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Deductions and Reductions Decoding Syllogistic Mnemonics

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Abstract: The syllogistic mnemonic known by its first two words Barbara Celarent introduced a constellation of terminology still used today. This concatenation of nineteen words in four lines of verse made its stunning and almost unprecedented appearance around the beginning of the thirteenth century, before or during the lifetimes of the logicians William of Sherwood and Peter of Spain, both of whom owe it their lasting places of honor in the history of syllogistic. The mnemonic, including the theory or theories it encoded, was prominent if not dominant in syllogistics for the next 700 years until a new paradigm was established in the 1950s by the great polymath Jan Łukasiewicz,

Entelekya Logico-Metaphysical Review

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a scholar equally at home in philosophy, classics, mathematics, and logic. Perhaps surprisingly, the then-prominent syllogistic mnemonic played no role in the Łukasiewicz work. His 1950 masterpiece does not even mention the mnemonic or its two earliest champions William and Peter. The syllogistic mnemonic is equally irrelevant to the post-Łukasiewicz paradigm established in the 1970s and 1980s by John Corcoran, Timothy Smiley, Robin Smith, and others. Robin Smith's comprehensive 1989 treatment of syllogistic does not even quote the mnemonic's four verses. Smith's work devotes only 2 of its 262 pages to the mnemonic. The most recent translation of *Prior Analytics* by Gisela Striker in 2009 continues the post-Łukasiewicz paradigm and accordingly does not quote the mnemonic or even refer to the code—although it does use the terminology. Full mastery of modern understandings of syllogistic does not require and is not facilitated by ability to decode the mnemonic. Nevertheless, an understanding of the history of logic requires detailed mastery of the syllogistic mnemonic, of the logical theories it spawned, and of the conflicting interpretations of it that have been offered over the years by respected logicians such as De Morgan, Jevons, Keynes, and Peirce. More importantly, an understanding of the issues involved in decoding the mnemonic might lead to an enrichment of the current paradigm—an enrichment so profound as to constitute a new paradigm. After presenting useful expository, bibliographic, hermeneutic, historical, and logical background, this paper gives a critical exposition of Smith's interpretation.

Keywords: Syllogistic, mnemonics, deduction, reduction, *Prior Analytics*, Robin Smith.

Overview

It is evident too that all imperfect syllogisms are perfected through the first figure. For they are all brought to a conclusion either ostensively or through the impossible, and in both cases the first figure comes about. 29a30

But one can also reduce all syllogisms to the universal ones in the first figure. 29b1[1]

Aristotle's syllogistic is restricted to arguments involving only propositions of the four forms known today by the letters A, E, I, and O, sometimes lowercase a, e, i, and o. Aristotle considered arguments with two or more premises. The fact that he seems to say that nothing follows from a single premise (and thus that all one-premise arguments are invalid) is an embarrassment to his admirers. In contrast, some take pride in his discussion of multi-premise arguments and even ones with infinitely many premises. However, at the core of Aristotle's syllogistic are 256 two-premise argument forms, 24 of which are "valid", more properly *omnivalid*, i.e., have only valid instances. The remaining 232 are *nullovalid*, i.e., have only invalid instances.

Although Aristotle did not explicitly identify all 24, the deduction system Aristotle presented establishes validity for each of the 24 by means of direct and indirect deductions that obtain the conclusions from the respective premises in a step-by-step way using eight formally specified rules of deduction. The direct and indirect deductions use as two-premise rules four of the 24 forms—those four known today as Barbara, Celarent, Darii, and Ferio. As one-premise rules they uses repetition and the three known as conversions.

The direct and indirect deductions are explicitly goal-directed: after the premises are identified, the conclusion is identified as a goal to be deduced. After that, deductions are completed by chains of reasoning that show the conclusion to be a consequence of the premises. In a direct deduction the first step in

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[1]   Aristotle, *Prior Analytics Book I*, trans. Gisela Striker (Oxford: Oxford University Press, 2009), 12.

the chain of reasoning is obtained by applying a rule. In an indirect deduction the first step in the chain of reasoning is the assumption of the contradictory of the conclusion.

Every deduction shows that its conclusion follows from its premise set. But of course, the deduction per se does not show that its conclusion is true. The premises need not be true and, even if they are true, they need not be known to be true—as required for demonstration. As in modern logic, Aristotle distinguishes deductions from demonstrations, which do produce knowledge of their conclusions. Aristotle's successors—whether ancient, medieval, or modern—do not always recognize Aristotle's deduction/demonstration distinction or incorporate it into their deliberations. This oversight leads to confusion.

Aristotle's syllogistic originated about 350 BCE as part of a theory of demonstrative knowledge. After Aristotle's substantial beginnings, early progress in developing syllogistic had been slow. Some historians think neither the number of forms, 256, nor the number of valid forms, 24, were established until about 2000 years later; some say around the time of Leibniz (1646-1716). Knowledge of the number of forms and the number of valid forms was not widespread until at least the late 1800s.

Anyway, much earlier, probably around 1200 there was a major notational and expository innovation—we call *the syllogistic mnemonic*—created by a mysteriously anonymous logician whose identity continues to elude historians. The substance of the innovation was soon reported by William of Sherwood (fl. 1250) and Peter of Spain (fl. 13th century). To start with, the A-E-I-O notation was introduced and the remaining letters at the beginning of the Latin alphabet, B, C, D, and F, were used as initial letters of names of Aristotle's four two-premise rule forms—the same names still used today: Barbara, Celarent, Darii, and Ferio.

The innovation did not end with these useful stipulations. Rather, the notations for the four categorical proposition-forms and four first-figure argument-forms were made the basis of an ingeniously intricate mnemonic system that assigned names—

such as Baroco, Cesare, Disamis, and Felapton—to most of the 20 non-rule two-premise valid forms. Moreover, that assignment also named processes reflecting a way of relating non-rule two-premise valid forms to the four rule forms, e.g., Baroco to Barbara, Cesare to Celarent, Disamis to Darii, and Felapton to Ferio. The processes were indicated by a third foursome of letters: C, M, P, and S. Some later logicians uncomfortable with the dual use of C replace it with K in the process use—turning Baroco into Baroko, for example. Each non-rule form name begins with the first letter of the name of the rule form it relates to. This paper investigates what that "way of relating" has been taken to be. That "way of relating" is explained in different ways by different decodings of the mnemonic names.

For example, *deductivists*, as we call them, decode the code name Bocardo as signifying a certain five-step indirect *deduction* of an O-conclusion from an O-major and A-minor. The deduction uses Barbara as a two-premise rule. In contrast, *reductivists* decode Bocardo as signifying a one-step indirect reduction that transforms a second-figure syllogism into Barbara, a first-figure syllogism. These are given in detail below.

For another example, *deductivists* decode the code name Camestres as signifying a certain three-step direct *deduction* of an E-conclusion from an A-major and E-minor. The deduction uses Celarent as a two-premise rule. Roughly, from the premises of Camestres the premises of Celarent are deduced and then Celarent is used to deduce a conclusion from which Camestres's conclusion is deduced. In the deduction, Celarent comes in the middle: after Camestres's premises have been given but before its conclusion has been deduced.

In contrast, *reductivists* decode Camestres as signifying a three-step direct reduction that transforms Camestres, a second-figure syllogism into Celarent, a first-figure syllogism. In the reduction, Celarent comes at the end after three steps: one transforming Camestres into another argument, one transforming that into still another argument, and one transforming that into

Celarent. These too are given in detail below.

We focus on three opinions: (1) On the deductivist opinion of the distinguished Aristotle scholar Robin Smith expressed in Appendix I of his masterful 1989 translation of Aristotle's *Prior Analytics*, (2) on the contrasting reductivist opinion of Peter of Spain, and (3) on the combined deductivist-reductivist opinion of Augustus De Morgan. Other opinions are also investigated.

The issue between the deductivists and the reductivists concerns how the four mnemonic verses are to be decoded. If suitable rules can be found or devised, there is no aprior reason why both cannot be 'right'; the issue would be one of subjective preference. Anyway, the issue does not concern the intentions of its anonymous creator.

Perhaps the issue is analogous to the question of how a certain device is to be used, a question to which the inventor's intention is irrelevant. Moreover, the issue is likewise independent of the content of *Prior Analytics.* Nevertheless, understanding the background of the mnemonic verses, requires awareness, as is widely known, that deduction and reduction are two processes recognized in *Prior Analytics*, Book A, Chapter 7.[2]

Smith 1989 brings deduction to our attention repeatedly but he recognizes reduction as a separate process without, however, attempting to give Aristotle's rules for it. In Chapter A 7, he translates Aristotle: "It is furthermore evident that all the incomplete deductions are completed through the first figure" (29a30). For Smith completing an incomplete deduction (*sullogismos*) is deduction which is distinguished from reduction, a "process of transforming [sc. Incomplete] deductions from one figure to another".[3]

Similarly, Striker 2009 also separates the two processes of

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[2]   See John Corcoran, "Deduction and Reduction: Two Proof-Theoretic Processes in *Prior Analytics* I," *Journal of Symbolic Logic* 48 (1983); Aristotle, *Prior Analytics*, trans. Robin Smith (Indianapolis: Hackett Publishing Company, 1989); Aristotle, *Prior Analytics Book I.*

[3]   Aristotle, *Prior Analytics*, 161.

deduction and reduction. In this chapter she translates Aristotle: "But one can also reduce all syllogisms to the universal ones in the first figure" (29b1). Without explicitly identifying the transformational nature of reduction as Smith did, she did give convincing textual evidence for the separation. It is worth quoting her in full (Striker 2009, page 109). Commenting on 29b1, she wrote: "The word 'also' indicates that [...] all imperfect moods can also be reduced to those of the first figure. Hence it is tempting to treat the verb 'to reduce' (*anagein*, literally, to lead back) as a synonym of 'to perfect', as was indeed done from the ancient commentators on. Yet this assumption turns out to be unwarranted, as the following paragraph shows: there are cases of reduction of a mood to another mood that are not cases of perfection—as in the reduction of the first-figure moods Darii and Ferio, which are already perfect, to second-figure moods".

Although interpretation of *Prior Analytics* is irrelevant to this article, it would be misleading to omit mentioning the fact that several deductions and their rules are readily identifiable in the text of *Prior Analytics*. See Smith's Introduction and Appendix I. In contrast, it would be misleading to suggest that reductions and their rules are readily identifiable in the text of *Prior Analytics*. We know of no clear examples. Smith thinks there are none.

For purposes of exposition we need a neutral word for whatever it is that the 15 "imperfect" mnemonic names encode, more precisely, for the things constructed by following the instructions encoded by those 15 names. The word 'derivation' seems suitable. Accordingly, deductivists take derivations to be deductions. For example, deductivists take Camestres to encode instructions for deducing the conclusion from the premises of a syllogism in the form known as Camestres. In contrast, reductivists take derivations to be reductions. For example, reductivists take Camestres to encode instructions for reducing the syllogism in the form known as Camestres to one in the form known as Celarent.

Unfortunately, the sharp distinction between (1) deductions

(of conclusions from premises) and (2) reductions (of arguments to arguments) is not yet standard in the literature. Some scholars use 'deduction' in the general sense of "derivation"; some use 'reduction' in that sense; and some use two or all three words interchangeably.

For example, in speaking of Aristotle's treatment of Bocardo on page 36, Parsons uses 'reduction by reductio' to refer to an indirect deduction.[4] Parsons insightfully distinguishes indirect deductions from indirect reductions on page 53 where he takes the name Bocardo to decode an indirect reduction, without using 'deduction' and 'reduction' as contrasting words. For an example of Parsons using 'deduction' for a reduction of an argument to an argument, see the first paragraph of page 39 of the same book.[5]

Introduction

There are then [nineteen] forms of syllogism [...]. I now put them down, with their derivations, [...], figures into which they fall, and the magic words by which they have been denoted for many centuries, words which I take to be more full of meaning than any that ever were made. — Augustus De Morgan, 1847, 150.[6]

William of Sherwood (c. 1200-1272) gave the oldest known version of the mnemonic.[7] Below we quote from the only known manuscript: Bibliothèque Nationale MS. Lat. 16617, more briefly, BN 16617. William's quoted version contains 19 names in four lines with the explicit auxiliary stipulation that "The first two lines are devoted to the first figure, four words of the third line to the second figure, and all the other words to the third figure". The first 4 of the 19 names are Barbara, Celarent, Darii, and Ferio—the earliest known logical use of these four words.

William's book had not used any of these 19 names earlier.

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[4]   Terence Parsons, *Articulating Medieval Logic* (Oxford: Oxford University Press, 2014), 36.

[5]   Parsons, *Articulating Medieval Logic*, 39.

[6]   Augustus De Morgan, *Formal Logic or The Calculus of Inference, Necessary and Probable* (London: Taylor and Walton, 1847), 150.

[7]   See William Sherwood, *William of Sherwood's Introduction to Logic*, trans. Norman Kretzmann (Minneapolis: University of Minnesota Press, 1966), 66ff.

Moreover, before presenting the mnemonic, and of course without using the mnemonic names, William had described conversions, the 4 perfect syllogisms, and the 15 imperfect syllogisms. Moreover he also presents derivations for the 15. Some were deductions using the 4 as rules (with conversions, of course). However, in presenting a deduction for a mood he routinely said that the mood "reduces" to one of the first four moods.[8] Some were reductions to the four; two were indirect even though the rule they used had not been mentioned before. Nothing was said about the lists of arguments later logicians called reductions.[9] We quote BN 16617:[10]

> *Barbara celarent darii ferio baralipton*
>
> *Celantes dabitis fapesmo frisesomorum*
>
> *Cesare campestres festino baroco darapti*
>
> *Felapton disamis datisi bocardo ferison*

A little later, Peter of Spain (fl. 13th century) gave a similar list with the same figure stipulation. We quote Parsons:[11]

> *Barbara Celarent Darii Ferio Baralipton*
>
> *Celantes Dabitis Fapesmo Frisesomorum*
>
> *Cesare Cambestres Festino Barocho Darapti*
>
> *Felapton Disamis Datisi Bocardo Ferison*

William and Peter differ on the spellings of Camestres and Baroco. More importantly, both present four-verse poems in classical dactylic hexameter, a form made famous by Homer in Greek and by Virgil and Ovid in Classical Latin. This suggests that the anonymous creator of the mnemonic was schooled in poetry over and above, as we will see, being masterful in his knowledge of Aristotle and imaginative in logic. Anyway, he was as attentive to the appearance of his creation as he was to its substance. His

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8 Sherwood, *Introduction to Logic*, 64ff.
9 Corcoran, "Deduction and Reduction."
10 Compare Lambertus Marie De Rijk, *Logica Modernorum* (Assen: Koninklijke Van Gorcum & Company, 1967), 401.
11 Parsons, *Articulating Medieval Logic*, 51.

patience, taste, learning, and imagination set him above many who discussed his work later.

Some later versions interpolate words usefully indicating groupings into figures but destroying the classical metric beauty. Others destroy the metric by rearranging the words or moving one word from one verse to another. Others contain alternative spelling such as Ferion and Ferioque for Ferio. Some reflect badly on the education of the author. For example, the word *Ferioque* was used by knowledgeable Latin writers but not as a name of Ferio: *que* is a conjunction and *Ferioque* means "and Ferio". People who copy things they do not understand are more likely to miscopy or to make what they mistaken regard as innovative improvements. On this point, Kneale and Kneale present what they called the first appearance of the mnemonic verses in William of Sherwood.[12] But they actually give Peter's version except that Cambestres is misspelled Campestres—substituting the mnemonically significant p for the mnemonically insignificant b. In addition, like the Parsons rendering of Peter's version, they capitalize all nineteen code names thereby giving the misleading impression that capitalization is mnemonically significant. Today it is conventional to use the capitalized forms whether or not the insignificance of the capitalization is noted.

We use the notation established in Corcoran 2009. In particular, Asp, Esp, Isp, and Osp are respectively the universal affirmative, universal negative, existential affirmative, and existential negative propositions with s as subject and p as predicate. As can be seen, we avoid the clutter of special notation for use-mention except where required by the context.

Arguments, i.e., premise-conclusion arguments, are presented by listing the premises vertically in a column, drawing a horizontal line, and listing the conclusion below. For typing convenience, the line is drawn by underlining the last premise.

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[12] William Kneale and Martha Kneale, *Development of Logic* (London: Clarendon Press, 1962), 232.

Using this notation, Barbara, Celarent, Darii, and Ferio are as follows.

Amp	Emp	Amp	Emp
Asm	Asm	Ism	Ism
Asp	Esp	Isp	Osp

In addition to the above, vertical column notation, we will also use a horizontal row notation which lists the premises in a row followed by a slash before the conclusion. Using the row notation, Barbara, Celarent, Darii, and Ferio are as follows.

Amp, Asm /Asp

Emp, Asm /Esp

Amp, Ism / Isp

Emp, Ism / Osp

In presenting an argument, as opposed to asserting the premises followed by an assertion of the conclusion as an inference, it would be misleading to replace the separating slash / by the conjunction 'therefore'. Likewise misleading would be to end the presentation with a period suggesting that it is a sentence.

 Using the syllogistic mnemonics, Ferio-1, Festino-2, and Felapton-3 are the following three syllogisms.

Emp	Epm	Emp
Ism	Ism	Ams
Osp	Osp	Osp

The first vowel in a code name indicates the type [A, E, I, O] of the major premise; the second indicates the type of the minor; and the third indicates the type of the conclusion. Neither William nor Peter identifies anything in the names Ferio, Festino, and Felapton indicating the figures assigned by the auxiliary stipulation: first, second, and third respectively.

Notice that without the full display of all names with explicit auxiliary figure stipulation the names would not indicate the figure: e.g., it would be unspecified whether the major of Ferio would be Emp or Epm, whether the minor of Ferio would be Ism or Ims, and whether the conclusion of Ferio would be Osp or

Ops. Where the auxiliary stipulation is not readily available the figure assignment must be done explicitly, e.g., by adding a number as Ferio-1, Festino-2, and Felapton-3. But that would be to deviate from mnemonic tradition.

Once a system of decoding is obtained, whether deductivist, reductivist, or other, it might be possible to use it to extract the figure from the code name, but we are not aware of any published sources about this. When we tried using one deductivist decoding and one reductivist decoding on a few examples, we succeeded.

According to logicians such as Smith,[13] the names Festino and Felapton encode instructions for constructing a deduction of the conclusion from the premises using Ferio as the two-premise rule—in the context of Aristotle's natural-deduction system.[14] The occurrence of *s* in Festino-2 indicates use of a one-premise rule of *"Simple conversion"* involving the component whose letter it follows: in this case deducing Epm from the major Emp. The occurrence of *p* in Felapton indicates use of a one-premise rule of *"Partial conversion"* involving the component whose letter it follows: in this case deducing Ism from the minor Ams.

1	Epm		1	Emp	
2	Ism		2	Ams	
?	Osp		?	Osp	
3	Emp	1, s	3	Ism	2, p
4	Osp	3, 2 F [Ferio]	4	Osp	1, 3 F
QED			QED		

The above deductions for Festino and Felapton are transcriptions of Aristotle's using the notation established in Corcoran 2009 and 2018 where the question mark indicates the goal, the conclusion to be reached. There are several reasons for leaving it without a line number: For example, no rule of inference is applied to it and thus numbering it would be pointless. For Aristo-

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[13] Aristotle, *Prior Analytics*, 229ff.

[14] Presented in Corcoran, "Completeness of an Ancient Logic," *Journal of Symbolic Logic* 37 (1972),

tle's deductions, where the conclusion to be reached is indicated before any deduction rules are applied.[15] Opinions like Smith's that take the names to describe deductions are called *deductivist*. The most recent deductivist opinion is that of Rini, who states: "The names of the syllogisms […] encode instructions for [sc. constructing] Aristotle's proofs".[16] For convenience we reproduce her only example of decoding: a deduction decoded from 'Darapti' and we juxtapose its transcription into our preferred notation.

(1)	A belongs to every C		1	Aca	
(2)	B belongs to every C		2	Acb	
(3)	C belongs to some B	A-Conversion 2	?	Iba	
(4)	A belongs to some B	Darii 1, 3	3	Ibc	2, p
			4	Iba	1, 3 D
			QED		

To be clear, although this is Rini's only example of decoding, two other deductions are given: Cesare and Datisi.[17] But nothing is said about obtaining those two deductions by decoding the words. Even more peculiar is the fact that despite the claim that "this chapter explains how to decode the medieval names of the syllogisms" nothing is said about transposition (indicated by m as in Disamis-3) or contraposition (indicated by c as in Baroco-2 and Bocardo-3).[18]

Below indirect deductions for Baroco-2 and Bocardo-3 are transcriptions of Aristotle's. As explained in Corcoran 2009 and Corcoran 2018, the X is read "A contradiction" and the numbers indicate the two lines comprising the contradiction.

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[15] See Aristotle, *Prior Analytics*, 7, 9 and 230.
[16] Adriane Rini, "Aristotle's Logic," *The History of Philosophical and Formal Logic: From Aristotle to Tarski*, eds. Alex Malpass and Marianna Antonutti Marfori (London: Bloomsbury Academic, 2017), 47.
[17] Rini, "Aristotle's Logic," 42-3.
[18] Rini, "Aristotle's Logic," 48, n. 3.

BAROCO-2

1	Apm	
2	Osm	
?	Osp	
3	@ Asp	
4	Asm	1, 3 B
5	X	4, 2
QED		

BOCARDO-3

1	Omp	
2	Ams	
?	Osp	
3	@ Asp	
4	Amp	3, 2 B
5	X	4, 1
QED		

As a guard against confusion, it is important to realize (with Aristotle) that every direct deduction transforms readily into an indirect deduction of the same conclusion from the same premises simply by two operations: (1) inserting the reductio assumption between the goal and the first step, (2) noting that the last step is the contradictory of the reductio assumption, thus completing an indirect deduction. Here we give the results of transforming direct deductions of Festino and Felapton into indirect deductions.

1	Epm		1	Emp	
2	Ism		2	Ams	
?	Osp		?	Osp	
3	@ Asp		3	@ Asp	
4	Emp	1, s	4	Ism	2, p
5	Osp	4, 2 F [Ferio]	5	Osp	1, 4 F
6	X	5, 3	6	X	5, 3
QED			QED		

The above indirect deductions for Festino and Felapton are obtained using Aristotle's instructions at 45b1-5.[19]

In contrast to logicians who take the mnemonic names to encode instructions for deducing conclusions from premises, logicians such as Eaton,[20] take the names to encode instructions for constructing a "reduction"—a list of arguments transforming the named syllogism (Festino and Felapton in these two cases) to one in the first figure (Ferio in these cases). Here the letter *s* after a premise or conclusion designation may indicate transforming that proposition into its simple converse to get the next argument. The occurrence of *p* in Felapton indicates transforming the component whose letter it follows, the minor Ams, into its *partial converse* Ism.

Epm, Ism /Osp		Emp, Ams /Osp	
Emp, Ism /Osp	s 1st premise	Emp, Ism /Osp	p 2nd prem.

The above reductions of Festino and Felapton to Ferio are transcriptions of Eaton's. Each reduction consists of two arguments: the first reduction is Festino-2 followed by Ferio-1; the second is Felapton-3 followed by Ferio-1.[21] One contrast between deductions and reductions is that although in deductions, except for the intended conclusion, any previous line or line pair is usable in transitioning to the next line (so numbering lines is useful), in reductions only the last line entered can be used in transitioning to the next line (so numbering lines is useless). For a succinct contrast between deduction and reduction, see Corcoran's 1983 lecture abstract.[22]

Corresponding to indirect deductions there are reductions traditionally called indirect.[23] Indirect reductions are those that use a rule, actually either of two rules, traditionally known as

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[19] Aristotle, *Prior Analytics*, 47 and 155.
[20] Ralph Eaton, *General Logic: An Introductory Survey* (London: Charles Scribners' Sons, 1931), 103ff.
[21] Eaton, *General Logic*, 125ff. and 123.
[22] Corcoran, "Deduction and Reduction."
[23] See Eaton, *General Logic*, 128ff.

contraposition, that carry one two-premise argument into another sharing one premise and having the other premise replaced by the contradictory of the conclusion while taking as its conclusion the contradictory of the replaced premise. We call the two rules *major contraposition* and *minor contraposition*. To illustrate how these two transformations work, we apply them to the invalid argument Amp, Ams /Asp.

Major contraposition	Minor contraposition
Amp Ams /Asp	Amp Ams /Asp
Osp Ams /Omp	Amp Osp /Oms

Leibniz and others thought of contraposition as combining two operations: (1) take one premise's contradictory and take the conclusion's contradictory, (2) replace that premise with the conclusion's contradictory and the conclusion with the premise's contradictory.

Major contraposition	Minor contraposition
Amp, Ams /Asp	Amp, Ams /Asp
Omp, Osp	Oms, Osp
Osp, Ams /Omp c major	Amp, Osp /Oms c minor

The indirect reductions we know of from the literature have only one contraposition application, but there is no consensus definition ruling out multiple applications. Our introduction to indirect reduction would be incomplete without the classic stock examples: reductions of Baroco-2 and Bocardo-3 to Barbara-1.

Apm Osm /Osp	Omp Ams /Osp
Apm Asp /Asm c minor	Asp Ams /Amp c major

The above reductions of Baroco and Bocardo to Barbara are transcriptions of Bocheński's.[24] Notice that an indirect deduction contains a contradiction and is thus properly called by names such as "deduction ad impossibile". In contrast, an indirect reduction is free of contradiction and thus should never be referred to by an expression suggesting otherwise such as "reduc-

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[24] Joseph Bocheński, *History of Formal Logic*, Trans. Ivo Thomas (Notre Dame: University of Notre Dame Press, 1961), 260.

tion ad impossibile"—without adequate disclaimers. The fact that indirect deductions contain contradictions but indirect reductions typically don't is clearly noted by Parsons where he attributes the observation to Peter of Spain.[25] Parsons also notes that it was inappropriate for Peter to call such a reduction 'a reduction by impossibility'.

The fact that indirect reduction uses two rules, one replacing the major and one the minor, is reflected in the placement of the code letter c : after major's letter as in Bocardo or after the minor's as in Baroco. This rare observation about the significance of the placement of the c code was made by Kneale and Kneale and by De Rijk.[26] For example, De Morgan 1847 omits it on pages 151ff where the decoding is treated and Parsons 2014 fails to mention it on pages 51ff where the mnemonic is treated.

There is no *locus classicus* we know of about transforming arbitrary direct reductions into corresponding indirect reductions, i.e., of the same initial argument to the same final argument—whether by Aristotle, a commentator, a medieval, or a traditional logician. Eaton mentioned two cases, though not in Aristotle's syllogistic as understood by Smith 1989 and the present writers.[27] However, Leibniz showed that all twelve valid two-premise categorical arguments in figures two and three can be reduced indirectly to one of the six in the first figure. Here are indirect reductions of Festino and Felapton to Celarent and Barbari.

Epm, Ism /Osp		Emp, Ams /Osp	
Epm, Asp /Esm	c, 2nd premise	Asp, Ams /Imp	c 1st prem.

The above reductions of Festino-2 and Felapton-3 to Celarent-1 and Barbari-1 respectively are transcriptions of those attributed to Leibniz by Bocheński.[28]

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[25] Parsons, *Articulating Medieval Logic*, 53.
[26] Kneale and Kneale, *Development of Logic*, 233; De Rijk, *Logica Modernorum*, 401.
[27] Eaton, *General Logic*, 129f
[28] Bocheński, *History of Formal Logic*, 259ff.

So far we have seen two approaches to decoding syllogistic mnemonics: one exemplified by Smith which we call *deductivist*, one exemplified by Eaton which we call *reductivist*. There is a major disagreement between deductivists and reductivists, even though in many cases deductivists are unaware or barely aware of the process of reduction and in many cases reductivists are unaware or barely aware of the process of deduction. There is no active debate between deductivists and reductivists. There are also major disagreements among deductivists and major among reductivists, as we indicate below.

However, there is one important agreement between the deductivist and the reductivist: both hold that the mnemonic names of the syllogistic forms not only denote argument forms; the names also encode sequences of operations. From the deductivist perspective, one difference between 'Barbara' and 'Baroco' is that the former names an argument form without giving an algorithm for deducing its conclusion from its premises, so to speak, whereas the latter does both. From the reductivist perspective, one difference between 'Barbara' and 'Baroco' is that the former names an argument form without giving an algorithm for reducing it to another argument form, whereas the latter does both.

The semantic differences between 'Barbara' and 'Baroco' resemble somewhat those between '9' and '((3 + 3) +3)'. One difference between '9' and '((3 + 3) +3)' is that the former names a number without giving an algorithm for computing it from a smaller number, so to speak, whereas the latter does both.

Along with the disagreements between deductivists and reductionists, there are many differences between the process of deduction and the process of reduction. Some have been described before.[29] But an important philosophical difference has not been mentioned in print before. To grasp this, notice that not all deduction produces knowledge of truth of their conclusions;

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[29]   See Corcoran, "Deduction and Reduction."

but demonstrative deduction does. Likewise, notice that not all reductions allegedly produce knowledge of validity of their initial arguments; but syllogistic reductions allegedly do, where a syllogistic reduction reduces incomplete forms to complete forms.

The alleged cognition-flow direction of syllogistic reduction is opposite from that of demonstrative deduction. We come to know that a conclusion is true by demonstratively deducing it from premises known to be true. The cognition-flow in demonstrative deduction is from known to unknown. Demonstration creates knowledge.

According to several of our sources, reduction has a cognition-producing function.[30] Allegedly, we come to know that an argument is valid by syllogistically reducing it to an argument known to be valid. The cognition-flow in reduction is from unknown to known. Reduction annihilates ignorance. But none of our sources explain how reduction produces knowledge. In fact none of them even attempts to make this obscure claim plausible. None of us, the authors of this article, can see how a reduction can bring about knowledge of validity or how a reduction can destroy ignorance of it. To us reduction is an interesting formal process whose epistemic significance, if any, remains to be established. We need an epistemology of reduction. Although it is easy to see that deductions, and in particular Aristotle's deductions, produce knowledge of validity of arguments. We have all been faced with an argument whose validity we did not know and then, after being shown a deduction of the conclusion from the premise, acquired knowledge of its validity.[31]

Knowing how to deduce is one form of operational knowledge, "know how". Deducing a conclusion from premises produce knowledge that the argument is valid, which is a form of propositional knowledge, "know that".

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[30] See, for example, Parsons, *Articulating Medieval Logic*, 51ff.; De Rijk, *Logica Modernorum*, 401; Sherwood, *Introduction to Logic*, 58ff.

[31] See Corcoran, "Argumentations and Logic," *Argumentation* 3 (1989), 17-43.

Knowing how to reduce is another form of operational knowledge, "know how". Reducing a given argument whose validity or invalidity is not known to one whose validity is known is supposed to produce knowledge that the given argument is valid. We, the authors, have never had this experience.[32]

Moreover, we have never seen a plausible answer to the question of what is learned by reducing a given argument whose validity or invalidity is not known to another whose validity or invalidity is not known. In fact, we have never seen a plausible answer to the question of what is learned by reducing one given argument to another.

Let the above introductory remarks suffice so we may proceed to one of the main goals of this paper: to analyze, criticize, and correct Smith's 1989 account of the mnemonic [Appendix I, pp. 229ff.]

Some Accounts of the Coded Processes

The third paragraph below is Smith's entire account verbatim. We have numbered selected sentences, clauses, and phrases in braces for convenience. Smith supplied no references and no indications of where he got his information. He did not say who created the mnemonic he uses, or whether there are or were alternatives. Likewise Smith does not reveal whether his mnemonic came into existence all at once or whether it evolved. Moreover, Smith does not say who constructed the deductions the mnemonic names encode. In particular, in contrast his fellow deductivist Rini says that they encode deductions Aristotle presented in *Prior Analytics*.[33]

More importantly, he does not say that the four lowercase vowels, a, e, i, and o, stand respectively for the four propositional

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[32] See Corcoran and Idris Samawi Hamid, "Investigating Knowledge and Opinion," *The Road to Universal Logic: Festschrift for 50th Birthday of Jean-Yves Béziau*, eds. Arnold Koslow and Arthur Buchsbaum (Dordrecht: Springer, 2014), 95-126.

[33] Rini, "Aristotle's Logic," 47.

kinds: universal affirmative, universal negative, particular affirmative, and particular affirmative. Likewise missing is indication that the four uppercase consonants, B, C, D, and F, stand for the four perfect, or complete, syllogisms, or deductions (to use Smith's terminology) in the first figure: Barbara, Celarent, Darii, and Ferio—in which the first vowel stands for the major, the second for the minor, and the third for the conclusion.

Smith's entire account.

{1} The traditional names for the incomplete forms actually encode instructions for carrying out proofs. {2} The first letter of the name (B, C, D, F) indicates the first-figure form to which the proof appeals; {3} 's' following a vowel indicates that the corresponding premise (always an *e* or *i*) is to be converted (*conversio simplex*); {4} 'p' following 'a' indicates 'conversion by limitation' (*conversio per accidens*) of a universal premise, i. e., {5} conversion into a particular premise (*a* into *i* , *e* into *o*); {6} 'r' indicates proof through impossibility; and {7} 'm' indicates that the premises must be interchanged. {8} (Other letters, such as 'l' and 'n,' have no significance.) {9}Thus, the name *Camestres* tells us that a proof that an *e* conclusion follows from an *a* major premise and an *e* minor may be constructed by {10} converting the first premise (*Camestres*) and {11} interchanging the premises (*Camestres*) {12}, giving the first-figure form **C**elarent, (***C***amestres) then {13} converting the conclusion (*Camestres*); and, that {14} a proof through impossibility is also possible (*Camestres*).

For comparison we present the medieval accounts by William and by Peter and the modern account by William and Mary Kneale—but only those sentences relating to the process code.

William's account of the process code.[34]

In these lines […]'s' [signifies] simple conversion [*conversio simplex*], 'p' conversion by limitation [*conversio per accidens*], 'm' transposition of the premisses, and 'b' and 'r' when they are in the same word signify reduction *per impossibile*.

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[34]   Sherwood, *Introduction to Logic*, 67.

COMMENTS: William's account has two errors in the quoted passage alone. (1) His instruction for decoding P does not cover Baralipton either for deductivist or reductivist decodings. The I proposition, indicated by the small letter preceding the P in Baralipton, does not convert accidentally. Other logicians make the same mistake. Jevons makes this mistake in an otherwise flawless and revealing account.[35] As an example on the next page, Jevons tries to reduce Bramantip-4 to Barbara and seems not to realize that he failed. As will be noted below, Smith makes it and another error in his account of the per accidens rule.[36] (2) William's instruction for encodings requiring indirect reduction fits Baroco and Bocardo but not Baralipton. How he arrived at this is a mystery. Besides, even adding a lame patch such as "except Baralipton" does not give enough information for the reader to handle Baroco and Bocardo differently as the different placements of C require—again, either for deductivist or reductivist decodings. Where William said simply that M indicates transposing the premises, Peter is more explicit. Peter says, "Wherever M is put, it signifies that a transposition in premises is to be done, and a transposition is making a minor out of a major, and the converse." This will appear to be a mistake to readers of Striker 2009 and Smith 1989, not to mention De Morgan, Jevons, and many others[37]—all of whom take an argument's major premise to be the one containing its conclusion's predicate and take an argument's minor premise to be the one containing its conclusion's subject. With that definition, transposition could not be making a minor out of a major. The only way of making a minor out of a major is to convert the conclusion.

However, Peter does not define an argument's major and minor premises at all. Rather he defines an argument presentation's major and minor premises to be those coming first and

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[35] W. Stanley Jevons, *Elementary Lessons in Logic: Deductive and Inductive* (London: Macmillan, 1870), 146.

[36] Aristotle, *Prior Analytics*, 230.

[37] De Morgan, *Formal Logic*, 148; Jevons, *Elementary Lessons in Logic*, 128.

second respectively. Thus, Peter is meticulously accurate—"transposition is making a minor out of a major, and the converse".

In contrast, in De Morgan's account the M rule is erroneously described making an argument's major premise of its minor and conversely.[38] Other modern logicians make the same mistake, e.g. Jevons.[39]

Incidentally, William does not give even one example of decoding one of the 15 coded instruction sets. As said above, before giving the mnemonic William gives derivations for his 15 imperfect moods but he never says how they are encoded or how they are obtained using his instructions.

Peter's account of the process code.[40]

"Also, wherever an S put in these words, it signifies that the proposition understood by the immediately preceding vowel is to be converted simply. And by P it signifies that the proposition is to be converted accidentally. Wherever M is put, it signifies that a transposition in premises is to be done, and a transposition is making a minor out of a major, and the converse. Where C is put, however, it signifies that the mood understood by that word is to be confirmed by impossibility."

COMMENT: Peter's account has two errors in the quoted passage alone. (1) His instruction for decoding P does not cover Baralipton either for deductivist or reductivist decodings. Parsons tries to excuse this erroneous instruction by saying: "These instructions work perfectly provided that conversion by limitation is used in the correct order; from universal to particular in premises, and from particular to universal in conclusions (the verse is written so as to require this)".[41] The I proposition, indicated by the small letter preceding the P in Baralipton, does not convert accidentally. Parsons sentence is an oxymoron or a tau-

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38 De Morgan, *Formal Logic*, 148 and 151.
39 Jevons, *Elementary Lessons in Logic*, 128 and 146.
40 Parsons, *Articulating Medieval Logic*, 52.
41 Parsons, *Articulating Medieval Logic*, 52.

tology. (2) Peter's instruction for decoding C does not give enough information for the reader to handle Baroco and Bocardo differently as the different placements of C require—again, either for deductivist or reductivist decodings.

Where William said simply that M indicates transposing the premises, Peter is more explicit. Peter says, "Wherever M is put, it signifies that a transposition in premises is to be done, and a transposition is making a minor out of a major, and the converse." This will appear to be a mistake to readers of Striker 2009 and Smith 1989, not to mention De Morgan[42] and many others—all of whom take an argument's major premise to be the one containing its conclusion's predicate and take an argument's minor premise to be the one containing its conclusion's subject. With that definition, transposition could not be making a minor out of a major. The only way of making a minor out of a major is to convert the conclusion. However, Peter does not define an argument's major and minor premises at all. Rather he defines an argument presentation's major and minor premises to be those coming first and second respectively. Thus, Peter is meticulously accurate. In contrast, in De Morgan's account the M rule is erroneously described making an argument's major premise of its minor and conversely.[43]

Incidentally, Peter does not give even one example of decoding one of the 15 coded instruction sets. Before giving the mnemonic Peter gives derivations for some imperfect moods but he never says how they are encoded or how they are obtained using his instructions.

The Kneales account of the process code.[44]

Here [...] s appearing immediately after a vowel indicates that the corresponding proposition is to be converted simply during reduction, while *p* in the same position indicates that the proposition is to be converted partially or *per accidens*, and *m*

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[42]   De Morgan, *Formal Logic*, 148.
[43]   De Morgan, *Formal Logic*, 148 and 151.
[44]   Kneale and Kneale, *Development of Logic*, 232ff.

between the first two vowels of a formula indicates that the premisses are to be transposed; *c* appearing after one of the first two vowels indicates that the corresponding premiss is to be replaced by the negative of the conclusion for the purpose of a reduction *per impossibile.*

COMMENTS: The Kneales account has at least three errors in the quoted passage alone. (1) As in William's account and in Peter's account, the instruction for decoding P does not cover Baralipton either for deductivist or reductivist decodings. The I proposition, indicated by the small letter preceding the P in Baralipton, does not convert accidentally. (2) The instruction for M has a new error—not in William's or Peter's, and not in Smith's. Inexplicably, it gratuitously restricts itself to occurrences of M between the first two vowels as in Camestres-2. Thus it leaves the Ms in Fapesmo-4, Frisesomorum-4, and Disamis-3.

This account can be credited for recognizing that the position of C is significant. But it can be faulted for referring to the negative of the conclusion instead of the contradictory opposite: there is nothing negative about the contradictory opposites of negative conclusions. Moreover, (3) from the deductivist perspective it is an error to say that a premise is replaced in an indirect deduction or for that matter in any deduction: once the premises are set they remain in place regardless of what is added to complete the deduction. Also, from the reductionist perspective it is an error not to say that the conclusion is replaced by the contradictory opposite of the replaced premise.

Deductions and Reductions for Camestres-2

1	Apm	
2	Esm	
?	Esp	
3	Ems	2, s
4	Eps	3, 1 C [Celarent]
5	Esp	4, s
QED		

The above direct deduction for Camestres-2 is a transcription of Aristotle's using the notation established in Corcoran 2009 and 2018.[45]

1   Apm

2   Esm

?   Esp

3   @ Isp

4   Ism   1, 3 D [Darii]

5   X   2, 4

QED

The above indirect deduction for Camestres-2 using the two-premise rule Darii is in the notation established in Corcoran 2009 and 2018. Aristotle says that Camestres can be completed indirectly,[46] but he does not give the indirect deduction nor does he say which of the four two-premise rules he used.

According to logicians such as Keynes,[47] the names encode instructions for "reducing" (transforming) the named syllogism to one in the first figure: Celarent in these two cases.

Here the letter *s* before a premise or conclusion designation may indicate transforming that proposition into its simple converse to get the next line. The letter *m*, for "mutation", meaninglessly redundant in deductions, indicates interchanging the premises in reduction—a bookkeeping operation required by the convention that in the initial and final lines of a reduction the major premise comes first.

The letter *c* indicates indirect reduction transforming the named syllogism by a "double-reversing" process of replacing a premise by the contradictory of the conclusion and replacing the conclusion by the contradictory of the replaced premise—a process known as *contraposition* since the 1200s.

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[45] Aristotle, *Prior Analytics*, xxi and 7.

[46] See Aristotle, *Prior Analytics*, 27a14ff.

[47] John Neville Keynes, *Studies and Exercises in Formal Logic* (London: Macmillan & Co., 1906), 318ff.

The below is a direct reduction (left) of Camestres to Celarent juxtaposed with an indirect reduction (right) of Camestres to Ferio.

Apm, Esm/ Esp		Apm, Esm/ Esp	
Esm, Apm / Esp	m	Isp, Esm / Opm	c [mjr contrap.]
Ems, Apm / Esp	s 1st	Ips, Esm / Opm	s 1st
Ems, Apm / Eps	s conclusion	Esm, Ips / Opm	m

The above direct reduction (left) of Camestres to Celarent is a transcription of Keynes.[48] The above indirect reduction (right) of Camestres to Ferio is Corcoran's. Compare Leibniz's one-step indirect reduction Camestres to Darii.[49]

 Notice that at lines 2 and 3 in the indirect deduction the minor is the first premise. Moreover, at line 4, the same proposition that was previously a minor becomes the major—and without doing anything to the premises. Converting the conclusion reverses majority and minority. To secure this point that otherwise careful writers stumble over, notice that there is no way to reverse majority and minority without reversing subject and predicate in the conclusion.

Critiquing Smith's Account

Our critique is organized as follows. The main item critiqued is quoted for ready reference. Our comments are labeled A, B, C, etc. followed by the numbers of the relevant items in braces.

{1} The traditional names for the incomplete forms actually encode instructions for carrying out proofs.

Comment A {1}: Instead of "carrying out proofs", this should say something like "completing the incomplete form after the premises are expressed and the conclusion is set as the goal to be reached". For example, Smith's intention is to say that the name 'Camestres' encodes instructions for completing the following incomplete deduction.

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[48] Keynes, *Studies and Exercises in Formal Logic*, 320.
[49] Bocheński, *History of Formal Logic*, 260.

1   Apm
2   Esm
?   Esp

To be as explicit as this context requires, Smith takes the 9-character name 'Camestres' to be an encoding of instructions for going from the above 3-line incomplete deduction to the below 5-line complete deduction.

1   Apm
2   Esm
?   Esp
3   Ems   2, s
4   Eps   3, 1 C [Celarent]
5   Esp   4, s
QED

Comment B {1, 6, 14}: There are problems reconciling {1} with {6}, {14}, and the example 'Camestres'. {1} says the names encode instructions for completing a deduction but {6} says *r* indicates proof through impossibility, i.e. an indirect deduction.

Indicating an indirect deduction is not giving instructions for constructing one. Smith's account is entirely devoid of instructions for indirect deduction. For example, where is there any indication of which premise to use with the contradictory of the conclusion? That would be the major in our indirect deduction for Camestres above. Moreover, where is there any indication of which perfect deduction is to be used? In this case that would be Darii as in Leibniz's indirect deduction for Camestres above.

Without the r, 'Camestres' gives adequate directions for a direct deduction. According to {14} the r says that there is also an indirect deduction. To the best of our knowledge no other commentator in the history of logic took the r in Camestres the way Smith does. William's unfortunate b-and-r instruction is remotely similar. See Comment J below.

{2} The first letter of the name (B, C, D, F) indicates the first-figure form to which the proof appeals [...].

Comment C {2}: Smith needs to say that each of the encoded deductions has only one application of only one two-premise rule. As it stands, his expression 'the first-figure form to which the proof appeals' is a nonsequitur. Again, 'proof' should be 'deduction', 'completed deduction', or something of the sort. The topic here is deduction, not demonstration. Moreover, {2} has (B, C, D, F) being names: the names are Barbara, Celarent, etc. Finally, {2} does not tell the first-time reader what first-figure form the letter indicates.

Rewriting {2}: The first letter (B, C, D, or F) of the name is the first letter of the first-figure form (Barbara, Celarent, Darii, or Ferio) which the deduction uses. For example, Camestres uses Celarent.

Comment D {3, 13}: Smith's text {3} is: 's' following a vowel indicates that the corresponding premise (always an e or i) is to be converted (conversio simplex).

This reads like a first draft or worse. To clear the air we rewrite it: 's' follows only e and i and it indicates that the corresponding premise is to be converted (conversio simplex), that is, to be used as the premise in an application of the appropriate simple conversion rule [and not to be replaced by its own simple converse].

Smith evidently overlooked the fact that i occurs after conclusion indicators. Here are all relevant occurrences: *Celantes Dabitis Fapesmo Frisesomorum Cesare Camestres Festino Disamis Datisi Ferison*. Smith's rule does not cover *Celantes, Dabitis, Camestres,* and *Disamis*.

It is incoherent, a nonsequitur, to instruct someone to apply simple conversion to a deduction line that has not been reached yet.

Fortunately for us one of the untreated cases, viz., Camestres, is the one Smith used to exemplify his decoding scheme. His explanation is lucid until he reaches the last occurrence of s. There after the Celarent rule is applied he says at {13} that s tells you to

convert the conclusion—meaning the conclusion of the rule application.

Comment E {4}: {4} 'p' following 'a' indicates 'conversion by limitation' (*conversio per accidens*) of a universal premise.

Smith's expression 'of a universal premise' must mean "of a universal affirmative premise" because that is what the letter a would be indicating and because Aristotle—however awkwardly, mysteriously, and arbitrarily—did not recognize partial conversion of universal negatives.[50]

The p occurs in Fapesmo, Darapti, Felapton, and Baralipton. Smith's treatment overlooks the occurrence of p in Baralipton in two ways: because it follows an i and because it follows a conclusion indicator. This raises the question of how a deductivist can deal with the omitted case and in such a way that the code can be applied to deductions other than those already encoded. No solution appears in the literature as far as we now know.

To preserve the viability of the deductivist reading we propose: p following an i in the conclusion position means that the final conclusion is reached from a previously occurring A proposition by partial conversion.

Comment F {4}: {4} 'p' following 'a' indicates 'conversion by limitation' (*conversio per accidens*) of a universal premise, i. e., {5} conversion into a particular premise (*a* into *i, e* into *o*)

In the first place, in deduction the result of conversion of a premise—whether simple or partial—is not into another premise. The occurrence of 'premise' in {5} should be changed to 'sentence'. In the second place, in Smith's reconstruction of Aristotle's deductions there is no rule of E-to-O conversion. The occurrence in {5} of '(*a* into *i, e* into *o*)' should read '(*a* into *i*)'. In the third place, nothing is said about p following i as in Baralipton. The list of scholars who have made this mistake is long; besides

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50   See Corcoran and Kevin Tracy, "Review of Joray, Pierre, 'A Completed System for Robin Smith's Incomplete Ecthetic Syllogistic'," *Mathematical Reviews*, MR3681098, 2018.

Smith it includes Peirce,[51] Rini,[52] Peter of Spain (see above), and others.

Comment G {7}: {7} 'm' indicates that the premises must be interchanged. If one is discussing generating argument presentation from argument presentations, it makes perfect sense to move from one to another by interchanging premises. But in deducing a conclusion from premises, interchanging premises makes no sense. There is no rule for transposing premises in any categorical deduction system we know of.

Once the premises and conclusion goal have been set, no changes can be made. The important point is that a rule of transposition makes perfect sense for transforming one argument presentation into another, but such a rule has no role in deducing conclusions from premises.

As an aside that applies not only to Smith but also to several other logicians, we point out that in Frisesomorum the second occurrence of m does not instruct retransposing the transposed premises. Somewhere each decoding must say or imply that the last four letters are to be ignored in Frisesomorum.

Comment H {8}: {8} (Other letters, such as 'l' and 'n,' have no significance.) In the first place, we are talking about non-initial occurrences in codings for imperfect moods. In the second place, the r that Smith took to indicate indirect deduction is the most used of the insignificant letters, viz., lowercase non-initial d (as in Bocardo), l, n, r, and t.

Comment I {9}: {9}Thus, the name *Camestres* tells us that a proof that an *e* conclusion follows from an *a* major premise and an *e* minor may be constructed by [...].

This might be Smith's worst nonsequitur. In the first place, the name *Camestres* does not tell us *that* anything; it tells us *how*

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[51] Charles Sanders Peirce, *Writings of Charles S. Peirce: Chronological Edition. Volume 5 (1884-1886),* eds. Nathan Houser et al. (Bloomington: Indiana University Press, 1993), 348ff.

[52] Rini, "Aristotle's Logic," 48.

to do something. In the second place, it is not about a proof of a semantic metatheorem, viz., "that an *e* conclusion follows from an *a* major premise and an *e* minor". It is about a deduction of an *e* conclusion from an *a* major premise and an *e* minor. In the third place, what Smith needs the name *Camestres* to tell us is much more specific than what Smith says. Smith needs the name *Camestres* to tell us how to deduce the conclusion of an argument in the form named *Camestres* from its premises.

To see how far off this passage is imagine a proof that an *e* conclusion follows from an *a* major premise and an *e* minor, more specifically, a proof that an *e* conclusion of an argument in Camestres follows from its *a* major premise and its *e* minor.

Comment J {6, 14}: {6} 'r' indicates proof through impossibility; {14} a proof through impossibility is also possible (Camestres).

Without clause {14} clause {6} would be taken to instruct us to do an indirect deduction for each form whose coding contained an R. But that would have been an error on Smith's part because telling someone to do an indirect deduction does not tell them how to proceed after assuming the contradictory opposite of the conclusion. What is the next step? This error is not exonerated by {14}: telling someone that an indirect deduction is possible does not instruct them how to proceed. Moreover, {14} introduces a new error: if 'r' says that an indirect deduction is possible, then all fifteen codings should contain an occurrence of 'r'—because every direct deduction is transformable into an indirect deduction of the same conclusion from the same premises. See above.

Conclusions

After carefully considering the evidence, we conclude that the reductivist decoding of the original fifteen encodings fits much better than the deductivist.

Both do equally well with (1) the initial letter—B, C, D, F— indicating for the reductivist the destination of the reduction or

for the deductivist the two-premise rule used, (2) the s for simple conversion as reductivist argument-presentation transformations or as deductivist one-premise rule applications, and (3) the c for contraposition as reductivist argument-presentation transformations or as deductivist indirect deduction instructions. Moreover, the letter p works equally well in the last two of its three occurrences: Baralipton, Fapesmo, and Darapti.

However, the two deductivists we studied, Smith and Rini, had nothing to say about p following i. We cannot imagine a plausible deductivist decoding of Baralipton or any other mood name having a p following an i in the conclusion position. This is no problem for a reductivist.

Similarly embarrassing for deductivists is the letter m: there is no rule for transposing premises in any categorical deduction system we know of. Again this is no problem for reductivists.

We are confident that the mnemonic does not readily admit a deductivist interpretation. In an important sense, this is a disappointing conclusion. Of the two processes, deduction is the clearer, the most useful, and the most important philosophically, scientifically, and historically. After two millennia it is still not clear what reduction accomplishes. Until this is known, the enormous attention devoted to reduction and the mnemonic verses could turn out to have been a useless distraction, a red herring in the development of logic.

On a positive note, the reductivist theory underlying the syllogistic-mnemonic verses emphasizes an aspect of *Prior Analytics* overlooked by both the Łukasiewicz paradigm and the Corcoran-Smiley paradigm thereby highlighting their common deficiency. As such, it could lead to a new paradigm that incorporates the Łukasiewicz theory of terms, the Corcoran-Smiley natural-deduction logic, and the medieval reduction system.

Acknowledgements

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