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THREE-DIMENSIONAL NUMERICAL SIMULATION OF TSUNAMI WAVES BASED ON THE NAVIER-STOKES EQUATIONS

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ABSTRACT

A numerical algorithm of solving the three-dimensional system of Navier-Stokes equations to simulate free surface waves and flows with gravity is presented. The main problem here is to ensure that the gravity force is properly accounted in the presence of discontinuities in the medium density. The task is made more complicated due the use of unstructured computational grids with collocated placement of unknown quantities and splitting algorithms based on SIMPLE-type methods. To obtain correctly the hydrostatic pressure, it is suggested that the contribution of the gravitational force in the equation for pressure should be distinguished explicitly; the latter being calculated by using the solution of the two-phase medium gravitational balance problem. To ensure the balance of the gravity force and the pressure gradient in the case of rest an algorithm in which the pressure gradient in the equation of motion is replaced by a modification considering the gravitational force action is suggested. This method is demonstrated by the example of tsunami wave propagation in the real water area of the World Ocean.

1. INTRODUCTION

At present, there are several methods used for modeling multiphase flows with a free surface, which differ in the way the latter is calculated. The first method is based on the 'Lagrangian' approach, in which a free surface is tracked either by moving the grid nodes or by particles (Shuvalov et al., 2012). The second method is based on the 'Euler' approach, in which special markers are introduced to track the free surface. Either particles (Lucy, 1977) or spatial marker functions (Harlow and Welch, 1965; Daly, 1969; Hirt and Nichols, 1981) that obey the convective transfer equation can act in the role of the above-mentioned markers. The second method is the most applicable in practice. It uses the volume fraction of fluid (VOF – Volume-of-fluid) as a marker function (Hirt and Nichols, 1981; Ubbink, 1977). The fluid-gas system in this approach is regarded as a single one-velocity medium with variable physical properties. This method is easily generalized is case of arbitrary unstructured grids and an arbitrary number of phases (Ubbink, 1977).

The determining force for waves and fluid flows with a free surface is the gravity force. The gravity force experiences a discontinuity on the free surface. It happens due to a sharp change in the medium density, resulting in a rupture in the pressure gradient magnitude, which in the case of the medium rest completely balanced the action of gravitational forces (Landau and Lifshitz, 1987).

The construction of a numerical algorithm correctly taken into account the gravity force and the pressure gradient value calculations is a non-trivial task. This is especially true for grids with a 'collocated' arrangement of unknown quantities, which is mainly used in practice, but leads to a weak coupling between the velocity and pressure fields (Ferziger and Peric, 2002; Jasak, 1996). Using the collocated arrangement of unknown quantities implies pressure and velocity determination in the same place (usually the cell center), leading to the appearance of even-odd oscillations, which can be eliminated by the use of the Rhie-Chow type method (Rhie and Chow, 1983). Papers (Gu et al., 1991; Mencinger, 2012; Majumdar, 1988) deal with the construction of a numerical algorithm ensuring the absence of numerical oscillations in the case of non-homogeneous gravity field. The volume force in the momentum conservation equation, in which the gravity force can act as the volume force, is analyzed in (Khrabry et al., 2010). To exclude oscillations in the velocity and pressure fields, it is proposed to use a correction of the Phie-Chow type (Rhie and Chow, 1983). However, this paper does not discuss problems with a strong discontinuity in the volume force field when arises the problem of the correct pressure gradient calculation, the value of which must completely balance the volume force when the medium is at rest. The theoretical analysis of the gravity force allowance is presented in (Mencinger, 2012). In addition to ideas borrowed from (Gu et al., 1991), an expression to interpolate the volume force and pressure on the inner faces of the computational grid, which provides the state of balance, is proposed. However, the considered examples demonstrate only the absence of oscillations in the velocity field but not the analysis of the obtained free surface forms. Similarly, when using arbitrary unstructured grids, the effectiveness of the algorithm is not considered. In (Khrabry et al., 2010), is presented an effective scheme to calculate the pressure gradient in the presence of gravitational forces. The scheme is based on the interpolation of the pressure gradient value with the medium density in the adjacent cells taken into account. This algorithm allows eliminating non-physical oscillations in the velocity field near the free surface. The algorithm efficiency, however, is demonstrated only on the orthogonal grids whose lines are parallel to the gravity direction.

This paper formulates a new algorithm to construct the pressure equation based on the Rhie-Chow method. The algorithm is constructed by replacing the gravity force by its direct discrete analogue in the equation. The expressions for a direct discrete analogue are formulated on the basis of hydrostatic approximation, which provides a correct pressure field on an arbitrary unstructured grid. To ensure the balance of the gravity force and the pressure gradient when the medium is at rest, an algorithm based on the replacement of the pressure gradient in the equation of motion by its modification considering the gravity action is proposed. The effectiveness of the proposed solution is explored on the example of the numerical simulation of tsunami wave propagation.

2. THE 3D NUMERICAL ALGORITHM

The VOF method unifies the continuity and the momentum conservation equations for all phases and solves for the resulting medium, whose properties linearly depend on the volume fraction of each phase.

The general system of multiphase medium equations has the form (Ferziger and Peric, 2002):

$$\begin{cases} \rho \dfrac{\partial u_i}{\partial t} + \dfrac{\partial \rho u_i u_j}{\partial x_j} - u_i \dfrac{\partial \rho u_j}{\partial x_j} = -\dfrac{\partial p}{\partial x_i} + \dfrac{\partial}{\partial x_j}\left(\mu\left(\dfrac{\partial u_i}{\partial x_j} + \dfrac{\partial u_j}{\partial x_i}\right)\right) + \rho g_i, \\ \dfrac{\partial u_i}{\partial x_i} = 0, \\ \dfrac{\partial F_j}{\partial t} + u_i \dfrac{\partial F_j}{\partial x_j} = 0, \quad j = 1 \ldots N-1, \quad F_N = 1 - \sum_{j=1}^{N-1} F_j, \end{cases} \quad (1)$$

where $\rho = \sum_{j=1}^{N} F_j \rho_j$ is the medium resulting density, $\mu = \sum_{j=1}^{N} F_j \mu_j$ is the resulting medium viscosity, p is the pressure, u_i is the velocity component vector, N is the number of phases in the problem, F_j is the phase j volume fraction, g_i is the gravity vector. In the system (1), the equation of motion is written in the form giving the best results in the numerical solution of problems with a free surface (Landau and Lifshitz, 1987).

To solve the system (1), the classical SIMPLE/PISO-type splitting algorithms (Ubbink, 1997; Ferziger and Peric, 2002; Jasak, 1996) are used, as well as the completely implicit algorithm (Chen, Z.J., Przekwas, 2010; Darwish et al., 2009; Kozelkov et al., 2016b), the common feature of which is the derivation of the pressure equation by substituting a discrete analogue of the equation for the velocity into equation continuity. The present paper considers a discrete analogue of the velocity calculation equation with a purpose of formulating an algorithm allowing us to obtain a correct hydrostatic pressure in the cell centers in the case of the density field discontinuity. With finite-volume discretization this algorithm takes the form:

$$a_{PP}u_{i,P} + \sum_k a_{PN}u_{i,N} = R_{i,P} - \left(\frac{\partial p}{\partial x_i}\right)_P V_P + \rho g_i V_P \,, \tag{2}$$

where a_{PP} is the diagonal coefficient for the P cell, a_{PN} is the coefficient at the velocity value in the cell N adjacent to the face k, the summation is done over all internal faces, V_P is the cell volume (Fig. 1).

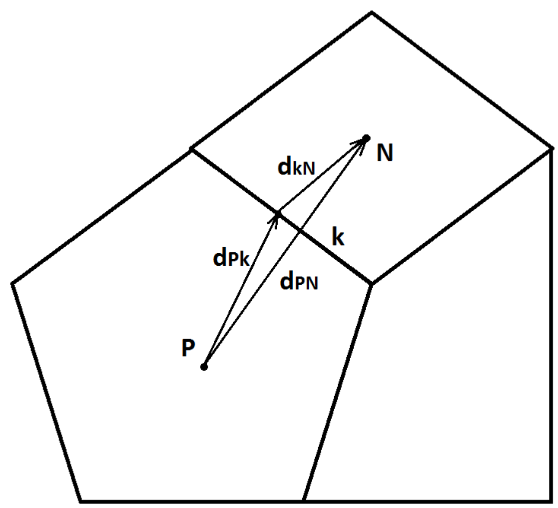

Figure 1. Two adjacent cells of the computational grid

From equation (2), the flow velocity is expressed as:

$$u_{i,P} = \frac{1}{a_{PP}}\left(R_{i,P} - \sum_k a_{PN}u_{i,N}\right) - \frac{V_P}{a_{PP}}\left(\frac{\partial p}{\partial x_i}\right)_P + \frac{V_P}{a_{PP}}\rho g_i = H_{i,P} - \frac{V_P}{a_{PP}}\left(\frac{\partial p}{\partial x_i}\right)_P + \frac{V_P}{a_{PP}}\rho g_i \,. \tag{3}$$

The velocity on the face $u_{i,k}$ is calculated by interpolating the expression (3) from the cell centers to the center of the face k with the weight λ_k . Exception is made for the pressure gradient whose contribution is replaced by its direct discrete analogue (Ubbink, 1997; Mencinger, 2012). The obtained expression for the velocity is substituted into the discrete continuity equation (1):

$$\sum_k u_{i,k} n_i S_k = \sum_k \left[\acute{I}_{\;k} n_i S_k - A_k \frac{p_N - p_P}{d_{PN}} S_k + A_k \left(\rho g_i\right)_k n_i S_k\right] = 0 \,, \tag{4}$$

where $\acute{I}_k = \lambda_k H_{i,P} + (1-\lambda_k) H_{i,N}$, $A_k = \lambda_k \frac{V_P}{a_{PP}} + (1-\lambda_k)\frac{V_N}{a_{NN}}$, $u_{i,k}$ is velocity on the face k, n_i is the normal of the face k, S_k is the area of the face, the summation being carried out over all the faces of the cell P. The weight λ_k in practice is determined in various ways, for example, starting from the geometric distances from the cell center to the face center (Jasak, 1996), the values of the diagonal coefficients a_{PP} and a_{NN} (Rhie and Chow, 1983), or $\lambda_k = 0.5$ is accepted.

Equation (4) determines the pressure field in the each cell centre. If the system is balanced, the pressure field must have a hydrostatic distribution (Landau and Lifshitz, 1987). In the presented equation entry for calculating the pressure (4) there is the gravity magnitude interpolated to the face – $\rho g_{i.}$ In order to determine its value, which allows us to obtain a correct pressure distribution, we consider a model one-dimensional problem of a resting system consisting of two fluids with different densities in the gravitational force field (Fig. 2).

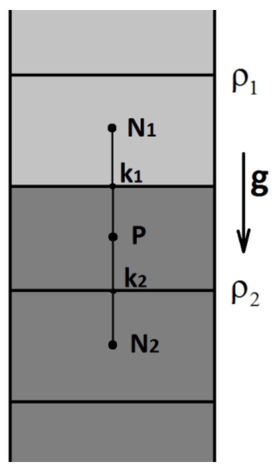

Figure 2.The system balance in the gravity field

In a state of balance, the velocities in the system are zero, so the equation for the pressure (4) takes the form:

$$A_{k_1}\left(\frac{p_{N_1}-p_P}{d_{PN_1}} - (\rho g)_{k_1} S_{k_1}\right) - A_{k_2}\left(\frac{p_{N_2}-p_P}{d_{PN_2}} - (\rho g)_{k_2} S_{k_2}\right) = 0. \quad (5)$$

The equation (5) shows that the term $(\rho g)_{k_1}$ determines the pressure drop between the cell centers P and N_1, and the term $(\rho g)_{k_1}$ does the same between the cell centers P and N_2. These pressure differences are calculated analytically (Landau and Lifshitz, 1987) under the assumption of a constant density within the cells:

$$p_{N_1} - p_P = g\left(d_{N_1k_1}\rho_1 + d_{k_1P}\rho_2\right), \tag{6}$$

where $d_{N_1k_1}$ is the distance from the cell N_1 center to face k_1, d_{k_1P} is the distance from face k_1 to the cell P center.

Taking this into account, the equation (5) takes the form:

$$A_{k_1}\left(\frac{p_{N_1} - p_P}{d_{PN_1}} - g\frac{\left(d_{N_1k_1}\rho_1 + d_{k_1P}\rho_2\right)}{d_{PN_1}}\right) - A_{k_2}\left(\frac{p_{N_2} - p_P}{d_{PN_2}} - g\frac{\left(d_{N_2k_2}\rho_1 + d_{k_2P}\rho_2\right)}{d_{PN_2}}\right) = 0\,. \tag{7}$$

Obviously, the solution of the equation (7) leads to a pressure drop corresponding to the hydrostatic solution (6).

In the three-dimensional case, the hydrostatic pressure drop between the adjacent cells P and N is given by:

$$p_N - p_P = d_{i,Nk}g_i\rho_N + d_{i,kP}g_i\rho_P \quad . \tag{8}$$

Taking into account (8), the equation (4) can be written in the form:

$$\sum_k u_{i,k}n_iS_k = \sum_k\left[Í_{\ k}n_iS_k - A_k\frac{p_N - p_P}{d_{PN}}S_k + A_kd_{i,Nk}g_i\rho_N\right] = 0 \quad . \tag{9}$$

The equation (9) allows us to ensure the calculation of the correct hydrostatic pressure field in the cell centers for any method of interpolating the vector H_i on the face of the calculated model.

3. THE PRESSURE GRADIENT ALGORITHM CALCULATION

To balance gravity by the pressure gradient when the free surface is at rest, it is also necessary to ensure the correct pressure gradient calculation near the free surface. It was shown in (Mencinger, 2012) that the pressure gradient calculation done by conventional methods, for example, by the Gauss method (Jasak, 1996) or by the least squares method, leads to the incorrect result due to the existence of a pressure field kink in the free surface. To solve this problem, (Khrabry et al., 2010) suggests using the Gauss method with a modified expression to interpolate the pressure value on the inner faces of the cell P:

$$\left(\frac{\partial p}{\partial x_i}\right)_P = \frac{1}{V_P}\sum_k p_kS_{i,k}\ ,$$

where

$$p_k = \xi\left(\frac{p_P d_{kN}\rho_N + p_N d_{Pk}\rho_P}{d_{kN}\rho_N + d_{Pk}\rho_P}\right) + (1-\xi)\left(\frac{p_P d_{kN} + p_N d_{Pk}}{d_{kN} + d_{Pk}}\right), \tag{10}$$

a $\xi = \cos^2\alpha$, α is the angle between the normal of the face k and the direction of the gravitational force. This approach allows us to estimate correctly the pressure gradient near the free surface in the case of using an orthogonal computational grid with grid lines parallel to the direction of the gravitational force. However, as shown below, the use of expression (10) in the case of an unstructured grid leads to an unsatisfactory result.

In this paper, we propose to use another method that allows obtaining good results on any computational grid type. The main idea of the method is as follows: it is not necessary to calculate the pressure gradient in order to solve the motion equation. For this purpose it is enough to calculate the expression representing the simultaneous contribution of the pressure gradient and the force of gravity. To calculate such an expression, we introduce the variable p^*, which is a modified pressure field:

$$\frac{\partial p^*}{\partial x_i} = \frac{\partial p}{\partial x_i} - \rho g_i . \tag{11}$$

To calculate the right-hand side of the equation (3) with the newly introduced variable, we write the integral expression (11) for the cell volume P:

$$\int_{V_P} \frac{\partial p^*}{\partial x_i} dV = \int_{V_P} \frac{\partial p}{\partial x_i} dV - \int_{V_P} \rho g_i dV .$$

The transition to the surface integral gives:

$$\int_{V_P} \frac{\partial p^*}{\partial x_i} dV = \int_{S_P} (p - G) n_i dS , \tag{12}$$

where G is the antiderivative of the function ρg_i, which, assuming the density inside the cells constant, can be written as:

$$G = \rho g_i r_i + C,$$

where r_i is the radius vector, C is an arbitrary constant. The presence of the constant C allows us to select the reference point of the vector r_i in an arbitrary way, for the cell P we choose it in the center of the cell, then:

$$G = \rho g_i \left(r_i - r_{P,i} \right). \tag{13}$$

The substitution of (13) into (12) and finite-volume discretization (13) yields:

$$\int_{V_P} \frac{\partial p^*}{\partial x_i} dV = \int_{S_P} \left(p - \rho g_i \left(r_i - r_{P,i} \right) \right) n_i dS \approx \sum_k \left[p - \rho g_i \left(r_i - r_{P,i} \right) \right]_k n_{k,i} S_k \ .$$

The second term in the square brackets of the last expression is the contribution of hydrostatic pressure to the total pressure. This contribution is calculated with respect to the center of the cell P. At the center of the cell P it goes to zero, and at the center of the next cell N it is calculated according to the expression (8) which is used to compose the equation for pressure. Using the linear interpolation with weight λ_k to calculate the values on the face, we get:

$$\begin{aligned} \int_{V_P} \frac{\partial p^*}{\partial x_i} dV &\approx \sum_k \left[\lambda_k \left[p - \rho g_i \left(r_i - r_{P,i} \right) \right]_P + (1-\lambda_k) \left[p - \rho g_i \left(r_i - r_{P,i} \right) \right]_N \right] n_{k,i} S_k = \\ &= \sum_k \left[\lambda_k p_P + (1-\lambda_k) \left(p - d_{i,Nk} g_i \rho_N - d_{i,kP} g_i \rho_P \right)_N \right] n_{k,i} S_k . \end{aligned} \tag{14}$$

The physical meaning of the expression (14) is reduced to the fact that the gravitational force ρg_i subtracts from the pressure gradient only the part that it contributed when forming the equation for the pressure (9). This allows ensuring the pressure gradient and the force of gravity balance when the medium is at rest.

The numerical results demonstrated the effectiveness of the proposed numerical algorithms for calculating pressure and the pressure gradient in problems with a free surface are presented in (Efremov et al., 2017). Numerical experiments were carried out using the LOGOS code, which is intended for solving conjugate three-dimensional problems of convective heat and mass transfer, aerodynamics and hydrodynamics on parallel computers (Betelin et al., 2014; Deryugin et al., 2015). The LOGOS code successfully underwent verification and showed fairly good results in a series of various hydrodynamic problems, including non-stationary turbulent flow (Kozelkov et al., 2015a, 2015b, 2016b), as well as geophysical phenomena based on multiphase Navier-Stokes equations (Kozelkov et al., 2015c, 2015d, 2016a; 2016c).

4. MODELLING OF THE 2003 MONTSERRAT TSUNAMI

The presented method is of great significance when considering gravity in the numerical modeling of flows with a free surface. This significance is particularly evident in the modeling of tsunami wave propagation over large distances. In comparison with the size of the water area where tsunami wave propagates, the source is small enough: - a tsunami can spread thousands of kilometers, and the source size is only a few kilometers (for landslide tsunami) or several tens of kilometers (for tsunami of seismic origin). In this case, the ocean surface outside the tsunami wave zone should not fluctuate during the entire numerical calculation. In practice, numerical solution oscillations for a free surface are allowed to be much smaller than the amplitude of the propagating wave. Such numerical oscillations should not grow. The proposed method makes it possible to achieve a stable count and control of the numerical oscillation amplitude on arbitrary unstructured grids.

This method was used to model the 2003 tsunami generated by the pyroclastic flow descent into the water, resulting from the Soufriere volcano eruption on the island of Montserrat in the Caribbean Sea (Pelinovsky et al., 2004). In (Pelinovsky et al., 2004) two approaches were used to model it. In the first case, a hydrodynamic source in the form of a cone was used as the initial approximation, and the propagation was computed by using the shallow water code TUNAMI (Goto et at., 1997), recommended by UNESCO for tsunami studies. In the second case, the pyroclastic flow was generated using the model described in (Watts and Waythomas, 2003), and the wave propagation was computed using the FUNWAVE code (Kirby et al., 1998), based on the nonlinear-dispersion theory. Later, after adding a block to calculate various initial disturbances, this code was called GEOWAVE. A rather significant difference in the results obtained with the help of these approaches is shown in (Pelinovsky et al., 2004). This difference can be observed both in the fundamental wave height prediction and in the wave pattern as a whole. It should be noted that the use of the nonlinear-dispersion theory is the most appropriate. As the hydrodynamic source in the form of a cone (Fig. 3a) was used as the initial condition for both calculations in (Pelinovsky et al., 2004), the source generated by the model described in (Watts and Waythomas, 2003) is taken for an adequate comparison of the tsunami distribution. The source obtained by this model also has the form of a cone (Fig. 3b), the geometric parameters of which correspond to the descending pyroclastic flow. The initial tsunami wave amplitude in the source is 1.26 meters. The distance between the calculated grid nodes is equal to 500 meters.

Figure 4 shows the results of computations the tsunami propagation within the framework of the Navier-Stokes equations with the help of the LOGOS code in comparison with the calculation results employing the non-linear dispersion theory using the GEOWAVE code. As can be seen, the figures are almost identical qualitatively. The quantitative comparison of the tide gauge records presented in Fig. 5 can also be considered very satisfactory. The first incoming waves to the north-western part of the island of Guadeloupe and to the island of Antigua are almost identical. It can be seen that the first three waves are well predicted by both methods, although their heights are somewhat different. Subsequent waves are different to a greater extent, and the Navier-Stokes equations lead to a more pronounced vibrational character. The waves calculated from the nonlinear-dispersion model, however, are more rapidly damped. These differences can be associated with many factors and require additional research. These factors are primarily the models themselves; they are different and are solved by using various numerical approaches, namely, finite differences and finite volumes. Secondary factors include the grid model resolution and the numerical approximation schemes used

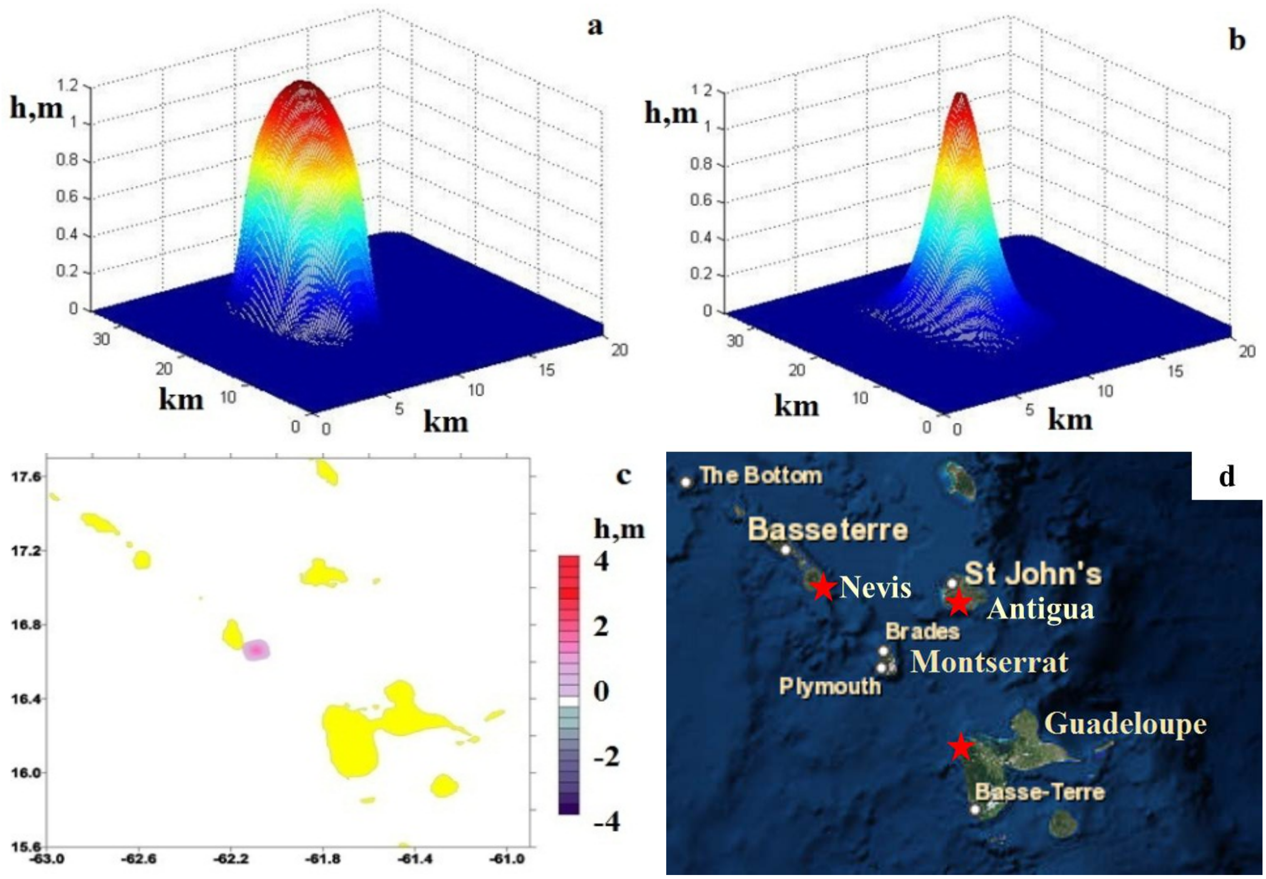

Figure 3. Initial tsunami disturbance: *a* – hydrodynamic source, *b* – the profile obtained by using the GEOWAVE code, *c* – calculation area, *d* – tide gauge positioning

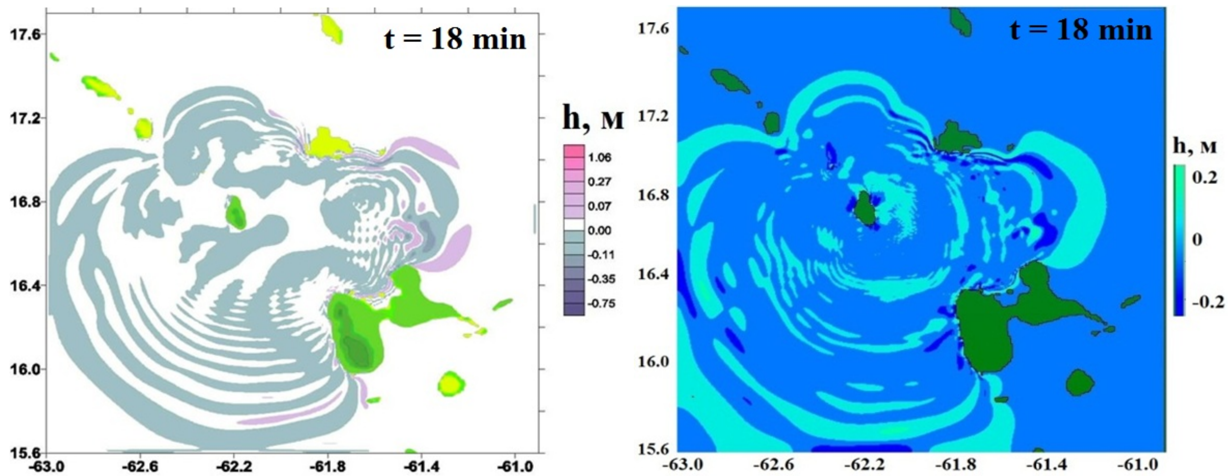

Figure 4. Tsunami propagation snapshots: GEOWAVE (left) and LOGO (right)

5. CONCLUSION

The given paper is concerned with the problem of constructing a numerical algorithm which ensures the correct calculation of the gravitational force and the pressure gradient value. The calculation is carried out in the case of medium density discontinuities, which are always present in the problems with a free surface. To obtain the correct field of hydrostatic pressure, when compiling the equation for pressure and its calculation, it is proposed to use the algorithm to isolate the gravity force contribution. In doing so, the solution of the problem of a two-phase medium gravitational equilibrium is used. The correct pressure gradient calculation in the event of gravitational force field discontinuities is ensured by using an algorithm that allows us to obtain good results on any computational grid type. The main idea of the algorithm consists in direct calculating the contribution of the pressure gradient and the gravity force to the equation of motion.

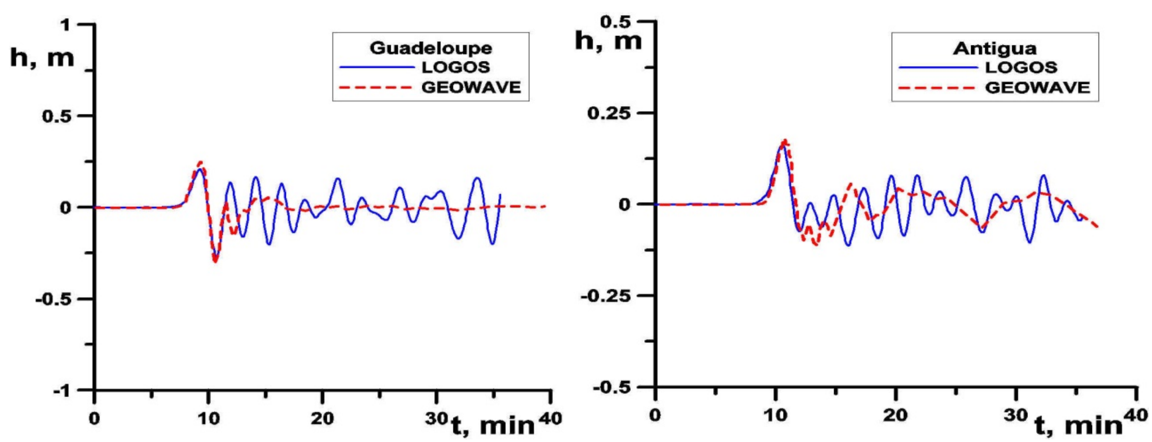

Figure 5. Comparison of tide gauge records on the islands of Guadeloupe and Antigua

The possibility of the proposed algorithm in tsunami problem use is demonstrated on the example of the tsunami simulation that occurred in the pyroclastic flow descent during the volcano eruption on the island of Montserrat in the Caribbean in 2003.

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REFERENCES

1. Betelin, V.B., Shagaliev, R.M., Aksenov, S.V., Belyakov, I.M., Deryuguin, Yu.N., Kozelkov, A.S., Korchazhkin, D.A., Nikitin, V.F., Sarazov, A.V., and Zelenskiy, D.K. Mathematical simulation of hydrogen–oxygen combustion in rocket engines using LOGOS code. Acta Astronautica, 2014, vol. 96, 53-64.
2. Chen, Z.J., and Przekwas, A.J. A coupled pressure-based computational method for incompressible/compressible flows. Journal of Computational Physics, 2010, vol. 229, 9150-9165.
3. Daly, B.J. A technique for including surface tension effects in hydrodynamic calculations. J. Comput. Phys, 1969, vol. 4, 97-117.
4. Darwish, M., Sraj, I., and Moukalled, F. A coupled finite volume solver for the solution of incompressible flows on unstructured grids. Journal of Computational Physics, 2009, vol. 228, 180-201.
5. Deryugin, Yu.N., Zhuchkov, R.N., Zelenskiy, D.K., Kozelkov, A.S., Sarazov, A.V., Kudimov, N. F., Lipnickiy, Yu.M., Panasenko, A.V., and Safronov, A.V. Validation results for the LOGOS multifunction software package in solving problems of aerodynamics and gas dynamics for the lift-off and injection of launch vehicles. Mathematical Models and Computer Simulations, 2015, vol. 7, no. 2, 144-153.
6. Efremov, V.R., Kozelkov, A.S., Kornev, A.V., Kurkin, A.A., Kurulin, V.V., Strelets, D.Yu., and Tarasova, N.V. Method for taking into account gravity in free-surface flow simulation. Computational Mathematics and Mathematical Physics, 2017, vol. 57, no. 10, 1720-1733.
7. Ferziger, J.H., and Peric, M. Computational Method for Fluid Dynamics. Springer-Verlag. New York, 2002.
8. Goto, C., Ogawa, Y., Shuto, N., and Imamura, N. Numerical method of tsunami simulation with the leap-frog scheme (IUGG/IOC Time Project), IOC Manual, UNESCO, no. 35, 96 p., 1997.
9. Gu, C.Y., Taylor C., and Chin, J.H. Computation of flows with large body forces. Numerical Methods in Laminar and Turbulent Flow, Pineridge Press, Swansea, 1991, 294-305.
10. Harlow, F.H., and Welch, J.E. Numerical calculation of time-dependent viscous incompressible flow. Phys. Fluids. 1965, vol. 8, 2182-2189.
11. Hirt, C.W., and Nichols, B.D. Volume of fluid (VOF) method for the dynamics of free boundaries. Journal of computational physics. 1981, vol. 39, 201-226.

13. Jasak, H. Error Analysis and Estimation for the finite volume method with applications to fluid flows. PhD Thesis, Department of Mechanical Engineering, Imperial College, 1996.
14. Khrabry, A.I., Smirnov, E.M., and Zaytsev, D.K. Solving the Convective Transport Equation with Several High-Resolution Finite Volume Schemes: Test Computations. Computational Fluid Dynamics 2010 – Proceedings of the 6th International Conference on Computational Fluid Dynamics, ICCFD 2010, 535-540.
15. Kirby, J., Wei, G., Chen, Q., Kennedy, A., and Dalrymple, R. Fully Nonlinear Boussinesq Wave Model Documentation and Users Manual. Center for Applied Coastal Research Department of Civil Engineering University of Delaware, Newark DE 19716, Research Report №. CACR-98-06, 1998.
16. Kozelkov, A.S., Krutyakova, O.L., Kurkin, A.A., Kurulin, V.V., and Tyatyushkina, E.S. Zonal RANS–LES Approach Based on an Algebraic Reynolds Stress Model. Fluid Dynamics, 2015b, vol. 50, no. 5, 621-628.
17. Kozelkov, A.S., Kurkin, A.A., and Pelinovskii, E.N. Effect of the Angle of Water Entry of a Body on the Generated Wave Heights. Fluid Dynamics, 2016c, vol. 51, no. 2, 288-298.
18. Kozelkov, A.S., Kurkin, A.A., Pelinovskii, E.N., and Kurulin, V.V. Modeling the Cosmogenic Tsunami within the Framework of the Navier–Stokes Equations with Sources of Different Types. Fluid Dynamics, 2015c, vol. 50, no. 2, 306-313.
19. Kozelkov, A.S., Kurkin, A.A., Pelinovskii, E.N., Kurulin, V.V., and Tyatyushkina, E.S. Modeling the Disturbances in the Lake Chebarkul Caused by the Fall of the Meteorite in 2013. Fluid Dynamics, 2015d, vol. 50, no. 6, 134-149.
20. Kozelkov, A.S., Kurkin, A.A., Pelinovsky, E.N., Tyatyushkina, E.S., Kurulin, V.V., and Tarasova, N.V. Landslide-type tsunami modelling based on the Navier-Stokes Equations. Science of Tsunami Hazards, 2016a, vol. 35, no. 3, 106-144.
21. Kozelkov, A., Kurulin, V., Emelyanov, V., Tyatyushkina, E., and Volkov, K. Comparison of convective flux discretization schemes in detached-eddy simulation of turbulent flows on unstructured meshes. Journal of Scientific Computing, 2015a, no. 89, 1-16.
22. Kozelkov, A.S., Kurulin, V.V., Lashkin, S.V., Shagaliev, R.M., and Yalozo, A.V. Investigation of Supercomputer Capabilities for the Scalable Numerical Simulation of Computational Fluid Dynamics Problems in Industrial Applications. Computational Mathematics and Mathematical Physics, 2016b, vol. 56, no. 8, 1506-1516.
23. Landau, L.D., and Lifshitz, E.M. Fluid Mechanics, Second Edition. Pergamon Press, 1987.
24. Lucy, L.B. A numerical approach to the testing of the fission hypothesis, Astron. J., 1977, vol. 82 (12), 1013-1024.
25. Majumdar, S. Role of under-relaxation in momentum interpolation for calculation of flow with nonstaggered grids. Numer. Heat Transfer., 1988, vol. 13, 125-132.
26. Mencinger, J. An Alternative Finite Volume Discretization of Body Force Field on Collocated Grid. Finite Volume Method - Powerful Means of Engineering Design, 2012, 101-116.
27. Pelinovsky, E., Kozelkov, A., Zahibo, N., Dunkly, P., Edmonds, M., Herd, R., Talipova, T., and Nikolkina, I. Tsunami generated by the volcano eruption on July 12-13, 2003 at Montserrat, Lesser Antilles. Science of Tsunami Hazards, 2004, vol. 22, no. 1, 44-57.
28. Rhie, C.M., and Chow, W.L. Numerical study of the turbulent flow past an airfoil with trailing edge separation, AIAA Journal, 1983, vol. 21, 1525-1532.

29. Shuvalov, V.V., Dypvik, H., Kalleson, E., Setsa, R., and Riis, F. Modeling the 2.7 km in Diameter, Shallow Marine Ritland Impact Structure. Earth Moon Planets, 2012, no. 108, 175-188.
30. Ubbink, O. Numerical prediction of two fluid systems with sharp interfaces. PhD Thesis, Imperial College, 1997.
31. Watts, Ph., and Waythomas, C.F. Theoretical analysis of tsunami generation by pyroclastic flows. Journal of Geophysical Research, 2003, vol. 108(B12), 1-21.