THE REDUCTION OF STRESS CONCENTRATION IN A UNI-AXIALLY LOADED INFINITE WIDTH RECTANGULAR ISOTROPIC/ORTHOTROPIC PLATE WITH CENTRAL CIRCULAR HOLE BY COAXIAL AUXILIARY HOLES

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ABSTRACT: A comprehensive plane stress finite element study is made for reduction of stress concentration factor (SCF) in a uni-axially loaded infinite width rectangular isotropic/orthotropic plate with central circular hole. The finite element formulation is carried out by the ANSYS package. With the help of present work, stress concentration can be reduced up to 24.4 % in an isotropic and 31 % in an orthotropic plate by introducing four coaxial auxiliary holes on either side of main hole. The study reveals that the introduction of these holes helps to smooth flow of the tensile stresses passes through the main hole and results, a reduction in stress concentration factor. With such reduction in maximum stress levels, the improvement in fatigue life of a component can be significant.

ABSTRAK: Satu kajian komprehensif tentang unsur terhingga telah dibuat keatas factor pemusatan tegangan bagi plat isotropic/ortotropik segi empat tepat lebar infinit yang dimuat secara uni-aksial berlubang pusat bulat. Rumusan unsur finit telah dibuat menggunakan pakej ANSYS. Dengan kajian masa kini, pemusatan tegangan dapat dikurangkan sehingga 24.4 % dalam plat isotropik and 31 % dalam plat ortotropik dengan membuat empat lubang auksiliari sepaksi di kedua belah lubang utama. Kajian ini menunjukkan penggunaan empat lubang ini telah melicinkan aliran tegasan tegangan melalui lubang utama. Ini mengakibatkan pengurangan factor pemusatan tegangan. Dengan pengurang tahap tegangan maksimum ini peningkatan hayat kelesuan komponen boleh menjadi signifikan.

KEYWORDS: *stress concentration factor; finite element method; plate; composite*

1. INTRODUCTION

An infinite width rectangular isotropic/orthotropic plate with central circular hole have found widespread applications in various fields of engineering such as aerospace, marine, automobile and mechanical. Stress concentration arises from any abrupt change in geometry of plate under loading. As a result, stress distribution is not uniform throughout the cross section. Failures such as fatigue cracking and plastic deformation frequently occur at points of stress concentration. Hence, for the design of a plate with central circular hole, stress concentration factor plays an important role and accurate knowledge of stresses and stress concentration factor at the edges of hole under in plane or transverse loading are required. Analytical solutions are available in the literature for prediction of

SCF in different types of abrupt changes in shape, but there are limited methods for reduction in SCF.

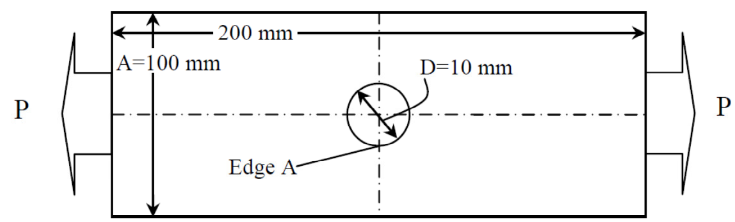
Meguid [1] presented a technique for reducing stress concentration factor by introducing defence hole system in a uni-axially loaded plate with two coaxial holes by finite element method, where reduction in SCF ranging from 7.5 to 11 % maximum have been achieved. But to achieve the maximum possible reduction, these auxiliary holes have to be introducing very near to the original holes, which is not practically possible in all cases. Giare et al. [2] presented a method for the reduction of stress concentration around the main hole in an isotropic plate by using a ring of composite material around the main hole, method is not suitable for all designing conditions and also no techniques were presented for reduction in SCF for orthotropic plate. Providakis and Sotiropoulos [3] have developed a boundary element approach to the reduction in stress concentration factor in visco-plastic plates by multiple holes. Tenchev et al. [4] presented a design procedure for reducing stress concentration around holes in laminated composite by increasing the thickness in this area. Younis [5] investigated by reflected photoelasticity method that the assembly stress contributes to reducing stresses around the holes in a plate. Mahiou and Bekaou [6] studied for local stress concentration and for the prediction of tensile failure in unidirectional composites. Toubal et al. [7] studied experimentally for stress concentration in a circular hole in composite plate. Wu and Mu [8] analysed stress concentrations for isotropic/orthotropic plates and cylinders with a circular hole by finite element method. Peterson [20] has developed good theory and charts on the basis of mathematical analysis and presented excellent mythology in graphical form for evaluation of stress concentration factors, but no techniques were presented for reduction of SCF.

This paper is concerned with the reduction of SCF in isotropic and orthotropic infinite width rectangular plates with central circular hole under in-plane static loading. In general stress concentration factor can be reduced up to 12-18 % by two coaxial auxiliary holes on either side of the main hole, to achieve the maximum possible reduction; these auxiliary holes are to be introducing very near to the original hole, which is not practically possible. It has been seen that the reduction in SCF at the edges of main hole become directly proportional to diameter of auxiliary holes, increasing the diameter of auxiliary holes results in reduction of normal stress (σ_x) at the edges of main hole, but this is also accompanied by an increase of σ_x at the edges of auxiliary holes. At a certain diameter of auxiliary holes, σ_x at the edges of auxiliary holes attained more than σ_x at the edges of main hole i.e. maximum value of σ_x is shifted from edges of main hole to edges of auxiliary holes. This problem can avoid by introducing two more coaxial holes on either side of first two coaxial holes and the diameter of first two auxiliary holes can increase more. This results more reduction in stress concentration. The work has been done firstly for isotropic plates, and then study is further expanded for orthotropic plates.

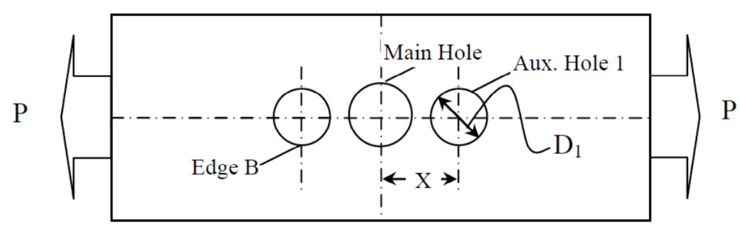
This work is most appreciated in aerospace and marine industry where the introduction of holes will not only reduce the stress concentration around main hole, but also reduce the weight of the components. This study will also provide guidelines technique to designer for reduction of stress concentration up to maximum level.

2. DESCRIPTION OF PROBLEM

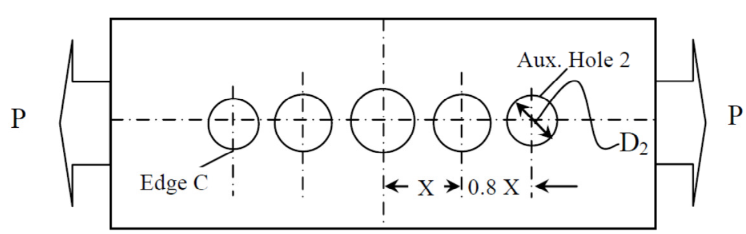
To study the influence of auxiliary holes upon the stress concentration around the main hole, three models *A, B* and *C* are described in Fig. 1, where model *A* is basic model (a plate with central circular hole of diameter *D*), model *B* is modification of model *A* (plate with two coaxial auxiliary holes of diameter D_1, numbered as auxiliary hole 1) and model *C* is modification of model *B* (plate with four coaxial auxiliary holes) i.e. two more coaxial auxiliary holes of diameter D_2 (numbered as auxiliary hole 2) are created in model *B*. The centre distance between main hole and auxiliary hole 1 is taken as *X* and the centre distance between auxiliary hole1 and auxiliary hole 2 is taken as 0.8 *X*. All the details of dimensions are described in Fig 1. A plate under in-plane static uniformly distributed load of *P,* is analyzed by finite element method for *D/A* ratio of 0.1, where *A* is plate width. The dimensions are selected such a way that it can be considered as infinite width plate.



(a) Model *A* (Basic plate with central circular hole of diameter *D*).



(b) Model *B*



(c) Model *C*

Fig. 1: Details of three models analysed in this study.

3. FINITE ELEMENT ANALYSIS

An eight nodded quadrilateral element with element length of 1 mm was selected and used through out the study. Each node has two degrees of freedom, making a total 16 degrees of freedom per element. In order to construct the graphical image of the geometries of the different models of plate examined using the ANSYS (Advanced Engineering Simulation), it was necessary to input the basic geometric elements such as points, lines and arcs. Due to the symmetric nature of different models investigated, it was only necessary to discretize and analyse the one quarter of the problem of each model for finite element analysis. Main task in finite element analysis is selection of suitable element type. Number of checks and convergence test are made for selection of suitable element type from different available element. Results were then displayed by using post processor of ANSYS programme.

4. RESULTS AND DISCUSSION

Numerical results are presented for isotropic and orthotropic plates with central circular hole under in-plane static loading. The SCF is calculated on nominal area in all cases. The reduction in SCF is achieved firstly for isotropic plates and then work is extended for orthotropic plates.

4.1 Isotropic Plate

In the basic model (model *A*) maximum stress concentration is always occurred at the edges of main hole. At the edges of main hole; normal stress in *X*- direction (σ_x) is placed 3.003 times of *P* and SCF (Calculate at nominal area) is appeared as 2.70, which is very close to theoretical value of 2.72.

Stress concentration at the edge *A* is reduced by two coaxial auxiliary holes of diameter D_1 in model *B*. The analysis for reduction in stress concentration at the edges of main hole has been done for different centre distances of main hole and auxiliary hole 1. It was very necessary to analyse, the influence of auxiliary holes on stress concentration around main hole as well as around of auxiliary holes.

In model *B*, edges of main hole (edge *A*) and edges of auxiliary hole 1 (edge *B*) are zone of stress concentration. Table 1 shows the effect of D_1/D on σ_x/P at the edges *A* and *B* for X/D=1.2, 1.5, 1.8 and 2.0 respectively in model *B*. For all cases of X/D, results illustrate clearly that increasing D_1/D results in continuously reduction of stress concentration at the edges of main hole, but this is also accompanied by an increase in stress concentration at the edges of auxiliary holes.

The following observations can be made from Table 1. For X/D=1.2; increasing D_1/D from 0.4 to 0.85 results in reduction of σ_x/P at edge *A* from 2.895 to 2.451, this is also accompanied by an increase of σ_x/P at edge *B* from 2.058 to 2.435. But when D_1/D increases from 0.85 to 0.86, σ_x/P is increased from 2.435 to 2.441 at edge *B* and attained more than σ_x/P at edge *A* i.e. maximum value of σx is shifted from edge *A* to edge *B*. The critical value of D_1/D is 0.85 for this case.

Table 1: Variation of σ_x/P at the edges of main hole and auxiliary hole 1 with D_1/D for different values of X/D in model *B*.

***X/D*=1.2**			***X/D*=1.5**			***X/D*=1.8**			***X/D*=2.0**		
D₁/D	**Edge *A***	**Edge *B***	***D₁/D***	**Edge *A***	**Edge *B***	***D₁/D***	**Edge *A***	**Edge *B***	***D₁/D***	**Edge *A***	**Edge *B***
0.40	2.895	2.058	0.40	2.892	2.320	0.40	2.904	2.503	0.40	2.912	2.598
0.60	2.747	2.215	0.60	2.751	2.439	0.60	2.779	2.581	0.60	2.800	2.645
0.80	2.521	2.407	0.80	2.551	2.507	0.70	2.698	2.589	0.70	2.728	2.660
0.84	2.466	2.436	0.82	2.530	2.523	0.74	2.663	2.606	0.76	2.681	2.666
0.85	2.451	2.435	0.84	2.505	2.537	0.78	2.626	2.600	0.78	2.664	2.661
0.86	2.437	2.441	-----	-----	-----	0.80	2.607	2.609	0.80	2.646	2.661

If D_1/D increases beyond critical value, maximum stress concentration occurs at edge B and increases with increase of D_1/D. For *X/D*=1.5; increasing D_1/D from 0.4 to 0.82 results in reduction of σ_x/P at edge *A* from 2.892 to 2.530, this is also accompanied by an increase of σ_x/P at edge *B* from 2.320 to 2.523. But when D_1/D increases from 0.82 to 0.84, σ_x/P is increased from 2.523 to 2.537 at edge *B* and attained more than σ_x/P at edge *A* i.e. maximum value of σ_x is shifted from edge *A* to edge *B*. The critical value of D_1/D is 0.82 for this case.

For *X/D*=1.8; increasing D_1/D from 0.4 to 0.78 results in reduction of σ_x/P at edge *A* from 2.904 to 2.626, this is also accompanied by an increase of σ_x/P at edge *B* from 2.503 to 2.600. But when D_1/D increases from 0.78 to 0.80, σ_x/P is increased from 2.600 to 2.609 at edge *B* and attained more than σ_x/P at edge *A* i.e. maximum value of σ_x is shifted from edge *A* to edge *B*. The critical value of D_1/D is 0.78 for this case.

For *X/D*=2.0; increasing D_1/D from 0.4 to 0.78 results in reduction of σ_x/P at edge *A* from 2.912 to 2.664, this is also accompanied by an increase of σ_x/P at edge *B* from 2.598 to 2.661. But when D_1/D increases from 0.78 to 0.80, σ_x/P at edge *B* is attained more than at edge *A* i.e. maximum value of σ_x is shifted from edge *A* to edge *B*. The critical value of D_1/D is 0.78 for this case.

Table 2 shows the effect of D_1/D and D_2/D on σ_x/P at the edges *A, B* and *C* for *X/D*=1.2, 1.5, 1.8 and 2.0 respectively in model *C*. For all cases of *X/D*, Model *C* is prepared by creating two more coaxial auxiliary holes in model *B* (with critical value of D_1/D).

The following observations can be made from Table 2. For *X/D*=1.2; increasing D_2/D from 0.4 to 0.74, while maintaining the D_1/D unchanged as 0.85 (critical value in model *B*), results in reduction of σ_x/P at edge *A* from 2.434 to 2.359, this is also accompanied by a decrease of σ_x/P at edge *B* from 2.362 to 2.142 and an increase of σ_x/P at edge *C* from 1.943 to 2.343. But when D_2/D increases beyond 0.74, σ_x/P at edge *C* is attained more than σ_x/P at edge *A* i.e. maximum value of σx is shifted from edge *A* to edge *C*. Again, increasing D_1/D from 0.85 to 0.92, while maintaining the D_2/D unchanged as 0.74, results in reduction of σ_x/P at edge *A* from 2.359 to 2.271, this is also accompanied by an increase of σ_x/P at edge *B* from 2.142 to 2.241 and a decrease of σ_x/P at edge *C* from 2.343 to 2.255.

For *X/D*=1.5; increasing D_2/D from 0.4 to 0.7, while maintaining the D_1/D unchanged as 0.82 (critical value in model *B*), results in reduction of σ_x/P at edge *A* from 2.507 to 2.454, this is also accompanied by a decrease of σ_x/P at edge *B* from 2.440 to 2.226 and an increase of σ_x/P at edge C from 2.226 to 2.448. But when D_2/D increases beyond 0.7, σ_x/P

at edge *C* is attained more than σ_x/P at edge *A* i.e. maximum value of σx is shifted from edge *A* to edge *C*. Again, increasing D_1/D from 0.82 to 0.90, while maintaining the D_2/D unchanged as 0.7, results in reduction of *σx/P* at edge *A* from 2.454 to 2.369, this is also accompanied by an increase of *σx/P* at edge *B* from 2.226 to 2.330 and a decrease of *σx/P* at edge *C* from 2.448 to 2.365.

For *X/D*=1.8; increasing D_2/D from 0.4 to 0.6, while maintaining the D_1/D unchanged as 0.78 (critical value in model *B*), results in reduction of σ_x/P at edge *A* from 2.603 to 2.574, this is also accompanied by a decrease of σ_x/P at edge *B* from 2.520 to 2.421 and an increase of σx/P at edge C from 2.461 to 2.563. But when D_2/D increases beyond 0.6, *σx/P* at edge *C* is attained more than σ_x/P at edge *A* i.e. maximum value of σ_x is shifted from edge *A* to edge *C*. Again, increasing D_1/D from 0.78 to 0.88, while maintaining the D_2/D unchanged as 0.6, results in reduction of σ_x/P at edge *A* from 2.574 to 2.482, this is also accompanied by an increase of σ_x/P at edge *B* from 2.421 to 2.474 and a decrease of σ_x/P at edge *C* from 2.563 to 2.457.

Table 2:
Variation of σ_x/P at the edges of main hole, auxiliary hole 1 and auxiliary hole 2 with D_1/D and D_2/D for different values of *X/D* in model *C*.

X/D	D_1/D	D_2/D	Edge *A*	Edge *B*	Edge *C*
1.2	0.85	0.40	2.434	2.362	1.943
	0.85	0.60	2.400	2.257	2.181
	0.85	0.70	2.371	2.181	2.281
	0.85	0.74	2.359	2.142	2.343
	0.88	0.74	2.324	2.195	2.297
	0.90	0.74	2.298	2.205	2.282
	0.92	0.74	2.271	2.241	2.255
1.5	0.82	0.40	2.507	2.440	2.226
	0.82	0.60	2.476	2.333	2.400
	0.82	0.70	2.454	2.226	2.448
	0.84	0.70	2.433	2.279	2.415
	0.86	0.70	2.413	2.308	2.411
	0.90	0.70	2.369	2.330	2.365
1.8	0.78	0.40	2.603	2.520	2.461
	0.78	0.60	2.574	2.421	2.563
	0.80	0.60	2.557	2.431	2.537
	0.82	0.60	2.539	2.440	2.518
	0.86	0.60	2.502	2.467	2.474
	0.88	0.60	2.482	2.474	2.457
2.0	0.78	0.40	2.642	2.587	2.585
	0.78	0.50	2.631	2.539	2.583
	0.78	0.56	2.623	2.514	2.616

	0.80	0.56	2.606	2.521	2.599
	0.82	0.56	2.590	2.532	2.579
	0.84	0.56	2.574	2.539	2.542
	0.86	0.56	2.556	2.546	2.536

For *X/D*=2.0; increasing D_2/D from 0.4 to 0.56, while maintaining the D_1/D unchanged as 0.78 (critical value in model *B*), results in reduction of σ_x/P at edge *A* from 2.642 to 2.623, this is also accompanied by a decrease of σ_x/P at edge *B* from 2.587 to 2.514 and an increase of σ_x/P at edge *C* from 2.585 to 2.616. But when D_2/D increases beyond 0.56, σ_x/P at edge *C* is attained more than σ_x/P at edge *A* i.e. maximum value of σ_x is shifted from edge *A* to edge *C*. Again, increasing D_1/D from 0.78 to 0.86, while maintaining the D_2/D unchanged as 0.56, results in reduction of σ_x/P at edge *A* from 2.623 to 2.556, this is also accompanied by an increase of σ_x/P at edge *B* from 2.514 to 2.546 and a decrease of σ_x/P at edge *C* from 2.616 to 2.536.

Figure 2 illustrates the variation of SCF at the edges of main hole (edge *A*) in model *B* and *C* with D_1/D for different values of *X/D*. These are maximum SCF in plate, attaining at the edges of main hole.

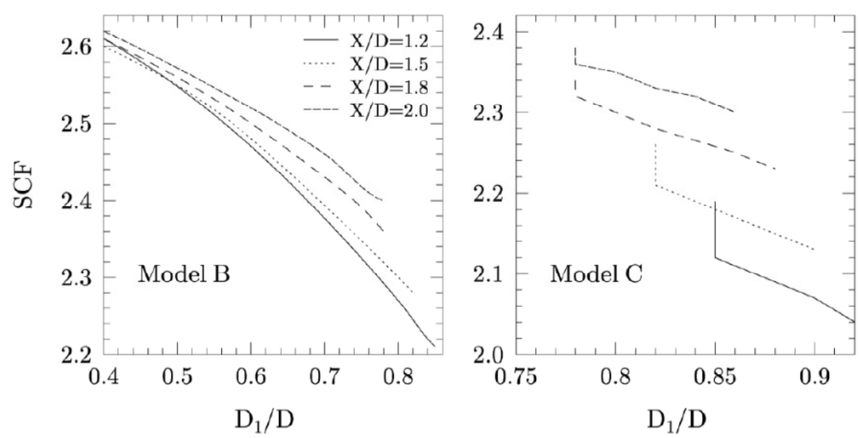


Fig. 2: Effects of D_1/D on SCF at the edges of main hole (edge A) in Model B and C.

In model *B*, SCF is reduced up to 2.21, 2.28, 2.36 and 2.4 for *X/D* = 1.2, 1.5, 1.8 and 2.0 respectively. Where, in model C, SCF is reduced up to 2.04, 2.13, 2.23 and 2.3 for *X/D* = 1.2, 1.5, 1.8 and 2.0 respectively.

Figure 3 shows the % reduction in SCF at the edges of main hole in model *B* and *C* with D_1/D for different values of *X/D*. In model *B*; 18.39, 15.75, 12.57 and 11.31 % reduction in SCF is achieved for *X/D* = 1.2, 1.5, 1.8 and 2.0 respectively.

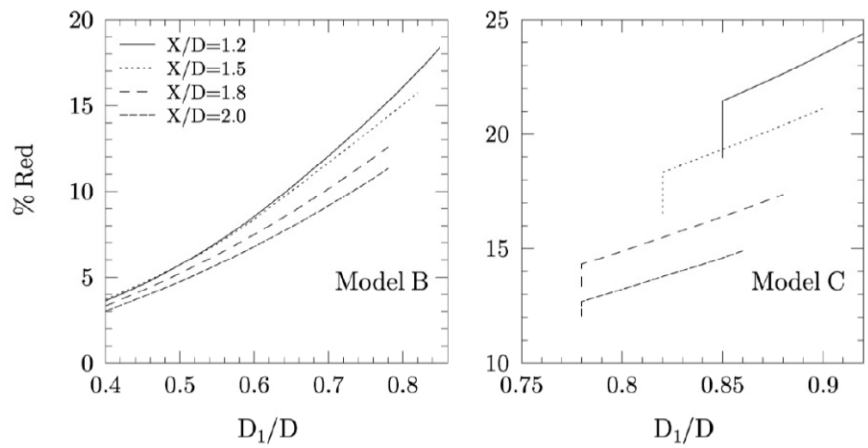


Fig. 3: Effects of D_1/D on % reduction in SCF at the edges of main hole (edge A) in Model B and C.

Where, in model *C*; 24.39, 21.13, 17.36 and 14.90 % reduction in SCF is achieved for *X/D* = 1.2, 1.5, 1.8 and 2.0 respectively. Results indicate clearly that maximum possible % reduction in SCF in model *C* is almost 6.0 % greater than model *B* for all values of *X/D*. It has been observed that maximum possible % reduction in SCF decreases with increase of *X/D* in model *B* and *C* both.

4.2 Orthotropic Plate

Thirteen different composite materials are selected for analysis of influence of auxiliary holes in model *B* and *C*, upon reduction of stress concentration around main hole in an orthotropic plate.

SCF in model *A* and % reduction in SCF in model *B* and *C* for *X/D*=1.2 and 1.5, are listed in Table 3. The elastic properties of different composite materials used in study are also listed in Table 3.

Table 3: SCF in model *A* and achieved % reduction in SCF in model *B* and *C* for orthotropic plates.

Material Type	E_1	E_2	G_{12}	μ	**SCF**	**% Reduction in SCF**			
						***X/D*=1.2**		***X/D*=1.5**	
						Model *B*	**Model *C***	**Model *B***	**Model *C***
E-glass/epoxy	39	8.6	3.8	0.28	3.816	21.13	29.60	19.36	27.15
S-glass/epoxy	43	8.9	4.5	0.27	3.789	21.20	29.58	19.40	27.10
Woven glass/epoxy	29.7	29.7	5.3	0.17	3.169	19.83	27.11	17.61	24.23
Kelvar/epoxy	87	5.5	2.2	0.34	5.017	18.70	29.42	17.73	27.95
Carbon/epoxy	142	10.3	7.2	0.27	4.488	20.81	30.55	19.52	28.60
Carbon/PEEK	131	8.7	5	0.28	4.696	20.11	30.27	18.96	28.51
Carbon/epoxy	177	10.8	7.6	0.27	4.637	20.46	30.48	19.27	28.66
Carbon/polyimide	216	5	4.5	0.25	5.384	17.97	29.15	17.18	27.95

Graphite/epoxy	294	6.4	4.9	0.23	5.585	16.89	28.28	16.13	27.26
Boron/epoxy	201	21.7	5.4	0.17	4.889	18.80	29.36	17.77	27.83
Boron/aluminum	235	137	47	0.3	3.169	20.08	27.20	17.82	24.28
SiC/Al	204	118	41	0.27	3.175	20.10	27.23	17.84	24.31
SiC/ceramic	121	112	44	0.2	2.783	18.73	24.95	16.17	21.74

The reduction in SCF is calculated with critical value of D_1/D and D_2/D of isotropic plate. Following observations can be made from these results.

For *X/D*=1.2; in case of model *B*, reduction in SCF is between the range of 17 to 21 % for all materials, maximum in case of S-glass/epoxy with 21.20 % and minimum in case of Graphite/epoxy with 16.89 %.

In case of model *C*, reduction in SCF is between the range of 25 to 31 % for all materials, maximum in case of Carbon/epoxy (AS4/3501-6) with 30.55 % and minimum in case of Sic/ceramic with 24.95 %.

For *X/D*=1.5; in case of model *B*, reduction in SCF is between the range of 16 to 20 % for all materials, maximum in case of Carbon/epoxy (AS4/3501-6) with 19.52 % and minimum in case of Graphite/epoxy with 16.13 %.

In case of model *C*, reduction in SCF is between the range of 22 to 29 % for all materials, maximum in case of Carbon/epoxy (IM6SC1081) with 28.66 % and minimum in case of Sic/ceramic with 21.74 %.

5. CONCLUSION

The reduction in SCF depends on diameter and position of auxiliary holes in isotropic or orthotropic plates, two parameters *X/D* and D_1/D in model *B* and three parameters *X/D*, D_1/D and D_2/D in model *C* influence the reduction in SCF. For an isotropic plate; in case of model *C*, maximum reduction in SCF is achieved for *X/D*=1.2 as 24.39 %, where, in case of model *B*, maximum reduction in SCF is achieved for *X/D*=1.2 as 18.39 % only. For a convenient design, for *X/D*=1.5, up to 21.13 % reduction in SCF can be achieve in model *C*, but in case of model B, it can be only 15.75 %. In case of orthotropic plate; maximum reduction in SCF is between the range of 25.0 to 31.0 % for different materials in model C, where in model *B*, maximum reduction in SCF is between the range of 17.0 to 21.0 % for different materials. It has been observed that maximum possible % reduction in SCF decreases with increase of *X/D* in model *B* and *C* both, for isotropic and orthotropic plates. It has been also seen that maximum possible reduction in SCF in an orthotropic plate depends on elastic constants. In case of orthotropic materials like Graphite epoxy, Kelvar/epoxy, carbon/polyimide, where SCF is approaching more than 5.0, reduction in SCF can be achieved more than 28.0 % in model *C*, where in model *B*, reduction in SCF can be achieved up to 18.0 % only. For an isotropic and an orthotropic plate, such reductions in maximum SCF improve fatigue life of component. It must be remembered that only three parameters *X/D*, D_1/D and D_2/D in model *C* affect the reduction in SCF, and these should be selected as per requirement and convenience of designer.

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