbf{r}_1 - \mathbf{r}_2|, \qquad k = \sqrt{\frac{2m\varepsilon}{\hbar^2}}. \tag{8}$$

Finally, for the leading term in $\hbar$ in the semiclassical inertia in units of that for irrotational flow of the hydrodynamical model, by using (4) and (8) one obtains

$$\frac{M}{M_{irr}} \propto \rho\left(\frac{b_v}{\varepsilon_F}\frac{KR}{b_s r_0}\right)^2 \frac{(k_F R)^4}{A} \propto A,$$

$$\rho = \rho_0\left(1 + \frac{6\,b_s}{K\,A^{1/3}}\right). \tag{9}$$

Here, ρ is the particle density accounting for a semiclassical surface correction to the value $\rho_0 = 0.16$ fm^{-3} of the infinite nuclear matter. Other notations are used for standard nuclear parameters, namely, the energy of particle separation from nuclear matter $b_v = 16$ MeV, the incompressibility modulus $K = 220$ MeV, the energy surface constant $b_s = 18$ MeV, $r_0 = (3/4\pi\rho_0)^{1/3}$. We found that the inertia value is much larger than the irrotational flow one by factor of about $M/M_{\text{irr}} \approx 4 - 7$ for the quadrupole vibrations and similarly, for octupole modes, $M/M_{\text{irr}} \approx 4 - 9$ for particle numbers $A = 100$ - 200. Smaller enhancement for the quadrupole excitations was found in [13] within the stohastic response function approach [14] without account for the consistency condition (2). Notice that the collective-consistent inertia parameter M, see (9), depends on the diffuseness of the mean-field potential edge through the phenomenological energy surface constant b_s, see [9] and [25, 26] therein. Other contributions of the finite nuclear diffuseness are expected to be negligibly small for heavy enough nuclei within our approach because of the double integration over the nuclear volume in (4). The stiffness is approximated by the standard sum of the liquid drop surface value and Coulomb correction [1]

$$C \approx C_{LD}^{(S)} + C_{LD}^{(coul)},$$

$$C_{LD}^{(S)} = \frac{b_s}{4\pi r_0^2}(L-1)(L+2)R^2,$$

$$C_{LD}^{(coul)} = -\frac{3(L-1)}{2\pi(2L+1)}\frac{Z^2 e^2}{R}, \tag{10}$$

where Ze is the nuclear charge.

Energies, transition probabilities and sum rules

The collective vibration energies were calculated as poles of the response function (3). The main term of the excitation energies differs essentially from the well-known results of the hydrodynamical model,

$$\hbar\omega_L = \hbar\sqrt{\frac{C}{M}} = \frac{D_L}{A}\left(1 - \frac{2.3}{A^{1/3}} + \frac{1.3}{A^{2/3}}\right),$$

$$D_L = \bar{D}_L\sqrt{1 + \frac{C_{LD}^{(coul)}}{C_{LD}^{(S)}}}, \quad L = 2, 3, \ldots \tag{11}$$

The surface $\left(\propto A^{1/3}\right)$ and curvature $\left(\propto A^{2/3}\right)$ corrections are mainly coming from the ETF relation (7) between particle number and semiclassical parameter. Parameter $\bar{D}_L$ does not depend on particle number, $\bar{D}_L \propto (b_s\,\varepsilon_F)^{3/2}/b_v K$, $\bar{D}_2 \approx$ ≈ 100 MeV; $\bar{D}_3 \approx 180$ MeV.

We may analytically evaluate also the contribution of the first low-lying excitation into the energy-weighted sum rule $S_{L,1}$, see (5), (9),

$$\frac{S_{L,1}}{S_{L,cl}} = \frac{\hbar\kappa^2}{2M} = \frac{M_{irr}}{M},$$

$$S_{L,cl} = \frac{3L\,\varepsilon_F}{4\pi(k_F R)^2}AR^{2L}, \tag{12}$$

where $S_{L,cl}$ is the classical sum rule independent of the model [1]. The enhancement of the inertia M corresponds to a decrease of the sum rule contribution as compared to that of the hydrodynamic model. Similarly, as for the energies (11), for the sum rules (12), one has the analytical expression for the A-dependence with the surface and curvature corrections,

$$\frac{S_{L,1}}{S_{L,cl}} = \frac{\bar{S}_L}{A}\left(1 - \frac{4.6}{A^{1/3}} + \frac{7.9}{A^{2/3}}\right), \tag{13}$$

where $\bar{S}_L$ is the constant independent of the particle number, $\bar{S}_L \propto \left(\varepsilon_F\,k_F\,b_s\,r_0/b_v K\right)^2$, $\bar{S}_2 \approx 7$, $\bar{S}_3 \approx 6$.

The semiclassical reduced transition probabilities are expressed through the sum rule S_{L0} of (5) and (3),

$$B(EL) \approx B_{scl}(EL) = (2L+1)\left(\frac{eZ}{A}\right)^2 S_{L,0} \approx$$

$$\approx (2L+1)\left(\frac{eZ}{A}\right)^2 \frac{\hbar\kappa^2}{2M\,\omega_L}. \tag{14}$$

From comparison of the transition probability (14) and the EWSR (12), one has the expected approximate relationship between these quantities and energies (11),

$$B_{scl}(EL) \equiv (2L+1)\left(\frac{eZ}{A}\right)^2 \frac{S_{L,1}}{\hbar\omega_L} = \frac{S_{L,cl}}{\hbar\omega_L}. \quad (15)$$

The reduced probability in the single-particle units is really large enough, as it must be for the collective excitations,

$$\frac{B_{scl}(E2)}{B_{s.p.}(E2)} \approx 50-70, \quad A = 100-200, \quad (16)$$

for quadrupole, and $50-60$ for octupole modes.

Comparison with experiment data

The local Thomas - Fermi approach (TF, dots) to the low-lying collective quadrupole excitation energies without surface and curvature corrections are compared with the experimental data [15] for almost spherical (even-even) nuclei in Fig. 2. The TF results for smooth vibration energies are significantly improved with respect to the hydrodynamic (HD, dash-dotted) behavior with the

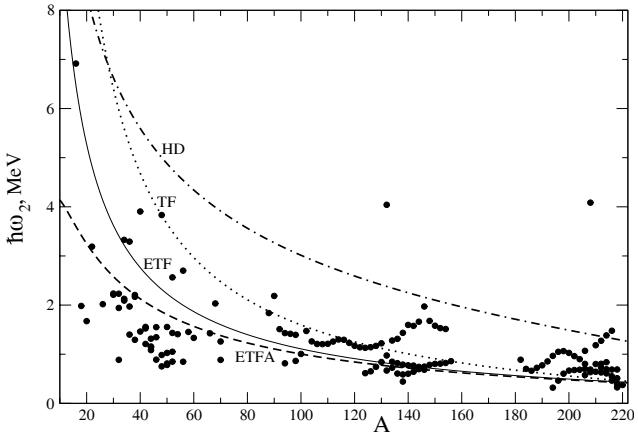

Fig. 2. Low-lying quadrupole vibration energies $\hbar\omega_2$ vs particle number A. Heavy full dots are the experimental data [15] for almost spherical nuclei with quadrupole deformations $q_2 < 0.05$, $\rho_0 = 0.16$ fm^{-3}, $b_v = 16$ MeV, $b_s = 18$ MeV, $K = 220$ MeV, $b_{\rm sym} = 60$ MeV, other notations are explained in the text.

More complete extended Thomas - Fermi approach with accounting for the surface and curvature corrections are shown as ETF solid curve. Comparison with experimental data, except for several doubly-closed-shell (magic) nuclei, is essentially improved by these corrections mainly for smaller particle numbers A. The reason of better agreement of the ETF approach, as compared to the HD model, vs experimental data for non-magic nuclei can be explained by significantly larger ETF inertia than that of the irrotational flow for enough heavy nuclei. As seen from this Figure, the explicit analytical asymptotics [ETFA, dashed, Eq. (11)] with the surface and curvature corrections originating from the ETF particle number conservation relation (7) is good enough for larger particle numbers.

Figs. 3 and 4 show the semiclassical reduced probability (14) and the lifetime $t_L \propto 1/B(EL)\omega^{2L+1}$ as compared with experimental data versus particle number for the quadrupole collective transitions ($L = 2$) in the low-lying energy region for the same (almost spherical) nuclei. As displayed in these Figures, one has a rather good agreement between the averaged semiclassical reduced transition probabilities (lifetimes) and a global behavior of their experimental data (besides of magic nuclei). The surface and the curvature correction effects improve much our semiclassical smooth A - systematic results toward the allowance data. The agreement between the full ETF (thin solid) and the analytical asymptotics ETFA (thick dashed) for larger particle numbers with the dominating surface and curvature corrections is really perfect. As seen from comparison of the ETF* and ETF curves in Fig. 3, one may really neglect the friction

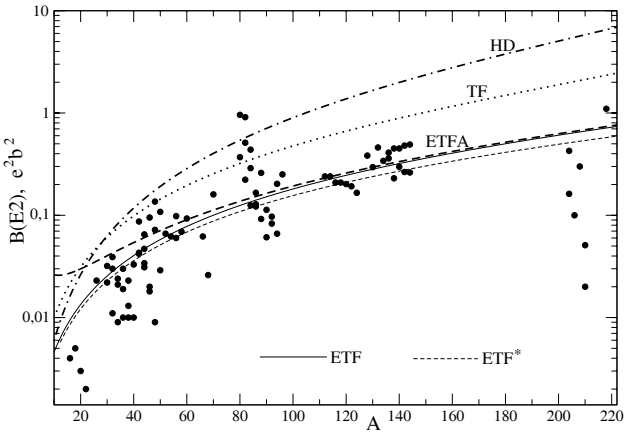

Fig. 3. The reduced probabilities $B(E2)$ for the transition $0^+ \to 2^+$ in standart units of $e^2 b^2$; full heavy points are experimental data [15]; ETF* accounts for the friction correction [9]; other notations are the same as in Fig. 2.

Fig. 5 shows agreement of the EWSR contribution (12) of the low-lying quadrupole state into the total value with experimental data [15] by the same reason of enhancement of the inertia with

respect to the irrotational flow value, especially better with accounting for surface and curvature $\hbar$-corrections. See also a good EWSR analytical

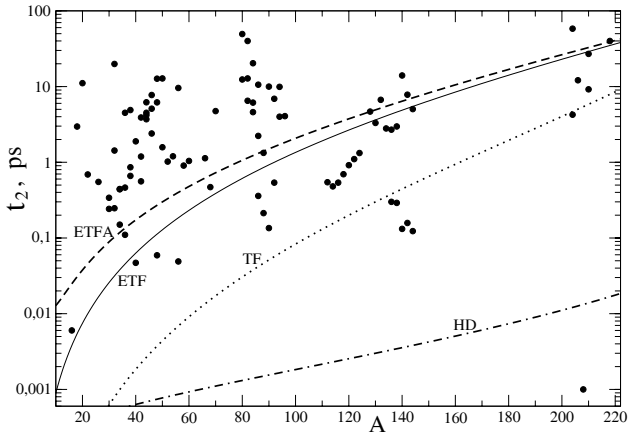

Fig. 4. The lifetime t_2 vs particle number A. Notations are the same as in Figs. 2 and 3.

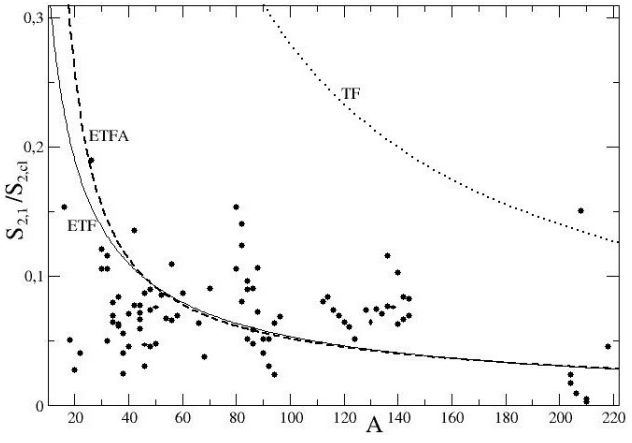

Fig. 5. The quadrupole EWSR $S_{2,1}$ in units of $S_{2,cl}$, see (12); full heavy points are $\hbar\omega_2 B(E2)$ with the experimental vibration energies $\hbar\omega_2$ and reduced probabilities $B(E2)$ of [15] shown in Figs. 2 and 3, respectively; ETFA is given by Eq. (13); other notations are the same as in Fig. 2 - 4.

Conclusions

For low-lying nuclear collective excitations within a few lowest orders of the EGA in $\hbar$ corresponding to the extended Thomas - Fermi approximation, we derived smooth inertia for the vibrations near a spherical shape of a mean edge-like field. The consistent collective ETF inertia is significantly larger than that of irrotational flow. Smooth low-lying collective vibration energies in almost spherical (besides of doubly-closed-shell) nuclei might roughly satisfy the A^{-1} particle-number dependence with the $A^{-1/3}$ surface and $A^{-2/3}$ curvature corrections for heavy enough nuclei, in contrast to the mainly $A^{-1/2}$ behavior predicted by the HD model and $A^{-1/3}$ dependence obtained in [8]. The smooth ETF energies, transition probabilities and EWSR differ from the statistically averaged experimental data for quadrupole [15] and octupole [16] low-lying states because of non-linear shell effects in the transport coefficients and coupling constants, see [11]. The quantum surface and curvature $\hbar$-corrections, coming mainly from the ETF dependence of the semiclassical parameter $k_F R$ on particle number A, are important in comparison with experimental data for the quadrupole and the octupole vibration energies and their EWSR contributions [9]. As the ETF inertia M is significantly larger than M_{irr} for the irrotational hydrodynamic flow, our vibration energies, the reduced transition probabilities and contributions into the EWSR are basically in much better agreement with their experimental data than those found in the HD approach for large enough (non magic) particle numbers. We proved semiclassically that the reduced transition probabilities in Weisskopf units for the low-lying vibration excitations are large sufficiently in order to refer them to the collective states. We found simple analytical asymptotics for the vibration energies (11), the reduced probabilities (14) and the EWSR (13) with explicit A-dependence for larger particle numbers A in good agreement with more complete ETF approach. As shown in [11], the shell corrections to the inertia and the stiffness improve agreement with the experimental data.

We are pleased to thank Profs. V. I. Abrosimov, F. A. Ivanyuk, H. Hofmann, V. M. Kolomietz, V. A. Plujko for many valuable discussions. We also gratefully acknowledge our colleagues V. A. Plujko and O. M. Gorbachenko for providing the files with the experimental data on the low-lying vibration energies and transition probabilities.

REFERENCES

1. *Bohr A., Mottelson B.* Nuclear structure. - N.-Y.: Benjamin Inc., 1975. - Vol. 2: Nuclear deformations. - 748 p.
2. *Migdal A. B.* Theory of finite Fermi systems and applications to the atomic nuclei. - N.-Y.: Wiley, 1967; New edition in Russian. - 1983. - 430 p.
3. *Hofmann H.* A quantal transport theory for nuclear collective motion: The merits of a locally harmonic approximation // Phys. Rep. - 1997. - Vol. 284, No. 4 - 5. - P. 137 - 380.
4. *Strutinsky V. M., Magner A. G.* Quasiclassical theory of nuclear shell structure // Sov. J. Part. Nucl. - 1976. -

Vol. 7, No. 2. - P. 356 - 418.
5. *Gutzwiller M.* Chaos in classical and quantum mechanics. - N.-Y.: Springer-Verlag, 1990. - 432 p.
6. *Brack M., Bhaduri R. K.* Frontiers in physics: Semiclassical physics. - Boulder: Westview Press, 2003. - 458 p.
7. *Magner A. G., Vydrug-Vlasenko S. M., Hofmann H.* Gross-shell effects in nuclear response functions // Nucl. Phys. A. - 1991. - Vol. 524, No. 1. - P. 31 - 64.
8. *Magner A. G., Gzhebinsky A. M., Fedotkin S. N.* Semiclassical inertia of nuclear collective dynamics // Phys. At. Nucl. - 2007. - Vol. 70, No. 4. - P. 647 - 657.
9. *Gzhebinsky A. M., Magner A. G., Fedotkin S. N.* Low-lying collective excitations of nuclei as a semiclassical response // Phys. Rev. C. - 2007. - Vol. 76. - P. 064315-1 - 064315-16.
10. *Ivanyuk F. A., Hofmann H., Pashkevich V. V., Yamaji J.* Transport coefficients for shape // Phys. Rev. C. - 1997. - Vol. 55, No. 4. - P. 1730 - 1746.
11. *Magner A. G., Gzhebinsky A. M., Fedotkin S. N.* Shell-srtucture inertia for slow collective motion // Phys. At. Nucl. - 2007. - Vol. 70, No. 11. - P. 1859 - 1872.
12. *Magner A. G., Gzhebinsky A. M., Fedotkin S. N.* Shells, orbits and transport coefficients of the nuclear collective dynamics // Scientific Papers of the Institute for Nuclear Research. - 2005. - Vol. 1(14). - P. 7 - 20.
13. *Kolomietz V. M., Kondratjev V. N.* Response function and kinetic coefficients of chaotic systems // Z. Phys. A. - 1992. - Vol. 344. - P. 125 - 133.
14. *Wilkinson M.* A semiclassical sum rule for matrix elements of classically chaotic systems // J. Phys. A: Math. Gen. - 1987. - Vol. 20. - P. 2415 - 2423.
15. *Raman S., Nestor Jr. C. W., Tikkanen P.* Transition probability from the ground to the first-excited 2^+ state of even-even nuclides // At. Data Nucl. Data Tables. - 2001. - Vol. 78. - P. 1 - 128.
16. *Kibedi T., Spear R. H.* Reduced electric-octupole transition probabilities $B(E3;0_1^+ \to 3_1^-)$ - an update // At. Data Nucl. Data Tables. - 2002. - Vol. 80, No. 1. - P. 35 - 82.

КВАЗІКЛАСИЧНЕ НАБЛИЖЕННЯ ДЛЯ НИЗЬКОЛЕЖАЧИХ КОЛЕКТИВНИХ ЗБУДЖЕНЬ В ЯДРАХ

А. М. Гжебінський, О. Г. Магнер, С. М. Федоткін

Отримано вираз для масового параметра у випадку низьколежачих колективних збуджень у рамках квазікласичного наближення Гутцвіллера до одночастинкової функції Гріна в найнижчому порядку по $\hbar$. Пораховані енергії збуджень, приведені ймовірності електромагнітних переходів і енергетично зважене правило сум узгоджуються з експериментальними даними.

КВАЗИКЛАССИЧЕСКОЕ ПРИБЛИЖЕНИЕ ДЛЯ НИЗКОЛЕЖАЩИХ КОЛЛЕКТИВНЫХ ВОЗБУЖДЕНИЙ В ЯДРАХ

А. Н. Гжебинский, А. Г. Магнер, С. Н. Федоткин

Получено выражение для массового параметра в случае низколежащих коллективных возбуждений в рамках квазиклассического приближения Гутцвиллера к одночастичной функции Грина в нижайшем порядке по $\hbar$. Рассчитанные энергии возбуждения, приведенные вероятности электромагнитных переходов и энергетически взвешенное правило сумм находятся в согласии с экспериментальными данными.

Received 23.06.08,
revised - 23.07.08.