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INTEGRATED LOAD OPTIMIZATION OF ELASTIC-PLASTIC AXISYMMETRIC PLATES AT SHAKEDOWN

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Abstract. An elastic-plastic axisymmetric steel bending plate subjected to a repeated variable load (RVL) is considered. The solution to the load optimization problem at shakedown is complicated because the stress-strain state of the dissipative systems (e.g. the plate plastic deforming) depends on their loading history. A new algorithm for the load optimization problem combining von Mises and Tresca yield criterion based on the Rosen project gradient method is proposed. The optimization results are obtained by integrating the existing software and that created by the authors.

Keywords: elastic-plastic plates, shakedown, energy principle, Mises and Tresca yield criterion, mathematical programming.

1. Introduction

An elastic-plastic axisymmetric steel bending plate subjected to a repeated variable load (RVL) $\mathbf{F}(t)$ is considered in this paper. The RVL is the system of loads where each of which can independently vary within the time t independent lower and upper bounds of the forces $\mathbf{F}_{inf}$, $\mathbf{F}_{sup}$ ($\mathbf{F}_{inf} \le \mathbf{F}(t) \le \mathbf{F}_{sup}$). An ideal elastic-plastic structure subjected by RVL can exceed its constructive requirements due to a failure caused by its incremental collapse and/or its alternating plasticity. Both cases are usually referred to as cyclic plastic collapse. The shakedown plates are investigated in this paper. The plastic strains $\mathbf{\Theta}_p$ developed in the initial loading cycle produce the residual moments $\mathbf{M}_r$ which ensure the purely elastic response of the plates during the following loading cycles. Load shakedown analysis via numerical and mathematical programming methods is relevant for civil engineering. This has been confirmed by the growing number of investigations in this field (Mróz *et al.* 1995; Weichert *et al.* 2002; Kaliszky and Lógó 2002; Pham 2003; Atkočiūnas *et al.* 2004; Merkevičiūtė and Atkočiūnas 2006; Stonkus *et al.* 2009; Žilinskaitė and Žiliukas 2008).

The solution of load optimization at shakedown is complicated because the stress-strain state of dissipative systems (e. g. the plate deforming) depends on their loading history (Lange-Hansen 1998). The load optimization problem is formulated by integrating extreme energy principles and methods of mathematical programming theory. A new algorithm for the problem combining Mises and Tresca yield criterion for adapted flexural plates optimization based on the Rosen project gradient method is proposed in this paper (Čyras and Atkočiūnas 1984; Atkočiūnas *et al.* 2007a, 2007b, 2008). The algorithm is based on the linear Tresca yield criterion. When the optimal solution is obtained, the von Mises yield criterion is applied in the latest step. The proposed algorithm simplifies the numerical solution of the complicated optimization problem when the Mises yield criterion is applied.

2. The main dependencies of a discrete plate

The discrete model of a symmetric round plate in the polar coordinate system $\boldsymbol{x} = (\rho, \theta)^T$ is obtained by dividing the plate into $k = 1, 2, ..., s$ ($k \in K$) circular finite elements with s_k nodes $l = 1, 2, s_k = 3$ $(l \in L)$, where the master nodes are numbered 1 and 3, respectively (see Fig. 1). The polar coordinate system is located in the center of the plate. It is enough to investigate only one radius of the plate because of the internal forces and the displacements do not depend on the coordinate Θ. Consequently, the second order circular element (the internal forces approximated by a second order polynomial) with three nodes, distributed along the radius ρ, is used. The finite elements are numbered along the radius in a consecutive order, starting from the center of the plate.

The circular plate can be subjected by a uniformly distributed load and linearly distributed load located on the plate's boundaries. The properties of the material (modulus of elasticity E and Poisson coefficient ν), thickness t and intensity of the distributed load q remain constant in the whole finite element. The functions

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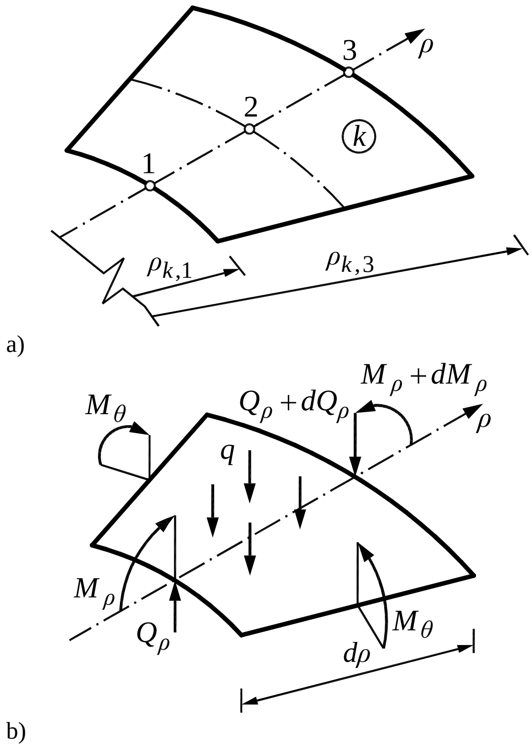

Fig. 1. The finite element of a round plate (a); the positive directions of internal forces (b)

of the internal forces distribution can have discontinuities (in the place of master nodes) when the equilibrium of finite elements are applied (Belytschko 1972; Belytschko *et al.* 2000; Gallager 1975; Faccioli and Vitiello 1973; Kalanta 1995) for elastic-plastic plates. Therefore, the finite elements have their own master nodes and sections under investigation and are indexed by the double index kl ($k \in K$, $l \in L$) or by common section index $i = 1, 2, ..., \zeta = s \times s_k$ ($i \in I$) for the discrete plate model. The vectors of internal forces of the finite element k are:

$$\mathbf{M}_k = \left(M_{\rho,k1}, M_{\theta,k1}, M_{\rho,k2}, M_{\theta,k2}, M_{\rho,k3}, M_{\theta,k3}\right)^T =$$

$$\left(\mathbf{M}_{k1}, \mathbf{M}_{k2}, \mathbf{M}_{k3}\right)^T = \left(\mathbf{M}_{kl}\right)^T . \quad (1)$$

Here, $\mathbf{M}_{k1} = (M_{\rho,kl}, M_{\Theta,kl})^T$, and the indexes ρ and Θ denote the radial and angular internal moments, respectively; the positive directions are shown in Fig. 1b.

The bending moments' interpolation function, in applying the finite element k shape function $\mathbf{N}_k(\rho)$ is:

$$\mathbf{M}_k(\rho) = \mathbf{N}_k(\rho)\mathbf{M}_k . \quad (2)$$

The functions (2) do not satisfy the plate element equations:

$$\left(-\frac{d^2}{d\rho^2} - \frac{2}{\rho}\frac{d}{d\rho}\right)M_\rho + \frac{1}{\rho}\frac{d}{d\rho}M_\Theta = q \quad \text{or}$$

$$\mathsf{A}\boldsymbol{M}(\rho) = q . \quad (3)$$

Therefore, equilibrium for the plate elements is assured for the elements and master nodes (Karkauskas 1994). The algebraic equilibrium equation for the finite element is obtained after differentiating the expression (3) which was applied (2):

$$\mathbf{A}_k(\rho)\mathbf{M}_k = q_k , \quad (4)$$

where

$$\mathbf{A}_k(\rho) = \mathsf{A}\mathbf{N}_k(\rho). \quad (5)$$

The separate elements are joined to a system by writing the equilibrium equations for the master nodes of the adjacent elements. Thus, the continuity of the radial moments M_ρ and the shear forces Q_ρ are ensured. The set of plate equilibrium equations while the boundary conditions are applied are:

$$[\mathrm{A}]\mathbf{M} = \mathbf{F} \ \text{ or } \sum_k [\mathrm{A_k}]\mathbf{M}_k = \mathbf{F} . \quad (6)$$

The dimension of the matrix $[\mathrm{A}]$ is $(m \times n)$, where $n = \zeta \times 2$. The geometrical equations for the discrete plate model are obtained by applying the virtual stress principle:

$$\delta\mathbf{F}^T\mathbf{u} = \sum_k \int_{A_k} \delta\mathbf{M}_k^T(\rho)\mathsf{D}\mathbf{M}_k(\rho)dA . \quad (7)$$

and by using equations (2) and (6):

$$\sum_k \delta\mathbf{M}_k^T[\mathrm{A}]_k^T\mathbf{u} = \sum_k \delta\mathbf{M}_k^T[\mathrm{D_k}]\mathbf{M}_k . \quad (8)$$

Here, the symmetric flexibility matrix $[\mathrm{D_k}]$ of the element k is calculated by the formula:

$$[\mathrm{D_k}] = \int_{A_k} \mathbf{N}_k^T(\rho)\mathsf{D}\mathbf{N}_k(\rho)dA . \quad (9)$$

The geometrical equations for the finite element are:

$$[\mathrm{A_k}]^T\mathbf{u} - [\mathrm{D}_k]\mathbf{M}_k = \mathbf{0} \quad (10)$$

and for whole discrete plate model:

$$[\mathrm{A}]^T\mathbf{u} - [\mathrm{D}]\mathbf{M} = \mathbf{0} . \quad (11)$$

Here, $[\mathrm{D}]$ is the quasidiagonal flexibility matrix of the elements. The sequence of the equilibrium equations $[\mathrm{A}]\mathbf{M} = \mathbf{F}$ determine the physical meaning of the components of the displacements vector $\mathbf{u}$.

If the transition to the plastic state is described via the nonlinear Mises-Huber yield condition:

$$M_\rho^2 - M_\rho M_\Theta + M_\Theta^2 \le (M_0)^2 . \quad (12)$$

The plasticity condition is verified in all the nodes of the finite element:

$$\mathbf{M}_{kl}^T[\Pi_{kl}]\mathbf{M}_{kl} \le (M_{0k})^2, \ k \in K, \ l \in L . \quad (13)$$

Here, $[\Pi_{kl}]$ is the matrix of the Mises-Huber plasticity condition for the bending circular plate

$$\left[\Pi_{kl}\right] = \begin{bmatrix} 1 & -0{,}5 \\ -0{,}5 & 1 \end{bmatrix}. \tag{14}$$

The plasticity condition is often expressed in the following form:

$$\varphi_{kl} = \left(M_{0k}\right)^2 - \mathbf{M}_{kl}^T \left[\Pi_{kl}\right]\mathbf{M}_{kl} \geq 0 . \tag{15}$$

The bending moment limit is constant in the entire finite element: $M_{0k} = const$. If the linear Tresca plasticity condition is applied, the equation (15) is described as:

$$\varphi_{kl} = \mathbf{C}_{kl} - \mathbf{\Phi}_{kl}\mathbf{M}_{kl} \geq \mathbf{0} \cdot \tag{16}$$

The Tresca plasticity condition matrix $\mathbf{\Phi}_{kl}$ is:

$$\mathbf{\Phi}_{kl} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \\ -1 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \tag{17}$$

The vector of the limit moments C_{kl} match the matrix $\mathbf{\Phi}_{kl}$. For the sake of simplicity, the calculation sections will be indexed as $i = 1, 2, ..., \zeta$, $i \in I$.

3. The main dependencies in the case of cyclic loading

In the practice of engineering, it is necessary to know the deformed state of the plate under plastic deformation just before its cyclic plastic failure (plate geometry, limit moments M_0 and load $\mathbf{F}$ are known) (Kalanta *et al.* 2009; Jankovski and Atkočiūnas 2008). Such a type of structural mechanics problem is referred to as an analysis problem (Cyras 1983). In such a case, it is useful to separate the elastic moments M_e and residual moments M_r : $\mathbf{M}_i = \mathbf{M}_{ei} + \mathbf{M}_{ri}$, $i \in I$. The elastic moments can be calculated by the formula $\mathbf{M}_e = [\alpha]\mathbf{F}$, where the moments influence matrix $[\alpha]$ have the following dimensions $(n \times m)$. When the load $\mathbf{F}(t)$ is a function of time t :

$$\mathbf{M}_i(t) = \mathbf{M}_{ei}(t) + \mathbf{M}_{ri} , \ i \in I . \tag{18}$$

If RVL is described by their variation boundaries as $\mathbf{F}_{inf}$, $\mathbf{F}_{sup}$, it is possible to determine the possible load combination count p ($j = 1, 2, ..., p$; $j \in J$) and the equation (18) is rewritten as:

$$\mathbf{M}_{ij} = \mathbf{M}_{ei,j} + \mathbf{M}_{ri} , \ i \in I . \tag{19}$$

The determination of $\mathbf{M}_{ei,j}$ is described in the work (Pham 2003). Then, the Mises-Huber plasticity condition (15) is rewritten as follows:

$$\varphi_{ij} = \left(M_{0k}\right)^2 - \mathbf{M}_{ij}^T \left[\Pi_i\right]\mathbf{M}_{ij} \geq 0 , \ i \in I , \ j \in J . \tag{20}$$

Thus, in the analysis of shakedown structures, it is the convenient separate residual moments $\mathbf{M}_r$, residual displacements $\mathbf{u}_r$ and deformations $\mathbf{\theta}_r = [\mathrm{D}]\mathbf{M}_r + \mathbf{\theta}_p$. Then, the equilibrium equations (6) and geometrical equations (11) are described by mentioned terms:

$$[\mathrm{A}]\mathbf{M}_r = \mathbf{0} \ \text{or} \ \sum_k [\mathrm{A}]_k \mathbf{M}_{rk} = \mathbf{0} \tag{21}$$

and

$$[\mathrm{A}]^T \mathbf{u}_r = [\mathrm{D}]\mathbf{M}_r + \mathbf{\theta}_p . \tag{22}$$

The components of the plastic deformation's vector $\mathbf{\theta}_p = \left(\mathbf{\theta}_{p,i}\right)$ are calculated by formula:

$$\mathbf{\theta}_{p,i} = \sum_j \left[\nabla \varphi_{ij}\left(\mathbf{M}_{ei,j} + \mathbf{M}_{ri}\right)\right]^T \lambda_{ij} ,$$

$$\lambda_{ij} \geq 0 , \ i \in I , \ j \in J . \tag{23}$$

Here, λ_{ij} is the plastic multiplier vector; $\left[\nabla \varphi_{ij}\right]$ – a matrix composed from the gradients of the plasticity conditions (20).

4. The mathematical models of the analysis problem

The static formulation of the analysis problem is based on the additional energy minimum principle and in the case of Mises plasticity conditions:

find

$$\min \frac{1}{2} \sum_k \mathbf{M}_{rk}^T \left[\mathrm{D}_k\right]\mathbf{M}_{rk} , \tag{24}$$

when

$$\sum_k \left[\mathrm{A}_k\right]\mathbf{M}_{rk} = \mathbf{0} , k \in K , \tag{25}$$

$$\varphi_{ij} = \left(M_{0i}\right)^2 - \left(\mathbf{M}_{ei,j} + \mathbf{M}_{ri}\right)^T \left[\Pi_i\right]\left(\mathbf{M}_{ei,j} + \mathbf{M}_{ri}\right) \geq 0 ,$$
$$i \in K , \ j \in J . \tag{26}$$

The optimal solution of the problem (24)–(26) is $\mathbf{M}_r^*$.

The kinematic formulation of the problem under analysis is created in accordance with the mathematical programming duality theory:

find

$$\max \left\{ -\frac{1}{2}\mathbf{M}_{rk}^T \left[\mathrm{D}_k\right]\mathbf{M}_{rk} - \sum_i \sum_j \lambda_{ij} \left[\nabla \varphi_{ij}\right]\mathbf{M}_{ri} - \sum_i \sum_j \lambda_{ij} \left[\left(M_{0i}\right)^2 - \mathbf{M}_{ij}^T \left[\Pi_i\right]\mathbf{M}_{ij}\right] \right\}, \tag{27}$$

when

$$\left[\mathrm{D}_k\right]\mathbf{M}_{rk} + \sum_j \left[\nabla \varphi_{kj}\right]^T \mathbf{\lambda}_{kj} - \left[\mathrm{A}_k\right]^T \mathbf{u}_r = \mathbf{0} , \tag{28}$$

$$\mathbf{\lambda}_{kj} \geq \mathbf{0} , \ k \in K , \ i \in I , \ j \in J . \tag{29}$$

The optimal solution of the kinematic formulation (27)–(29) is $\mathbf{M}_r^*$, $\boldsymbol{\lambda}_{kj}^*$, $\mathbf{u}_r^*$.

In the case of the Tresca plasticity condition, only equation (26) should be changed:

$$\varphi_{ij} = \mathbf{C}_i - [\Phi_i](\mathbf{M}_{ei,j} + \mathbf{M}_{ri}) \geq \mathbf{0}. \quad (30)$$

The vector $\mathbf{C}_i$ contains the limit moments of the corresponding finite element.

5. The influence matrixes of the residual displacements and residual moments

If the solution of the static (24)–(26) and kinematic (27)–(29) analysis problem is unknown, then it can be obtained from the nonlinear set of equations:

$$[\mathrm{A}]\mathbf{M}_r = \mathbf{0}, \quad (31)$$

$$\varphi_{ij} = (M_{0k})^2 - \mathbf{M}_{ij}^T[\Pi_i]\mathbf{M}_{ij}, \quad (32)$$

$$\lambda_{ij}\left[(M_{0k})^2 - \mathbf{M}_{ij}^T[\Pi_i]\mathbf{M}_{ij}\right] = 0, \ \lambda_{ij} \geq 0, \quad (33)$$

$$[\mathrm{D}]\mathbf{M}_r + \sum_j [\nabla\varphi_j]^T \boldsymbol{\lambda}_j - [\mathrm{A}]^T \mathbf{u}_r = \mathbf{0}, \quad (34)$$

$$\boldsymbol{\lambda}_j \geq (\lambda_{ij}), \ i \in I, \ j \in J. \quad (35)$$

The equation set is composed of the constraints of the static formulation problem (24)–(26) and the Kuhn–Tucker conditions (Bazaraa *et al.* 2004). When the plastic deformations $\boldsymbol{\theta}_p^*$ are known, then from the set of equations

$$\boldsymbol{AM}_r^* = \mathbf{0},$$

$$\boldsymbol{DM}_r^* + \boldsymbol{\theta}_p^* - \boldsymbol{A}^T \boldsymbol{u}_r^* = \mathbf{0}$$

it is possible to find the right values of $\mathbf{M}_r^*$ and $\mathbf{u}_r^*$:

$$\mathbf{u}_r^* = \left([\mathrm{A}][\mathrm{D}]^{-1}[\mathrm{A}]^T\right)^{-1}[\mathrm{A}][\mathrm{D}]^{-1}\boldsymbol{\theta}_p^* = [\overline{\mathbf{H}}]\boldsymbol{\theta}_p^*, \quad (36)$$

$$\mathbf{M}_r^* = \left[[\mathrm{D}]^{-1}[\mathrm{A}]^T\left([\mathrm{A}][\mathrm{D}]^{-1}[\mathrm{A}]^T\right)^{-1}[\mathrm{A}][\mathrm{D}]^{-1}\right]\boldsymbol{\theta}_p^*;$$

$$\mathbf{M}_r^* = [\overline{\mathbf{G}}]\boldsymbol{\theta}_p^*. \quad (37)$$

The vectors $\mathbf{u}_r^*$ and $\mathbf{M}_r^*$, calculated by formulas (36) and (37), respectively, coincide with the optimal ones calculated by the mathematical models (24)–(26) and (27)–(29).

The residual displacement and residual moments influence matrixes $[\overline{\mathrm{H}}]$ and $[\overline{\mathrm{G}}]$, and in the case of Tresca plasticity conditions, do not depend on internal forces $\mathbf{M}_j$:

$$\mathbf{u}_r^* = [\overline{\mathrm{H}}][\Phi]^T \boldsymbol{\lambda}^* = [\mathrm{H}]\boldsymbol{\lambda}^*, \ \mathbf{M}_r^* = [\overline{\mathrm{G}}][\Phi]^T \boldsymbol{\lambda}^* = [\mathrm{G}]\boldsymbol{\lambda}^*. \quad (38)$$

This feature has an important significance for the creation of the mathematical models for the load optimization problem: initially, the Tresca yield condition is applied and only in the latest step is the Mises plasticity criterion applied.

6. The algorithm of RVL optimization

The shakedown plate is safe in respect to plastic collapse, but it can exceed the requirements of serviceability (i.e. stiffness constraints). Therefore, in the mathematical model of the plate load, optimization should not only be included in the requirements of the strength (plasticity), but the constraints for displacements, too. The mathematical model in the case of Tresca plasticity conditions is:

find

$$\max\left(\mathbf{T}_{sup}^T \mathbf{F}_{sup} + \mathbf{T}_{inf}^T \mathbf{F}_{inf}\right) \quad (39)$$

when

$$\varphi_{ij} = \mathbf{C}_i - [\Phi_i](\mathbf{M}_{ei,j} + [\mathrm{G}]\boldsymbol{\lambda}) \geq \mathbf{0}, \quad (40)$$

$$\boldsymbol{\lambda}_{ij}\left[\mathbf{C}_i - [\Phi_i](\mathbf{M}_{ei,j} + [\mathrm{G}]\boldsymbol{\lambda})\right] = 0, \quad (41)$$

$$\boldsymbol{\lambda} = (\boldsymbol{\lambda}_{ij}), \ i \in I, \ j \in J, \quad (42)$$

$$\mathbf{u}_{min} \leq [\mathrm{H}]\boldsymbol{\lambda} + \mathbf{u}_{e,inf}, \quad (43)$$

$$[\mathrm{H}]\boldsymbol{\lambda} + \mathbf{u}_{e,sup} \leq \mathbf{u}_{max}. \quad (44)$$

Here, $\mathbf{u}_{e,sup}$ and $\mathbf{u}_{e,inf}$ are the maximal and minimal elastic displacements, respectively. They, summarized together with the residual displacements $\mathbf{u}_r$, should not exceed the prescribed maximal and minimal displacements boundaries, $\mathbf{u}_{max}$ and $\mathbf{u}_{min}$. The solution of the optimization problem is $\mathbf{F}_{sup}^*$, $\mathbf{F}_{inf}^*$, $\boldsymbol{\lambda}^*$. The algorithm of the load optimization problem illustrating the switch from Tresca to the Mises plasticity condition is shown in Fig. 2.

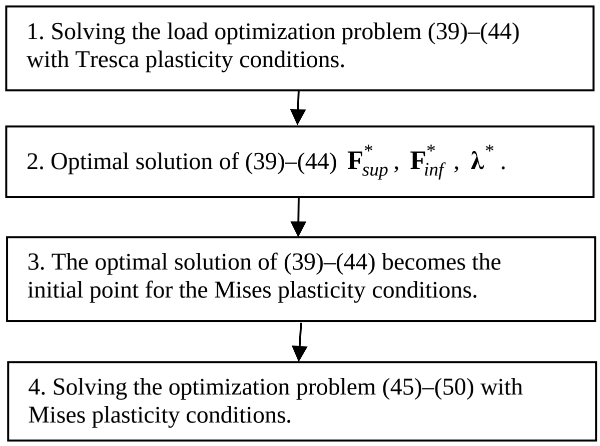

Fig. 2. The algorithm of load optimization with Tresca and Mises plasticity conditions

The mathematical model of the load optimization problem in the case of Mises plasticity conditions is composed using the influence matrixes $[\mathrm{G}]$ and $[\mathrm{H}]$:

find

$$\max \left(\mathbf{T}_{sup}^{T}\mathbf{F}_{sup} + \mathbf{T}_{inf}^{T}\mathbf{F}_{inf}\right), \quad (45)$$

when

$$\varphi_{ij} = (M_{0i})^2 - \left(\mathbf{M}_{ei,j} + [\mathrm{G}]\boldsymbol{\lambda}\right)^T [\Pi_i]\left(\mathbf{M}_{ei,j} + [\mathrm{G}]\boldsymbol{\lambda}\right) \geq 0, \quad (46)$$

$$\boldsymbol{\lambda}_{ij}\left[(M_{0i})^2 - \left(\mathbf{M}_{ei,j} + [\mathrm{G}]\boldsymbol{\lambda}\right)^T [\Pi_i]\left(\mathbf{M}_{ei,j} + [\mathrm{G}]\boldsymbol{\lambda}\right)\right] = 0, \quad (47)$$

$$\lambda_{ij} > 0, \; \boldsymbol{\lambda} = (\boldsymbol{\lambda}_{ij}), \; i \in I, \; j \in J, \quad (48)$$

$$\mathbf{u}_{min} \leq [\mathrm{H}]\boldsymbol{\lambda} + \mathbf{u}_{e,inf}, \quad (49)$$

$$[\mathrm{H}]\boldsymbol{\lambda} + \mathbf{u}_{e,sup} \leq \mathbf{u}_{max}. \quad (50)$$

The graphical illustration of the switch from Tresca to Mises plasticity conditions is shown in Fig. 3.

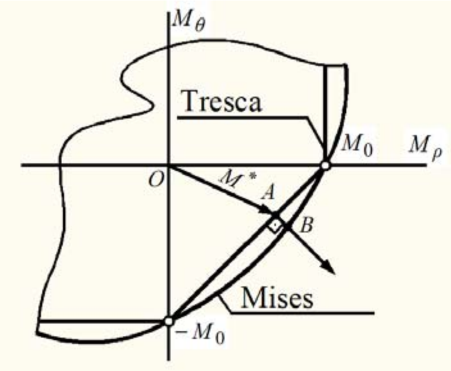

Fig. 3. The fragment of the switch from Tresca plasticity conditions to Mises plasticity conditions

7. Numerical example

The proposed calculation technique is illustrated by the example of a circular plate with a hole in the middle (Fig. 4). The supports are applied in the outside boundary of plate.

Radius of plate $R = 1.0$ m, height $h = 0.025$ m, diameter of hole $d = 0.30$ m. The material – steel, $E = 210$ GPa, $\nu = 0.3$, $\sigma_y = 235$ MPa. The limit moment of the plate $M_0 = \frac{1}{4}\sigma_y t^2 = 36.719$ kNm.

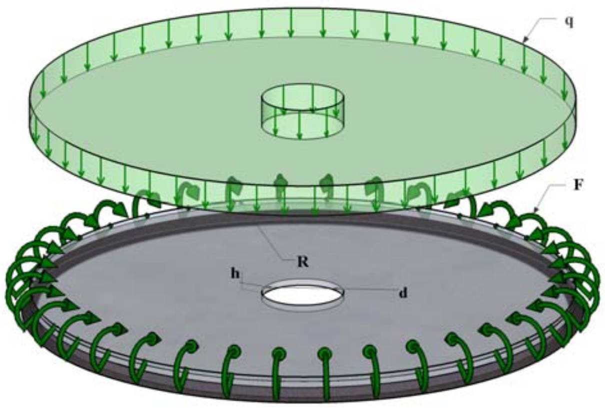

Fig. 4. The geometry of the round plate and boundary conditions

The outside boundary of the plate is loaded by the uniformly distributed linear moment $M = 5.0$ kNm/m, and the surface of the plate is subjected to a uniformly distributed load q, which is an unknown of the optimization problem. The displacement variations have boundaries which are $u_{min} = 0.$ m, $u_{max} = 0.037$ m in the place of the hole. When the problem (39)–(44) was solved, the optimal load of $q^* = 131.246$ kPa was obtained. In the case of the Mises plasticity condition, the following more optimal solution was obtained: $q^* = 140.747$ kPa.

8. Conclusions

1. The influence matrixes of residual moments and displacements do not depend on the residual moments of $\mathbf{M}_r$.

2. In the case of Mises plasticity conditions, the influence matrixes should be formulated using the gradients of plasticity conditions, which themselves depend on $\mathbf{M}_r$. The main load optimization problem, in the case of Mises, becomes practically not realizable, even with applied computer algebra methods.

3. One of the possible resolutions of the load optimization problem with a Mises plasticity condition is the application of an analogous problem solution obtained with Tresca plasticity conditions.

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INTEGRUOTAS TAMPRIAI PLASTINĖS SIMETRINĖS PRISITAIKANČIOS PLOKŠTĖS APKROVOS OPTIMIZAVIMAS

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S a n t r a u k a

Nagrinėjama tampriai plastinė simetrinė lenkiama plokštė, veikiama kintamosios kartotinės apkrovos. Prisitaikančių konstrukcijų įtempių ir deformacijų būvis priklauso nuo apkrovimo istorijos. Plokštės apkrovos optimizavimo uždavinio matematiniame modelyje naudojamos stiprumo ir standumo sąlygos. Į apkrovimo istoriją atsižvelgiama, pasitelkiant ekstremines įražų ir įlinkius ribojančias jų normines reikšmes. Remiantis Rozeno projektuojamųjų gradientų metodu sukurtas naujas apkrovos optimizavimo algoritmas, derinantis Mizeso ir Treska takumo sąlygas. Skaitinio pavyzdžio rezultatai gauti originalia autorių kompiuterine programa.

Reikšminiai žodžiai: prisitaikymas, ekstreminiai energetiniai principai, tampriai plastinė plokštė, Mizeso ir Treska takumo sąlygos, matematinis programavimas.

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