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MONOHARMONIC APPROACH TO INVESTIGATION OF THE VIBRATIONS AND SELF-HEATING OF THIN-WALL INELASTIC MEMBERS

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Abstract. An approximate formulation is given to a dynamic coupled thermo-mechanical problem for physically non-linear inelastic thin-walled structural elements within the framework of a geometrically linear theory and the Kirchhoff–Love hypotheses. A simplified model is used to describe the vibrations and dissipative heating of inhomogeneous physically non-linear bodies under harmonic loading. Unsteady vibration self-heating problem is solved. The dissipative function obtained from the solution for steady-state vibrations is used to simulate internal heat sources. For the partial case of forced vibrations of a beam, the amplitude–frequency characteristics of the field quantities are studied within a wide frequency range. The temperature characteristics for the first and second resonance modes are compared.

Keywords: thin-wall structure, coupled thermo-mechanical problem, dissipative self-heating, monoharmonic approximation.

1. Introduction

The forced-vibration analysis of structures and their elements occupies a significant place in the dynamics of deformable systems. This research area attracts great interest because of the need for a deeper theoretical analysis (especially of non-linear systems) and purely practical requirements in various fields of engineering (Karkauskas 2004). Under intensive dynamic loads, e.g. resonance vibrations, there are several factors that determine the behaviour of a structure. Among them are inertia effects, non-linearity of material properties and the coupling of the mechanical and thermal fields. In particular, dynamic loading may cause plastic deformation of elements of damping systems, building structures, test specimens in low-cycle fatigue tests, etc. (Karkauskas 2007; Norkus and Karkauskas 2004).

This may result in elevated temperatures due to dissipative self-heating. The heating, in turn, may change the strength characteristics of the structure, deteriorate its performance, and, under adverse conditions, even cause fracture.

Cyclic (in particular harmonic) loading is one of the most widespread types of load on structural elements and equipment. Under high-level loading, the material of a structure may go over into a viscoplastic state, which, in long-term operation, may be accompanied by intensive dissipative heating. In most cases, this is the cause of altered mechanical characteristics and degraded performance of structural elements and equipment.

The interaction of the mechanical and thermal fields in inelastic bodies is investigated within the framework of a coupled thermo-mechanical problem (Allen and Haisler 1985). Recently, generalized flow theories (for example, Chan *et al.* 1990; Bodner and Partom 1975) have widely been used to describe the behaviour of materials. Within the framework of these models, the physical relations include a system of evolution equations for the internal state parameters, which are essentially non-linear. In the case of long-term inelastic deformation, the complexity of the solution is due to the necessity of storing a large body of data and performing extensive computations to allow for the deformation history. To overcome these difficulties in the specific case of harmonic loading, a simplified model of thermo-mechanically coupled processes was developed by Karnaukhov (1982), Senchenkov *et al.* (1997a, b). The model is based on the concept of complex moduli, determined by a modified technique of equivalent linearization (Karnaukhov 1982; Senchenkov *et al.* 1997a). In terms of these moduli, the initial problem is reduced to a scleronomous system of equations for complex amplitudes of mechanical field quantities – displacements, stresses, total and inelastic strains.

Here we use the simplified model of the coupled behaviour of physically non-linear bodies under harmonic loading to give an approximate formulation to the coupled dynamic problem for thin-walled structural elements. Within the framework of this formulation, an elementary thin-walled element – a beam – is used as an example to study the laws governing the forced vibrations and dissipative heating of thin-walled elements over a wide frequency range.

2. Coupled problem statement

The approximate model is based on the assumption that when elements of a solid are under harmonic loading

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$$e_{kl}(t) = e'_{kl}\cos\omega t - e''_{kl}\sin\omega t, \qquad (2.1)$$

the response of the stress and inelastic strain are also nearly harmonic,

$$s_{kl}(t) = s'_{kl}\cos\omega t - s''_{kl}\sin\omega t,$$
$$\varepsilon^p_{kl}(t) = \varepsilon^p_{kl}{}'\cos\omega t - \varepsilon^p_{kl}{}''\sin\omega t, \qquad (2.2)$$

where e_{kl} and s_{kl} are the deviators of the strain and stress tensors, respectively, $(\cdot)'$ and $(\cdot)''$ are the real and imaginary parts of complex amplitudes, $(\tilde{\cdot}) = (\cdot)' + i(\cdot)''$, and ω is the frequency of vibrations.

We determine the effective strain, stress, and inelastic strain (the intensities of the corresponding quantities) as follows:

$$e_i = \left[\frac{1}{2}\left(e'_{ij}e'_{ij} + e''_{ij}e''_{ij}\right)\right]^{1/2},$$
$$s_i = \left[\frac{1}{2}\left(s'_{ij}s'_{ij} + s''_{ij}s''_{ij}\right)\right]^{1/2}, \qquad (2.3)$$
$$\varepsilon^p_i = e^p_i = \left[\frac{1}{2}\left(\varepsilon^p_{ij}{}'\varepsilon^p_{ij}{}' + \varepsilon^p_{ij}{}''\varepsilon^p_{ij}{}''\right)\right]^{1/2}.$$

The procedure of harmonic linearization assumes that the complex amplitudes of the strain deviator $\tilde{e}_{ij} = e'_{ij} + ie''_{ij}$, inelastic-strain deviator $\tilde{\varepsilon}^p_{ij} = \varepsilon^p_{ij}{}' + i\varepsilon^p_{ij}{}''$ and the stress deviator $\tilde{s}_{ij} = s'_{ij} + is''_{ij}$ are related by means of the complex shear modulus $\tilde{G}$, $\tilde{G} = G' + iG''$ and the inelasticity factor $\tilde{\kappa}_p$, $\tilde{\kappa}_p = \kappa'_p + i\kappa''_p$ (Senchenkov *et al.* 1997a, b)

$$\tilde{s}_{ij} = 2\tilde{G}\tilde{e}_{ij}, \quad \tilde{\varepsilon}^p_{ij} = \tilde{\kappa}_p\tilde{e}_{ij}, \qquad (2.4)$$

where $\tilde{G}$ and $\tilde{\kappa}_p$ under proportional loading are functions of the effective strain (or stress), frequency and temperature,

$$\tilde{G} = \tilde{G}(e_i, \omega, \theta), \quad \tilde{\kappa}_p = \tilde{\kappa}_p(e_i, \omega, \theta). \qquad (2.5)$$

The components of the complex moduli $\tilde{G}$ and $\tilde{\kappa}_p$ are found from the condition of equality of dissipation rates and from the cyclic diagrams $s_i \sim e_i$ and $\varepsilon^p_i \sim e_i$ for the initial and approximate models. For a deformation cycle, we obtain

$$\langle D'\rangle = \left\langle s_{ij}\dot{\varepsilon}^p_{ij}\right\rangle = \left\langle s_{ij}\dot{e}_{ij}\right\rangle = 2\omega G'' e_i^2 = 2\omega\kappa''_p e_i^2 / G,$$
$$s_i / e_i = 2\left|\tilde{G}\right|, \quad \varepsilon^p_i / e_i = 2\left|\tilde{\kappa}_p\right|, \qquad (2.6)$$

where

$$\langle(\cdot)\rangle = \frac{1}{T_\omega}\int_0^{T_\omega}(\cdot)\,dt,\ T_\omega = \frac{2\pi}{\omega},\ \left|\tilde{G}\right| = \left(G'^2 + G''^2\right)^{1/2}.$$

To determine the complex characteristics appearing on the right-hand sides of relations (2.6), the dissipation rate D' and the cyclic curves $s_i \sim e_i$ and $\varepsilon^p_i \sim e_i$ are determined through direct integration of the Bodner–Partom equations (Chan *et al.* 1990; Bodner and Partom 1975) for torsion of a thin-walled cylinder. Introducing a cylindrical coordinate system $(Orz\varphi)$, where the Oz-axis coincides with the cylinder axis, we obtain

$$e_{z\varphi} = -e^0_{z\varphi}\sin\omega t,\ e = e^0_{z\varphi},\ s = \Delta\sigma_{z\varphi},\ \varepsilon_p = \Delta\varepsilon^p_{z\varphi}, \qquad (2.7)$$

where $\Delta\sigma_{z\varphi}$ and $\Delta\varepsilon^p_{z\varphi}$ are the ranges of stresses and inelastic strains in a cycle.

The real parts G' and κ'_p of the moduli are calculated by relations (2.6) and (2.7):

$$G'(e_i, \omega) = \left[\frac{s_i^2(e_i, \omega)}{4e_i^2} - G''^2(e_i, \omega)\right]^{1/2},$$
$$\kappa'_p(e_i, \omega) = \left[\frac{\varepsilon_i^{p2}(e_i, \omega)}{4e_i^2} - \kappa''^2_p(e_i, \omega)\right]^{1/2}. \qquad (2.8)$$

Using the fact that elastic and inelastic strains appear in the total strain additively and using Eqs (2.4), it is easy to show that the moduli $\tilde{G}$ and $\tilde{\kappa}_p$ are linked by the relation

$$\tilde{G} = G\left(1 - \tilde{\kappa}_p\right). \qquad (2.9)$$

In the approximate formulation of the coupled thermo-mechanic problem for physically non-linear inelastic bodies, the following heat-conductivity equation is used (Senchenkov *et al.* 1997b):

$$c_v\dot{\theta} + 3\alpha_0\theta K_V\left(\dot{\varepsilon}_{kk} - 3\alpha_0\dot{\theta}\right) - D' - \left(k\theta_{,i}\right)_{,i} - r = 0, \qquad (2.10)$$

which is averaged over the period of vibrations.

Here c_v is the heat capacity at constant volume, k and α_0 are the coefficients of thermal conductivity and linear thermal expansion, respectively, K_V – the bulk modulus of the material, and r – a given heat source.

Averaging Eq. (2.10) over the period of vibrations in view of $r \equiv 0$ and neglecting the thermo-elastic effects, $3\alpha_0 K_V\langle\theta\dot{\varepsilon}_{kk}\rangle$, $9\alpha_0\theta K_V / c_v \approx 0$, we obtain

$$c_v\dot{\theta} - \left(k\theta_{,i}\right)_{,i} - \langle D'\rangle = 0. \qquad (2.11)$$

The dissipative function can be represented as (2.6)

$$\langle D'\rangle = \left\langle s_{ij}\dot{\varepsilon}^p_{ij}\right\rangle = \left\langle \dot{W}^p\right\rangle. \qquad (2.12)$$

Reasoning as we did to derive relation (2.9), we can represent the first equation in (2.4) in an alternative form

$$\tilde{s}_{ij} = 2G\left(\tilde{e}_{ij} - \tilde{\varepsilon}^p_{ij}\right) = 2G\left(1 - \tilde{\kappa}_p\right)\tilde{e}_{ij}. \qquad (2.13)$$

Since a unique dependencies $s_i = s_i(e_i)$ exists for proportional loading with given ω and θ, the moduli $\tilde{G}$ and $\tilde{\kappa}_p$ can be considered as functions of s_i,

$$\tilde{G} = \tilde{G}(s_i, \omega, \theta), \quad \tilde{\kappa}_p = \tilde{\kappa}_p(s_i, \omega, \theta) \qquad (2.14)$$

Relations (2.5) and (2.14) are formally equivalent. However, it is more convenient, from the computational point of view, to use smooth relationships (2.5) for materials with a very weak cyclical hardening.

The equations for the volumetric components are

$$\tilde{\sigma}_{kk} = 3K_V\tilde{\varepsilon}_{kk}. \qquad (2.15)$$

If we neglect the mechanical transient, then the vibration equation takes the form

$$\tilde{\sigma}_{ij,j} + \rho\omega^2\tilde{u}_i = 0, \qquad (2.16)$$

where $\tilde{u}_k$ is the displacement amplitude, $\tilde{u}_k = u'_k + iu''_k$.

Supplementing Eqs (2.4), (2.11), (2.15), and (2.16) with boundary conditions and initial conditions for temperature, we obtain an approximate formulation of the coupled problem for physically non-linear bodies under harmonic loading.

The derived equations are valid for arbitrary 3-D solid.

There are some limitations for the applications of the developed monoharmonic approach. The limitations are imposed by the processes under consideration, types of the materials and phenomena studied. The structure response should be close to the monoharmonic one (the most vivid case is the case of resonance). The loading should provide the stress-strain state that is quite close to the proportional one. The objects of the investigation should be the amplitudes (ranges) of the main field variables and values averaged over the period (dissipated energy, for example). The approach developed is incapable in describing the peculiarities of the structure response within one separate cycle of vibration, sub- and super-harmonic resonance and chaotic scenarios.

The main advantage of the technique elaborated is significant reduction in the calculation time. Application of the generalized flow theories for simulating the physically non-linear response of the structural material leads to the necessity of following the complete response history. This process is extremely time-consuming, especially for high-cycle problems. However, having determined steady complex moduli, one can avoid the time integration of the significantly non-linear equations of the model describing inelastic behaviour of the material.

Furthermore, under some circumstances, the general equations developed above can be significantly simplified.

3. Problem statement for thin-wall structural members

For approximate formulation of the coupled dynamic problem for laminated thin-walled shells of revolution under harmonic loading, we adopt the model considered in Section 2. The equations for thin-walled elements of more general geometry are derived similarly.

Let a shell have a constant thickness H and consist of an arbitrary number, K, of layers of thickness h_i each. Initially, the material of the layers is isotropic. The layers are in perfect mechanical and thermal contact. Consider a cylindrical coordinate system $(Or\zeta\phi)$. The datum surface of the shell is referred to a curvilinear orthogonal coordinate system $(rz\phi)$. The meridian of the datum surface is described by the equation $r = r(z)$. The datum surface itself is bounded by the lines of principal curvatures $s = \text{const}$ and $\phi = \text{const}$, where s is the angular position of the meridian of the datum surface. The thickness coordinate is reckoned form the datum surface along the external normal. We assume that $\{u, v, w\} = \{u_1, u_2, u_3\}$ are the displacement components of an arbitrary point of the shell and consider the steady-state vibrations of the shell under harmonic force or kinematic loading. Let φ be an angle between the normal to datum surface and axis direction; R_s and R_φ are main radii of curvature. Let us also use the classical nomenclature of shell theory, as it has been adopted in Shevchenko and Prokhorenko (1981) and Grigorenko *et al.* (1986).

We also assume that all the kinematic and force characteristics vary harmonically:

$$\begin{aligned} u_k &= \mathrm{Re}\left(\tilde{u}_k e^{i\omega t}\right) = u'_k \cos\omega t - u''_k \sin\omega t, \\ \varepsilon_{kl} &= \mathrm{Re}\left(\tilde{\varepsilon}_{kl} e^{i\omega t}\right) = \varepsilon'_{kl} \cos\omega t - \varepsilon''_{kl} \sin\omega t, \\ \sigma_{kl} &= \mathrm{Re}\left(\tilde{\sigma}_{kl} e^{i\omega t}\right) = \sigma'_{kl} \cos\omega t - \sigma''_{kl} \sin\omega t. \end{aligned} \quad (3.1)$$

Using relations (2.13) and (2.15), we write Hooke's law for the material of the j^{th} layer as

$$\tilde{\sigma}_{kl} = 2\tilde{G}_j \left[\tilde{\varepsilon}_{kl} + \frac{\tilde{\nu}_j}{1 - 2\tilde{\nu}_j} \tilde{\varepsilon}_{kk} \delta_{kl} \right], \quad k,l = \overline{1,3}; \quad j = \overline{1,K}, \quad (3.2)$$

where $\tilde{\nu}_j$ is a complex-value Poisson ratio, $\tilde{\nu}_j = \nu'_j - i\nu''_j = \left(3K_{Vj} - 2\tilde{G}_j\right) / \left(6K_{Vj} + 2\tilde{G}_j\right)$.

According to (2.14) and (2.15), $\tilde{G}_j$ and $\tilde{\nu}_j$ are functions of the intensity e_i or s_i (2.3).

Relations (3.2) formally coincide with the linear elastic equations in the case of harmonic deformation with the only difference that Eqs (3.2) include complex quantities. Therefore, to construct an approximate model of shells, we will use the general procedure developed in Karnaukhov and Kirichok (1986), Shevchenko and Prokhorenko (1981), Grigorenko *et al.* (1986).

From the force hypothesis $\tilde{\sigma}_{zz} = 0$ and Eq. (3.2), we obtain equations for thickness deformation:

$$\tilde{\varepsilon}_{zz} = -\frac{\tilde{\nu}_j}{1 - \tilde{\nu}_j} \left(\tilde{\varepsilon}_{ss} + \tilde{\varepsilon}_{\varphi\varphi} \right). \quad (3.3)$$

Formally substituting (3.3) into the other equations in (3.2), we obtain

$$\begin{aligned} \tilde{\sigma}_{ss} &= -\frac{\tilde{E}_j}{1 - \tilde{\nu}_j^2} \left(\tilde{\varepsilon}_{ss} + \tilde{\nu}_j \tilde{\varepsilon}_{\varphi\varphi} \right), \\ \tilde{\sigma}_{\varphi\varphi} &= -\frac{\tilde{E}_j}{1 - \tilde{\nu}_j^2} \left(\tilde{\varepsilon}_{\varphi\varphi} + \tilde{\nu}_j \tilde{\varepsilon}_{ss} \right), \\ \tilde{\sigma}_{s\varphi} &= -\frac{\tilde{E}_j}{1 - \tilde{\nu}_j} \tilde{\varepsilon}_{s\varphi}. \end{aligned} \quad (3.4)$$

In view of the hypotheses adopted above, we obtain the following expressions for the intensities:

$$s_i = \left\{ \frac{1}{6} \left[\left[\left| \tilde{\sigma}_{ss} \right|^2 + \left| \tilde{\sigma}_{\varphi\varphi} \right|^2 + \left| \tilde{\sigma}_{\varphi\varphi} - \tilde{\sigma}_{ss} \right|^2 + 6 \left| \tilde{\sigma}_{s\varphi} \right|^2 \right] \right] \right\}, \quad (3.5)$$

$$e_i = \left\{ \frac{1}{6} \left[\left| \tilde{\varepsilon}_{ss} - \tilde{\varepsilon}_{zz} \right|^2 + \left| \tilde{\varepsilon}_{zz} - \tilde{\varepsilon}_{\varphi\varphi} \right|^2 + \left| \tilde{\varepsilon}_{\varphi\varphi} - \tilde{\varepsilon}_{ss} \right|^2 + 6 \left| \tilde{\varepsilon}_{s\varphi} \right|^2 \right] \right\}, \quad (3.6)$$

The manner of the hypothesis $\tilde{\sigma}_{zz} = 0$ implementation depends on whether the complex characteristics are functions of s_i or e_i. In the former case, we cannot exclude the strain $\tilde{\varepsilon}_{zz}$ explicitly. Therefore, the condition $\tilde{\sigma}_{zz} = 0$ is equivalent to non-linear relation (3.3). It is regarded as an additional non-linear equation that is satisfied in the course of the general iterative process of boundary-value solution. In the latter case, the equations of plane stress state follow from this hypothesis (3.4) without additional relations such as (3.3).

To derive the equations for complex forces and moments, we substitute the stresses into the classical formulas for forces and moments per unit arc length of the datum surface and then use the geometrical equations of the shell theory (Karnaukhov and Kirichok 1986; Shevchenko and Prokhorenko 1981; Grigorenko *et al.* 1986). As a result, the complex analogue of the plastic relations can be written as (Karnaukhov and Kirichok 1986):

$$\begin{aligned} \tilde N_s &= \tilde C_1\tilde\varepsilon_{11} + \tilde C_2\tilde\varepsilon_{22} + \tilde K_1\tilde\kappa_{11} + \tilde K_2\tilde\kappa_{22}, \\ \tilde N_\varphi &= \tilde C_2\tilde\varepsilon_{11} + \tilde C_1\tilde\varepsilon_{22} + \tilde K_2\tilde\kappa_{11} + \tilde K_1\tilde\kappa_{22}, \\ \tilde N_{s\varphi} &= \tilde C_{12}\tilde\varepsilon_{12} + \tilde K_{12}\tilde\kappa_{12}, \\ \tilde M_s &= \tilde K_1\tilde\varepsilon_{11} + \tilde K_2\tilde\varepsilon_{22} + \tilde D_1\tilde\kappa_{11} + \tilde D_2\tilde\kappa_{22}, \\ \tilde M_\varphi &= \tilde K_2\tilde\varepsilon_{11} + \tilde K_1\tilde\varepsilon_{22} + \tilde D_2\tilde\kappa_{11} + \tilde D_1\tilde\kappa_{22}, \\ \tilde M_{s\varphi} &= \tilde K_{12}\tilde\varepsilon_{12} + \tilde D_{12}\tilde\kappa_{12}. \end{aligned} \tag{3.7}$$

Here $\tilde\varepsilon_{ij}$ and $\tilde\kappa_{ij}$ ($i,j=1,2$) are the complex analogues of the deformation characteristics of classical shell theory, $\tilde C_{1,2}$, $\tilde K_{1,2}$, $\tilde D_{1,2}$, $\tilde C_{12}$, $\tilde K_{12}$ and $\tilde D_{12}$ are determined from the corresponding formulas of shell theory, in which the parameters E_j and ν_j are replaced by the complex characteristics $\tilde E_j$ and $\tilde\nu_j$ depending on s_i or e_i, i.e.

$$\tilde C_1 = \sum_{j=0}^{J-1}\int_{z_j}^{z_{j+1}} \frac{\tilde E_j}{1-\tilde\nu_j^2}\,dz, \qquad \tilde C_2 = \sum_{j=0}^{J-1}\int_{z_j}^{z_{j+1}} \frac{\tilde\nu_j\tilde E_j}{1-\tilde\nu_j^2}\,dz,$$

$$\tilde K_1 = \sum_{j=0}^{J-1}\int_{z_j}^{z_{j+1}} \frac{\tilde E_j z}{1-\tilde\nu_j^2}\,dz, \qquad \tilde K_2 = \sum_{j=0}^{J-1}\int_{z_j}^{z_{j+1}} \frac{\tilde\nu_j\tilde E_j}{1-\tilde\nu_j^2}z\,dz,$$

$$\tilde D_1 = \sum_{j=0}^{J-1}\int_{z_j}^{z_{j+1}} \frac{\tilde E_j}{1-\tilde\nu_j^2}z^2\,dz, \quad \tilde K_2 = \sum_{j=0}^{J-1}\int_{z_j}^{z_{j+1}} \frac{\tilde\nu_j\tilde E_j}{1-\tilde\nu_j^2}z\,dz,$$

$$\tilde C_{12} = \sum_{j=0}^{J-1}\int_{z_j}^{z_{j+1}} \frac{\tilde E_j}{1+\tilde\nu_j}\,dz, \qquad \tilde K_{12} = \sum_{j=0}^{J-1}\int_{z_j}^{z_{j+1}} \frac{\tilde E_j}{1+\tilde\nu_j}\,zdz,$$

$$\tilde D_{12} = \sum_{j=0}^{J-1}\int_{z_j}^{z_{j+1}} \frac{\tilde E_j}{1+\tilde\nu_j}\,z^2dz.$$

It should be noted that Eqs (3.7) have no terms $N_{\alpha\beta}^{P,\theta}$ and $M_{\alpha\beta}^{P,\theta}$, accounting for the thermal and inelastic strains (Shevchenko and Prokhorenko 1981; Grigorenko *et al.* 1986). The components of plastic strains are determined from

Eqs (2.4) for each j^{th} layer:

$$\tilde\varepsilon_{\alpha\beta}^{p} = \tilde\kappa_p^j\tilde e_{\alpha\beta}, \quad \alpha,\beta = s,\varphi; \quad \tilde\varepsilon_{zz}^{p} = -\tilde\varepsilon_{ss}^{p} - \tilde\varepsilon_{\varphi\varphi}^{p}. \tag{3.8}$$

Based on (3.1), we derive the vibration equations for shells by means of the substitution

$$\{\ddot u, \ddot v, \ddot w\} \to \{-\omega^2\tilde u, -\omega^2\tilde v, -\omega^2\tilde w\},$$

$$\{N_\alpha, N_{\alpha\beta}, Q_\alpha, M_\alpha, M_{\alpha\beta}, q_\gamma, m_\alpha\} \to$$

$$\{\tilde N_\alpha, \tilde N_{\alpha\beta}, \tilde Q_\alpha, \tilde M_\alpha, \tilde M_{\alpha\beta}, \tilde q_\gamma, \tilde m_\alpha\},$$

$$\alpha,\beta = s,\varphi, \quad \alpha\ne\beta; \quad \gamma = s,\varphi,z$$

to classical equation (Shevchenko and Prokhorenko 1981; Grigorenko *et al.* 1986).

As a result, the equations of motion take the form (Zhuk and Senchenkov 2002; Zhuk *et al.* 2004):

$$\frac{\partial r\tilde N_s}{\partial s} + \frac{\partial \tilde N_{s\varphi}}{\partial\varphi} - \tilde N_\varphi\cos\phi + \frac{r}{R_s}\tilde Q_s + r\tilde q_s + \omega^2 rH_s\bar\rho\tilde u = 0,$$

$$\frac{\partial r\tilde N_{\varphi s}}{\partial s} + \frac{\partial \tilde N_\varphi}{\partial\varphi} + \tilde N_{s\varphi}\cos\phi + \tilde Q_\varphi\sin\phi_s + r\tilde q_\varphi + \omega^2 rH_s\bar\rho\tilde v = 0,$$

$$\frac{\partial r\tilde Q_s}{\partial s} + \frac{\partial \tilde Q_\varphi}{\partial\varphi} - \frac{r}{R_s}\tilde N_s - \tilde N_\varphi\sin\phi + r\tilde q_z + \omega^2 rH_s\bar\rho\tilde w = 0,$$

$$\frac{\partial r\tilde M_s}{\partial s} + \frac{\partial \tilde M_{s\varphi}}{\partial\varphi} - r\tilde Q_s - \tilde M_\varphi\cos\phi + r\tilde m_s + \omega^2 rH_s^2\bar{\bar\rho}\tilde u = 0,$$

$$\frac{\partial r\tilde M_{\varphi s}}{\partial s} + \frac{\partial \tilde M_\varphi}{\partial\varphi} - r\tilde Q_\varphi + \tilde M_{s\varphi}\cos\phi + r\tilde m_\varphi + \omega^2 rH_s^2\bar{\bar\rho}\tilde v = 0,$$

$$\tilde N_{s\varphi} - \tilde N_{\varphi s} + \frac{\tilde M_{s\varphi}}{R_\varphi} - \frac{\tilde M_{\varphi s}}{R_s} = 0.$$

$$\bar\rho = H_s^{-1}\sum_{j=0}^{J-1}\int_{z_j}^{z_{j+1}}\rho_j\,dz, \quad \bar{\bar\rho} = H_s^{-2}\sum_{j=0}^{J-1}\int_{z_j}^{z_{j+1}}\rho_j z\,dz,$$

where ρ_j are densities of the layer materials.

The dissipative function (2.12) in the heat-conductivity equation

$$\begin{aligned} c_v\dot\theta = k\frac{\partial^2\theta}{\partial x^2} - \frac{\alpha_{y1}-\alpha_{y2}}{h_y}(\theta-\theta_c) - \\ -\frac{\alpha_{z1}-\alpha_{z2}}{H}(\theta-\theta_c) - \langle D'\rangle \end{aligned} \tag{3.9}$$

for the j^{th} layer, in view of (3.4), takes the form

$$\begin{aligned} \langle D'\rangle = D_{0j} + zD_{1j} + z^2B_{2j}, \\ z_{j-1} < z < z_j, \quad j = \overline{1,K}. \end{aligned} \tag{3.10}$$

Here θ_c is the ambient temperature, $\alpha_{y1,2}$ and $\alpha_{z1,2}$ are the heat-transfer coefficients for the surfaces $y = \pm h_y/2$ and $z = \pm H/2$, respectively, h_y is the beam width, and

$$D_{0j} = \frac{\omega}{2}\Big[B_{1j}''\big(|\tilde\varepsilon_{11}|^2 + |\tilde\varepsilon_{22}|^2\big) + 2B_{2j}''(\varepsilon_{11}'\varepsilon_{22}' + \varepsilon_{11}''\varepsilon_{22}'') + \\ + 2B_{12j}''|\tilde\varepsilon_{12}|^2\Big],$$

$$D_{1j} = \omega\Big[B_{1j}''(\varepsilon_{11}'\kappa_{11}' + \varepsilon_{11}''\kappa_{11}'' + \varepsilon_{22}'\kappa_{22}' + \varepsilon_{22}''\kappa_{22}'') + \\ + B_{2j}''(\varepsilon_{11}'\kappa_{22}' + \varepsilon_{11}''\kappa_{22}'' + \varepsilon_{22}'\kappa_{11}' + \varepsilon_{22}''\kappa_{11}'') + \\ + B_{12j}''(\varepsilon_{12}'\kappa_{12}' + \varepsilon_{12}''\kappa_{12}'')\Big],$$

$$D_{0j} = \frac{\omega}{2}\Big[B_{1j}''\big(|\tilde\kappa_{11}|^2 + |\tilde\kappa_{22}|^2\big) + 2B_{2j}''(\kappa_{11}'\kappa_{22}' + \kappa_{11}''\kappa_{22}'') + \\ + 2B_{12j}''|\tilde\kappa_{12}|^2\Big],$$

$$B''_{1j} = \mathrm{Im}\left(\frac{\tilde{E}_j}{1-\tilde{\nu}_j^2}\right), \quad B''_{2j} = \mathrm{Im}\left(\frac{\tilde{\nu}_j \tilde{E}_j}{1-\tilde{\nu}_j^2}\right),$$

$$B''_{12j} = \mathrm{Im}\left(\frac{\tilde{E}_j}{1-\tilde{\nu}_j}\right),$$

The following dissipative function averaged over the thickness gives a good approximation for single-layered shells made of materials with good heat conductivity:

$$\langle \overline{D}' \rangle = \int_0^H \langle D' \rangle dz = \frac{\omega}{2}\Big[N''_s \varepsilon'_{11} - N'_s \varepsilon''_{11} + N''_\varphi \varepsilon'_{22} - N'_\varphi \varepsilon''_{22} + 2\left(N''_{s\varphi}\varepsilon'_{12} - N'_{s\varphi}\varepsilon''_{12}\right) + M''_s \kappa'_{11} - M'_s \kappa''_{11} + M''_\varphi \kappa'_{22} - M'_\varphi \kappa''_{22} + 2\left(M''_{s\varphi}\kappa'_{12} - M'_{s\varphi}\kappa''_{12}\right)\Big]. \quad (3.11)$$

Thus, the approximate coupled dynamic problem for thin-walled elements under harmonic loading is reduced to complex analogues of 5 equilibrium equations, 6 complex geometrical equations, 6 inelastic relations (3.7), and heat-conductivity equations (2.11) for 18 unknowns $\tilde{\varepsilon}_{ss}$, $\tilde{\varepsilon}_{\varphi\varphi}$, $\tilde{\varepsilon}_{s\varphi}$, $\tilde{\kappa}_{11}$, $\tilde{\kappa}_{22}$, $\tilde{\kappa}_{12}$, $\tilde{N}_s$, $\tilde{N}_\varphi$, $\tilde{N}_{s\varphi}$, $\tilde{M}_s$, $\tilde{M}_\varphi$, $\tilde{M}_{s\varphi}$, $\tilde{Q}_s$, $\tilde{Q}_\varphi$, $\tilde{u}$, $\tilde{v}$, $\tilde{w}$, and θ.

Once the problem is solved, the inelastic strains $\tilde{\varepsilon}^p_{\alpha\beta}$ are determined by formulae (3.8). The boundary conditions on the shell periphery are formulated in an ordinary manner (Karnaukhov and Kirichok 1986; Shevchenko and Prokhorenko 1981; Grigorenko *et al.* 1986), including complex variables (Senchenkov *et al.* 1997a, b).

4. Formulation of the problem for a beam

Let us study some laws governing the coupled thermomechanical behaviour of thin-walled elements by an example of the flexural vibrations of a beam $0 \le x \le L$ of thickness H. In this case, the system of mechanical equations (2.4), (2.11), and (2.15) is reduced to

$$\frac{d\tilde{u}_x}{dx} = \tilde{Q}_1\tilde{N}_x - \tilde{Q}_2\tilde{M}_x, \quad \frac{d\tilde{u}_z}{dx} = -\tilde{\vartheta},$$
$$\frac{d\tilde{M}_x}{dx} = \tilde{Q}_x, \quad \frac{d\tilde{N}_x}{dx} = -\rho\omega^2 H\tilde{u}_x, \quad (4.1)$$
$$\frac{d\tilde{Q}_x}{dx} = -\rho\omega^2 H\tilde{u}_x, \quad \frac{d\tilde{\vartheta}}{dx} = -\tilde{Q}_2\tilde{N}_x + \tilde{Q}_3\tilde{M}_x,$$

where $\tilde{\vartheta}$ is the complex angle of rotation,

$$\{\tilde{v}, \tilde{v}, \tilde{w}\} = \{\tilde{u}_x, \tilde{u}_y, \tilde{u}_z\},$$
$$\tilde{Q}_1 = \tilde{D}_1 / \left(\tilde{C}_1\tilde{D}_1 - \tilde{K}_1^2\right),$$
$$\tilde{Q}_2 = \tilde{K}_1 / \left(\tilde{C}_1\tilde{D}_1 - \tilde{K}_1^2\right),$$
$$\tilde{Q}_3 = \tilde{C}_1 / \left(\tilde{C}_1\tilde{D}_1 - \tilde{K}_1^2\right).$$

The vibration of the beam are excited by moments applied at the ends $x = 0, L$ and varying in time as

$$M = -\frac{\sigma^0_{xx} H^2}{6}\sin\omega t = M_0 \sin\omega t,$$

where σ^0_{xx} is the maximum amplitude of the stress distributed linearly along the beam's ends $x = 0, L$. The corresponding complex conditions have the form

$$\tilde{N}_x = \tilde{Q}_x = 0,$$
$$\tilde{M}_x = -iM_0 = -\frac{i\sigma^0_{xx}H^2}{6} \quad \text{for} \quad x = 0, L. \quad (4.2)$$

To solve the thermal problem, we use a more accurate two-dimensional formulation rather than shell relations (3.9–3.11):

$$c_v\dot{\theta} = \frac{\partial}{\partial x}\left(k\frac{\partial\theta}{\partial x}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial\theta}{\partial z}\right) - \frac{2\alpha_3\theta}{h_y} + \langle D'(x,z)\rangle \quad (4.3)$$

with the boundary conditions

$$\pm k\frac{\partial\theta}{\partial x} = \alpha_1\theta, \quad x = 0, L,$$
$$\pm k\frac{\partial\theta}{\partial z} = \alpha_2\theta, \quad z = \pm H/2, \quad (4.4)$$

and the initial condition

$$\theta(x,z) = 0, \quad t = 0. \quad (4.5)$$

In (4.3–4.5), $\overline{\theta} = \theta - \theta_0$ is the redundant temperature, α_1, α_2 and α_2 are the heat-transfer coefficients for the surfaces $x = 0, L$, $z = \pm H/2$, and $y = \pm h_y/2$, respectively.

Thus, in such a formulation, we solve an unsteady heat conductivity problem in which internal heat sources are simulated by a dissipative function calculated in the steady-state vibration problem.

5. Numerical analysis

Let a beam be made of *AMg-6* aluminum alloy. How to determine the parameters of the approximate model is described in Senchenkov *et al.* (1997a), Zhuk *et al.* (2001) in detail. Specific parameters corresponding to the chosen material are taken from Zhuk and Senchenkov (2000, 2001).

The mechanical problem (4.1), (4.2) formulated in Section 4 is solved by an iterative method similar to the method of variable elastic parameters (Shevchenko and Prokhorenko 1981). In linearizing the problem at the nth iteration, the mechanical properties of the material are determined from the amplitudes of the strain intensity calculated at the $(n-1)^{\text{th}}$ iteration. To accelerate the convergence process, the Steffensen–Aitken algorithm is used, according to which an improved approximation is constructed as a linear combination of the two previous ones.

After stationary solution of the mechanical problem is found, the unsteady heat-conductivity problem (4.3–4.5) is solved using the finite-difference method with an explicit difference scheme (Tikhonov and Samarskii 1972). The expediency of using difference methods is determined, first of all, by practical considerations – the simplicity of computational algorithms and their adequate accuracy in solving Eq. (4.3) with non-linearity of quite general form for bodies of arbitrary shape. If one wants to account for the temperature dependence of the material properties, one should use a step-by-step (in time) solution scheme.

The calculations were performed for the following geometrical parameters of the beam:

$$c_v = 2.479 \cdot 10^2 \ \mathrm{J/m^3 \cdot K}, \quad k = 90 \ \mathrm{W/m \cdot K},$$

$$\alpha_1 = \alpha_2 = \alpha_3 = 100 \ \mathrm{W/m \cdot K}, \quad \theta_c = 20 \ ^\circ C.$$

We analyzed the following basic mechanical characteristics: the amplitude of deflections $\Delta u_z = |\tilde{u}_z|$, the amplitudes of the intensities of stress deviator s_i, the total-strain deviator e_i, and the inelastic-strain deviator e_i^p (see relations (2.3)), the dissipated energy $\langle \overline{D'} \rangle$ averaged over the period of vibrations, the stored energy $\langle W \rangle_V$ maximum in the period and averaged over the volume, the absorption coefficient $\psi = 2\pi \langle \overline{D'} \rangle_V / \omega \langle W \rangle_V$, and the redundant temperature $\overline{\theta}$. The expressions for $\langle \overline{D'} \rangle_V$ and $\langle W \rangle_V$ and the formula for W in terms of complex-value amplitudes are given below:

$$\langle \overline{D'} \rangle_V = \frac{1}{V} \int_V \overline{D'} dV,$$

$$W = \left[\frac{\tilde{\sigma}_{kk} \tilde{\sigma}_{kk}^*}{18 K_V} + \frac{\tilde{s}_{ij} \tilde{s}_{ij}^*}{4G} \right], \tag{5.1}$$

$$\langle W \rangle_V = \frac{1}{V} \int_V W dV,$$

where $\tilde{a}^*$ – complex conjugate to $\tilde{a}$, $\tilde{a}^* = a' - ia''$.

Fig. 1 shows the amplitude–frequency characteristics of the forced vibrations of the beam for $\sigma_{xx}^0 = 50$ MPa. Curves 1–4 represent the quantities Δu_z, e_i, e_i^p, and s_i, respectively. The frequency interval of interest includes the first and second resonances of the beam $f_{r1} = 248$ Hz and $f_{r1} = 1348$ Hz (symmetric modes). Near the first resonance, the field quantities have high amplitudes. The second resonance extends, however, over a wider frequency range.

The magnitudes of the stress intensity for the first and second resonance are the same. It can be explained by the nature of the physical non-linearity considered.

Cyclic behaviour of the aluminum alloy stress-strain diagram in terms of amplitudes is very close to the diagram for elastic-perfectly plastic material diagram. Under these circumstances, the peaks in stress intensity are "cut" at the value of cyclic yield stress. As a result, plateaus in the stress amplitude-frequency characteristics are formed (see line 4 in Fig. 1).

The accuracy of the results is estimated by comparing them with data obtained by other methods. This can be treated as the verification of the technique elaborated.

Table 1 presents, for a frequency of 250 Hz, the finite-element solution of the problem (the first column).

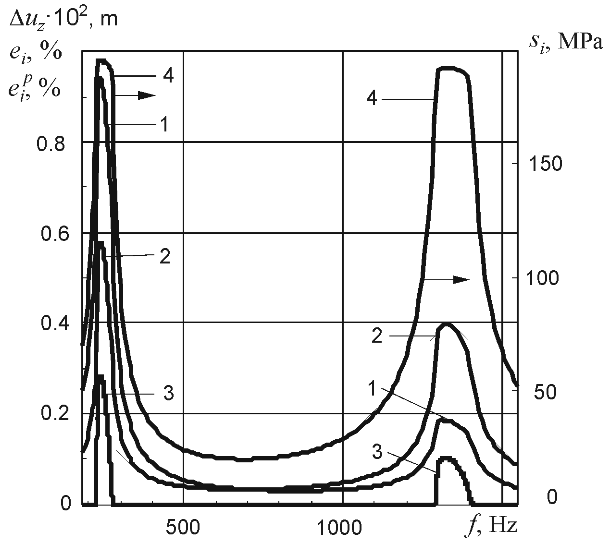

Fig. 1. Amplitude–frequency characteristics of the forced vibrations of the beam for $\sigma_{xx}^0 = 50$ MPa

Table 1. Accuracy estimation

Model	1	2	3
s_i, MPa	196.9	195.4	195.2
e_i, %	0.552	0.488	0.533
e_i^p, %	0.251	0.192	0.238
$\langle \overline{D'} \rangle$, MWt/m^3	933.8	738.4	920.9
W, MJ/m^2	0.714	0.702	0.700
$\Delta u_z \cdot 10$, m	0.950	0.880	0.885

This problem was solved in an unsteady formulation and the behaviour of the material was described by the Bodner–Partom model (Chan *et al.* 1990; Bodner and Partom 1975). The second column contains the finite-element solution by an approximate model based on the concept of complex moduli developed in Karnaukhov (1982), Senchenkov *et al.* (1997a, b). The third column represents the solution by the approach developed here. The tabulated values correspond to the point $x = 0.413$ m, $z = 0.015$ m. The data are in a good agreement.

Fig. 2a shows how the intensities s_i (curves 1) and e_i^p (curves 2) are distributed along the length of the beam on its surface. The results correspond to frequency $f_1 = 250$ Hz and $\sigma_{xx}^0 = 50$ MPa.

The same dependencies calculated for $f_{r1} = 1350$ Hz are presented in Fig. 2b. Near those two frequencies, the amplitude–frequency characteristics have peaks within the first and second resonance domains, respectively. An analysis of these results reveals that the vibration mode determines the nature of those distributions. In particular, the domains of inelastic deformation are located near the regions of the stress maximum.

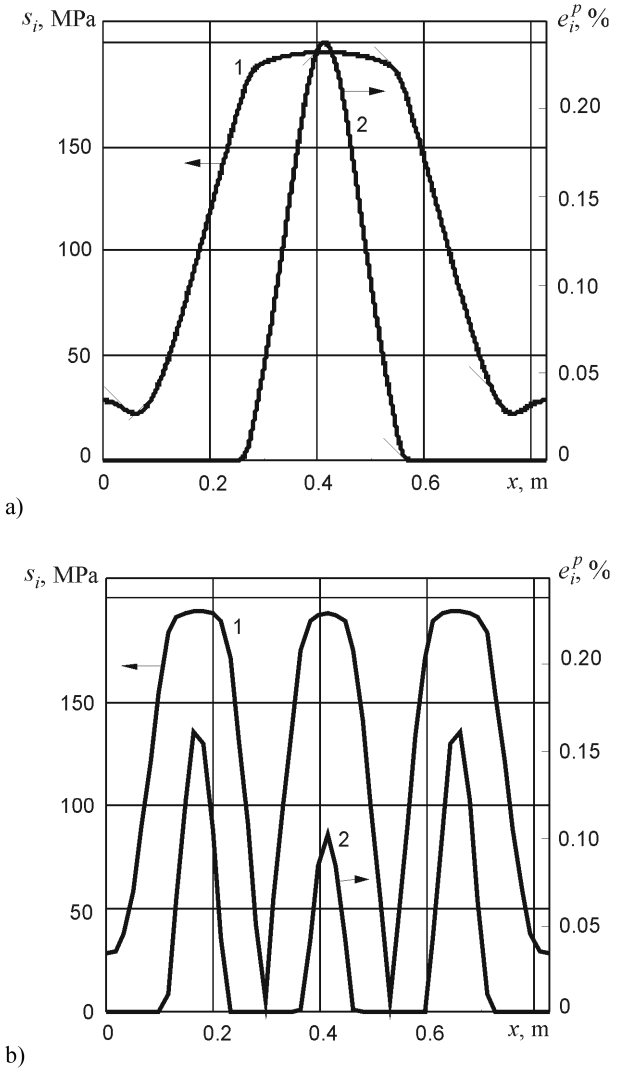

Fig. 2. Stress and inelastic strain intensities distributions along the length of the beam: a) $f = f_1 = 250$ Hz and $\sigma_{xx}^0 = 50$ MPa; b) $f = f_2 = 1350$ Hz and $\sigma_{xx}^0 = 50$ MPa

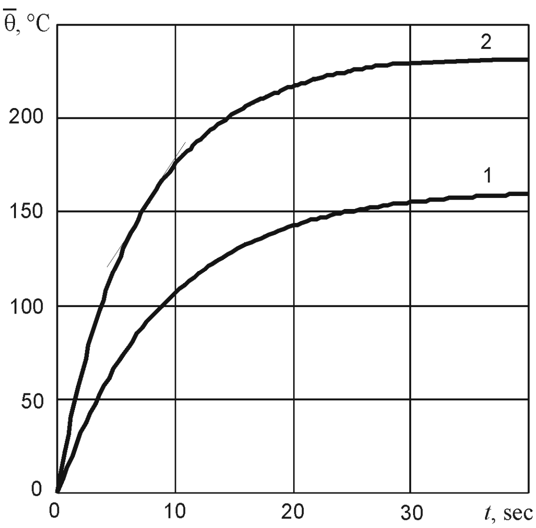

Fig. 3. Time evolution of the maximum over the volume redundant temperature

The energy aspect of vibrations at the frequencies f_1 and f_2 may be evaluated from the coefficient of absorption of mechanical energy ψ. At 250 Hz and 1350 Hz, it is equal to 0.27 and 0.12, respectively. Thus, under the given conditions, the first-mode vibrations are characterized by intensive internal loss. Significant values of ψ are one of the reasons for employing the plastic mechanism to damp vibrations of high amplitude (Chiba and Kobayashi 1990).

The results of the vibration self-heating study are reflected in Figs 3 and 4.

Fig. 3 shows the evolution in time of the maximum (within the volume) redundant temperature at frequencies of 250 Hz (curve 1) and 1350 Hz (curve 2) $\sigma_{xx}^0 = 16$ MPa. It is worth to emphasize that in both cases curves demonstrate the saturation type behaviour of the temperature. The highest heating temperature is less than 250°C. Therefore, it is well founded that temperature dependencies of the material parameters are not taken into account under those conditions. The heating level is not enough to change material properties significantly.

Low heating levels are caused by 2 main factors. The first one is relatively high heat conductivity of the metal alloy. The second factor is the heat convection at the beam faces.

For plastic structural elements, self-heating can significantly affect the strength, durability and performance mainly because the heat conductivity of the most plastics is poor.

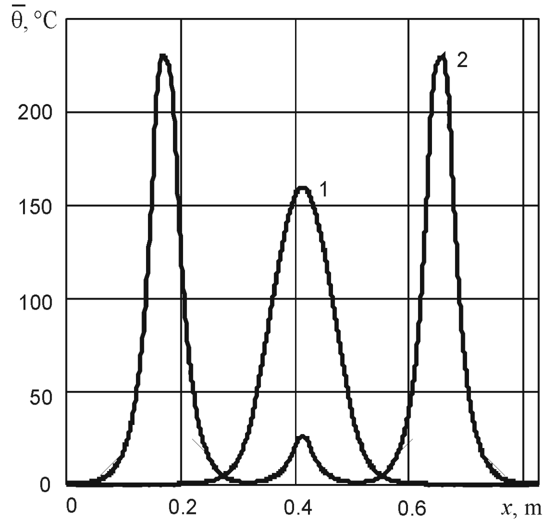

Fig. 4. Steady state temperature distributions along the beam length

Fig. 4 demonstrates the stationary distributions of the temperature along the beam length. The line numbers correspond to those shown in Fig. 3. An analysis of the results reveals that under the given conditions, the first-mode vibrations are accompanied by smaller heating, and the spatial distribution of temperature, as well as the mechanical field characteristics, is determined by the vibration mode.

6. Conclusions

1. The coupled thermo-mechanical problem statement is used to investigate the interaction of the mechanical and thermal fields in the inelastic solids. Recently, generalized flow theories have been widely used to describe the elastic-viscoplastic behaviour of materials. Within the framework of these models, a set of internal variables is used to describe spectrum of inelastic effects. However, the physical relations include a system of evolution equations for the internal state variables, which are essentially non-linear. In the case of long-term inelastic deformation, the complexity of the solution leads to the necessity of storing an extensive information and performing vast computations to allow for the deformation history.

2. To overcome these difficulties in the specific case of harmonic loading, a simplified model of thermo-mechanically coupled processes is developed.

3. The model is based on the concept of complex-value moduli, which are determined by a modified technique of equivalent linearization. In terms of these moduli, the initial problem is reduced to a scleronomous system of equations for complex-value amplitudes of mechanical field variables – displacements, stresses, total and inelastic strains.

4. The simplified model of the coupled behaviour of physically nonlinear bodies under harmonic loading is used to develop an approximate formulation of the coupled dynamic problem for thin-walled structural elements.

5. Within the framework of this formulation, for the partial case of forced vibrations of a beam, the amplitude–frequency characteristics of the main field variables are studied within a wide frequency range. The temperature characteristics for the first and second resonance modes are compared.

6. Approximate monoharmonic approach provides quite reliable and accurate results for the considered class of processes under harmonic loading and described conditions.

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MONOHARMONINIS BŪDAS TIRIANT PLONASIENIŲ NETAMPRIŲ ELEMENTŲ SVYRAVIMUS IR DISIPACINĘ ŠILUMĄ

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S a n t r a u k a

Naudojant geometrinio tiesiškumo ir Kirchhofo ir Love hipotezes, pateikiama apytikslė jungtinė dinamikos ir disipacinės šilumos uždavinio formuluotė fiziškai netiesiniams konstrukciniams elementams. Apytikslis modelis taikomas harmonine apkrova veikiamo nehomogeninio fiziškai netiesinio kūno svyravimams ir išskiriamai šilumai aprašyti. Sprendžiamas neharmoninių svyravimų disipacinės šilumos uždavinys. Pasitelkus harmoninių svyravimų uždavinį, gaunama disipacijos funkcija, kuri naudojama vidinės šilumos šaltiniams modeliuoti. Esant priverstiniams sijos svyravimams, plačiai nagrinėjamos amplitudės ir dažnio charakteristikos. Lyginamos temperatūros charakteristikos, atitinkančios pirmojo ir antrojo rezonanso formas.

Reikšminiai žodžiai: plonasienė konstrukcija, jungtinis temperatūrinis ir mechaninis uždavinys, disipacinė šiluma, monoharmoninė aproksimacija.

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