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## SURFACE ELECTROMAGNETIC WAVES IN A GAP BETWEEN TWO LEFT-HANDED MATERIALS

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The slow surface electromagnetic waves that propagate along the planar waveguide structure that contains two identical isotropic left-handed material with the vacuum (or air) gap has been considered. The possibility of propagation of slow surface electromagnetic waves of TM and TE polarizations in the frequency range at which the dielectric permittivity and magnetic permeability are negative are shown. By selecting a frequency, one, two or three modes can be excited simultaneously. The group and phase velocities of the TE polarization waves are directed in opposite directions. The frequency range of the existence of the TE mode with an antisymmetric field distribution lies above the frequency range of the existence of the TE mode with a symmetric field distribution. The TM wave, depending on the wavelength, can be either forward or backward, or have zero group velocity. The wave of TM-polarization has a slightly varying group velocity in a sufficiently wide frequency range. A rich set of different properties of these waves make them promising in applications.

**KEYWORDS:** left-handed materials, surface electromagnetic waves, dispersion, forward and backward waves

### ПОВЕРХНЕВІ ЕЛЕКТРОМАГНІТНІ ХВИЛІ У ЩІЛИНІ МІЖ ДВОМА ЛІВОСТОРОННІМИ МАТЕРІАЛАМИ

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Розглянуто повільні поверхневі електромагнітні хвилі, які розповсюджуються уздовж планарної хвилеводної структури, що утворена двома ідентичними ізотропними лівосторонніми матеріалами з вакуумним (або повітряним) зазором. Показано можливість поширення повільних поверхневих електромагнітних хвиль ТМ- і ТЕ-поляризацій в області частот, за яких діелектрична і магнітна проникності від'ємні. Вибором частоти можливо збудити одну, дві абож три моди одночасно. Групова та фазова швидкості хвиль ТЕ-поляризацій мають протилежні напрями. Область частот існування ТЕ-моди з антисиметричним розподілом поля знаходиться вище області частот існування ТЕ-моди з симетричним розподілом поля. ТМ-хвиля, в залежності від довжини хвилі, може бути або прямою, або зворотною, абож мати нульову групову швидкість. Хвиля ТМ-поляризації має слабо змінну групову швидкість в досить широкій області частот. Багатий набір різних властивостей цих хвиль роблять їх перспективними в застосуваннях.

**КЛЮЧОВІ СЛОВА:** лівосторонні метаматеріали, поверхневі електромагнітні хвилі, дисперсія, прямі та зворотні хвилі

### ПОВЕРХНОСТНЫЕ ЭЛЕКТРОМАГНИТНЫЕ ВОЛНЫ В ЩЕЛИ МЕЖДУ ДВУМЯ ЛЕВОСТОРОННИМИ МАТЕРИАЛАМИ

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Рассмотрены медленные поверхностные электромагнитные волны, которые распространяются вдоль планарной волноводной структуры, которая содержит два идентичных изотропных левых материала с вакуумным (или воздушным) зазором. Показана возможность распространения медленных поверхностных электромагнитных волн ТМ- и ТЕ-поляризацій в области частот, при которых диэлектрическая и магнитная проницаемости отрицательны. Выбором частоты возможно возбуждение одной, двух или трёх мод одновременно. Групповая и фазовая скорости волн ТЕ-поляризации направлены в противоположные стороны. Область частот существования ТЕ-моды с антисимметричным распределением поля находится выше области частот существования ТЕ-моды с симметричным распределением поля. ТМ-волна, в зависимости от длины волны, может быть прямой, либо обратной, либо иметь нулевую групповую скорость. Волна ТМ-поляризации обладает слабо меняющейся групповой скоростью в достаточно широкой области частот. Богатый набор различных свойств этих волн делают их перспективными в применении.

**КЛЮЧЕВЫЕ СЛОВА:** левосторонние метаматериалы, поверхностные электромагнитные волны, дисперсия, прямые и обратные волны

Theoretical and experimental studies of the artificial materials (metamaterials or double-negative, or left-handed materials, LHM, with  $\varepsilon(\omega) < 0$  and  $\mu(\omega) < 0$ ) have been carried out during the last 15 years. Many extraordinary electromagnetic features have been investigated [1]. This opened the new horizons on science and technology.

In the paper [2] it was searched the bulk electromagnetic waves in the vacuum (or air) gap between two LHM media. Aim of our work is to show the possibility of existing the eigen electromagnetic waves of surface type in such structure and to search the dispersive properties of these waves. The surface waves play crucial role in the interactions

of electromagnetic fields with different materials (including biological) at the short distances from its boundary. The knowledge of these waves properties will give rise the new possibilities for both a diagnostics and modification of these objects.

### TASK SETTINGS

Let us consider the surface electromagnetic waves that propagate along the planar waveguide structure that contains two identical isotropic left-handed material with the vacuum (or air) gap of a thickness  $D$ . The left-handed material will be characterized by effective permittivity  $\varepsilon(\omega)$  and permeability  $\mu(\omega)$  that depend on the wave frequency  $\omega$  and commonly expressed with the help of experimentally obtained expressions [3]:

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \quad \mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2}, \quad (1)$$

where  $\omega_p$  is effective plasma frequency,  $\omega_0$  is the characteristic frequency of LHM,  $F$  - constant.

We are interested on the slow surface type electromagnetic waves at the frequency intervals where simultaneously  $\varepsilon(\omega) < 0$  and  $\mu(\omega) < 0$ . We will look for solutions for wave fields between two left-handed materials in the form

$$E, H \propto [F_1 \exp(\kappa x) + F_2 \exp(-\kappa x)] \exp[i(\beta z - \omega t)], \quad (2)$$

and choose an exponentially decreasing (from both boundaries) solutions for fields in the left-handed materials:

$$E, H \propto \exp(-h\rho) \exp[i(\beta z - \omega t)], \quad (3)$$

where  $F_{1,2}$  are the functions of  $(\omega, \beta)$ ,  $\kappa = \sqrt{\beta^2 - k^2} \geq 0$ ;  $h = \sqrt{\beta^2 - \varepsilon(\omega)\mu(\omega)k^2} \geq 0$ ,  $\rho$  is a distance from the media interfaces;  $k = \omega/c$ , where  $c$  is the speed of light in vacuum.

In our case the system of Maxwell equations splits into two sub-systems of equations. One of them describes the waves of TM-polarization  $\{H_y, E_x, E_z\}$ , and another – waves of TE-polarization  $\{E_y, H_x, H_z\}$ .

By matching the appropriate tangential components at the media interfaces  $x=0$  and  $x=D$ , we obtained the following appropriate dispersive equation for TM-mode:

$$2\varepsilon(\omega)\kappa h + (\kappa^2 \varepsilon^2(\omega) + h^2) \tanh(\kappa D) = 0 \quad (4)$$

and for TE-modes

$$2\mu(\omega)\kappa h + (\kappa^2 \mu^2(\omega) + h^2) \tanh(\kappa D) = 0. \quad (5)$$

### RESULTS

The results of numerical calculation of dispersion equations for TM- and TE-waves (4,5) for the selected set of task parameters are shown at Fig. 1.

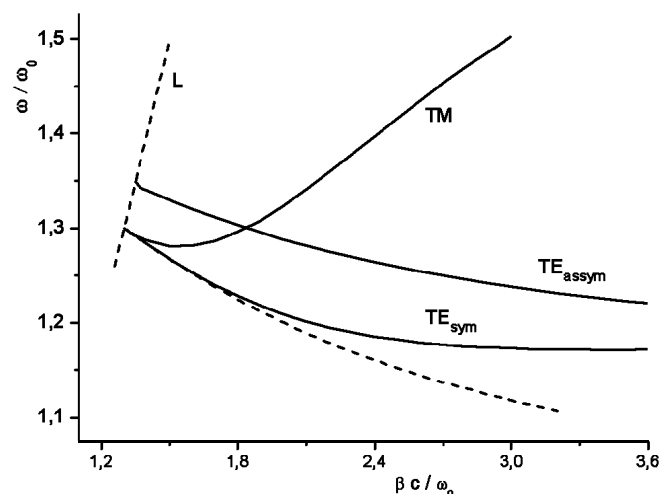


Fig. 1. Dispersive curves for slow surface electromagnetic waves.

In our study it was considered the LHM with  $\omega_p / 2\pi = 10$  GHz and  $\omega_0 / 2\pi = 4$  GHz,  $F = 0.56$  [2] and the thickness of the vacuum gap  $D = 0.717$  cm. As a result the conditions  $\varepsilon(\omega) < 0$  and  $\mu(\omega) < 0$  fulfilled in the frequency intervals in which  $1 < \omega / \omega_0 < 1.5$ .

The inclined straight line (L) corresponds to “light” line  $\omega = c\beta$  and the lower dashed curve corresponds to the condition  $h(\beta, \omega) = 0$ . These lines separate region, where the slow surface electromagnetic wave can exist.

There are three surface modes which can propagated in such gap structure in the frequency range under consideration. Two surface modes have the TE-polarization and are backward. One of these waves is more high frequency mode with antisymmetric field distribution. Another TE-mode has symmetric field distribution. The TM-mode can be either backward or forward in dependence on value of wave vector.

As we can see it is possible to excite one or more modes by the appropriate choice of working frequency. There is a frequency interval in which the simultaneous existence of both forward and backward waves is possible.

The spatial distribution of the amplitude of the transverse magnetic field of TM-mode is as follows:

$$H_y(x, z, t) = \exp[i(\beta z - \omega t)] \begin{pmatrix} \exp(hx); & x \leq 0 \\ C_1 e^{\kappa x} + C_2 e^{-\kappa x}; & 0 \leq x \leq D \\ R \exp(-h(x - D)); & x \geq D \end{pmatrix}, \quad (6)$$

where

$$C_1 = (1/2)[1 + h/(\kappa\varepsilon(\omega))]; C_2 = (1/2)[1 - h/(\kappa\varepsilon(\omega))]; R = (1/2)\{[1 + h/(\kappa\varepsilon(\omega))]\exp(\kappa D) + [1 - h/(\kappa\varepsilon(\omega))]\exp(-\kappa D)\}$$

and is presented in the inserts at Fig.2 for corresponding places on the dispersion curve. On all inserts, the interval of variation of coordinate is the same  $-4, 5 \leq x\omega_0 / c \leq +4, 5$ . The electric fields of this surface mode are as follows :

$$E_x(x, z, t) = \left(\frac{-i}{\kappa\varepsilon(\omega)}\right) \frac{\partial H_y}{\partial z}; E_z(x, z, t) = \left(\frac{i}{\kappa\varepsilon(\omega)}\right) \frac{\partial H_y}{\partial x}. \quad (7)$$

The TM-polarization mode possesses the most interesting dispersion features (Fig.2). It is worth noting the presence of a pair  $(\omega, \beta)$  for which  $V_{gr} = d\omega/d\beta = 0$ . The TM mode group velocity has an almost unchanged value 0.65 c for short wavelengths if the frequency varies from 1.3  $\omega_0$  to 1.5  $\omega_0$  at the selected parameter set of our task.

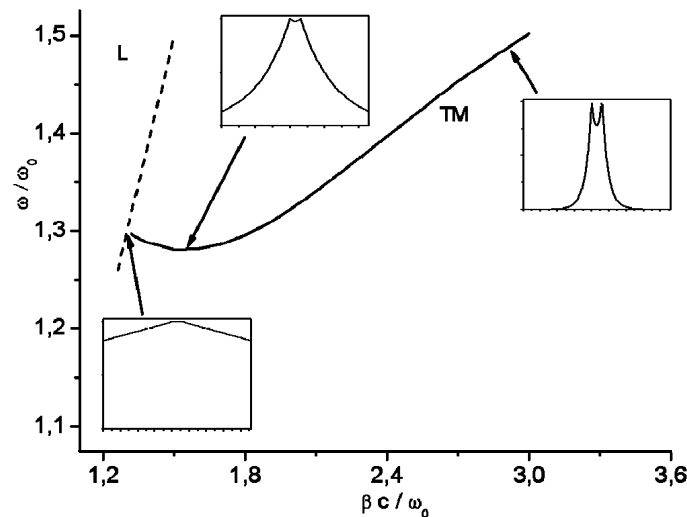


Fig. 2. Evolution of transverse magnetic field distribution along dispersive curve for TM mode

At the inserts of Fig. 3,4 we presented the spatial distribution of the amplitude of the transverse electric field of TE-modes for corresponding places on the dispersion curves:

$$E_y(x, z, t) = \exp[i(\beta z - \omega t)] \begin{pmatrix} \exp(hx); & x \leq 0 \\ B_1 e^{\kappa x} + B_2 e^{-\kappa x}; & 0 \leq x \leq D \\ T \exp(-h(x - D)); & x \geq D \end{pmatrix}, \quad (8)$$

where

$$B_1 = (1/2)[1 + h/(\kappa\mu(\omega))];$$

$$B_2 = (1/2)[1 - h/(\kappa\mu(\omega))];$$

$$T = (1/2)\{[1 + h/(\kappa\mu(\omega))]\exp(\kappa D) + [1 - h/(\kappa\mu(\omega))]\exp(-\kappa D)\}.$$

The magnetic fields of these surface TE-modes may be calculate in this way:

$$H_x(x, z, t) = \left( \frac{i}{k\mu(\omega)} \right) \frac{\partial E_y}{\partial z}; \quad H_z(x, z, t) = \left( \frac{-i}{k\mu(\omega)} \right) \frac{\partial E_y}{\partial x} \quad (9)$$

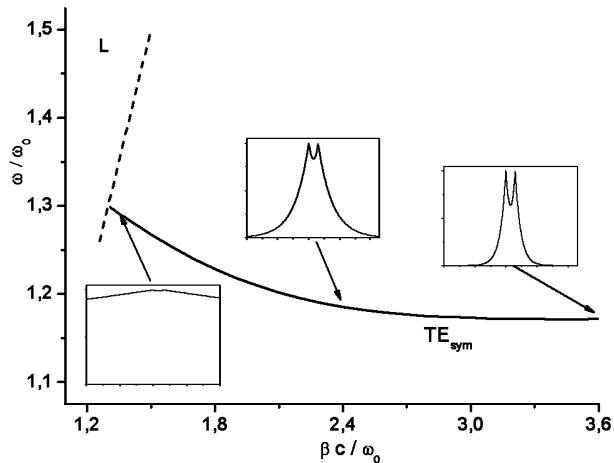


Fig. 3. Evolution of transverse electric field distribution along dispersive curve for symmetric TE mode.

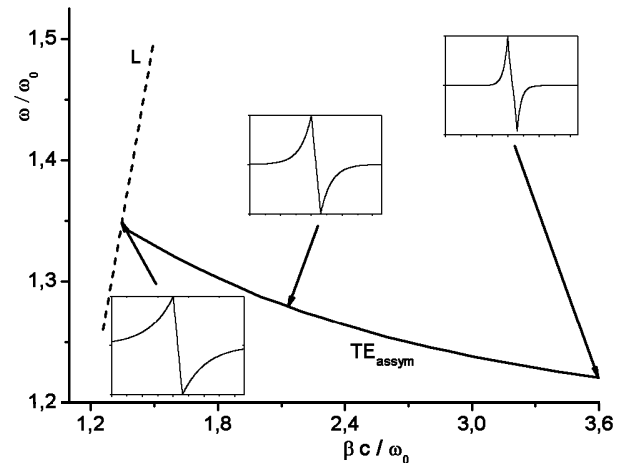


Fig. 4. Evolution of transverse electric field distribution along dispersive curve for antisymmetric TE-mode

Both TE- modes are backward, i.e. their group velocities are directed to the opposite (in relation to the phase velocities) direction (Fig.3,4). At the decreasing of wave frequency the fields are more localized near the surfaces. The antisymmetric mode has the point where transverse electric field is zero. When a point on the dispersion curve approaches the light dispersion line L, the TM-mode and the symmetric TE-mode become in fact the bulk waves but not the surface ones.

## CONCLUSIONS

We have demonstrated the possibility of propagation of slow surface electromagnetic waves in a gap between two left-handed metamaterials. These waves have reach variety of properties. They may be both forward and backward waves and even with zero group velocity.

The obtained results can be useful for the diverse practical applications of metamaterials in science and technology.

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