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Optimal ordering policy with non- increasing demand for time dependent deterioration under fixed life time production and permissible delay in payments

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***Abstract* —** Most of the items in the universe deteriorate over time. Many items such as pharmaceuticals, high tech products and readymade food products also have their expiration dates. This paper developes an economic order quantity model for retailer in which demand rate is linearly time dependent and non increasing function of time, deterioration rate is time dependent having expiration dates under trade credits..We then show that the total average cost is sensitive with respect to the key parameters. Furthermore, we discuss several sub- special cases. Finally, numerical examples and sensitivity analysis is provided to illustrate the results. Mathematica 5.2 software is used to find numerical results.

Keywords — Inventory, expiration dates, deterioration, optimality, trade credit, time-dependent demand

1. INTRODUCTION

At present, it is common that, the vendor often provides to his/ her customer a trade credit period to reduce inventory and stimulate sales. Thus trade credit is beneficial for both vendor and buyer point of view. Goyal [1] is the first author who has established the retailer's optimal economic order quantity under permissible delay in payments. Aggarwal and Jaggi [2] extended model [1] for deteriorating items. Stochastic EOQ model for deteriorating items under permissible delay in payments was developed by Shah [3]. Shinn *et al.* [4] extended model [1] considering quantity discount for freight cost. Chu *et al*. [5] also extended model [1] for deteriorating items. Khanra *et al.* [6] established an EOQ (Economic Order Quantity) model for a deteriorating item with time-dependent demand under trade credits. Teng [7] modified Goyal's [1] model for the fact that unit selling price is necessarily higher than purchase cost. Lou and Wang [8] established an EPQ (Economic Production Quantity model for a manufacturer (or wholesaler) with defective item when its supplier offers an up- stream trade credits M while it turn provides its buyers a down-stream trade credit N. Huang [9] established an EOQ model for a supply chain in which supplier offers the wholesaler the permissible delay period M and the wholesaler in turn provides the trade credit period N to its retailers. Soni and Shah [10] developed an EOQ model with an inventory- dependent demand under increasing payment scheme. Teng and Chang [11] presented optimal manufacturer's replenishment policies under two levels of trade credit financing. Tripathi [12] presented an inventory model for seller with exponential demand under permitted credit period by the vendor. Many related research papers can be found in Chung [13], Devis and Gaither [14], Chung and Liao [15], Huang and Hsu [16], Ouyang *et al.* [17], Skouri *et al.* [18], Yang *et al.* [19] and their citations.

Many products like medicines, green vegetables, volatile liquids, milk, bread and others deteriorate continuously but also have their expiration dates. However, few researchers have considered the expiration date of deteriorating items. Kreng and Tan [20] established the optimal replenishment decision in an economic production quantity model with defective item under permissible delay in payment. Wu *et al.* [21] proposed an economic order quantity model for retailer where (i) the supplier provides an up-stream trade credit and the retailer also offers a down-stream trade credit, (ii) the retailer's down-stream trade credit to the buyer not only increases sales and revenue but also opportunity cost and default risk and (iii) deteriorating items having their expiration dates. Ghare and Schrader [22] established an inventory model by considering an exponentially decaying inventory. Dave and Patel [23] developed an economic order quantity (EOQ) model for deteriorating items with linearly non decreasing demand with no shortages. The model [22] is extended by Sachan [24] to allow for shortages. Hariga [25] established inventory models for deteriorating items with time- dependent demand. Goyal and Giri [26] studied a survey on the recent trends in modelling of deteriorating items. Teng *et al.* [27] developed inventory model to allow for partial backlogging. Skouri *et al.* [28] presented inventory models with ramp type demand rate and Weibull

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deterioration rate. Mahata [29] considered an economic production quantity (EPQ) model for deteriorating items under trade credits. Dye [30] developed an inventory model for the effect of technology investment on deteriorating item. Recently, Wang *et al.* [31] proposed an EOQ models for a seller by incorporating the facts (i) deteriorating products not only deteriorate continuously but also have their maximum life and (ii) permissible delay period increases with demand and default risk.

The remaining part of the paper is framed as follows. Section 2 presents assumptions and notations followed by mathematical formulation for different situations. Optimal solution is determined in section 4. Numerical examples and sensitivity analysis is discussed in section 5. At last conclusion and future research is provided in section 6.

2. ASSUMPTION AND NOTATIONS

The following assumptions are used throughout the manuscript.

1. The demand Rate is time dependent and non increasing function of time
2. The deterioration rate is time dependent and $\theta(t) = \frac{1}{1+ \text{m-t}}$, $0 \le \text{t} \le T \le m$.

In case of time approaches to expiration date m, deterioration rate close to 1.
Deterioration becomes zero for very large expiring date, i.e. $m \to \infty$ and $\theta(t) \to 0$.

3. If cycle time T is longer than retailer's trade credit period M then retailer pays for the interest charges on item in stock with interest charge I_c during time [M, T]. If cycle time T is shorter than retailer's trade credit period then interest charges is zero in whole cycle. But if seller's permissible delay period (M) is greater than N, the retailer can accumulate revenue and earn interest in [N, M] with rate I_e.
4. Renewal rate is instantaneous.
5. Time horizon is infinite.
6. Selling price is necessarily greater than purchase cost.

In addition the following notations are adopted throughout the manuscript:

h	: Unit stock holding cost / year in dollars excluding interest charges.
A	: Ordering cost / order.
c	: Unit purchase cost .
s	: Selling price per unit time , $s > c$
M	: Retailer's permissible delay period offered by the supplier.
N	: Customer's permissible delay period offered by the retailer.
$D \equiv D(t) = a\text{-}bt$	: Demand rate , where $a > 0$ and $0 \le b \le 1$, a is the initial demand.
I_e	: Interest earned /dollar.
I_c	: Interest charged / \$ in stocks per year.
t	: The time in years.
$I(t)$	: Inventory level at time t.
$\theta(t) = \frac{1}{1+ m - t}$	: The deterioration rate at time t, $0 \le \theta(t) \le 1$
m	: The expiration date of item.
T	: Renewal time (in years).
Q	: Order quantity.
$TC(T)$	: Total cost/ year.
T^*	: Optimal replenishment time (in years).
Q^*	: Optimal order quantity.
TC^*	: Optimal total cost/year (in dollars).

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3. MATHEMATICAL FORMULATIONS

The inventory level *I(t)* decreases to meet time dependent demand and time dependent deterioration. The differential equation of states at *I(t)* during the replenishment cycle [*0,T*] is given by

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -(a - bt),\ 0 \le t \le T\ . \tag{1}$$

With the condition $I(T) = 0$. The solution of (1) is

$$I(t) = b(1+m-t)(T-t) + \{(m+1)b-a\}(1+m-t)\log\left(\frac{1+m-T}{1+m-t}\right) \tag{2}$$

The Retailer' order quantity is

$$Q = I(0) = (1+m)bT + \{(m+1)b-a\}(1+m)\log\left(\frac{1+m-T}{1+m}\right) \tag{3}$$

The total annual cost contains the following elements:

1. Ordering cost is = $\dfrac{A}{T}$
2. Purchase cost per cycle is equal to

$$(1+m)bc + \frac{c\{(m+1)b-a\}(1+m)}{T}\log\left(\frac{1+m-T}{1+m}\right) \tag{4}$$

3. Stock holding cost is = $\dfrac{h}{T}\displaystyle\int_0^T I(t)dt$

$$= \frac{h}{T}\left[\frac{bT(1-m)^2}{2} + \frac{b(1+m-T)^3}{6} - \frac{(1+m)^3}{6} + \{(m+1)b-a\}\left\{\frac{(1+m)^2}{2}\log\left(\frac{1+m-T}{1+m}\right) + \frac{T(1+m)}{2} - \frac{T^2}{4}\right\}\right] \tag{5}$$

The two cases may arise to calculate the annual capital opportunity cost i.e. (i) $N < M$ and (ii) $N \ge M$.

Case 1: $N < M$

We discuss two possible sub- cases based on the values of M and $T + N$. If $T + N \ge M$, then the retailer takes the full revenue at the time $T + N$ and pay off total purchase cost at retailer's credit period M. The two sub cases are depicted in the following Figures 1 and 2.

Sub-case 1. (i): $M \le T + N$

In this situation, the last payment of retailer in time $T + N$ is longer than retailer's credit period, therefore the retailer financed all item sold after $M - N$ at an interest charge I_c

Interest charged per year $= \dfrac{cI_c}{T}\displaystyle\int_0^{T+N-M} t(a-bt)dt$

$$= \frac{cI_c}{T}(T+N-M)^2\left\{\frac{a}{2} - \frac{b}{3}(T+N-M)\right\} \tag{6}$$

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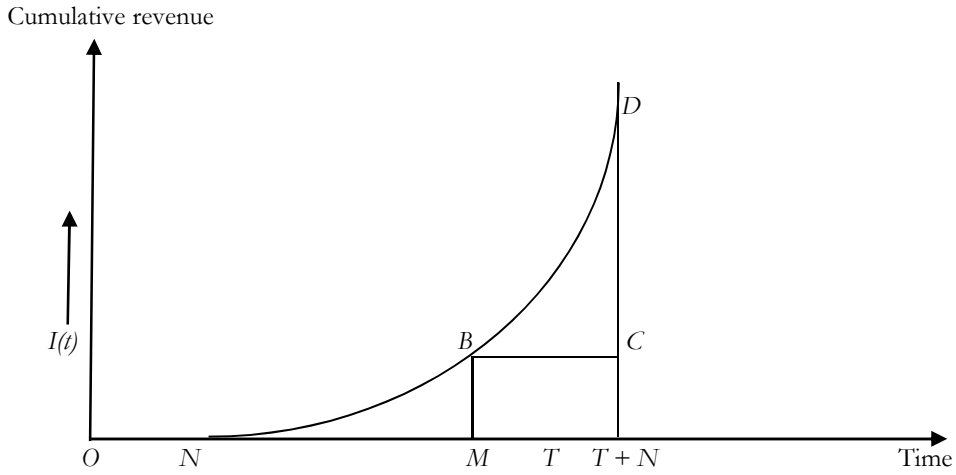

Fig 1: $N < M$ and $M \leq T + N$.

The retailer sells deteriorating goods at the beginning but receives money at time N. During N to M, the retailer accumulate revenue in an account that earns I_e /\$/year. Hence, the interest earned/ year is

$$= \frac{sI_e}{T}\int_0^{M-N} t(a-bt)dt = \frac{sI_e}{T}(M-N)^2\left\{\frac{a}{2}-\frac{b}{3}(M-N)\right\} \tag{7}$$

Therefore, the annual capital opportunity cost is

$$= \frac{cI_c}{T}(T+N-M)^2\left\{\frac{a}{2}-\frac{b}{3}(T+N-M)\right\} - \frac{sI_e}{T}(M-N)^2\left\{\frac{a}{2}-\frac{b}{3}(M-N)\right\} \tag{8}$$

Total annual cost for the retailer can be expressed as follows:

$$\begin{aligned} TC_1(T) = {} & \frac{A}{T} + (1+m)cb + \frac{\mathrm{c\{(m+1)b\text{-}a\}(1+m)}}{\mathrm{T}}\log\left(\frac{\mathrm{1+m\text{-}T}}{\mathrm{1+m}}\right) + \frac{h}{T}\left[\frac{bT(1-m)^2}{2} + \frac{b(1+m-T)^3}{6}\right. \\ & \left. - \frac{(1+m)^3}{6} + \{(m+1)b-a\}\left\{\frac{(1+m)^2}{2}\log\left(\frac{1+m-T}{1+m}\right) + \frac{T(1+m)}{2} - \frac{T^2}{4}\right\}\right] \\ & + \frac{cI_c}{T}(T+N-M)^2\left\{\frac{a}{2}-\frac{b}{3}(T+N-M)\right\} - \frac{sI_e}{T}(M-N)^2\left\{\frac{a}{2}-\frac{b}{3}(M-N)\right\} \end{aligned} \tag{9}$$

$$\begin{aligned} & = \frac{c}{T}\left[(1+m)bT + \frac{\{(m+1)b-a\}(m^2-T^2+2m+1)}{2(1+m)}\right] + \frac{A}{T} + \frac{h}{T}\left[\frac{bT(1-m)^2}{2} + \frac{b(1+m-T)^3}{6} - \right. \\ & \left. \frac{b(1+m)^3}{6} + \frac{\{(m+1)b-a\}(m^2-T^2+2m+1)}{4} + \frac{\{(m+1)b-a\}T(1+m)}{2} - \frac{\{(m+1)b-a\}T^2}{4}\right] \\ & + \frac{cI_c(T+N-M)^2}{T}\left\{\frac{a}{2}-\frac{b}{3}(T+N-M)\right\} - \frac{sI_e}{T}(M-N)^2\left\{\frac{a}{2}-\frac{b}{3}(M-N)\right\} \end{aligned} \tag{10}$$

Sub-case1.2: $M \geq T + N$

In this situation retailer credit period is longer than time at which the retailer receives the payout from the customer, retailer receives the whole revenue and there is no interest charge,

and Interest earned / year $= \frac{sI_e}{T}\left\{\int_0^T t(a-bt)dt + \int_{T+N}^M t(a-bt)dt\right\}$

$$= \frac{sI_e}{T}\left\{\frac{a}{2}\left(M^2-N^2\right) - \frac{b}{3}\left(M^3-N^3\right) + TN\left(a+bT+bN\right)\right\} \tag{11}$$

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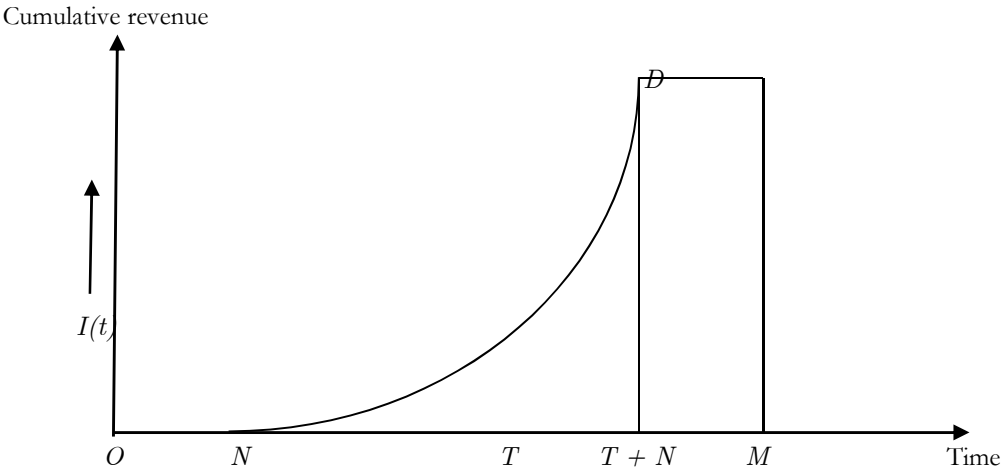

Fig 2: $N < M$ and $M > T + N$.

Therefore, the total annual cost of retailer

$$TC_{1.2}(T) = \frac{A}{T} + (1+m)cb + \frac{c\{(m+1)b-a\}(1+m)}{T}\log\left(\frac{1+m-T}{1+m}\right) + \frac{h}{T}\left[\frac{bT(1-m)^2}{2}\right.$$
$$+ \frac{b(1+m-T)^3}{6} - \frac{(1+m)^3}{6} + \{(m+1)b-a\}\left\{\frac{(1+m)^2}{2}\log\left(\frac{1+m-T}{1+m}\right) + \frac{T(1+m)}{2} - \frac{T^2}{4}\right\}\Bigg] \tag{12}$$
$$\left. - \frac{sI_e}{T}\left[\frac{a}{2}\left(M^2 - N^2\right) + \frac{b}{3}(N^3 - M^3) + TN\{a + b(T+N)\}\right]\right.$$
$$= \frac{c}{T}\left[(1+m)bT + \frac{\{(m+1)b-a\}(m^2 - T^2 + 2m + 1)}{2(1+m)}\right] + \frac{A}{T} + \frac{h}{T}\left[\frac{bT(1-m)^2}{2} + \frac{b(1+m-T)^3}{6}\right.$$
$$\left. - \frac{b(1+m)^3}{6} + \frac{\{(m+1)b-a\}(m^2 - T^2 + 2m + 1)}{4} + \frac{\{(m+1)b-a\}T(1+m)}{2} - \frac{\{(m+1)b-a\}T^2}{4}\right] \tag{13}$$
$$\left. - \frac{sI_e}{T}\left[\frac{a}{2}\left(M^2 - N^2\right) + \frac{b}{3}(N^3 - M^3) + TN\left\{a + b(T+N)\right\}\right]\right.$$

Case 2: $N \geq M$

In this case, customer's credit period is longer than retailer's credit period, the retailer doesn't earned interest, but charged interest during N to $T + N$.

Therefore, the interest charged per year

$$= cI_c\left\{(N-M)(a-bT) + \frac{1}{T}T\{a - \frac{b}{2}(T-2N)\}\right\} = cI_c\left\{(N-M)(a-bT) + \frac{1}{T}\int_N^{T+N}(a-bt)dt\right\} \tag{14}$$

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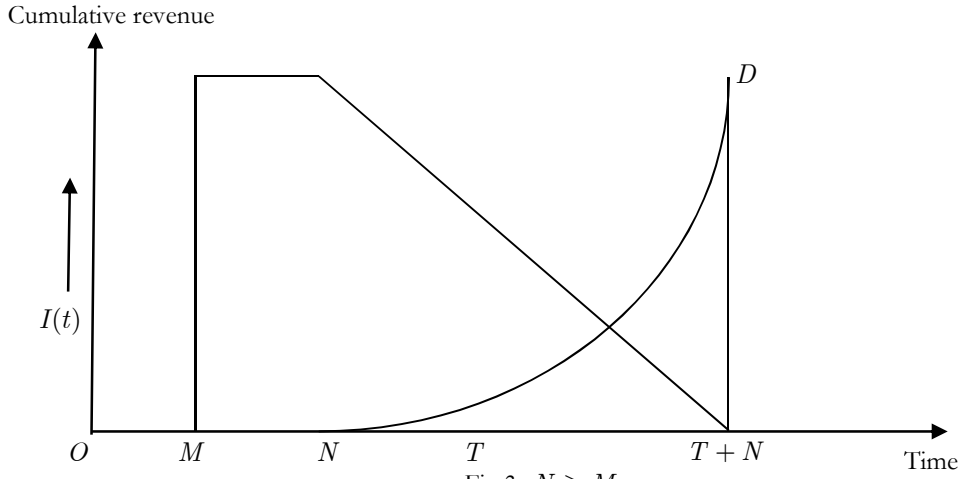

Fig 3: $N \geq M$

$$TC_2(T) = \frac{A}{T} + (1+m)cb + \frac{c\{(m+1)b-a\}(1+m)}{T}\log\left(\frac{1+m-T}{1+m}\right) + \frac{h}{T}\left[\frac{bT(1-m)^2}{2} + \frac{b(1+m-T)^3}{6} - \frac{(1+m)^3}{6} + \{(m+1)b-a\}\left\{\frac{(1+m)^2}{2}\log\left(\frac{1+m-T}{1+m}\right) + \frac{T(1+m)}{2} - \frac{T^2}{4}\right\}\right] + cI_c\left\{(N-M)(a-bT) + T\left(a+\frac{b}{2}T-bN\right)\right\} \tag{15}$$

or

$$TC_2(T) = \frac{c}{T}\left[(1+m)bT + \frac{\{(m+1)b-a\}(m^2-T^2+2m+1)}{2(1+m)}\right] + \frac{A}{T} + \frac{h}{T}\left[\frac{bT(1-m)^2}{2} + \frac{b(1+m-T)^3}{6} - \frac{b(1+m)^3}{6} + \frac{\{(m+1)b-a\}(m^2-T^2+2m+1)}{4} + \frac{\{(m+1)b-a\}T(1+m)}{2} - \frac{\{(m+1)b-a\}T^2}{4}\right] + cI_c\left\{(N-M)\ (a-bT) + T\left(a+\frac{b}{2}T-bN\right)\right\} \quad \text{(Approximately)} \tag{16}$$

4. DETERMINATION OF THE OPTIMAL SOLUTION

For case 1: $N < M$ **Sub-case1.1:** $M \leq T+N$, **Sub-case1.2:** $M \geq T+N$ and **Case 2:** $N \geq M$
Differentiating Equations (10), (13) and (16) with respect to T, two times, we get

$$\frac{dTC_1(T)}{dT} = \frac{-c\{(m+1)b-a\}(m^2+T^2+2m+1)}{2(1+m)T^2} - \frac{A}{T^2} - \frac{hbT(1+m-T)^2(1+m+2T)}{6T^2} + \frac{b(1+m)^3}{6T^2} - \frac{h\{(m+1)b-a\}(m^2+T^2+2m+1)}{4T^2} - \frac{h\{(m+1)b-a\}}{4} - \frac{cI_c a(T+N-M)(N-M-T)}{2T^2} + \frac{cI_c b(T+N-M)^2(N-M-2T)}{3T^2} + \frac{1}{T^2}sI_e(M-N)^2\left\{\frac{a}{2} - \frac{b}{3}(M-N)\right\} \tag{17}$$

$$\frac{dTC_{1.2}(T)}{dT} = \frac{-c\{(m+1)b-a\}(m^2+T^2+2m+1)}{2(1+m)T^2} - \frac{A}{T^2} - \frac{hbT(1+m-T)^2(1+m+2T)}{6T^2} + \frac{b(1+m)^3}{6T^2} - \frac{h\{(m+1)b-a\}(m^2+T^2+2m+1)}{4T^2} - \frac{h\{(m+1)b-a\}}{4} - \frac{sI_e a(M^2-N^2)}{2T^2} - \frac{sI_e b(N^3-M^3)}{3T^2} - SI_e bN \tag{18}$$

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$$\frac{\mathrm{dTC}_2(T)}{dT} = \frac{-c\{(m+1)b-a\}(m^2+T^2+2m+1)}{2(1+m)T^2} - \frac{A}{T^2} - \frac{hbT(1+m-T)^2(1+m+2T)}{6T^2} + \frac{b(1+m)^3}{6T^2}$$
$$-\frac{h\{(m+1)b-a\}(m^2+T^2+2m+1)}{4T^2} - \frac{h\{(m+1)b-a\}}{4} - cI_c b(N-M) + cI_c(a+bT+bN) \tag{19}$$

$$\frac{d^2TC_1(T)}{dT^2} = \frac{1}{T^3}\left[\left\{(m+1)b-a\right\}(m+1)\left(c+\frac{h}{2}\right) + 2A + \frac{hbT\left(1+m-T\right)^2}{3} - \frac{b\left(1+m\right)^3}{3} + \right.$$
$$\left. cI_c\left\{a(N-M-T) - \frac{2b(T+N-M)(N-M-2T)}{3}\right\}(T+N-M) - 2sT_e\left(M-N\right)^2\left\{\frac{a}{2}-\frac{b}{3}\left(M-N\right)\right\}\right] \tag{20}$$
$$+\frac{hb(1+m-T)}{6}\left\{8-\frac{(1+m)}{T}-\frac{(1+m)^2}{T^2}\right\} + \frac{cI_c}{T}\left\{a-2b(T+N-M)\right\}.$$

$$\frac{d^2TC_{12}(T)}{dT^2} = \frac{1}{T^3}\left[\left\{(m+1)b-a\right\}(m+1)\left(c+\frac{h}{2}\right) + 2A + \frac{hbT\left(1+m-T\right)^2}{3} - \frac{b\left(1+m\right)^3}{3} + \right.$$
$$\left. +sI_e(M-N)\left\{a(M-N) + \frac{2b\left(N^2+MN+M^2\right)}{3}\right\}\right] + \frac{hb(1+m-T)}{6}\left\{8-\frac{(1+m)}{T}-\frac{(1+m)^2}{T^2}\right\} \tag{21}$$

and

$$\frac{d^2TC_2(T)}{dT^2} = \frac{1}{T^3}\left[\left\{(m+1)b-a\right\}(m+1)\left(c+\frac{h}{2}\right) + 2A + \frac{hbT\left(1+m-T\right)^2}{3} - \frac{b\left(1+m\right)^3}{3} + bcI_c\right. . \tag{22}$$

The necessary condition for $\mathrm{TC}_1(T)$, $\mathrm{TC}_{1.2}(T)$ and $\mathrm{TC}_2(T)$ to be minimum is $\frac{dTC_1(T)}{dT}=0$, $\frac{dTC_{1.2}(T)}{dT}=0$, $\frac{dTC_2(T)}{dT}=0$, provided $\frac{d^2TC_1(T)}{dT^2}>0$, $\frac{\mathrm{d}^2\mathrm{TC}_{1.2}(T)}{dT^2}>0$ and $\frac{d^2TC_2(T)}{dT^2}>0$, which is clear from (20) , (21) and (22) that all second derivatives are positive. Putting, $\frac{dTC_1(T)}{dT}=0$, $\frac{dTC_{1.2}(T)}{dT}=0$, and $\frac{dTC_2(T)}{dT}=0$, we get

$$\frac{c\{(m+1)b-a\}(m^2+T^2+2m+1)}{2(1+m)} + A + \frac{hbT(1+m-T)^2(1+m+2T)}{6} - \frac{b(1+m)^3}{6} +$$
$$\frac{h\{(m+1)b-a\}(m^2+T^2+2m+1)}{4} + \frac{hT^2\{(m+1)b-a\}}{4} + \frac{cI_c a(T+N-M)(N-M-T)}{2} \tag{23}$$
$$-\frac{cI_c b(T+N-M)^2(N-M-2T)}{3} - sI_e(M-N)^2\left\{\frac{a}{2}-\frac{b}{3}(M-N)\right\} = 0,$$

$$\frac{c\{(m+1)b-a\}(m^2+T^2+2m+1)}{2(1+m)} + A + \frac{hbT(1+m-T)^2(1+m+2T)}{6} - \frac{b(1+m)^3}{6}$$
$$+\frac{h\{(m+1)b-a\}(m^2+T^2+2m+1)}{4} + \frac{hT^2\{(m+1)b-a\}}{4} + \frac{sI_e a(M^2-N^2)}{2} \tag{24}$$
$$+\frac{sI_e b(N^3-M^3)}{3} + sI_e bNT^2 = 0,$$

and

$$\frac{c\{(m+1)b-a\}(m^2+T^2+2m+1)}{2(1+m)} + A + \frac{hbT(1+m-T)^2(1+m+2T)}{6} - \frac{b(1+m)^3}{6} +$$
$$\frac{h\{(m+1)b-a\}(m^2+T^2+2m+1)}{4} + \frac{hT^2\{(m+1)b-a\}}{4} + \tag{25}$$

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$$cI_c bT^2(N-M) - cI_c T^2(a+bT+bN) = 0\ .$$

5. NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS

Example: 1

Let us consider the parameter values $a = 5$ units/year, $b = 0.5$, $s = \$15$ per unit, $c = \$10$ per unit, $A = \$50$/order, $h = \$2$/unit/year, $M = 100/365$ year, $N = 50/365$year, $m = 1$ year, $I_c = \$0.17\ /\ \$\ /$ year, $I_e = \$0.10\ /\ \$$ year. Substituting these values in the equation (20), we get, the optimum solutions for $T = T^* = 0.322129$ year and corresponding optimum total annual cost $TC = TC^* = \$11.3159$.

Case 1: $N < M$

Sub-case1.1: $M \leq T + N$

Table 1.The sensitivity analysis will be helpful in decision making to analyze the effect of change of these variations. Using the same above data (Example 1) the sensitivity analysis of different parameters has been done. We study the effect of the variations in a single parameter keeping other system parameters same on the optimal solutions.

Changing parameters	change	T^*	TC^*
a	4.9	0.494728	17.2577
	4.8	0.632397	21.4523
	4.7	0.757551	24.7180
	4.6	0.877638	27.3512
s	17	0.324222	11.2856
	19	0.323462	11.2579
	21	0.322701	11.2292
	23	0.321938	11.2004
	9.5	0.469112	15.6140
c	9	0.587279	18.5632
	8	0.794300	22.4641
	7	0.988557	24.7753
A	53	0.515288	18.4356
	56	0.653646	23.5398
	59	0.768402	27.7268
	62	0.868848	31.3571
	2.1	0.288785	9.59314
h	2.2	0.248577	7.67585
	2.3	0.201936	5.46005
	2.4	0.142658	2.68382
	125/365	0.325699	10.7822
M	150/365	0.327035	9.75954
	175/365	0.329057	9.27845
	200/365	0.331827	9.27795
	0.9	0.464599	17.1607
m	0.8	0.565615	21.8798
	0.7	0.645577	26.1225
	0.6	0.710943	30.1416

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Example: 2

Let $a = 5$ units/year, $b = 0.5$, $s = \$15$ per unit, $c = \$10$ per unit, $A = \$50$ per order, $h = \$2$/unit/year, $M = 375/365$ years, $N = 50/365$ years, $m = 1$ year, $I_c = \$0.17 / \$$ year, $I_e = \$0.10 / \$$ year. Substituting these values in (21), we get, the optimum solutions for T* = 0.619185 year and corresponding optimum total annual cost $TC^* = \$\ 5.55998$.

Sub-case1.2: $M \geq T + N$

Table 2. The sensitivity analysis will be helpful in decision making to analyze the effect of change of theses variations. Using the same above data (Example 2) the sensitivity analysis of different parameters has been done. We study the effect of the variations in a single parameter keeping other system parameters same on the optimal solutions.

Changing parameters	*change*	*T**	*TC**
s	17	0.645715	5.16105
	19	0.671267	4.79179
	21	0.695946	4.44699
	23	0.719844	4.12275
	51	0.672408	8.03404
A	52	0.721797	10.2053
	53	0.768087	12.1523
	54	0.811806	13.9258
	2.1	0.591773	4.00894
h	2.2	0.563970	2.35804
	2.3	0.535674	0.590233
	2.4	0.506757	---------
	400/365	0.645695	5.30010
M	425/365	0.672592	5.05806
	450/365	0.699809	4.83274
	475/365	0.727290	4.62303
	0.95	0.670320	8.36814
m	0.90	0.715880	10.8927
	0.85	0.756805	13.2231
	0.80	0.793748	15.4163

Note: Dotted data shows the non feasible value.

Example: 3

Let us consider the parameter values $a = 5$units per year, $b = 0.5$, $s = \$15$ per unit, $c = \$10$ per unit, $A = \$50$ per order, $h = \$2$/unit/year, $M = 50/365$ years, $N = 100/365$ years, $m = 1$ year, $I_c = \$0.17 / \$$ year, $I_e = \$0.10 / \$$ year. Substituting these values in (22) the optimum solution for $T^* = 0.290311$ year and the corresponding optimum total annual cost $TC^* = \$14.8652$.

Case 2: $N > M$

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Table 3. The sensitivity analysis will be very helpful in decision making to analyze the effect of change of theses variations. Using the same above data (Example 3) the sensitivity analysis of different parameters has been done. We study the effect of the variations in a single parameter keeping other system parameters same on the optimal solutions.

Changing parameter	*change*	*T**	*TC**
a	4.9	0.370967	18.4641
	4.8	0.439593	21.3604
	4.7	0.501321	23.8154
	4.6	0.558600	25.9550
c	9.5	0.418399	19.5789
	9.0	0.522858	22.7987
	8.5	0.616741	25.1919
	8.0	0.705222	27.0191
	52	0.410014	20.5204
A	54	0.501651	24.8651
	56	0.578761	28.5313
	58	0.646588	31.7641
	2.1	0.258429	12.9934
	2.2	0.222757	10.9103
h	2.3	0.181043	8.50185
	2.4	0.127443	5.47454
	55/365	0.290240	14.7521
M	60/365	0.290169	14.6391
	65/365	0.290098	14.5261
	70/365	0.290027	14.4131
	0.9	0.415136	21.3080
m	0.8	0.505855	26.4788
	0.7	0.578344	31.0928
	0.6	0.638416	35.4257

All the above observations can be summed up as follows:
From Table 1, following inferences can be made:
(i). Increase of initial demand '*a*' , unit purchase cost '*c*', ordering cost '*A*' and expiration date 'm' will result increase in total annual cost *TC*. That is, change in *'a', 'c' 'A'* and '*m*' will lead positive change in *TC*.
(ii). Increase of unit selling price '*s*' unit stock holding cost '*h*' and retailer's permissible delay period '*M*' will lead decrease in total annual cost *TC* . That is change in *'s' , 'h'* and '*M'* will lead negative change in *TC*.
From Table 2, the following inferences can be made:
(i). Increase of ordering '*A*' and expiring time 'm' will lead increase in total annual cost TC. That is change, in '*A*' and '*m*' will cause positive change in *TC*.
(ii). Increase of unit selling price '*s*', unit stock holding cost '*h'* and retailer's credit period '*M'* will lead decrease in total annual cost *TC* . That is change in '*s', 'h'* and '*M*' will cause negative change in *TC*.
From Table 3, the following inferences can be made:
(i). Increase of initial demand '*a*', unit purchase cost '*c*', ordering '*A*' and expiring time '*m*'
will lead increase in total annual cost *TC*. That is change, in *'a', 'c', 'A'* and '*m*' will cause positive change in *TC*.
(ii). Increase of unit stock holding cost '*h'* and retailer's credit period '*M*' will lead decrease in total annual cost *TC* . That is change in '*h*' and '*M*' will cause negative change in *TC*.

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6. CONCLUSION AND FUTURE RESEARCH

This study was motivated by the observation of daily practice in the field of material and pharmaceutical management. In this paper, we have developed general approach to determine optimal ordering policy for decreasing demand with time dependent deterioration under fixed life time production and trade credits. By adopting the time dependent deterioration, the deterioration becomes zero for large expiring date and becomes one for time approaches to expiration date. In this paper, we have built up an optimal order quantity model to obtain optimal total annual cost considering (i) customer's trade credit period offered by the retailer is less than retailer's permissible delay period offered by the supplier (i.e. $N < M$) and (ii) customer's permissible delay period offered by the retailer is greater than or equal to retailer's trade credit period offered by the supplier (i.e. $N \geq M$). Mathematical models have derived to find optimal solution. Moreover, we have shown that the variation is quite sensitive with respect to different to key parameters
The possible extension of the present model could be allowable shortages and inflation. The model can also be generalized for adding the freight charges and others.

REFERENCES

1. Goyal,S.K. (1985). Economic order quantity under permissible delay in payments. *Journal of Operational Research Society*, 36, 335-338.
2. Aggarwal,S.P. and Jaggi,C.K.(1993). Ordering policies of deteriorating items under permissible delay in payments. *Journal of Operational Research Society*, 46, 658-662.
3. Shah,N.H. (1993). Probabilistic time- scheduling model for an exponentially decaying inventory when delay in payment is permissible. *International Journal of Production Economics,* 32(1), 77-82.
4. Shinn,S.W. Hwang,H.P. and Sung,S.(1996). Joint price and lot-size determination under conditions of permissible delay in payments and quantity discount for freight cost. *European Journal of Operational Research*, 91, 528-542.
5. Chu, P. Chung, K.J. and Lan, S.P. (1998). Economic order quantity of deteriorating items under permissible delay in payments. *Computers and Operations Research*, 25, 817-824
6. Khanra, S., Ghosh, S.K. and Chaudhuri, K.S. (2011). An EOQ model for deteriorating items with time-dependent quadratic demand under permissible delay in payments. *Applied Mathematics and Computation,* 218, 1-9.
7. Teng, J.T. (2002). On the economic order quantity under conditions of permissible delay in payments. *Journal of Operational Research Society*, 53, 915-918.
8. Lou, K.R. and Wang, L. (2013). Optimal lot-sizing policy for a manufacturer with defective item in a supply chain with up- stream and down-stream trade credits. *Computers and Industrial Engineering,* 66, 1125-1130.
9. Huang, Y.F. (2003). Optimal retailer's ordering policies in the EOQ model under trade credit financing. *Journal of Operational Research Society*, 54, 1011-1015.
10. Soni, N. and Shah, N.H. (2008). Optimal ordering policy for stock-dependent demand under progressive payment scheme. *European Journal of Operational Research*, 184, 91-100.
11. Teng, J.T. and Chang, C.T. (2009). Optimal manufacturers' replenishment policies in the EPQ model under two levels of trade credit policy. *European Journal of Operational Research*, 195(2), 358-363.
12. Tripathi, R.P. (2014). Optimal payment time for a retailer with exponential demand under permitted credit period by the wholesaler. *Applied Mathematics and Information Science Letter,* 2(3),91-101.
13. Chung, K.J., Chang, S.L. and Yang, W.D. (2001). The optimal cycle time for exponentially deteriorating products under trade credit financing. *The Engineering Economist*, 46, 232-242.
14. Davis, R.A. and Gaither, N. (1985). Optimal ordering policies under conditions of extended payment privilages. *Management Science*,31, 499-509.
15. Chung, K.J. Liao,J.J. (2004). Lot –sizing decisions under trade credits depending on the ordering quantity. *Computers and Operations Research*,31, 909-928.
16. Huang,Y.F. and Hsu,K.H. (2008). An EOQ model under retailer partial trade credit policy in supply chain. *International Journal of Production Economics*,112,655-664.
17. Ouyang,l.Y., Chang,C.T. and Shum,P. (2012). The EPQ with defective item and partially permissible delay in payments linked to order quantity derived algebraically. Central *European Journal of operations Research*, 20(1),141-160.
18. Skouri, K. Konstantaras, I. Papachristos, S., and Teng, J.T.(2011). Supply chain models for deteriorating products with ramp type demand rate under permissible delay in payments. *Expert systems with Applications*, 38, 14861-14869.
19. Yang,C.T. Pan,Q.H. ouyang,L.Y. and Teng,J.T. (2012). Retailer's optimal order and credit policy for deteriorating item

1813-713X Copyright © 2016 ORSTW

with stock-dependent demand and relaxed terminal condition under limited storage space. *Central European Journal of Operations Research,* 19(1), 139-153.

20. Kreng,V.B. and Tan, S.T.(2011).Optimal replenishment decision in an EPq model with defective item under supply chain trade credit policy. *Expert System with Applications*, 38, 9888-9899.
21. Wu,J. Ouyang,L.Y. Barron,L.E.C. and Goyal,S.K. (2014). Optimal credit period nd lot size for deteriorating items with expiration dates under two- level trade credit financing. *European Journal of operational Research*, 237,898-908.
22. Ghare,P.M.and Schrader,G.P. (1963). A model for an exponentially decaying inventory. *Journal of Industrial Engineering*,14, 238-243.
23. Dave, U. And Patel, L.K. (1981).(T,S_i) policy inventory model for deteriorating items with time proportional demand. *Journal of Operational Research Society*, 32(2), 137-142.
24. Sachan,R.S. (1984). On (T,S_i) policy inventory model for deteriorating item with time proportional demand. *Journal of Operational Research Society*, 35(1), 1013-1019.
25. Hariga,M.A. (1996). Optimal EOQ models for deteriorating item with time varying demand. *European Journal of operations Research*,134(1),1-16.
26. Goyal,S.K. and Giri,B.C.(2001). Recent trends in modelling of deteriorating inventory. *European Journal of Operational Research*, 134(1), 1-16.
27. Teng,J.T. (2002). On the economic order quantity under conditions of permissible delay in payment. *Journal of the Operational Research Society*, 53,915-918.
28. Skouri,K. Konstantaras,I. Papachristos,S. And Ganas,I. (2009). Inventory models with ramp type demand rate, partial backlogging and Weibull- deterioration rate. *European Journal of Operational Research*,192(1), 79-92.
29. Mahata,G.C. (2012). An EPQ based inventory model for exponentially deteriorating item under retailer partial trade credit policy in supply chain. *Expert Systems with Applications*, 39(3), 3537-3550.
30. Dye, C.Y. (2013). The effect of preservation technology investment on a non- instantaneous deteriorating inventory model. *Omega*, 41(5), 872-880.
31. Wang, W.C., Teng, J.T. and Lon, K,R. (2014). Seller's optimal credit period and cycle time in a supply chain for deteriorating items with maximum lifetime. *European Journal of Operational Research*, 232,315-321.

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