Research

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New Algorithms for $\delta\gamma$–Order Preserving Matching

Nuevos Algoritmos para Búsqueda de Orden $\delta\gamma$

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Abstract

Context: Order-preserving matching regards comparing the relative order of symbols within different strings. However, its application areas require more flexibility in the matching paradigm. We advance in this direction in this paper that extends our previous work [27].

Method: We define $\delta\gamma$-order preserving matching as an approximate variant of order-preserving matching. We devise two solutions for it based on segment and Fenwick trees: $segtreeBA$ and $bitBA$.

Results: We experimentally show the efficiency of our algorithms compared to the ones presented in [26] ($naiveA$ and $updateBA$). Also, we present applications of our approach in music retrieval and stock market analysis.

Conclusions: Even though the worst-case time complexity of the proposed algorithms (namely, $O(nm \log m)$) is higher than the $\Theta(nm)$-time complexity of $updateBA$, their $\Omega(n \log n)$ lower bound makes them more efficient in practice. On the other hand, we show that our approach is useful to identify similarity in music melodies and stock price trends through real application examples.

Keywords: String searching, experimental algorithm analysis, strings similarity metric.

Language: English.

Resumen

Contexto: El emparejamiento de cadenas según el orden compara la estructura de las cadenas de texto. Sin embargo, sus áreas de aplicación requieren mayor flexibilidad en el criterio de comparación. Este artículo avanza en esta dirección al extender [27].

Método: Se define la búsqueda de orden-$\delta\gamma$ como una variante aproximada del problema de emparejamiento de cadenas según orden. Se proponen dos soluciones basadas en árboles de segmentos y árboles Fenwick: $segtreeBA$ and $bitBA$.

Resultados: La eficiencia de los algoritmos propuestos se muestra experimentalmente comparándolos con los algoritmos presentados en [26] ($naiveA$ y $updateBA$). Además, se presentan aplicaciones.

Conclusiones: A pesar de que la complejidad en tiempo de peor-caso de los algoritmos propuestos (a decir, $O(nm \log m)$) es mayor que la complejidad de $updateBA$ ($\Theta(nm)$), su cota baja $\Omega(n \log n)$ los hace más efiecientes en la práctica. También se muestran aplicaciones del enfoque propuesto en recuperación de música y análisis del mercado de acciones con ejemplos reales.

Palabras clave: Análisis experimental de algoritmos, búsqueda de texto, métrica de similitud.

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1. Introduction

Stringology is the branch of computer science that is dedicated to the study of problems in which sequences are involved. One of the main problems of interest in stringology is *string matching*, which consists of finding the occurrences of a pattern P within a text T both defined over a given alphabet Σ. Let $T_{0\ldots n-1}$ represent a length-n string defined over Σ. The symbol at the position i of a string T is denoted as T_i. Also, $T_{i\ldots j}$ represents the substring of the text T from the position i to the position j, i.e. $T_{i\ldots j} = T_i T_{i+1} \cdots T_j$, where it is assumed that $0 \leq i \leq j < n$. In particular, we are interested in each length-m substring that starts at position i of the text, i.e. $T_{i\ldots i+m-1}$, $0 \leq i \leq n-m$, which we call *text window* and denote as T^i in the rest of the paper. Then, the output of the exact string matching problem should list all the positions i, $0 \leq i \leq n-m$, such that $P_j = T_{i+j}$ for all $0 \leq j \leq m-1$.

In this paper, two variants of the problem of exact search of patterns are combined: the $\delta\gamma$–matching problem and the order preserving matching problem. Both of them consider integer alphabets. The $\delta\gamma$–*matching* problem consists of finding all the text windows in T for which $\max_{0\leq j\leq m-1} |P_j - T_{i+j}| \leq \delta$ and $\sum_{j=0}^{m-1} |P_j - T_{i+j}| \leq \gamma$. This is denoted as $P \overset{\delta\gamma}{=} T^i$. We can see that δ limits the individual error of each position while γ limits the total error. Then, $\delta\gamma$–matching has applications in bioinformatics, computer vision and music information retrieval, to name some. Cambouropoulos et al. [3] was perhaps the first to mention this problem motivated by Crawford's work et al. [6]. Recently, it has been used to make flexible other string matching paradigms such as parameterized matching [20], [21], function matching [22] and jumbled matching [23], [24].

On the other hand, *order-preserving matching* considers the order relations within the numeric strings rather than the approximation of their values. Specifically, the output of this problem is the set of text windows whose natural representation match the natural representation of the pattern. The natural representation of a string is a string composed by the rankings of each symbol in such string. In particular, the ranking of symbol T_i of string $T_{0\ldots n-1}$ is:

$$rank_T(i) = 1 + |\{T_j < T_i : 0 \leq j, i < n \wedge i \neq j\}| + |\{T_j = T_i : j < i\}|.$$

Then, the natural representation of T is $nr(T) = rank_T(0) rank_T(1) \cdots rank_T(n-1)$. Therefore, order preserving matching consists of finding all the text windows T^i such that $nr(P) = nr(T^i)$. Note that this problem is interested in matching the internal structure of the strings rather than their values. Then, it has important applications in music information retrieval and stock market analysis. Specifically, in music information retrieval, one may be interested in finding matches between relative pitches; similarly, in stock market analysis the variation pattern of the share prices may be more interesting than the actual values of the prices [18]. Since Kim et al. [18] and Kubica et al. [19] defined the problem, it has gained great attention from several other researchers [4], [5], [7]–[9], [11], [14], [14], [15].

Despite the extensive work on order-preserving matching, the only approximate variant in previous literature, to the best of our knowledge, was recently proposed by Uznański and Gawrychowski [13]. In particular, they allow k mismatches between the pattern and each text window. Then, they regard the number of mismatches but not their magnitude. In this paper, we propose a different approach to approximate order-preserving matching that bounds the magnitude of the mismatches

through the $\delta\gamma$- distance. Specifically, δ is a bound between the ranking of each character in the pattern and its corresponding character in the text window; likewise, γ is a bound on the sum of all such differences in ranking. Thus, δ and γ respectively restrict the magnitude of the error individually and globally across the strings. We define *$\delta\gamma$–order-preserving matching* as the problem of finding all the text windows in T that match the pattern P under this new paradigm. This paper is an extended version of the work [27] presented in the Workshop on Engineering Applications 2017. Furthermore, some of its contents were developed in the Master's Thesis [25].

We first defined the notion of $\delta\gamma$–order preserving matching in [26]. Now, in this paper, we provide a more formal definition and two new algorithms for this problem in Section 2. Then, we present some experimental results of the proposed algorithms and discuss some applicacions in Section 3. Finally, the concluding remarks are presented in Section 4.

2. Methods

In Section 2.1 we formally define $\delta\gamma$–order preserving matching while we present its solutions in Section 2.2.

2.1. Definition of $\delta\gamma$–order preserving matching problem ($\delta\gamma$–OPMP)

The motivation to define $\delta\gamma$–order-preserving matching stems from the observation that the application areas of order-preserving matching, mainly stock market analysis and music information retrieval, require to search for occurrences of the pattern that may not be exact but rather have slight modifications in the magnitude of the rankings. For example, let us assume that the text T presented in Figure 1 is a sequence of stock prices and that we want to determine whether it contains similar occurrences of the pattern P (also shown in this figure). Under the exact order-preserving matching paradigm, there are no matches, but there are similar occurrences at positions and 1 and 11. In particular, $T_{1...8}$ and $T_{11...18}$ are similar, regarding order structure, to the pattern. This similarity can be seen even more clearly if we consider natural representations of these strings (also shown in Figure 1). Next we formally define these notions.

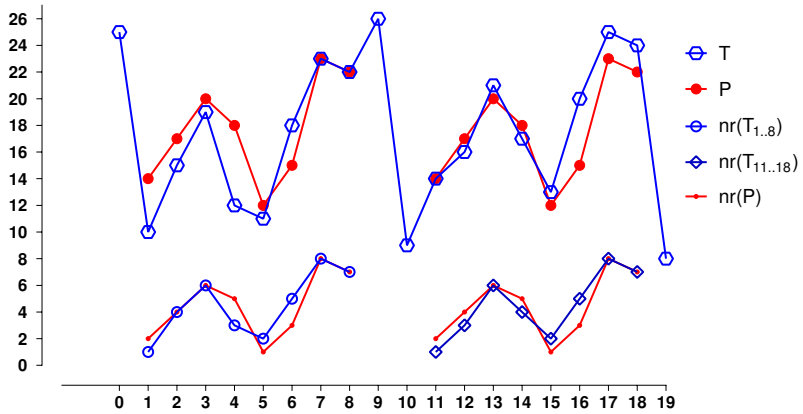


Figure 1: Order preserving matching under $\delta\gamma$ approximation example.

Definition 1 (**$\delta\gamma$–order-preserving match**) *Let $X = X_{0\ldots m-1}$ and $Y = Y_{0\ldots m-1}$ be two equal-length strings defined over Σ_σ. Also, let δ, γ be two given numbers ($\delta, \gamma \in \mathbb{N}$). Strings X and Y are said to $\delta\gamma$–order-preserving match, denoted as $X \overset{\delta\gamma}{\rightsquigarrow} Y$, **iff** $nr(X) \overset{\delta\gamma}{=} nr(Y)$.*

Given $\delta = 2$, $\gamma = 6$, $X = \langle 10, 15, 19, 12, 11, 18, 23, 22\rangle$ and $Y = \langle 14, 17, 20, 18, 12, 15, 23, 22\rangle$, $X \overset{\delta\gamma}{\rightsquigarrow} Y$ as $nr(X) = \langle 1, 4, 6, 3, 2, 5, 8, 7\rangle$, $nr(Y) = \langle 2, 4, 6, 5, 1, 3, 8, 7\rangle$ and $nr(X) \overset{\delta\gamma}{=} nr(Y)$.

Problem 1 (**$\delta\gamma$–order-preserving matching**) *Let $P = P_{0\ldots m-1}$ be a pattern string and $T = T_{0\ldots n-1}$ be a text string, both defined over Σ_σ. Also, let δ, γ be two given numbers ($\delta, \gamma \in \mathbb{N}$). The $\delta\gamma$–order-preserving matching problem is to calculate the set of all indices i, $0 \leq i \leq n - m$, satisfying the condition $P \overset{\delta\gamma}{\rightsquigarrow} T^i$. From now on $\delta\gamma$–**OPMP**.*

2.2. Algorithms for the $\delta\gamma$–OPMP

In this section, we present two algorithms that solve the $\delta\gamma$–OPMP: one that uses segment trees (Section 2.2.1) and the other utilizes Fenwick trees (Section 2.2.2).

2.2.1. Segment tree based algorithm ($segtreeBA$)

The segment tree is a powerful data structure that answers queries in ranges of an underlying array A [2], [10]. We use the segment tree data structure to solve the range minimum query (*RMQ*) problem, which consists in finding the index of the minimum value of the array in a given range, and we are able to change elements of the array. Building a segment tree to solve the *RMQ* problem for an array A of length $|A|$ takes $O(|A|)$ space and time. The update and query operations both take $O(\lg |A|)$. Based on this data structure, we propose the algorithm called $settreeBA$ (see Figure 2). It first calculates the natural representation of the pattern P (line 1 in Figure 2). Then, it iterates over all possible position and tries to find $\delta\gamma$-order preserving matches in every one of them. The process of finding a match at position i in T is as follows: First the algorithm finds the smallest number in the interval $[i, i+m-1]$ (line 8); this value has the rank 1 in the sliding window T^i. It then uses the natural representation of P to check the δ and γ restrictions for the rank 1 in the window T^i. Then it prepares the segment tree for the next iteration; this is done by changing the smallest value in the interval $[i, i + m - 1]$ to infinity, so in the next iteration of the first inner loop the operation $querySegTree(minIndex, i, i + m - 1)$ finds the second smallest value in the same interval. This process is done for all the rankings from 1 to m.

In the second inner loop (lines 17 and 18 in Figure 2), the values of T in the interval $[i, i+m-1]$ must be changed so that, in the next window, those contain the original values of T and no infinity. The arrays $oldValue$ and $changedIndex$ help in the process of restoring the segment tree. We are going to adapt the standard operations of the segment tree to this solution as follows:

- $buildSegTree(T, 0, n-1)$: Builds a segment tree with $T_0, T_1, \ldots, T_{n-1}$ and returns the root node. The complexity is $O(n)$.
- $updateSegTree(minIndex, i, x)$: Sets T_i to x. The complexity is $O(\lg n)$.
- $querySegTree(minIndex, i, j)$: Returns the index of the minimum value among T_i, T_{i+1}, $\ldots$, T_j. If there are several minimum values, the leftmost (smallest index) is chosen. The complexity is $O(\lg n)$.

Algorithm 1: $\delta\gamma$–OPMP **segtreeBA**

Input: $P = P_{0\ldots m-1}, T = T_{0\ldots n-1}, \delta, \gamma$
Output: $\{i \in \{0,\ldots,n-m\} : T^i \overset{\delta\gamma}{\rightsquigarrow} P\}$
1. **Create as Array:** $P^{nr} \leftarrow nr(P)$
2. **Create as Array of size** m**:** $oldValue$,$changedIndex$
3. **Create as Segment Tree:** $minIndex \leftarrow buildSegTree(T, 0, n-1)$
4. **Define:** $curDelta$,$curGamma$,$rank$,$idxT$,$idxP$,$nChanges$ **as integers**
5. $nChanges \leftarrow 0$
6. **for** $i = 0 \rightarrow n-m$ **do**
7. **for** $rank = 1 \rightarrow m$ **do**
8. $idxT \leftarrow querySegTree(minIndex, i, i+m-1)$
9. $idxP \leftarrow idxT - i$
10. $curDelta \leftarrow |rank - P^{nr}_{idxP}|$
11. $curGamma \leftarrow curGamma + curDelta$
12. **if** $curDelta > delta \vee curGamma > gamma$ **then break loop**
13. $changedIndex_nChanges \leftarrow idxT$
14. $oldValue_nChanges \leftarrow T_{idxT}$
15. $nChanges \leftarrow nChanges + 1$
16. $updateSegTree(minIndex, idxT, \infty)$
17. **for** $c = 0 \rightarrow nChanges - 1$ **do**
18. $updateSegTree(minIndex, changedIndex_c, oldValue_c)$
19. **if** $rank > m$ **then report** i
20. $nChanges \leftarrow 0$

Figure 2: Segment tree based algorithm: *segtreeBA*.

The total complexity of the algorithm is then $O(nm \lg n)$ with a lower bound of $\Omega(n \lg n)$.

2.2.2. Fenwick tree based algorithm ($bitBA$)

The Binary Indexed Tree (*BIT*) or Fenwick tree, is a data structure that can be used to maintain and query cumulative frequencies [12]. In particular, it is mainly used to efficiently calculate prefix sums in an array of numbers. Based on this data structure, we propose the algorithm called $bitBA$ (see Figure 3). The BIT data structure could be considered then as an abstraction of an integer array of size n indexed from 1, i.e., a bit encapsulate $A = A_1 A_2 \cdots A_n$. The version we are going to use has two operations:

- $sumUpTo(tree, i)$: Returns $A_1 + A_2 + \ldots + A_i$. The complexity is $O(\lg n)$.
- $addAt(tree, i, x)$: Sums x to A_i. The complexity is $O(\lg n)$.

The algorithm has a preprocessing phase in which the data structures needed to solve the $\delta\gamma$–OPMP are created. This is done with a complexity of $\Theta(n + n \lg n + m \lg m)$. The term n is due to the creation of the BIT. The term $n \lg n$ is due to the creation of T^{nr} and the term $m \lg m$ is due to the creation of P^{nr}. In the searching phase, it iterates over all possible positions in the text T to find the existing matches. For each position i to be considered, the algorithm uses the BIT to get the rank of every symbol in the searching window $T_{i\ldots i+m-1}$, and then each rank in the window is compared with each rank in P^{nr} to check if T^i is a $\delta\gamma$–order preserving match. This operation is evaluated using the function $isAMatch(P, T^i, \delta, \gamma)$; in particular, this function returns true ***iff***

$P \overset{\delta\gamma}{\leftrightsquigarrow} T^i$ and this takes $O(m \lg m + m)$. Each rank calculation using the BIT costs $O(\lg n)$. Then the total complexity of the algorithm is $O(nm \lg n)$, but with a lower bound of $\Omega(n \lg n)$.

In the preprocessing phase, the algorithm first creates the natural representations of the pattern P and the text T (P^{nr} and T^{nr}, respectively). Then, it creates a BIT which is an encapsulation of an array with n positions numbered from 1 to n. Then assigns 1 the positions T_0^{nr}, T_1^{nr}, $\ldots T_{m-2}^{nr}$ (Lines 1 to 5 in Figure 3). In the searching phase, for each candidate position i, the algorithm computes the rank of each symbol $T_{i+j}, 0 \leq j \leq m-1$ using $sumUpTo(i+j)$. After checking if there is a match at position i, the BIT must be updated in each iteration to consider symbol T_{i+m} (line 7 in Figure 3). And the BIT must be updated so it does not consider the position i in the next search window (line 9 in Figure 3).

Algorithm 2: $\delta\gamma$–OPMP **bitBA**

Input: $P = P_{0\ldots m-1}, T = T_{0\ldots n-1}, \delta, \gamma, \Sigma_\sigma$
Output: $\{i \in \{0, \ldots, n-m\} : T^i \overset{\delta\gamma}{\leftrightsquigarrow} P\}$
1. **Create as Array:** $T^{nr} \leftarrow nr(T)$
2. **Create as Array:** $P^{nr} \leftarrow nr(P)$
3. **Create as Array of size n:** bit
4. **for** $i = 0 \rightarrow m-2$ **do**
5. $\quad addAt(bit, T_i^{nr}, 1)$
6. **for** $i = 0 \rightarrow n-m$ **do**
7. $\quad addAt(bit, T_{i+m-1}^{nr}, 1)$
8. $\quad isAMatch(i, bit, T^{nr}, P^{nr}, \delta, \gamma)$ **then report** i
9. $\quad addAt(bit, T_i^{nr}, -1)$

Figure 3: BIT based algorithm: *bitBA*.

3. Results

In Section 3.1 we present experimental results on the proposed algorithms while we present applications for $\delta\gamma$-order preserving matching in Section 3.2.

3.1. Experiments on Artificial Data

In this section, we describe the experimental setup we designed to evaluate the performance of the proposed algorithms. We compare our algorithms with two baseline algorithms: The naive algorithm, which we call $naiveA$, and $updateBA$, presented in [26]. The former, whose time complexity is $\Theta(nm \lg m)$, considers all possible positions in the text and, for each one of them, verifies if there is a match in $\Theta(m \lg m + m)$ time. The latter algorithm, whose time complexity is $\Theta(nm)$, is based on linear update and verification.

We present the experimental framework (Section 3.1.1) and describe the data generation (Section 3.1.2). Then, we discuss the results obtained (Section 3.1.3). Finally we show the results of the experiments intended to study how the algorithms $segtreeBA$ and $bitBA$ behave in the worst-case scenario for all experiment instances (Section 3.1.4).

Table I: Experimental values of n m, δ, γ and σ.

	Varying n	Varying m	Varying δ	Varying γ	Varying σ
n	$[3000, 60000]$ $\Delta n = 3000$	10000	10000	10000	10000
m	40	$[30, 600]$ $\Delta m = 30$	40	40	40
δ	10	10	$[0, 228]$ $\Delta\delta = 12$	10	10
γ	60	60	60	$[0, 570]$ $\Delta\gamma = 30$	60
σ	100	100	100	100	$[12, 240]$ $\Delta\sigma = 12$

3.1.1. Experimental setup

Hardware and software: All the algorithms were implemented using C++. The computer used for the experiments was a Lenovo ThinkPad with a processor Intel(R) Core(TM) i7 4600u CPU @ 2.10GHz 2.69 GHz and installed RAM memory of 8GB. The computer was running 64-bit Linux Ubuntu 14.04.5 LTS. The C++ compiler version was g++ (Ubuntu 4.8.4-2ubuntu1 14.04.3) 4.8.4.

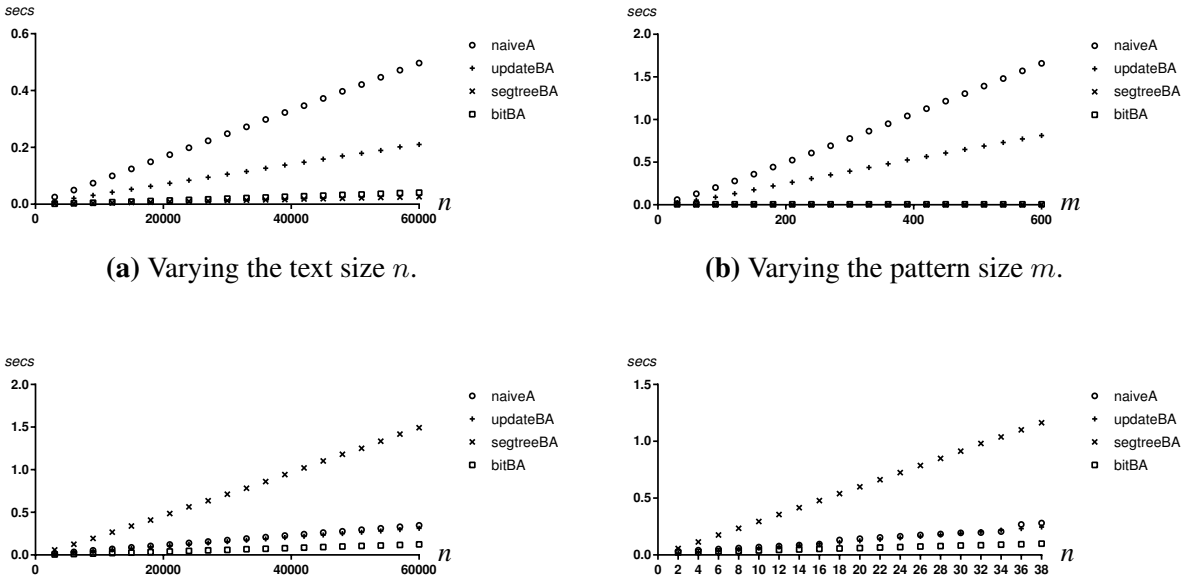
Parameters: To show how our solution behaves with different configuration of the different parameters, we perform five types of experiments. In each experiment, we vary one of the given parameters n, m, δ, γ and σ, and let the other four parameters fixed at a given value. We chose the fixed values after several attempts via try and error to find values that produced results varying from no matches to matches near the value of n. For each experiment type, we performed five different experiments and took the median as the value to plot, making the median of five experiments the representative value for a experiment configuration of values n m, δ, γ and σ. The variation of the parameter values for each experiment type is presented in Table I.

3.1.2. Random data generation

An experiment consists of two stages. The first stage is the pseudo-random generation of a text T of length n and the pattern P of length m. The second stage is the execution of the algorithms on the generated strings P and T. The random generation of each character of both the pattern P and the text T is done by calling a function that pseudo-randomly and selects a number between 1 and σ with an uniform probability distribution, i.e., all symbols have the same probability to appear in a position and for that reason, on average, every symbol in the alphabet will appear with the same frequency on an arbitrary generated string.

3.1.3. Experimental results and analysis

The first result to highlight is the fact that, in every experiment, the naive algorithm always has the worst performance, as expected. We found that the size of the alphabet and the parameters δ and γ have practically no impact on the execution time of any of the algorithms, they all show nearly constant time behavior. Figures 4a and 4b verify the theoretical complexity analysis that states that n and m are the parameters that really determine the growth in the execution time of all



(a) Varying the text size n.

(b) Varying the pattern size m.

(c) Varying n in the worst case.

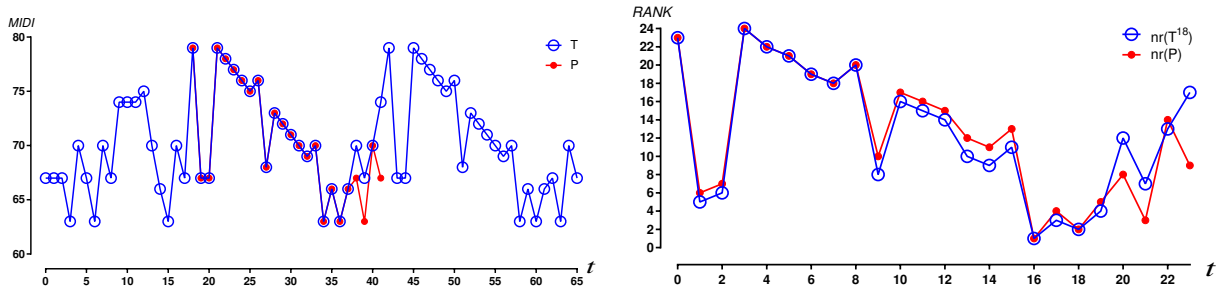
(d) Varying m in the worst case.

Figure 4: Experimental results of comparing the four algorithms by varying different parameters.

the algorithms. In Figure 4a, m is a constant and n is a variable while in Figure 4b, n is a constant and m is a variable. It is important to notice that, under these conditions, the graphs are expected to be linear and the experiments verify that.

In the figures where we show the result of varying the parameter n and the parameter m, (Figures 4a-4b), we can see that the best two algorithms are the based on data structures ($segtreBA$ and $bitBA$). This despite the fact that these two algorithms have a higher upper bound on their complexities in relation with the first two algorithms ($naiveA$ and $updateBA$). This result can be explained by the fact that the lower bound on the data structure based algorithms is considerably lower in comparison with the other two. The lower bound of the data structures based algorithms is $\Omega(n \lg n)$ and the lower bound of the $naiveA$ and $updateBA$ is the same as their upper bound meaning they are $\Theta(nm \lg m)$ and $\Theta(nm)$ respectively. This can be understood by taking into account that the first two algorithms check for a match after a natural representation of every window is completely obtained; on the contrary, data structure based algorithms break the calculation of a given natural representation of a window if at some point the δ or γ restriction do not hold.

Given the result of the experiments, it is safe to say that the algorithms based on data structures are faster in most cases, especially if they are going to be used in applications where very few matches are expected to appear. This is due to their lower bound of complexity. We test two different implementations of the segment tree data structure: one based on classes and pointers, and the other based on an array. Ultimately, we recommend to chose the array based as representative for the segment tree based solution and the experiments plots show their results. The array–based segment tree is almost twice time faster than the classes–based implementation.



(a) Pitches of T and P and match at position 18. **(b)** Natural representation of P and the match.

Figure 5: Darth Vader's theme from Star Wars by John Williams (Excerpt).

3.1.4. Worst case experiments on $segtreeBA$ and $bitBA$

Taking into account that the first two algorithms, $naiveA$ and $updateBA$ both have complexities in Big Theta notation, i.e. their worst case is the same as their best case, the experiments described so far are enough for their experimental analysis. For the data structures based algorithms a more particular kind of experiment is needed, i.e. the worst case experimental analysis. For this algorithms the worst case is when there is a match in every candidate position. An easy way to generate data for the worst case is when all the symbols in both the pattern P and the text T are the same. Other way to generate worst cases scenarios for this two algorithms is when either both P and T are strictly increasing or both are strictly decreasing. Results from this experiments show a fast degradation in experimental performance of the $segtreeBA$ algorithm, but a very slow degradation of the $bitBA$ algorithm. Results of this last experiments are shown in Figures 4c and 4d.

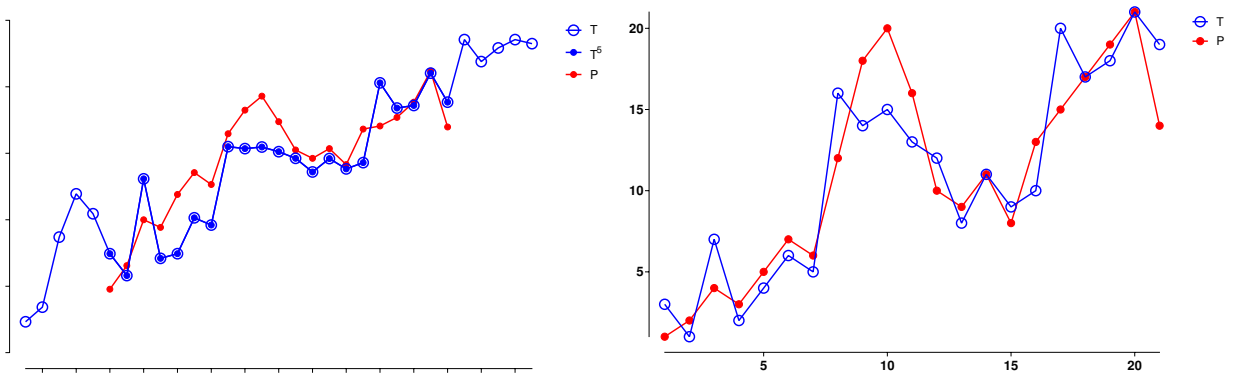
3.2. Applications

In this section we show a couple of applications of the defined problem, in music and finance.

3.2.1. Application in music

For the music application example, we choose the main theme from The Imperial March, soundtrack of the film series Star Wars [17] composed by John Williams [16] also known as the Darth Vader's theme because it represents him. This melody sounds every time this villain has a significant scene. Here we use an integer alphabet abstraction of a music piece, where each note of the melody is an integer. This abstraction of music takes into account only the pitch leaving out other aspects as silences, note duration, harmony, or instrumentation, but it gives a very good idea of the possible applications in music retrieval. The alphabet for music applications could be for example the given by the MIDI (Musical Instrument Digital Interface) technical standard [1], [28]. In the MIDI standard, the first note, 0 is a C note of the octave 0 (the lowest octave), note 1 is a $C\#$ of the same octave and so on. There could be up to note 127 which will be a G in the $10th$ octave.

We draw an example of $\delta\gamma$–OPM with the same musical piece. We consider a 66-pitch excerpt of the main melody as the text $T = T_{0...65}$, and from the same except we extract $T_{42...65}$ as the pattern P (see Figure 5a). For $\delta = 8$ and $\gamma = 32$, we found a match in position 18 which for professional musicians, and even non professional musicians, sounds very similar to the pattern. Namely,



(a) P and text fragment with a match. **(b)** Natural representations of P and the match found.

Figure 6: Stock price of the Facebook company from May 18 2012 to March 31 2017.

Gabriela Rojas, a professional musician from the National University of Colombia Conservatory found the match similar to the pattern. The parameters δ and γ were chosen by attempting different values of both starting from 0 and increasing them until more matches were found. Furthermore, we can see in Figure 5a how similar the pitches of the pattern and the match are. In Figure 5b we show the similarity of their natural representation . This gives us an idea of possible applications in musical retrieval of approximate string matching. This can be useful for the advanced music students in order to help them with the theoretical analysis of the scores so they can look for melodic similarities or differences either in the same piece or comparing different pieces.

3.2.2. Application in finance

For the finance application we choose to analyse the stock price of the Facebook company. We take the history of the stock price of Facebook from May 18 2012 to March 31 2017 as the text T (the size of this text is 1225). As the pattern, we take the 21-day period starting in February 28 2017 up to March 28 2017. Take into account that not all days the stock actions change, for that reason we choose 21 days which is approximately the amount of days the stock actions change in a month. In Figure 6a we show the pattern and the portion of the text that we found to be the most similar to the pattern. In this figure we omit the y and x axes labels because we want to show the similarity in shape of the pattern and the search window, not the similarity in absolute values which indeed is quite different. In fact the values in the pattern to search are values lower than 34 and the text window found has values greater than 100. Finally, in Figure 6b we show the natural representation (or ranks) of both the pattern P and the match found. We can see that they have a similar structure. We selected the pattern randomly and then attempted to find its matches with different values for δ and γ. Given that a 21 day period is not a short period, it was expected that we just found one match in the given text window.

4. Conclusions and Discussion

We define a new variant of the string matching problem, the $\delta\gamma$–order preserving matching problem ($\delta\gamma$–OPMP). This new variant provides the possibility of searching a pattern according to the relative order of the symbols as the order preserving matching problem. But it also gives more flex-

ibility to the search by allowing error in the individual ranking comparisons through the parameter δ. And also, the proposed problem gives a bound for the global error in the comparison of a pattern against a text window through γ. This new variant has at least the same applications as the order preserving matching problem.

Our experimental results on randomly generated data show that in most cases, given the uniformly data generation, the proposed algorithms work faster than the naive solution and $updateBA$. One question that remains open is if an algorithm with better worst case time complexity than $O(nm)$ can be designed; other question that also remains open is that if an algorithm with better lower bound than $\Omega(n \lg n)$ can be obtained. We show experimental results on the worst cases of the $bitBA$ and $segtreeBA$. We conclude that the degradation in performance in the $segtreeBA$ algorithm is much more notorious than the degradation of $bitBA$. It still remains open to prove empirically that we can device an experimental setup where the best worst–case algorithm, $updateBA$ experimentally beats the other three algorithms. Given the theory behind the big O notation, we can say that such experimental setup exist.

We show two applications with real data in music and finance. In music we use our findings to search for a portion of a melody in the melody itself. Those two portions of the melody are also very similar according to professional musicians consulted. For the financial application, we show how similar the changes in stock prices are despite the difference in their absolutes values. An aspect left to explore related to the applications is to establish whether, in the finance application, the tools presented here can be useful to device or complement algorithms/techniques to make predictive analysis of stock price changes. In music, our contributions can be useful to design tools for advanced music students in order to help them with the theoretical analysis of the scores so they can look for melodic similarities or differences either in the same piece or comparing different pieces. Also composers could see the $\delta\gamma$–OPMP, as a tool to check the perception they have about the similarity of musical ideas developed in different ways in one or several pieces of their own. For the musicologist this could be a way to track the development of one composer's musical ideas throughout their life and to analyze the way the composer evolves.

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