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Research Article

Inherent Irreversibility of Exothermic Chemical Reactive Third-Grade Poiseuille Flow of a Variable Viscosity with Convective Cooling

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Abstract. In this study, the analysis of inherent irreversibility of chemical reactive third-grade poiseuille flow of a variable viscosity with convective cooling is investigated. The dissipative heat in a reactive exothermic chemical moves over liquid in an irreversible way and the entropy is produced unceasingly in the system within the fixed walls. The heat convective exchange with the surrounding temperature at the plate surface follows Newton's law of cooling. The solutions of the dimensionless nonlinear equations are obtained using weighted residual method (WRM). The solutions are used to obtain the Bejan number and the entropy generation rate for the system. The influence of some pertinent parameters on the entropy generation and the Bejan number are illustrated graphically and discussed with respect to the parameters.

Keywords: Exothermic reaction; Third-grade fluid; Poiseuille flow; Variable viscosity; Convective cooling

1. Introduction

Theoretical analysis on non-Newtonian fluids flow has presently recieved impressive consideration due to its significant applications in the process industry. Because of their intricacy, non-Newtonian liquids can't fit into a solitary constitutive model and various models have been established for various classes of non-Newtonian fluids. The flow activity of these fluids can't be sufficiently clarified on the premise of the traditional viscous linear model. Among the several suggested models is the class of third-grade fluids. Broad studies on the fluids have been taken into consideration [1-3]. According to one of the previous studies [4], the overall details of the stability and uniqueness of thermodynamics constitutive models of the distinction type with third-grade fluid as an exceptional instance were examined. For problems relating to the heat transport for third-grade fluids, a whole thermodynamics analysis of the constitutive function was carried out [5].

Chemical non-Newtonian reactive flow of viscoelastic polymer such as lactic acid production can be very sophisticated [6-8]. An inclusive depiction of several complex polymers in diverse systems has been recently provided [9] and the significance of modifying mathematical models to define transport phenomena more correctly was stressed. A number of scholars have recently investigated the numerically and analytically solution of non-Newtonian chemical reaction in the systems processing. Among all, one of the sudies adopted numerical simulation to examine the Oldroyd-B viscoelastic flow with thermal convection and Arrhenius kinetics[10]. Furthermore, other research conducted and investigated the flow and stability of the combustible liquid by adopting a potent and swiftly convergent analytical approximation [11 & 12], and also investigated Newtonian cooling with the combustible flow gel propellant in a hybrid rocket chamber using a spectral quasilinearization numerical method [13]. They exploited the third-grade Rivlin-Ericksen model for viscoelasticity.

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However, it is known that fluid physical properties can change significantly with temperature. When the effects of variable viscosity are taken into consideration, the flow characteristics are significantly changed compared to constant physical properties. As a result, the influence of radiation and variable viscosity on the energy and mass transfer in MHD fluid flow past a permeable plate was examined [14]. Moreover, studied significance of variable viscosity and thermal conductivity of micropolar fluid in the presence of magnetic field was investigated [15]. Another study on the effect of thermal conductivity and variable viscosity on the hydromagnetic flow through a vertical plate was carried out [16]. It was found that the velocity distribution rises with the reduction in the thermal conductivity parameter. On the other hand, the radiative heat transfer of variable viscosity and thermal conductivity effects on the inclined magnetic field with dissipation in a non-Darcy medium was analyzed [17].

Currently, the entropy generation modeling is a lively area of thermal engineering sciences because heat moves through a liquid in an irreversible way that brings about changes in fluid particles entropy. This is due to the difference in the fluid particle temperature that causes disorderliness in the fluid particles motion. Obviously, this will diminish the system efficiency due to the fact that several thermal processes occur at a very high temperature; the main goal of the present study is to examine the thermal performance of the flow in a channel by ignoring time-dependent effects [18]. This is essential in measuring the productivity of the system since the amplified entropy reduces the energy level of the system. The target approach follows the second law of thermodynamics based on an original pioneering work [19], that has transformed the thermal optimization of engineering processes for non-Newtonian fluids [20].

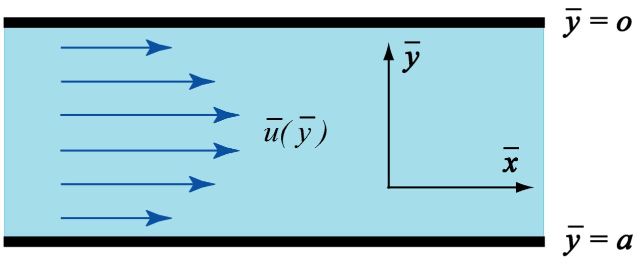
The third-grade fluid with Vogel viscosity to examine the entropy generation in a channel was conducted in a study [21]. It was found that diverse viscosity parameters can either amplify or reduce the entropy production in the channel. In a careful investigation, the computational solutions for the effect of entropy on the forced convection flow in a channel was carried out [22]. Recently, research on the entropy generation for several flow patterns is predictable through the results of some previous studies [23 - 26].

The main interest of this study is to examine the inherent irreversibility reactive fluid flow and variable viscosity with third-grade liquid in poiseuille flow conditions. The flow formulation model is provided in section 2. In section 3, a weighted residual method is developed and implemented for the solution process. In section 4, the entropy generation analysis is presented. In section 5, the graphical results are shown and discussed accordingly for various entrenched parameters in the flow.

2. Mathematical Formulation of the Flow

Consider laminar, the isotropic chemical reactive variable viscosity of an incompressible third-grade fluid flow within fixed parallel horizontal plates as illustrated in Figure 1. The non-Newtonian model is employed to cause the viscoelastic effects. The flow is simulated by Arrhenius chemical kinetics and assumed to be driven by the combined effect of the buoyancy force and the axial pressure gradient. The flow is considered to be in the direction of x with y - axis normal to the flow. The wall surfaces are subjected to exchange of heat convection at the ambient temperature. Ignoring time-dependent effects and the fluid reactive viscose consumption, the momentum and heat balance governing equations are as follows [14]:

$$\bar u = 0,\ k\frac{\partial \bar T}{\partial \bar y} = h(\bar T - T_0)$$



$$\bar u = 0,\ -k\frac{\partial \bar T}{\partial \bar y} = h(\bar T - T_0)$$

Fig. 1. Geometry of the flow

$$-\frac{d\bar P}{d\bar x}+\frac{d}{d\bar y}\left[\bar\mu(T)\frac{d\bar u}{d\bar y}\right]+6\gamma\frac{d^2\bar u}{d\bar y^2}\left(\frac{d\bar u}{d\bar y}\right)^2+\rho g\beta(T-T_0)=0 \tag{1}$$

$$k\frac{d^2T}{d\bar y^2}+\left(\frac{d\bar u}{d\bar y}\right)^2\left[\bar\mu(T)+2\gamma\left(\frac{d\bar u}{d\bar y}\right)^2\right]+QCA\left(\frac{KT}{\upsilon l}\right)^m e^{-\frac{E}{RT}}+Q_0(T-T_0)=0 \tag{2}$$

the impose boundary conditions are:

$$\bar{y} = a; \quad \bar{u} = 0, \ -k\frac{dT}{d\bar{y}} = h(T - T_0)$$
$$\bar{y} = 0; \quad \bar{u} = 0, \ k\frac{dT}{d\bar{y}} = h(T - T_0) \tag{3}$$

The temperature dependent viscosity ($\bar{\mu}$) is defined as $\bar{\mu}(T) = \mu_0 e^{-\varepsilon(T - T_0)}$. Introducing the dimensionless quantities (4) into Eqs. (1-3) along with the temperature dependent viscosity result in:

$$y = \frac{\bar{y}}{a}, u = \frac{\rho a\bar{u}}{\mu_0}, \theta = \frac{E(T - T_0)}{RT_0}, x = \frac{\bar{x}}{a}, \mu = \frac{\bar{\mu}}{\mu_0}, P = \frac{\bar{P}\rho a^2}{\mu_0^2}, G = -\frac{dP}{dx},$$
$$\sigma = \frac{\varepsilon R T_0^2}{E}, \delta = \frac{\gamma\mu_0}{\rho^2 a^4}, G_r = \frac{\rho^2 a^3 g\beta R T_0^2}{\mu_0^2 E}, r = \frac{RT_0}{E}, \omega = \frac{\mu_0^3 e^{\frac{E}{RT}}}{\rho^2 QAa^4 C}\left(\frac{vl}{RT_0}\right)^m \tag{4}$$
$$\lambda = \frac{QEAa^2Ce^{-\frac{E}{RT}}}{RKT_0^2}\left(\frac{KT_0}{vl}\right)^m, Q = \frac{Q_0 T_0^2 RKe^{\frac{E}{RT}}}{E^2 QAC}\left(\frac{vl}{KT_0}\right)^m, Bi = \frac{ah}{K}$$

Therefore, the governing equation reduces to:

$$G + e^{-\sigma\theta}\frac{d^2u}{dy^2} - \sigma e^{-\sigma e}\frac{d\theta}{dy}\frac{du}{dy} + 6\delta\frac{d^2u}{dy^2}\left(\frac{du}{dy}\right)^2 + Gr\theta = 0 \tag{5}$$

$$\frac{d^2\theta}{dy^2} + \lambda\left\{(1 + r\theta)^m e^{\frac{\theta}{1+r\theta}} + \omega\left(\frac{du}{dy}\right)^2\left[e^{-\sigma\theta} + 2\delta\left(\frac{du}{dy}\right)^2\right] + Q\theta\right\} = 0 \tag{6}$$

The corresponding boundary conditions are as follows:

$$u = 0, \frac{d\theta}{dy} = -Bi\theta, at, y = 1$$
$$u = 0, \frac{d\theta}{dy} = Bi\theta, at, y = 0 \tag{7}$$

3. Method of solution

The idea of weighted residual method [24, 25] is to look for an approximate result in the polynomial form of the differential equation as follows:

$$D\left[v\left(y\right)\right] = f \quad \text{in the domain } R, \quad A_\mu\left[v\right] = \gamma_\mu \text{ on } \partial R \tag{8}$$

where $D\left[v\right]$ represents a differential operator relating non-linear or linear spatial derivatives of the dependent variables v, f is the function of a known position, $A_\mu\left[v\right]$ denotes the approximate number of boundary conditions with R as the domain and ∂R as the boundary. By assuming an approximation to the solution $v\left(y\right)$, the following form is expressed:

$$v(y) \approx w(y, a_1, a_2, a_3 ... a_n) \tag{9}$$

which depends on a number of parameters $a_1, a_2, a_3 .. a_n$ in such a way that for arbitrary value $a_i's$ the boundary conditions are satisfied and the residual in the differential equation becomes:

$$E(y, a_i) = L(w(y, a_i)) - f(y) \tag{10}$$

The aim is to minimize the residual $E(y, a)$ to zero in an average sense over the domain. That is:

$$\int_Y E(y, a)W_i dy = 0, i = 1, 2, 3, ...n \tag{11}$$

where the number of weight functions W_i is exactly the same as the number of unknown constants a_i in w. Here, the weighted functions are chosen to be Dirac delta functions where $W_i(y) = \delta(y - y_i)$ and the error is zero at the chosen nodes y_i. The integration of Eq. 11 with $W_i(y) = \delta(y - y_i)$ results in $E(y, a_i) = 0$. By applying WRM to Eqs. (5-7), assuming a polynomial with unknown coefficients or parameters to be determined later, this polynomial is called the trial function which is defined as follows:

$$u(y)=\sum_{i=0}^{n}a_i y^i,\theta(y)=\sum_{i=0}^{n}b_i y^i \tag{12}$$

Imposing the boundary conditions (7) on the trial functions (12) as well as substituting the trial functions into Eqs. (5 & 6) to obtain the residual results in:

$$\begin{aligned}u_r &= G+e^{-\sigma\left(y^{10}b_{10}+y^9b_9+y^8b_8+y^7b_7y^6b_6+y^5b_5+y^4b_4y^3b_3+y^2b_2+yb_1+b_0\right)}(90y^8a_{10}+\\ &72y^7a_9+56y^6a_8+42y^5a_7+30y^4a_6+20y^3a_5+12y^2a_4+6ya_3+2a_2)-\\ &\sigma e^{-\sigma\left(y^{10}b_{10}+y^9b_9+y^8b_8+y^7b_7y^6b_6+y^5b_5+y^4b_4y^3b_3+y^2b_2+yb_1+b_0\right)}(10y^9a_{10}+9y^8a_9+...\end{aligned} \tag{13}$$

$$\begin{aligned}\theta_r &= 90y^8b_{10}+72y^7b_9+56y^6b_8+42y^5b_7+30y^4b_6+20y^3b_5+12y^2b_4+6yb_3\\ &2b_2+\lambda((r(y^{10}b_{10}+y^9b_9+y^8b_8+y^7b_7y^6b_6+y^5b_5+y^4b_4y^3b_3+y^2b_2\\ &yb_1+b_0)+1)^m e^{\frac{y^{10}b_{10}+y^9b_9+y^8b_8+y^7b_7y^6b_6+y^5b_5+y^4b_4y^3b_3+y^2b_2+yb_1+b_0}{r\left(y^{10}b_{10}+y^9b_9+y^8b_8+y^7b_7y^6b_6+y^5b_5+y^4b_4y^3b_3+y^2b_2+yb_1+b_0\right)+1}}+...\end{aligned} \tag{14}$$

The residual errors are minimized to zero at some collocation points at a regular interval within the domain when $G_r=2, m=0.5, Bi=1, \sigma=0.1, \omega=1, \lambda=0.5, Q^*=1, \delta=1$, and $G=0.5$. That is, $y_k=(b-a)k/N$, where $k=1,2,...,N-1$, and $a=0, b=1, N=10$, which are solved using MAPLE 2016 software to obtain the unknown coefficients. Therefore, the dimensionless velocity and heat equations are:

$$\begin{aligned}u &= -24.22541072y^{10}+121.1270536y^9-258.385123y^8+306.7781705y^7\\ &-222.6386435y^6+102.9259587y^5-30.74682807y^4+5.604150107y^3\\ &-0.8883527363y^2+0.4490251210y\end{aligned} \tag{15}$$

$$\begin{aligned}T &= 1.598167156\times10^{-37}y^{10}-8.019179894\times10^{-37}y^9+1.737351190\times10^{-36}y^8-\\ &2.127976190\times10^{-36}y^7+1.620173611\times10^{-36}y^6-7.937918162\times10^{-37}y^5+\\ &2.503503638\times10^{-37}y^4-4.937301587\times10^{-38}y^3-0.250y^2+0.250y+0.250\end{aligned} \tag{16}$$

The process of weighted residual method is repeated for different values of the parameters $G_r, m, Bi, \sigma, \omega, \lambda, Q, \delta$, and G.

4. Entropy Generation Analysis

The fluid physical properties can change significantly with temperature when the effects of variable viscosity are taken into consideration. The overall entropy generation equation for the non-Newtonian flow per unit volume is define as [19]

$$E_G=\frac{k}{T_0^2}(\nabla\bar{T})^2+\frac{\mu}{T_0}\phi \tag{17}$$

The heat transfer irreversibility is the first term of Eq. (17), while the second term is the viscous dissipation entropy generation. By Eq. (17), the dimensionless entropy generation number is obtained as:

$$N_S=\frac{E^2a^2E_G}{R^2T_0^2k}=\left(\frac{d\theta}{d\theta}\right)^2+\frac{\lambda\omega}{r}\left(\frac{du}{dy}\right)^2\left[e^{-\sigma\theta}+2\delta\left(\frac{du}{dy}\right)^2\right] \tag{18}$$

From Eq. (17), the first term is assigned as N_1 and the second term is assigned as N_2, i.e,

$$N_1=\left(\frac{d\theta}{dy}\right)^2,\ N_2=\frac{\lambda\omega}{r}\left(\frac{du}{dy}\right)^2\left[e^{-\sigma\theta}+2\delta\left(\frac{du}{dy}\right)^2\right] \tag{19}$$

The Bejan number (Be) is defined mathematically as follows:

$$Be=\frac{N_1}{N_S}=\frac{N_1}{N_1+N_2}=\frac{1}{1+\phi},\phi=\frac{N_2}{N_1} \tag{20}$$

The N_S and Be are respectively represented in the Figs. 7-12.

5. Results and Discussion

The associated graphical results of Eqs. (5-7) are presented for varying parameters values. Fig. 2 represents the response of the fluid flow to the variation in the variable viscosity parameter (σ). It can be seen that a rise in the parameter σ diminishes the fluid viscosity and thereby congruently weakens the fluid's opposition to the flow. This basically leads to amplification in the fluid velocity as illustrated in the following figure. The reaction of the fluid velocity to changes in the values of the non-Newtonian parameter δ is depicted in Fig. 3.

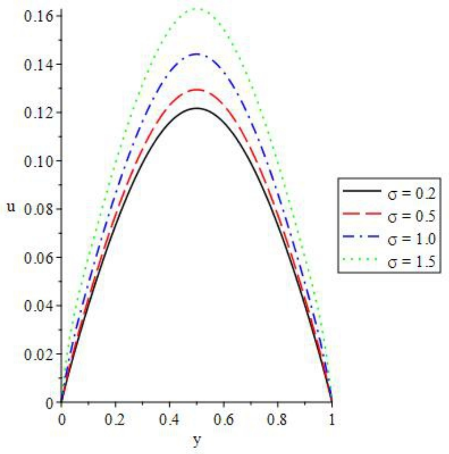


Fig. 2. Influences of σ on velocity

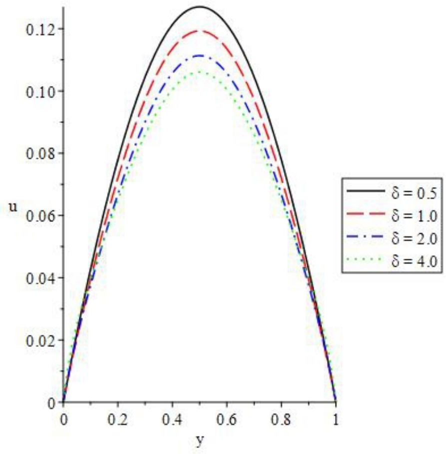


Fig. 3. Influences of δ on velocity

The parameter δ that is linked to the shear rate terms diminishes the influence of the source terms and consequently decreases the shear rates. The reason is that a rise in the values of δ increases the viscoelasticity of the fluid that in turn decreases the flow velocity as illustrated in the the following graph. Figs. 4 and 5 illustrate the influences of Biot number Bi on the flow and temperature fields.

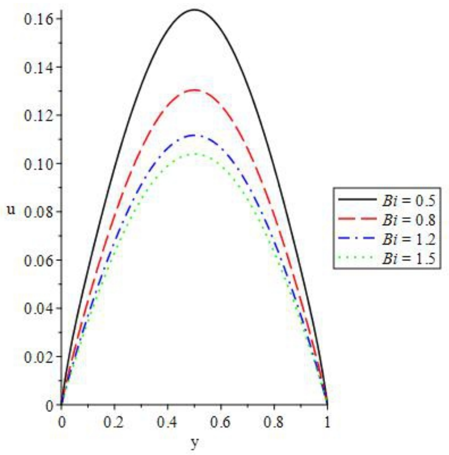


Fig. 4. Influences of Bi on velocity

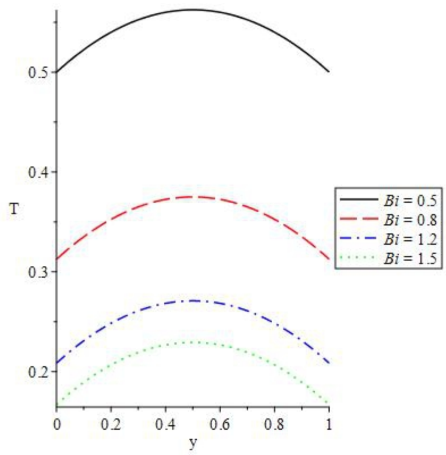


Fig. 5. Influences of Bi on temperature

As it can be seen, in the temperature boundary condition (7), the more the Biot numbers, the higher the convective cooling at the plate surfaces, and thereby correspondingly, the lower the temperatures at the surfaces and the bulk fluid. The whole temperature profile reduces with an increase in the Biot number as the fluid persistently regulates to the lower surface temperatures. The decreased temperatures congruently diminish the fluid viscosity that in turn reduces the fluid momentum over the viscosity coupling. As seen earlier, such a coupling depends on the parameters σ that results in the pronounce changes in the velocity and heat distribution. Fig. 6 shows the effect of the reaction parameter λ on the temperature field.

A rise in the parameter values λ causes a significant rise in the viscous heating source and the reaction rate, and consequently magnifies the temperature distribution. This is due to straightening in the momentum viscosity coupling that results in reasonable rises in the flow temperature. The response of entropy generation rate to the Biot number is displayed in Fig. 7. As it was observed, an increase in the convective cooling Bi decreases the entropy generation rate.

The reason is that the irreversible heat flows from the hot surfaces to the ambient in a way that supports the Newtonian law of cooling that leads to a reduction in the entropy of the fluid region near the cool surfaces. Smaller thermal conductivity is related to higher Biot number as well as substantial cooling. Fig. 8 represents the reaction of entropy generation rate to various values of the Frank-Kamenetskii parameter λ. The figure shows that a rise in the reaction parameter boosts the entropy generation rate in the channel.

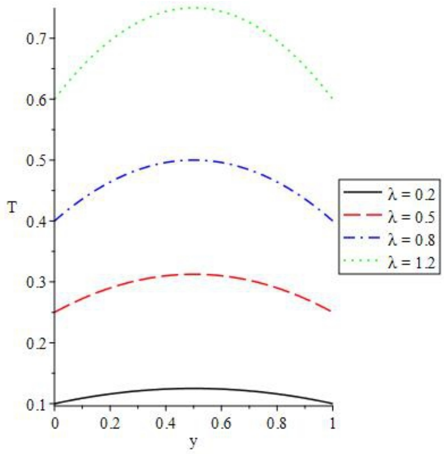


Fig. 6. Influences of λ on temperature

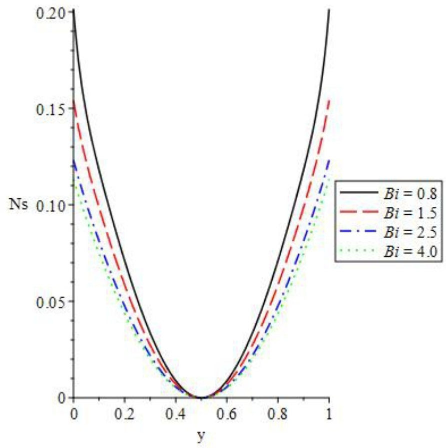


Fig. 7. Influences of Bi on entropy generation

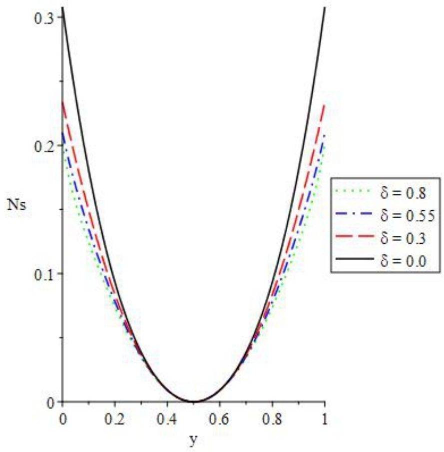


Fig. 8. Influences of λ on entropy generation

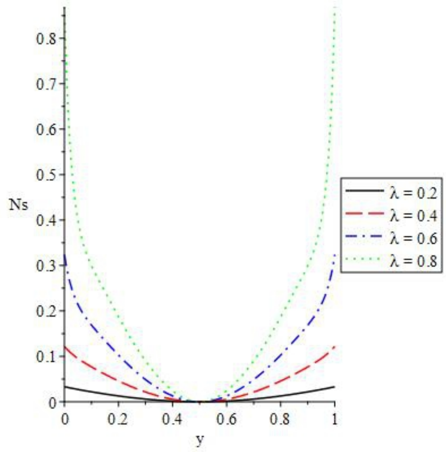


Fig. 9. Influences of δ on entropy generation

As is clear, when the internal heat generation rises, the reacting reagents is enhances as well. The exothermic chemical relation increases the heat transfer rate from the combustion region to the cool surface. Furthermore, heat is conveyed over the fluid to melt the fluid viscosity in other to enhance collision of particles. Accordingly, extra heat is generated by the interaction of viscous fluid particle that in turn increases the entropy generation. Figur. 9 depicts the influence of non-Newtonian parameter δ on the entropy generation rate. As can be seen, the entropy production reduces as the viscoelastic material parameter enhances. The reason is an increase in the fluid particle bonding force that makes the fluid to be more viscoelastic. Therefore, the entropy generation in the system reduces. The descending trend is due to the imbalance between the convective cooling and the nonlinear heat at the surfaces as the viscoelastic parameter increases. Fig. 10 shows the changes in Bejan number with difference values of Biot number. A rise in the convective cooling Bi amplifies the irreversibility of heat transfer in the system.

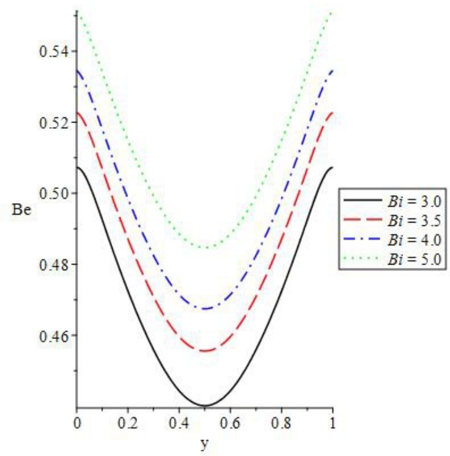


Fig. 10. Influences of Bi on the Bejan number

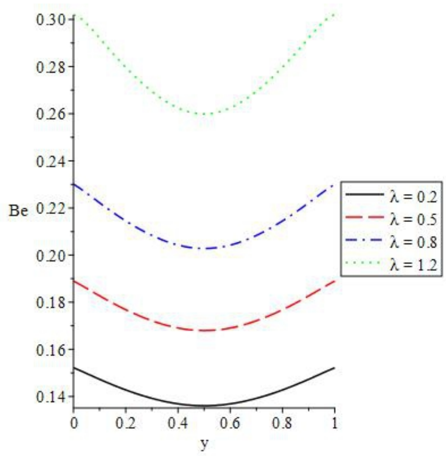


Fig. 11. Influences of λ on the Bejan number

The results indicate that heat transfer to the wall increases as Biot number increases. Therefore, irreversibility as a result of fluid friction relaxes over heat transfer as the Biot number rises. Accordingly, the Bejan number is enhances. Fig. 11 demonstrates the action of varification in the values of Frank-Kamenetskii parameter λ on the Bejan number. It can be observed that a rise in the parameter λ results in a high rise in the irreversibility of heat transfer due to an exothermic chemical reaction that takes place in the fluid flow through fixed walls. Therefore, heat transfer rules over the irreversibility of the fluid friction as the chemical kinetic influence is enhances, thereby increases the Bejan number profiles. Fig. 12 illustrates the effect of non-Newtonian parameter δ on the Bejan number. It is shown that the Bejan number increases as the non-Newtonian parameter rises. The reason is that the bonding force of the fluid particles rises as the viscoelastic parameter enhances. Therefore, heat transport irreversibility increases and hence causes an increase in the Bejan number.

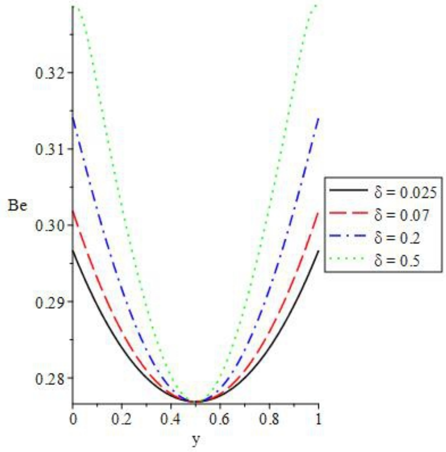


Fig. 12. Influences of δ on the Bejan number

6. Conclusion

In the present study, the irreversibility of heat in an exothermic chemical reaction of non-Newtonian fluid through fixed walls with convective cooling was examined. This analysis was carried out using the second law of thermodynamics. The dimensionless momentum and heat equations were solved using WRM. The solutions were used to obtain the entropy generation and the Bejan number. The investigation reveals that:

(i) An increase in the Frank-Kamenetskii parameter needs to be monitored because it contributes meaningfully to the ruin of the thermo-fluid in the system while the viscoelastic material effect as well as the Biot number diminish the entropy profile.

(ii) A rise in the Frank-Kamenetskii was observed to amplify the prevailing irreversibility of heat transfer over the fluid friction. Moreover, enhancing the Biot number parameter decreases the fluid friction and increases heat transfer irreversibility in the fixed walls.

Nomenclature

a	Channel characteristic length (m)	Ns	Entropy generation number
A	Reaction rate constant	$\bar{P}$	Pressure $(kgms^{-2})$
Bi	Biot number	Q	Heat of reaction (W)
C	Initial species concentration	Q_0	Heat generation $(Wm^{-3}K^{-1})$
E	Activation energy	R	Universal gas constant
g	Acceleration due to gravity (ms^{-1})	r	Activation energy parameter
G	Pressure gradient parameter	T	Temperature (K)
Gr	Thermal Grashof number	T_0	Wall temperature (K)
h	Heat transfer coefficient (W)	$\bar{u}$	Axial velocity (ms^{-1})
K	Boltzmanns constant	$\bar{y}$	Coordinate system (m)
k	Thermal conductivity $(Wm^{-1}K^{-1})$	y	Dimensionless coordinate system (m)
l	Plancks number		

Greek letters

β	Expansivity coefficient (K^{-1})	ε	Variation viscosity
θ	Dimensionless temperature	μ_0	Viscosity dynamic $(kgm^{-1}s^{-1})$

ρ	Density of the fluid (kgm^{-3})	λ	Frank Kamenetskii parameter
γ	Material coefficients	δ	Non-Newtonian parameter
ν	Vibration frequency	ω	Viscous heating
σ	Variable viscosity ($kgm^{-1}s^{-1}$)		

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