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Reliability of a Maintainable Manufacturing Network subject to Budget

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Abstract — Applying network analysis, a manufacturing system can be constructed as a manufacturing network by representing each workstation as an arc and each inspection station as a node. In particular, the capacity of each workstation is stochastic (i.e. multistate) due to the possibility of failure, partial failure, and maintenance. In practical cases, such a manufacturing network has to achieve a specified production level to satisfy the customers' orders. Hence, maintenance is necessary to guarantee a manufacturing network can retain a minimal production level. A maintenance model, namely maintainable manufacturing network (MMN), is proposed to evaluate whether the manufacturing system can provide sufficient capacity subject to maintenance budget or not. The maintenance reliability is further proposed to calculate the probability that the MMN provides a sufficient capacity level to meet the minimal production level under maintenance budget

Keywords — Maintainable manufacturing network (MMN), Maintenance reliability, Minimal production level.

1. INTRODUCTION

From the perspectives of both operations research and production management, production performance is dependent on the capacity of a manufacturing system. Meanwhile, the capacity of a manufacturing system is determined by capacities of all workstations in the system. Thus, it is a crucial task to retain the production level of a manufacturing system for comprehending whether it can satisfy customers' demand or not. To retain a minimal production level, maintenance should be involved to guarantee the manufacturing system can provide a sufficient capacity by Lin and Chang (2012). In addition, budget is always an important constraint to be considered in maintenance. Such being the case, it emerges a valuable issue to study the maintenance with budget constraint for keeping the minimal production level of a manufacturing system.

Network analysis in terms of AOA (activity-on-arc) diagram is an applicable approach to represent a manufacturing system by Lin (2007), Yeh (2008). That is, we may construct the manufacturing system as a manufacturing network for further analysis. For a practical manufacturing network, each arc can be regarded as a workstation consisting of identical or similar machines; while each node is an inspection station following each workstation. In particular, the capacity of each workstation in the manufacturing network is stochastic due to the possibility of failure, partial failure, and maintenance of machines. Therefore, the manufacturing network characterized by such workstations also has stochastic capacity and we can treat it as a genre of stochastic-flow network by Jane et al. (1993), Lin (2007), Zuo et al. (2007), Yeh (2008). In addition, defect rate of each workstation affects the performance of a manufacturing network and leads to defective products, in which defective products would be reworked or scrapped by Stevenson (2015). In other words, a practical manufacturing network has to be capable to deal with rework and scrap.

Several studies have been devoted to applying manufacturing network to investigate the performance of a manufacturing system. Lin and Chang (2013) proposed a typical manufacturing network model to consider rework and scrap. In Lin and Chang's study, the input flow for a specified demand is calculated to judge the minimal capacity needed to be provided by each workstation. Hence, the minimal capacity vectors for all workstations can be derived. In terms of such vectors, the system reliability (i.e. defined as probability of demand satisfaction) of a manufacturing network can be evaluated. Several extension works further consider other characteristics in production, such as time constraint by Lin et al. (2016), quality issue by Chang (2015), and labor intensive industry by Chang et al. (2015), for achieving practical needs. However, none of the above studies takes maintenance as a consideration.

This paper addresses a maintainable manufacturing network (MMN) in which the MMN has to guarantee a minimal production level so that it can produce at least d units of product per unit time under a maintenance budget B. The performance indicator, maintenance reliability, is defined as the probability that the MMN provides a sufficient capacity level to meet the demand under maintenance budget. The production manager could determine if the production level of the MMN can satisfy the customers' orders or not. The remainder of this paper is organized as follows. Problem description, assumptions, and notations are introduced in Section 2. The maintenance model with budget constraint is studied in Section 3. A numerical example is demonstrated in Section 4 to illustrate three proposed algorithm and how the maintenance reliability

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may be calculated. Conclusion of this paper is summarized in Section 5.

2. PROBLEM DESCRIPTION, ASSUMPTIONS AND NOTATIONS

An MMN is requested to provide a minimal production level to produce d units of products under the situation that WIP may be defective during processing. Therefore, the input raw materials I should be more than the output products O. To retain the production level, a budget B is allowed to maintain the MMN. This paper determines the minimal capacity vectors that an MMN should provide to satisfy the minimal production level. Subsequently, we evaluate the maintenance reliability that satisfies the production level in terms of minimal capacity vectors. To evaluate the maintenance reliability of the MMN, this paper is studied based on the following assumptions:

1. Each node (inspection station) is perfectly reliable.
2. The capacity x_i of each arc a_i (workstation) is a random variable takes value from $\{0, 1, 2, \ldots, M_i\}$ according to a given probability distribution.
3. The capacities of different arcs (workstations) are statistically independent.
4. Each defective WIP is reworked at most once by the same workstation. That is, if the defective WIP after reworking is still defective, then it is scrapped.

Notations

n	number of arcs (workstations)
a_i	ith arc, $i = 1, 2, \ldots, n$
A	$\{a_1, a_2, \ldots, a_n\}$: set of arcs
N	set of nodes (inspection stations)
G	(**N**, **A**): an MMN
d	minimal production level
I	input amount of raw materials
O	output amount
w_i	workload of a_i, $i = 1, 2, \ldots, n$
W	$(w_1, w_2, \ldots, w_n)$: workload vector
y_i	minimal capacity of a_i
Y	$(y_1, y_2, \ldots, y_n)$: minimal capacity vector
B	maintenance budget
$TC(X)$	total cost to maintain the arcs from the state X
c_i	per unit maintenance cost of a_i, $i = 1, 2, \ldots, n$
C	$\{c_1, c_2, \ldots, c_n\}$: set of maintenance cost
M_i	maximal capacity of a_i, $i = 1, 2, \ldots, n$
x_i	current capacity of a_i, $i = 1, 2, \ldots, n$
X	$(x_1, x_2, \ldots, x_n)$: capacity vector
M	$\{M_1, M_2, \ldots, M_n\}$: set of maximal capacity
δ_i	capacity of a machine in workstation a_i
Δ	$\{\delta_1, \delta_2, \ldots, \delta_n\}$: machine capacity vector
w_i	a vector in which the ith position is set as δ_i and others are 0
R_M	maintenance reliability
Φ	set to store minimal capacity vectors

Nomenclatures

MMN	maintainable manufacturing network
RSDP	recursive sum of disjoint products

3. MAINTENANCE MODEL

Let G = (**N**, **A**) represent an MMN, in which **N** is the set of nodes (inspection stations) and $\mathbf{A} = \{a_1, a_2, \ldots, a_n\}$ is the set of arcs (workstations). The capacity x_i of each workstation a_i for $i = 1, 2, \ldots, n$ is a random variable and thus the MMN also performs stochastic capacity. For a production level d, the amount of raw materials should be determined backward in advanced. Let I be the amount of raw materials needed to produce O units of product such that $O \geq d$. Set the defect rate q_i and success rate $p_i = 1 - q_i$ ($0 \leq p_i \leq 1$) of each a_i, the workload (denoted by w_i) can be derived in terms of I and q_i. The input I and workload pattern ($w_1, w_2, \ldots, w_n$) can be solved by several presented methods no matter rework is considered in an MMN (Lin and Chang, 2013) or not (Chang et al. 2015). Thereafter, the minimal capacity Y can be generated by some well-developed methods (Lin and Chang, 2013; Chang et al. 2015).

3.1 Construction of maintenance model

The MMN must be maintained to keep a minimal production level so that it can produce at least d units of product per unit time. That is, once we obtain the minimal capacity vector Y such that $V(Y) \geq d$, such Y is defined as the minimal production level. The maintenance is therefore taken into account to retain a sufficient capacity level to meet the demand subject to a maintenance budget B (i.e. the proposed model in Lin and Chang (2013) does not consider the maintenance budget, which implies that the budget is infinite). We measure the probability that the MMN can produce at least d under the maintenance budget B. Such a probability is henceforth referred to as the maintenance reliability, which is a performance indicator to identify a maintenance strategy so that the MMN satisfies the basic order requirement (i.e. minimal production level). The maintenance cost is calculated in terms of the amount of capacity that each workstation needs to be maintained to satisfy a minimal production level. This guarantees the production level is greater than the minimal production level and could be as high as the best performance level. Thus, the total cost to maintain the workstations in an MMN from the state X is

$$TC(X) = \sum_{i=1}^{n} c_i \frac{(M_i - x_i)}{\delta_i}, \tag{1}$$

where c_i is the per unit maintenance cost to maintain a machine and δ_i is the capacity of a machine in workstation a_i. Hence, $\frac{(M_i - x_i)}{\delta_i}$ is equivalent to the number of machines for a_i to recover from the current capacity x_i to its highest capacity M_i. The amount, $c_i \frac{(M_i - x_i)}{d_i}$, is the cost for such a maintenance action. For instance, given the current capacity vector $X = (25, 25, 20)$, the maximal capacity set $\mathbf{M} = \{M_1, M_2, M_3\} = \{35, 30, 32\}$, the unit maintenance cost $\mathbf{C} = \{c_1, c_2, c_3\} = \{100, 120, 150\}$, and the single machine capacity in workstations $\mathbf{\Delta} = \{\delta_1, \delta_2, \delta_3\} = \{5, 5, 4\}$. The total maintenance cost to restore an MMN from state X to its highest capacity is

$$\begin{aligned} TC(X) &= c_1 \frac{(M_1 - x_1)}{\delta_1} + c_2 \frac{(M_2 - x_2)}{\delta_2} + c_3 \frac{(M_3 - x_3)}{\delta_3} \\ &= 100 \frac{(35-25)}{5} + 120 \frac{(30-25)}{5} + 150 \frac{(32-20)}{4} = 770. \end{aligned}$$

However, it is not necessary to recover workstations to their highest capacities (i.e. maximal number of machines in each workstation). That is, we can restore to any specific capacity state instead of M_i, necessitating a simple modification to equation (1). The following constraint shows that the total maintenance cost can not exceed the budget B,

$$\sum_{i=1}^{n} c_i \frac{(M_i - x_i)}{\delta_i} \leq B \tag{2}$$

Under the maintenance budget B, the maintenance reliability R_M is the probability that the MMN can produce at least d units/time with maintenance action. Thus, the maintenance reliability is $\Pr\{X \mid V(X) \geq d$ and $TC(X) \leq B\}$. For the minimal capacity vector $Y = (y_1, y_2, \ldots, y_n)$, we has two cases to determine the state Y can be maintained under budget B or not. If $TC(Y) \leq B$, it is indeed that such Y is the minimal capacity vector fulfils both d and B. Otherwise, the workstation in the MMN should be maintained earlier rather than falling to the state Y. A branch-and-bound based adjusting algorithm is proposed to find all the minimal capacity vectors satisfy both d and B.

3.2 Algorithm to generate all minimal capacity vectors

Based on the minimal capacity vector Y, the system state to be restored from X can be determined with the following steps.

Step 0. Input Y and initialize $\Phi = \varnothing$, where Φ is the set to store the minimal capacity vectors fulfilling both d and B.

Step 1. Find the maintenance cost $TC(Y) = \sum_{i=1}^{n} c_i \frac{(M_i - x_i)}{\delta_i}$.

Step 2. If $TC(Y) \leq$ B, then $Y \in \Phi$ and go to Step 4.

Step 3. If $TC(Y) >$ B, do the following steps. //Adjusting procedure

3.1 For the unqualified Y, let $Y_i = Y + \omega_i$. In the equation, ω_i is a vector in which the i th position is set as δ_i and others are 0. If the capacity of a_i in Y_i is larger than M_i, remove the Y_i.

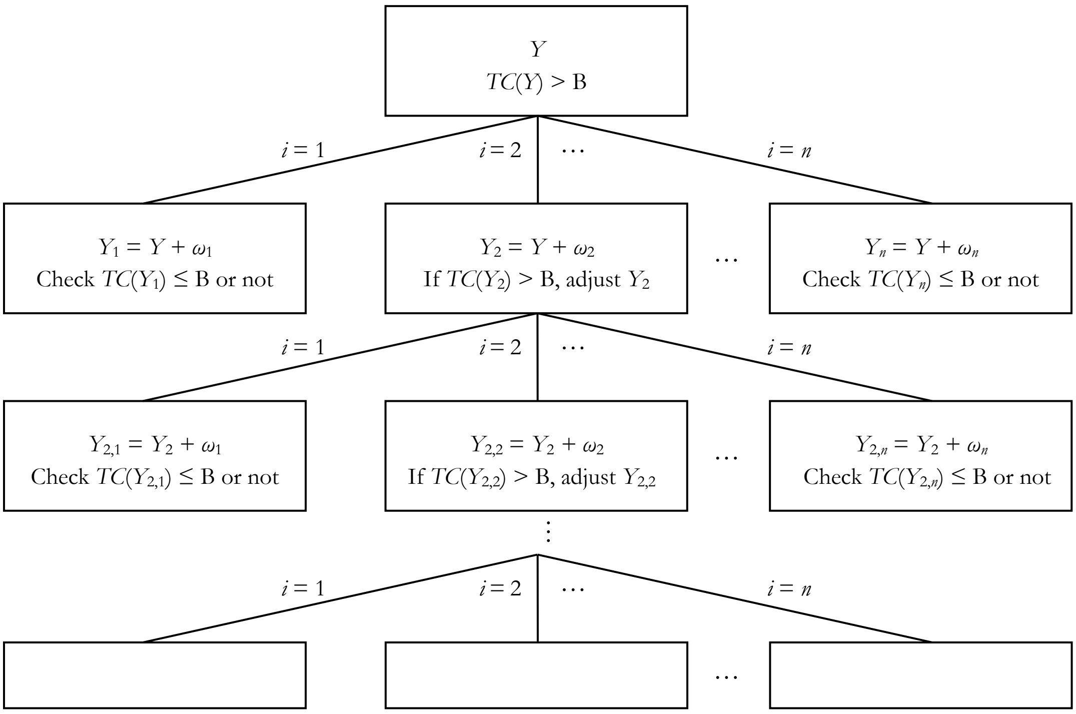
3.2 Compare each Y_i with $X \in \Phi$. If Y_i is larger than or equal to any X in Φ, delete Y_i; otherwise, $Y_i \in \Phi$. If Y_i is less than an X in Φ, delete that X from Φ.

3.3 Treat each Y_i as the role of Y and go to Step 1.

Step 4. All the $Y_i = (y_{i1}, y_{i2}, \ldots, y_{in}) \in \Phi$ are the minimal capacity vector for d and B.

Step 1 checks the total maintenance cost to restore the MMN from state Y is exceeding the budget or not. Step 2 shows that if $TC(Y) \le$ B, it is necessary that Y is the minimal capacity vector satisfies both d and B. For the Y such that $TC(Y) >$ B, Step 3 utilizes an adjusting procedure, which applies a branch-and-bound approach, to obtain the set Φ. Each adjusted $Y_i \in \Phi$ is generated from Y by adding ω_i to reduce the total cost. That is, the MMN is maintained just enough to prevent it from falling below the minimal production level.

Since we adjust only one a_i at a time, several Y_i are generated from the algorithm. For those Y_i still exceeding the budget B, each unqualified Y_i can be branched as $Y_{i,i}$ in further depth (see Figure 1). Suppose $Y_1, Y_2, \ldots, Y_h$ are all the minimal capacity vectors satisfy d and B. The reliability with maintenance can be represented as $R_M = Pr\{\bigcup_{v=1}^{h} D_v\}$, where $D_y = \{X \mid X \ge Y_y\}$ for v = 1, 2, …, h. To calculate $\bigcup_{v=1}^{h} D_v$, there are several methods can be adopted, such as the inclusion-exclusion principle (Hudson and Kapur, 1985; Lin, 2009; 2010), disjoint-event method (Hudson and Kapur, 1985; Yarlagadda and Hershey, 1991), state-space decomposition (Aven, 1985; Jane et al., 1993), and recursive sum of disjoint products (RSDP) algorithm (Zuo et al., 2007). Jane and Laih (2008) proved that the state-space decomposition performs a better efficiency in computation and storage space than inclusion-exclusion principle and disjoint-event method. In addition, Zuo et al. (2007) pointed out that the RSDP is more efficient than the state-space decomposition, especially for larger networks. Hence, the RSDP algorithm is beneficial to be applied for the reliability evaluation.



ω_i: ith position is set as δ_i and others are 0.

Figure 1 Search tree for all minimal capacity vectors.

4. A NUMERICAL EXAMPLE

An example is utilized to demonstrate the proposed algorithm in Section 3.2. A manufacturing network with six workstations (see Figure 2) that has to satisfy the minimal production level with d = 150 per unit time. The defect rate and capacity distribution of workstations are provided in Table 1.

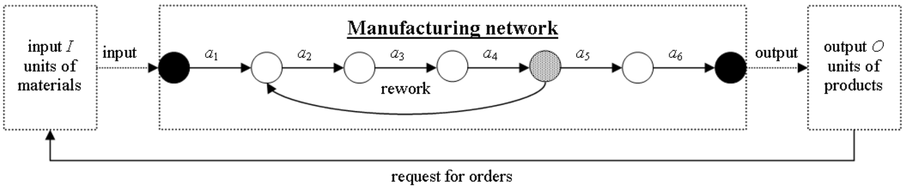


Figure 2 An MMN with six workstations.

Table 1. The workstation data of Figure 2.

Workstation	Defect rate	Capacity	Probability	Workstation	Defect rate	Capacity	Probability
		0	0.001			0	0.001
		50	0.002			40	0.002
		100	0.002			80	0.002
a_1	0.98	150	0.005	a_4	0.97	120	0.005
		200	0.010			160	0.015
		250	0.010			200	0.025
		300	0.975			240	0.950
		0	0.001			0	0.005
		50	0.002			40	0.005
		100	0.002			80	0.010
a_2	0.95	150	0.005	a_5	0.90	120	0.010
		200	0.010			160	0.010
		250	0.980			200	0.960
		0	0.002				
		40	0.003			0	0.005
		80	0.005			40	0.005
		120	0.005			80	0.005
a_3	0.96	160	0.010	a_6	0.93	120	0.010
		200	0.010			160	0.015
		240	0.015			200	0.960
		280	0.950				

According to the approach presented by Lin and Chang (2013), the workload vector $W = (w_1, w_2, w_3, w_4, w_5, w_6) = (201.210, 202.581, 192.452, 184.754, 179.211, 161.290)$. Subsequently, the minimal capacity vector satisfies W is, $Y = (y_1, y_2, y_3, y_4, y_5, y_6) = (250, 250, 200, 200, 200, 200)$. Note that, the maintenance budget is not considered yet. Here, we further consider that the MMN is maintainable and it has to guarantee a minimal production level with $d = 150$ with a budget B = 2000.

Step 0. Input $Y = (250, 250, 200, 200, 200, 200)$ and initialize $\Phi = \varnothing$.

Step 1. For Y, $TC(Y) = 20(300–250) + 20(250–250) + 15(280–200) + 15(240–200) + 15(200–200) + 10(200–200) = 2800$.

Step 2. $TC(Y) > \text{B} = 2000$, go to Step 3.

Step 3. Adjusting procedure for Y.

3.1 For $i = 1, 2, \ldots, 6$, do the following steps.

$Y_1 = Y + \omega_1 = (\underline{250}, 250, 200, 200, 200, 200) + (\underline{50}, 0, 0, 0, 0, 0)$
$= (\underline{300}, 250, 200, 200, 200, 200)$;

$Y_2 = Y + \omega_2 = (250, \underline{250}, 200, 200, 200, 200) + (0, \underline{50}, 0, 0, 0, 0)$
$= (250, \underline{300}, 200, 200, 200, 200)$;

$Y_3 = Y + \omega_3 = (250, 250, \underline{200}, 200, 200, 200) + (0, 0, \underline{40}, 0, 0, 0)$
$= (250, 250, \underline{240}, 200, 200, 200)$;

$Y_4 = Y + \omega_4 = (250, 250, 200, \underline{200}, 200, 200) + (0, 0, 0, \underline{40}, 0, 0)$
$= (250, 250, 200, \underline{240}, 200, 200)$;

$Y_5 = Y + \omega_5 = (250, 250, 200, 200, \underline{200}, 200) + (0, 0, 0, 0, \underline{40}, 0)$
$= (250, 250, 200, 200, \underline{240}, 200)$;

$Y_6 = Y + \omega_6 = (250, 250, 200, 200, 200, \underline{200}) + (0, 0, 0, 0, 0, \underline{40})$
$= (250, 250, 200, 200, 200, \underline{240})$.

The capacity of a_2 in Y_2 is larger than the maximal capacity $M_2 = 250$, so it is not feasible and should be removed. The same follows for Y_5 and Y_6, which are also removed.

3.2 Since $\Phi = \varnothing$, none of the Y_i is deleted in this step.

3.3 Treat each Y_i as the role of Y and go to Step 1, respectively.

Step 1a. For Y_1, $TC(Y_1)$ = 20(300–300) + 20(250–250) + 15(280–200) + 15(240–200) + 15(200–200) + 10(200–200) = 1800 < B = 2000. Then $\Phi = \Phi \cup \{Y_1\} = \{Y_1\}$.
Step 1b. For Y_3, $TC(Y_3)$ = 2200.
Step 2b. $TC(Y_3)$ = 2200 > B = 2000, go to Step 3.
Step 3b. Adjusting procedure for Y_3.

3.1b For i = 1, 2, …, 6, do the following steps.

$Y_{3,1} = Y_3 + \omega_1$ = (<u>300</u>, 250, 240, 200, 200, 200);
$Y_{3,2} = Y_3 + \omega_2$ = (250, <u>300</u>, 240, 200, 200, 200);
$Y_{3,3} = Y_3 + \omega_3$ = (250, 250, <u>280</u>, 200, 200, 200);
$Y_{3,4} = Y_3 + \omega_4$ = (250, 250, 240, <u>240</u>, 200, 200);
$Y_{3,5} = Y_3 + \omega_5$ = (250, 250, 240, 200, <u>240</u>, 200);
$Y_{3,6} = Y_3 + \omega_6$ = (250, 250, 240, 200, 200, <u>240</u>).

The capacity of a_2 in $Y_{3,2}$ is larger than the maximal capacity M_2 = 250, so it is not feasible and should be removed. The same follows for $Y_{3,5}$ and $Y_{3,6}$, which are also removed.

3.2b Since $\Phi = \{Y_1\}$ and $Y_{3,1} > Y_1$, $Y_{3,1}$ is removed.

3.3 Treat each $Y_{3,i}$ as the role of Y and go to Step 1, respectively.

⋮

The adjusting procedure and results are shown in Figure 3.

Step 4. $\Phi = \{Y_1, Y_{3,3}, Y_{3,4}\}$ are the minimal capacity vectors for d and B.

Three minimal capacity vectors, Y_1 = (300, 250, 200, 200, 200, 200), $Y_{3,3}$ = (250, 250, 280, 200, 200, 200), and $Y_{3,4}$ = (250, 250, 240, 240, 200, 200), satisfying d and B are derived. The maintenance reliability R_M = 0.84560 is derived by the RSDP (Zuo et al., 2007) algorithm afterwards.

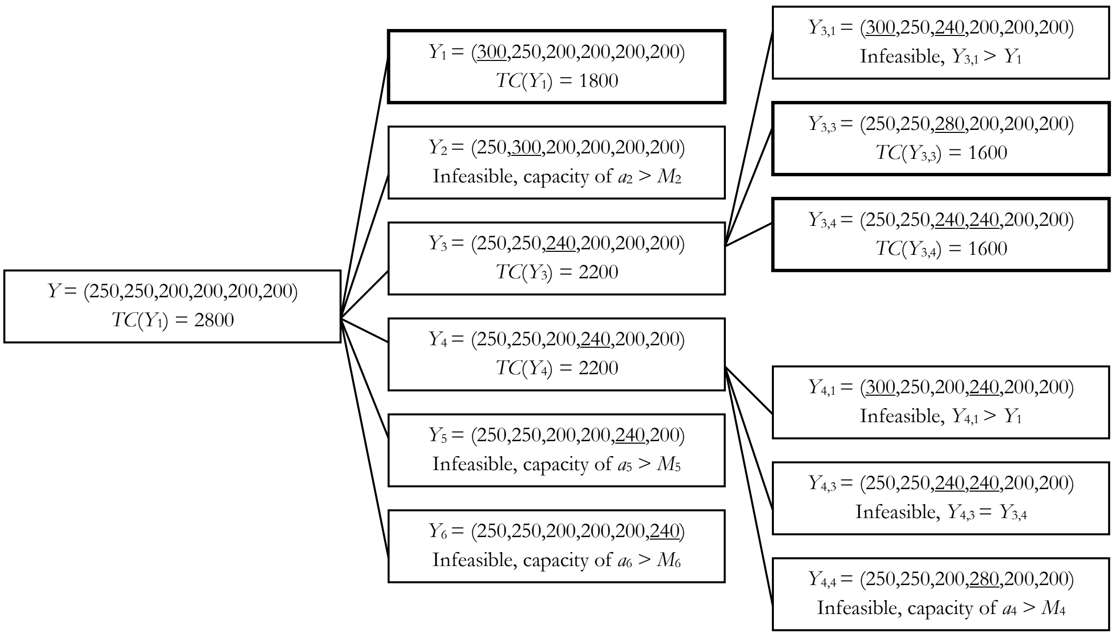


Figure 3 Adjustment process for the example.

5. CONCLUSION

To assess the robustness of a manufacturing system, this paper constructs an MMN model considering reworking action and defect rates for all workstations. In addition, a maintenance budget is considered to retain the MMN can satisfy a minimal production level. The probability that an MMN could fulfil production level d units of products, namely maintenance reliability, is evaluated to indicate the performance of the MMN. An algorithm is proposed to generate all minimal capacity vectors to produce sufficient products and satisfy minimal production level d. The proposed model and algorithm can be extended to more than one reworking actions cases intuitively. Based on the maintenance reliability, the production manager could conduct the sensitivity analysis to investigate the most important workstation in an MMN to improve the system to be more reliable.

REFERENCES

1. Aven, T. (1985). Reliability evaluation of multistate systems with multistate components. *IEEE Transactions on Reliability*, 34, 473-479.
2. Chang, P.C. (2015). On quality improvement of a multistate production network with service level consideration. *International Journal of Systems Science: Operations & Logistics* (article in press).
3. Chang, P.C., Lin, Y.K. and Chen JC. (2015). A fuzzy-based assessment procedure for a clothing factory with waste-prevention consideration. *Journal of Cleaner Production*. 108 484-493.
4. Hudson, J.C. and Kapur, K.C. (1985). Reliability bounds for multistate systems with multistate components. *Operations Research*, 33, 153-160.
5. Jane, C.C. and Laih, Y.W. (2008). A practical algorithm for computing multi-state two-terminal reliability. *IEEE Transactions on Reliability*, 57, 295-302.
6. Jane, C.C., Lin, J.S., and Yuan, J. (1993). On reliability evaluation of a limited-flow network in terms of minimal cutsets. *IEEE Transactions on Reliability*, 42, 354-361.
7. Lin, Y.K. (2007). Two-commodity reliability evaluation of a stochastic-flow network with varying capacity weight in terms of minimal paths. *Computers & Operations Research*, 36, 1050-1063.
8. Lin, Y.K. (2009). A MP-based algorithm for a multicommodity stochastic-flow network with capacity weights. *International Journal of Industrial Engineering: Theory Applications and Practice*, 16, 282-292.
9. Lin, Y.K. (2010). Spare routing reliability for a stochastic flow network through two minimal paths under budget constraint. *IEEE Transactions on Reliability*, 59, 2-10.
10. Lin, Y.K. and Chang, P.C. (2012). Evaluation of system reliabilities for a maintainable stochastic-flow network. *IEEE Transactions on Reliability*, 61, 398-409.
11. Lin, Y.K. and Chang, P.C. (2013). A novel reliability evaluation technique for stochastic flow manufacturing networks with multiple production lines. *IEEE Transactions on Reliability*, 62, 92-104.
12. Lin, Y.K., Huang, D.H. and Yeng, L.C. (2016). Reliability evaluation of a hybrid flow-shop with stochastic capacity within a time constraint. *IEEE Transactions on Reliability*, 65, 867-877.
13. Stevenson, W.J. (2015). *Operations Management: 12th Edition*. New York: McGrawHill.
14. Yarlagadda, R. and Hershey, J. (1991). Fast algorithm for computing the reliability of communication network. *International Journal of Electronics*, 70, 549-564.
15. Yeh, W.C. (2008). A simple minimal path method for estimating the weighted multi-commodity multistate unreliable networks reliability. *Reliability Engineering & System Safety*, 93, 125-136.
16. Zuo, M.J., Tian, Z. and Huang, H.Z. (2007). An efficient method for reliability evaluation of multistate networks given all minimal path vectors. *IIE Transactions*, 39, 811-817.