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STUDY OF FREE UNDAMPED AND DAMPED VIBRATIONS OF A CRACKED CANTILEVER BEAM

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Abstract

This paper deals with the analytical study on the free un-damped and damped vibration of Euler–Bernoulli beam containing edge cracks. In this study, the effects of the different crack parameters such as crack depth, crack location and crack angle on the dynamic responses of the beam are discussed. Research studies presented the effect of transverse cracks on the natural frequency. Earlier studies have not considered the effect of oblique cracks on the cantilever beam. Presence of cracks in various parts of machine changes its vibration parameters to a considerable degree i.e. natural frequency and damping factor. In this paper, the transverse cracks and oblique cracks are considered on a cantilever beam at different locations and depths to study its effects on the various vibration parameters. The information of the dynamic response i.e. changes in the natural frequency, is much needed in the health monitoring of the beam to determine the location and depth of the crack in the beam. The response of a cracked cantilever beam for a damping factor has been studied by the combination of finite element analysis and theoretical method. Tests were conducted on the cantilever beam for both an intact and cracked cases by using FFT Analyzer. ANSYS software is used to validate the experimental results. The results of this study suggest that the natural frequency of the beam decreases significantly, when crack depth increases to 80% of the depth of the beam and it is least affected, when depth of the crack is either 20% or less than 20% of the depth the beam. Further it is also observed that the value of damping factor is least affected when crack remains present in the beam more towards the free end, when compared to damping factor of an intact beam.

Keywords: Cracked beams, Natural frequency, Crack Angle, ANSYS, Damping factor.

Nomenclatures

A	Cross sectional area of the beam, m^2
a	Depth of the crack, m
a/H	Ratio between the crack depth and depth of the beam (crack depth ratio)
B	Breadth of the beam, (Fig. 1), m
E	Young's modulus , N/m^2
f	Frequency, Hz
f_n	Natural Frequency, Hz
H	Height or depth of the beam, m
I	Moment of inertia of the beam, m^4
L	Length of the beam (Fig. 2(a)), m
L_1	Location of the crack from the cantilevered end, m
L_1/L	Ratio between the location of the crack from fixed end and the length of the beam (crack location ratio)
r	Ratio between the natural frequency of cracked and un-cracked beam (natural frequency ratio)

Greek Symbols

ε	Damping factor
θ	Angle of crack, Fig. 2(b), deg.
λ	Frequency parameters
ρ	Density of the beam, kg/m^3

1. Introduction

Cracks are produced at the highly stressed region in the structures or machine parts due to the application of fatigue load. Defect like crack is generally produced in the material due its less fatigue strength. In many engineering applications i.e. turbine blades, cantilever bridges, automobile propeller shaft, the members like cantilever beam is widely used. Presence of cracks decreases the service life of the structure or any machine part. Cracks are likely to nucleate and cultivate in the tensile stress region of the beam. The main end result of the presence of crack is that it alters the vibration characteristics of the beam i.e. natural frequency, damping factor and stiffness of the beam. Thus the detection of any crack like small size crack or large size crack is very important to ensure safety of the structure.

Studies based on structural healthiness monitoring for crack detection deal with the changes in natural frequencies and mode shapes of the beam. Studies based on structural health monitoring have been performed for a long times and the almost all the concepts in a crack detection regard have been well established from mathematical theory [1] to impact echo method [2]. Cracks in a vibrating component can start terrible failures. So, there is a need of understanding the dynamics of the cracked structures [3]. When a structure carries defect like crack, its vibrant properties can change. Particularly, the presence of crack decreases the stiffness and natural frequencies of the beam and on the other hand increases the damping effects [4]. Chondros and Dimarogonas [5] conducted the number of experiments on aluminium cracked cantilever beam. They proved that the experiments agree with the mathematical formulae. Orhan [6] conducted the number of experiments on open-edge cracked cantilever beams to see the effect of crack on the vibration parameters. When the structure suffers from damage then its dynamic properties get changed due to the changes in the stiffness [7]. As these changes in the structure, leads to its crack detection.

In the present study, the vibrant behaviour of a cracked cantilever beam is studied. The effects of crack location, crack angle and crack depth on the vibration parameters are deeply investigated by FEA and the experimental method as such vibration parameters are used to detect the various crack parameters of the defective beam.

2. Theory

The beam with a crosswise edge crack is clamped at left end, free at other end; it has a uniform square cross-section. The Euler-Bernoulli beam model is assumed because length to width ratio of a beam is 18. The crack is assumed to be an open crack and the effect of small damping is considered in this study.

2.1. Governing equation of free vibration

The free bending vibration of an Euler -Bernoulli beam of a constant square cross section is given by the following differential equation [8].

$$EI\frac{d^4y}{dx^4} - m\omega_i^2 y = 0 \tag{1}$$

where m is the mass of the beam per unit length (kg/m), ω_i is the natural frequency of the ith mode (rad/s), E is the modulus of elasticity (N/m^2) and, I is the area moment of inertia (m^4).

By defining $\lambda^4 = \omega_i^2 m/EI$ Eq. (1) is rearranged as a fourth- order differential equation as follow:

$$\frac{d^4y}{dx^4} - \lambda_i^4 y = 0 \tag{2}$$

The general solution for Eq. (2)

$$y = A\cos\lambda_i x + B\sin\lambda_i x + C\cosh\lambda_i x + D\sinh\lambda_i x, \tag{3}$$

where *A*, *B*, *C*, *D* are constants λ_i is a frequency parameter. As the bending vibration is studied, edge crack is modeled as a rotational spring with a lumped stiffness. The crack is assumed open. Based on this modeling, the beam is divided into two segments: the first and second segments are left and right-hand side of the crack, respectively. When this equation is solved by applying beam boundary conditions and compatibility relations, the natural frequency of the ith mode for uncracked Eq. (4) and cracked Eq. (5) beams is finally obtained.

$$\omega_{i0} = c_i\sqrt{\frac{EI}{mL^4}} \tag{4}$$

$$\omega_i = r_i c_i\sqrt{\frac{EI}{mL^4}} \tag{5}$$

where ω_{i0} is the ith mode frequency of the uncracked beam and c_i is known constant depending on the mode number and beam end conditions (for clamped-free beam, c_i is 3.516 and 22.034 for the first and second mode, respectively). ω_i is i_{th} mode frequency of the cracked beam. r_i is the ratio between the natural frequencies of the cracked and uncracked beam. L is the length of the beam.

2.2. Half power band width method

Half power bandwidth method is well known method to determine the damping factor of the practical systems of a single degree freedom type, but it can be extended for the multi degree freedom system as well in which modes are clearly distinguished. It is assumed that the half the total power in this mode occurs in the frequency band between f_1 and f_2, where f_1 and f_2 are the frequencies corresponding to an amplitude of $f_{res}/2$. It is shown in standard texts formula of damping factor [9].

$$2\xi = \frac{f_2 - f_1}{f_n}$$

3. Experimental Study

The aim of the experimentation is to monitor the change in the natural frequency and damping in a cantilever beam due to the presence of crack.

3.1. Experimental setup

The instruments used for the experimental analysis are accelerometer, 8 channel Fast Fourier Transform (FFT) analyzer and related accessories, as shown in Fig. 1. Specimens of EN 47 material are used to study the effects of cracks on the vibration parameters. EN 47 materials is tested in ELCA Lab, Pune, to get material properties, i.e., Young's modulus and density ρ= 7800 kg/m^3, E= 1.95×10^{11} N/m^2, A= 0.02 m × 0.02 m, and L= 0.360 m. Wire EDM process is used to produce cracks on the specimens.

The beam is clamped at one end by a fixture and other end is free. An accelerometer is of piezo electric type and it is mounted on the beam to measure the acceleration of the vibrating body as shown in Fig. 1. Total 22 specimens are tested by using the FFT analyzer, out of 22 specimens, one specimen is crack-free specimen. Out of remaining 21 specimens, each 7 specimens have a transverse crack, oblique crack (10° crack), and again oblique crack (20° crack). A vibration of transverse waves is comparatively on the higher side than longitudinal waves [10]. Again pure bending mode is obtained at the first natural frequency only in the transverse direction. Owing to this reason, only the vibration in the transverse direction is considered in this study.

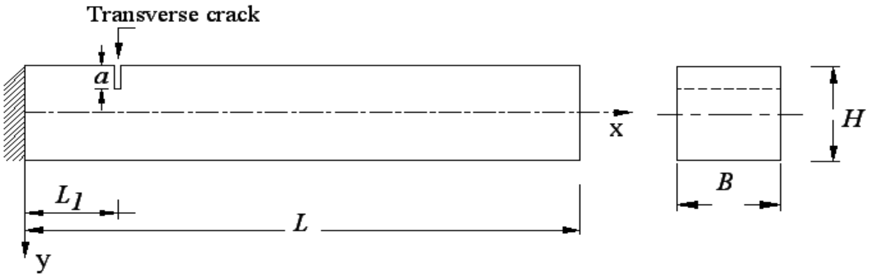
3.2. Crack configurations

Total 21 cracked specimens are used in this study to find out how the cracks affect the dynamic behaviour of a cantilever beam. In this study, 2 cases are considered.

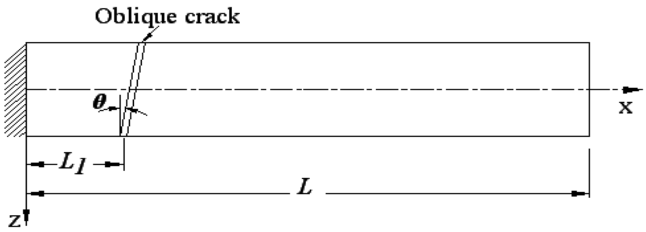
Case 1: In this case 12 specimens are considered. It is sub divided into 3 sub cases. In the first sub case, transverse cracks are considered as shown in Fig. 2(a), by keeping crack location constant at 60 mm from the cantilever end. At this location, crack depth is varied from 4 mm to 16 mm by an interval of 4 mm. Similarly, in the next two sub cases same configuration is used. Instead of transverse crack oblique cracks are considered. For second sub case cracks are taken at 10^0 and for third sub case it is taken at 20^0.

Case 2: In this case 9 specimens are considered. It is sub divided into 3 sub cases. In the first sub case, transverse cracks are considered by keeping crack depth constant at 16 mm and crack location is varied as 120 mm, 180 mm, and 240 mm. Similarly, in the next two sub cases same configuration is used. Only cracks are oblique having 10^0 crack angles for second sub case and 20^0 crack angles for third sub case.

Fig. 1. Experimental set-up.



(a) Cantilever Beam with Transverse Crack



(b) Cantilever Beam with Oblique Crack

Fig. 2. A schematic diagram of a cracked cantilever beam.

4. Finite Element Modelling and Analysis

ANSYS 12.1 [11] finite element program is used to determine natural frequencies and damping factors of the undamaged as well as cracked beams. For this purpose, a rectangle area is created (Fig. 3). This area is extruded in the third direction to get the 3 D model. Then at the required location, a small rectangular area of crack of 0.5 mm width and required depth is created and extruded. Then small volume of the crack is subtracted from a large volume of cantilever beam to obtain a cracked three dimensional model.

The width of crack is kept constant throughout its depth in this study. A 20 node structural solid element (solid 186) is selected for modelling the beam because of some special features i.e. stress stiffening, hyper elasticity, large strain, and large deflection. Finite element boundary conditions are applied on the beam to constrain the all degrees of freedom of the extreme left hand end of the beam. The Block Lanczos eigenvalue solver is used to calculate the natural frequencies of the beams because it is as accurate as subspace solver but, on the other hand it is quicker in Harmonic analysis, to get the graph between excitation frequency and resonant amplitude. Later, this graph is used along with band width method [9] to find out the value of damping factor. In harmonic analysis, full method is used instead of the reduced method with an excitation frequency sweep from 0-160 Hz and stepped boundary condition is applied. These procedures are repeated for an intact and all the defective beams.

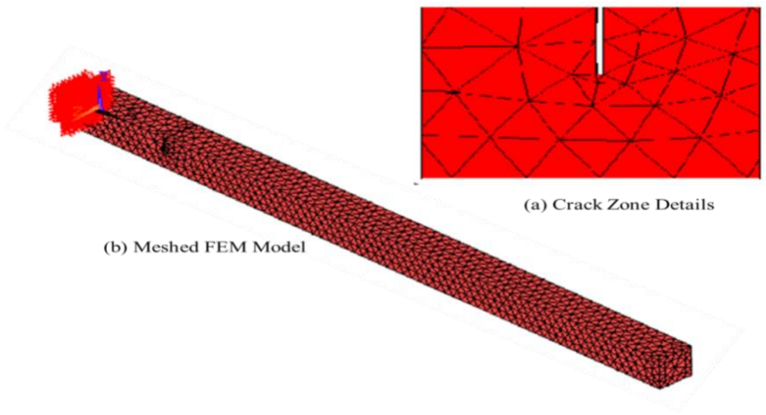


Fig. 3. A cracked beam finite element modelling.

5. Results

By finite element analysis, the natural frequencies and damping factors present in the cracked beams are found to confirm the experimental results. Experimentally values of natural frequencies and damping factors are determined by using FFT Analyzer. One experimental plot is as shown in Fig. 4. Also, by making use of harmonic analysis (FEM) and half band width method (as shown in Fig. 5), damping factor is determined as stated in [9].

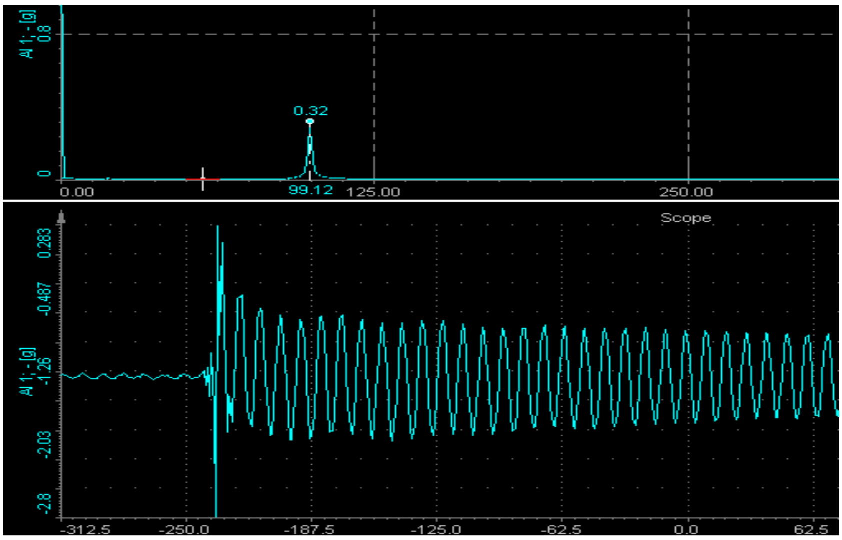


Fig. 4. Natural frequency and time-displacement plot obtained through FFT analyzer of a transverse cracked beam, location =60mm; crack depth= 8 mm.

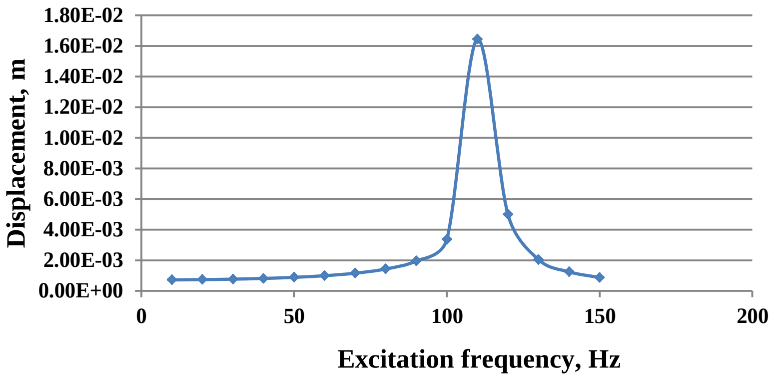


Fig. 5. Plot of determination of damping factor by a mixture of harmonic analysis and band width method.

From Fig. 6, it is found that as depth of the crack increases at any unique location then value of natural frequency of the beam decreases and is true for all the transverse cracked beams as well as oblique cracked beams, this is only due to the reduction in stiffness of the beam. The natural frequency of the beam decreases significantly, when crack depth increases to 80% of the depth of the beam and it is least affected, when depth of the crack is either 20% or less than 20% of the depth the beam. From Figs. 7-9, it is found that, as the value of crack angle increases, natural frequency also increases, it means that transverse cracked beam produces larger bending moment at the cantilever end as compared to oblique cracked beam. Therefore, it is cleared that change in crack depth is a function of natural frequency.

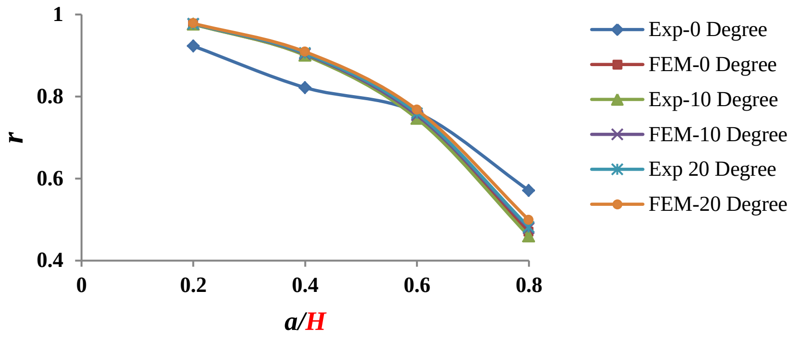


Fig. 6. Variation of natural frequency ratio with crack depth ratio.

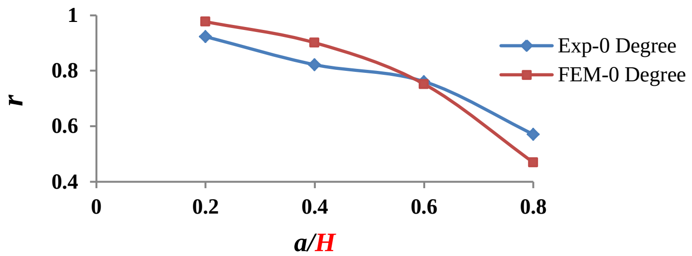


Fig. 7. Variation of Natural Frequency Ratio with Crack Depth Ratio.

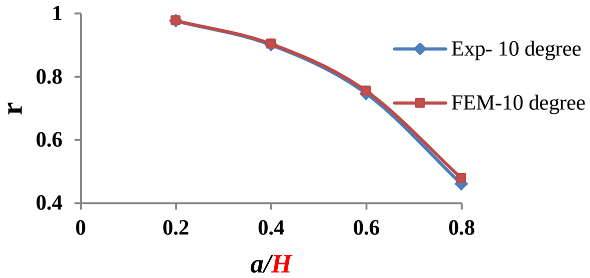


Fig. 8. Variation of natural frequency ratio with crack depth ratio.

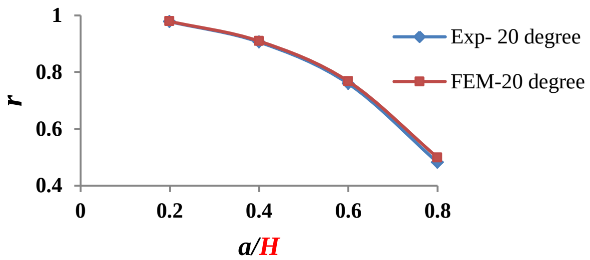


Fig. 9. Variation of natural frequency ratio with crack depth ratio.

From Fig. 10, it is found that as crack location increases from the fixed end, then natural frequency increases for transverse cracked beams as well as for oblique cracked beams. This is due to the fact that when crack is located far away from the cantilever end, then it decreases damping effect in the beam. Fundamental natural frequency is least affected, when crack is located more towards the free end. For the transverse cracked beams, values of natural frequencies are lower as compared to oblique cracked beams for the same configuration, since the trailing end of the oblique crack is more closed to the free end than the trailing end of the transverse crack.

From Figs. 11-13, it is found that as crack location increases, natural frequency also increases. Values of natural frequencies for oblique crack (20° crack) are maximum due to the above facts.

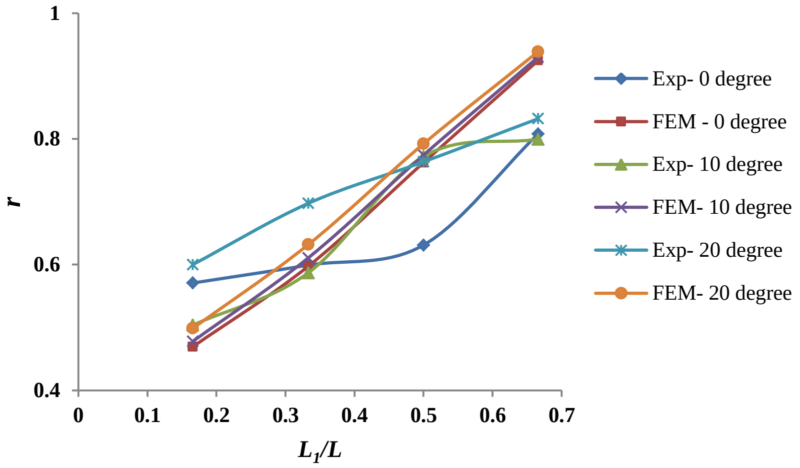


Fig. 10. Variation of natural frequency ratio with crack location ratio.

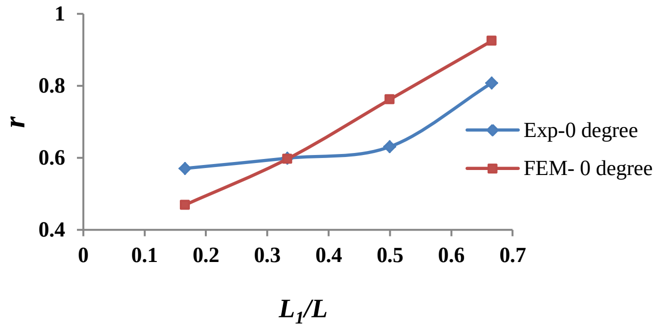


Fig. 11. Variation of natural frequency ratio with crack location ratio.

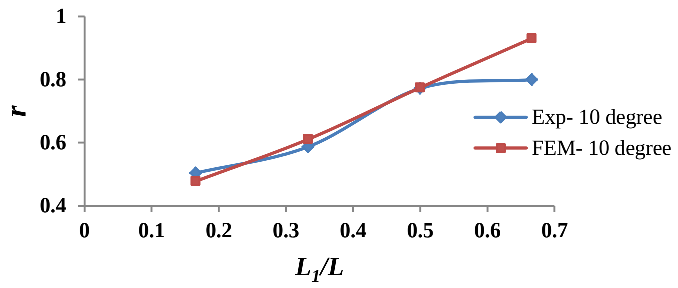


Fig. 12. Variation of natural frequency ratio with crack location ratio.

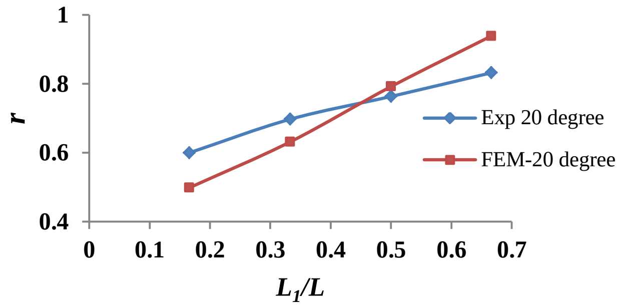


Fig. 13. Variation of natural frequency ratio with crack location ratio.

From Fig. 14, it is revealed that as the depth of the crack increases at any unique location then amount of damping in the beam increases and is true for all the transverse cracked beams as well as oblique cracked beams due to the reduction in the rigidity or stiffness of the beam.

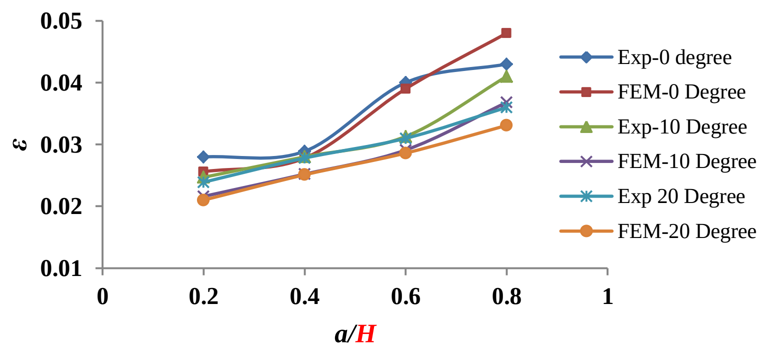


Fig. 14. Variation of damping factor with crack depth ratio.

From Figs. 15-17, it is found that as the value of crack angle increases, magnitude of damping in the beam decreases; it means that transverse cracked beam produces larger damping effect than oblique cracked beams at any location in a cantilever beam. Larger damping in a cantilever beam is only due to the larger bending moment at the crack.

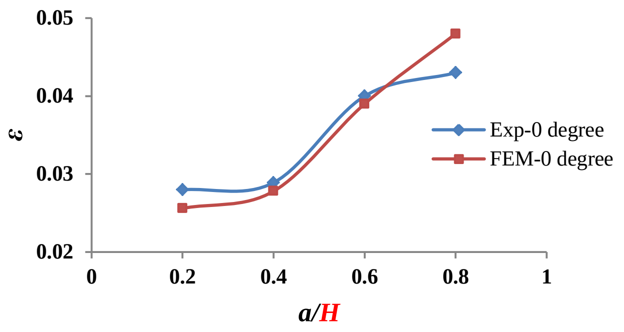


Fig. 15. Variation of damping factor with crack depth ratio.

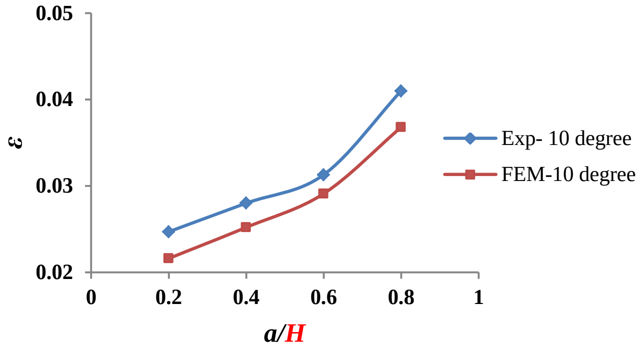


Fig. 16. Variation of damping factor with crack depth ratio.

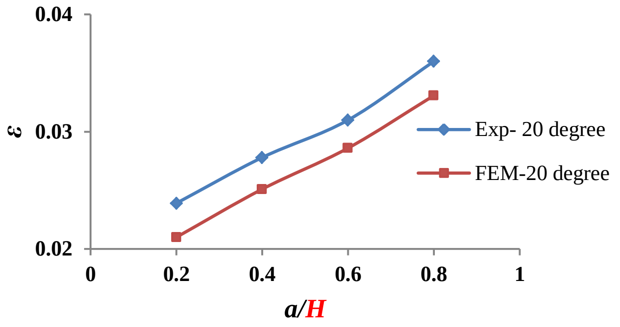


Fig. 17. Variation of damping factor with crack depth ratio.

From Fig. 18, it is found that as the crack location increases from the fixed end, then damping factor decreases for all the cracked beams. This is due to the fact that the crack which is far away from the cantilever end contributes least effect of damping in the cracked beam.

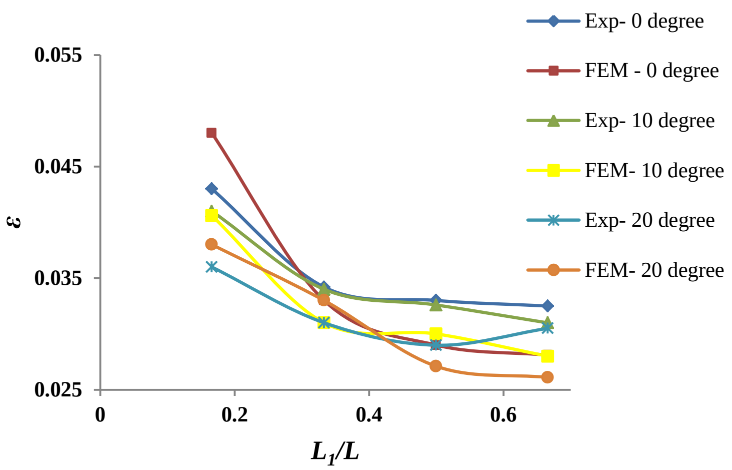


Fig. 18. Variation of damping factor with crack location ratio.

From Figs. 19-21, it is found that as crack location increases from the fixed end, then damping effect decreases, Amount of damping is found to be least for 20° cracked specimens.

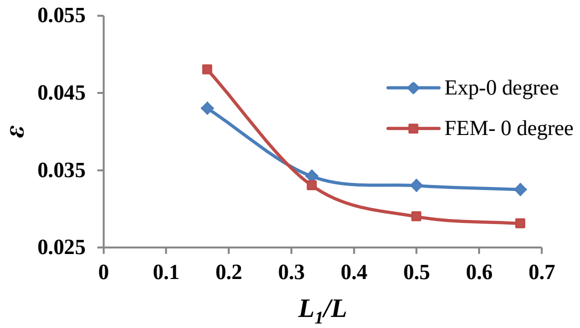


Fig. 19. Variation of damping factor with crack location ratio.

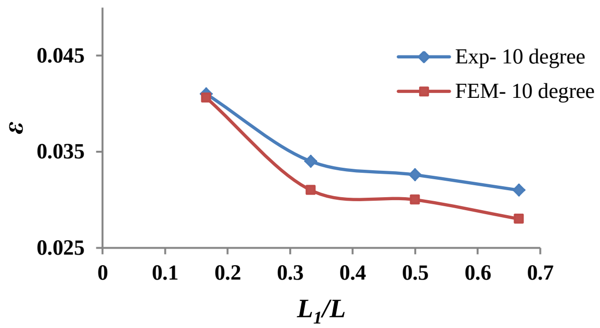


Fig. 20. Variation of damping factor with crack location ratio.

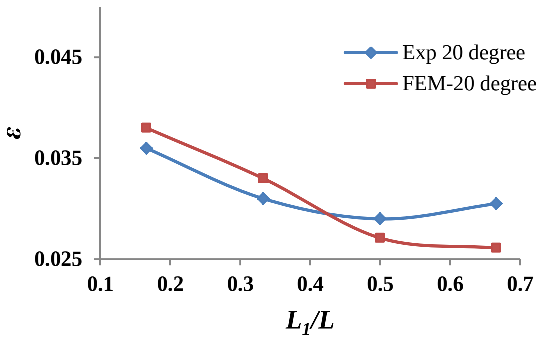


Fig. 21. Variation of damping factor with crack location ratio.

6. Conclusions

The following conclusion can be drawn from this study.

- When the location of the crack is kept constant and crack depth increases, then the natural frequency of the beam decreases.
- When the location of the crack on the beam is kept constant and crack depth increases for the transverse cracked as well as for oblique cracked beam then the natural frequency of the beam decreases, but for the oblique cracked specimens the decline in natural frequency is less abrupt than transverse crack.
- When the depth of the crack is kept constant and crack location is varied, then natural frequency of the beam increases. It is true for the transverse as well as oblique crack specimens, but increase in the natural frequency of the oblique cracked case is relatively more, it means that as the crack angle increases towards the free end of the beam, then the natural frequency increases.
- When the location of the crack is kept constant and crack depth increases, then amount of damping in the beam increases for transverse as well as oblique cracked beams.
- The effect of damping in the transverse cracked beam is found more than oblique cracked beam for the same configuration.
- When depth of the crack is kept constant and crack location is varied from fixed end, then damping factor of the beam decreases. Collapse in the value of damping factor is comparatively less for 20° oblique cracked beam than the transverse and 10° cracked beam of same configuration.
- Damping factor of the cracked beam decreases as crack angle increases.

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