JOURNAL OF CIVIL ENGINEERING AND MANAGEMENT
2009
16(2): 172–178

STRUCTURAL SAFETY OF ROLLED AND WELDED BEAMS SUBJECTED TO LATERAL-TORSIONAL BUCKLING

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Received 21 Oct. 2009; accepted 17 Jan. 2010

Abstract. The expediency of using probability-based approaches in the analysis of beams subjected to lateral-torsional buckling is discussed. The values of buckling resistance moments and their uncertainties for rolled and equivalent welded I sections as particular members of the designed structures are analyzed. The safety margins of buckling steel sections exposed to permanent and variable vertical loads are modeled. The survival probability and reliability index of sections exposed to lateral-torsional buckling are considered. The prediction of probability-based safety of rolled and welded beams in buildings and civil engineering works are provided and illustrated by numerical examples.

Keywords: lateral-torsional buckling, safety margin, survival probability, welded beams.

1. Introduction

The lateral-torsional buckling occurs with certain construction types of steel beams having deep I sections the compression flanges of which are insufficiently restrained against flexural action effects about the major axes of their sections caused by heavy gravity loads. Due to the buckled positions of deformed beams with open cross-sections (Fig. 1), tensile and compressive stresses develop in their top and bottom flanges, respectively. The dangerous values of these stresses caused by torsional and lateral flexure effects of vertically applied loads may cause the horizontal buckling of beam flanges because they cannot be completely prevented by beam webs. Beams with sufficient restrained to the compression flange are not susceptible to lateral-torsional buckling.

The reliability class (RC) for buckling steel beams as particular members of the structures must be designated in the same consequences class than for the entire structure of buildings civil engineering works. Failures and collapses of deep I sections may be caused not only by the gross human errors of designers or erectors but also by statistical uncertainties of sustained and extraordinary variable loads or some conditionalities of recommendations and directions presented in semi-probabilistic design codes and standards.

The lateral-torsional buckling criteria for unrestrained beams may be generally expressed as the critical values of either their compressive bending stresses or ultimate bending moments. However, the buckling resistance of beams depends not only on their geometric parameters, support conditions and torsional properties but also on mechanical and statistical features of rolled and equivalent welded sections. They may exert a significant influence on their structural safety. Regardless of fairly developed concepts of the theory of buckling resistances of beams, any semi-probabilistic analysis can lead to the groundless overestimation of the reliability indices of designed and erected important engineering structures.

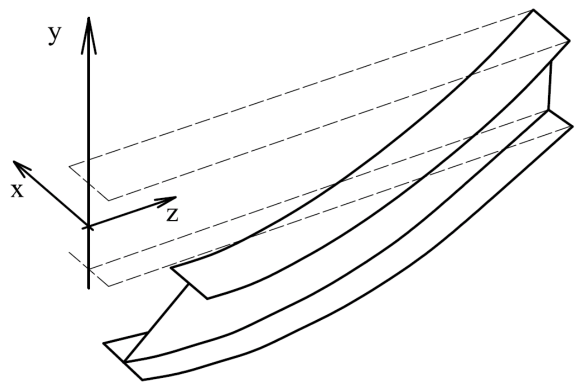

Fig. 1. Schematic representation of buckled beams

A wide range of applied reliability issues can be neither formulated nor solved within deterministic or semi-probabilistic approaches. Therefore, careful attention of designers must be given to the full-probabilistic analysis of buckling members. The probability-based analysis of beams exposed to lateral-torsional buckling may be inevitable in cases when their variable bending moments are caused by extreme extraordinary or recurrent static and dynamic loads. The probabilistic analysis of buckling members subjected to sustained variable loads is fairly unsophisticated. However, the structural safety prediction of members subjected to intermittent recurrent loads may be rather complicated due to some mathematical difficulties.

ISSN 1392–3730 print / ISSN 1822–3605 online
doi:10.3846/jcem.2010.18

The object of this paper is to assess the difference in reliability indices of rolled and welded beams with buckling flanges and to encourage designers having minimum appropriate skills and experience to use probability-based methods in their design practice.

2. Buckling resistance moment

According to EN 1993-1-1 (2004), the beams loaded in the plane of the web and subject to major axis bending (Fig. 1) should be verified against lateral-torsional buckling as follows:

$$M_{Ed} < M_{b,Rd}\,, \tag{1}$$

where M_{Ed} is a design bending moment,

$$M_{b,Rd} = \chi_{LT} W_y f_y / \gamma_{M1} \tag{2}$$

is a design buckling resistance moment expressed by its reduction factor χ_{LT}, appropriate section modulus W_y equal to $W_{pl,y}$, $W_{el,\,y}$ and $W_{eff,y}$ for Class 1 or 2, 3 and 4 cross-sections. When f_y is the nominal (characteristic) value of yield strength and γ_{M1}=1.0 is its particular partial factor, the design yield strength is equal to $f_y/\gamma_{M1} = f_y = f_{yk}$. Thus, a design buckling resistance moment is equal to its characteristic value, i. e. $M_{b,Rd} = M_{b,Rk}$.

For rolled and equivalent welded beams of constant cross sections, the values of reduction factors of buckling resistance moments may be determined from the equation:

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \left(\Phi_{LT}^2 - 0.75\bar{\lambda}_{LT}^2\right)^{0.5}}, \begin{cases} \le 1.0 \\ \le 1/\bar{\lambda}_{LT}^2 \end{cases}, \tag{3}$$

where the value to determine the reduction factor, Φ_{LT}, may be calculated as follows:

$$\Phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.4\right) + 0.75\bar{\lambda}_{LT}^2\right]. \tag{4}$$

It consists of the imperfection factor α_{LT} equal to 0.21 or 0.34 and 0.49 or 0.76 for rolled and welded sections, respectively, and their non-dimensional slenderness

$$\bar{\lambda}_{LT} = \left(W_y f_y / M_{cr}\right)^{0.5}, \tag{5}$$

when M_{cr} is the elastic critical moment for the lateral-torsional buckling. Increasing an elastic critical moment, the non-dimentional slenderness of a beam, $\bar{\lambda}_{LT}$, decreases and the value of its buckling resistance moment may be improved.

In the case of a beam of uniform cross-section that is symmetrical about the minor and major axis, the elastic critical moment for lateral-torsional buckling is given by the formula:

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(kL)^2} \left\{ \left[\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 G I_t}{\pi^2 E I_z} + \left[C_2 z_g\right]^2 \right]^{0.5} - C_2 z_g \right\}, \tag{6}$$

where C_1, C_2 – are factors depending on the loading and end restraint conditions, k and k_w – are effective length factors, E – modulus of elasticity, G – shear modulus, L is the length of the beam between points which have lateral restraint, I_t – the torsion constant, I_w – the warping constant, I_z – the second moment of area about the minor axis, $z_g = z_a - z_s$ when z_a is the coordinate of the point of load application, z_s is the ordinate of the shear centre.

When the parameters f_y and χ_{LT} of steel sections are treated as random statistically independent variables, their means and standard deviations may be expressed as:

$$f_{ym} = \frac{f_y}{1 - k_{0.95}\,\delta f_y}, \; \sigma f_y = \delta f_y \times f_{ym}, \tag{7}$$

$$\chi_{LTm} = \chi_{LT}, \; \sigma\chi_{LT} = \delta\chi_{LT} \times \chi_{LT}, \tag{8}$$

where the coefficients of variation of yield strength f_y and reduction factor χ_{LT} may be defines as $\boldsymbol{\delta} f_y \approx 0.08$ and $\boldsymbol{\delta}\chi_{LT,r} \approx 0.06$ or $\boldsymbol{\delta}\chi_{LT,w} \approx 0.08$–$0.10$. Therefore, the means and coefficients of variation of buckling resistance moments of rolled and welded sections may be expressed as:

$$M_{b,R_r m} = \chi_{LT,r} W_y f_{ym}, \tag{9}$$

$$\delta M_{b,R_r} = \sqrt{\delta^2 f_y + \delta^2 \chi_{LT,r}} = \sqrt{0.08^2 + 0.06^2} = 0.10, \tag{10}$$

$$M_{b,R_w m} = \chi_{LT,w} W_y f_{ym} = (0.85 - 0.9) M_{bR_r m}, \tag{11}$$

$$\delta M_{b,R_w} = \sqrt{\delta^2 f_y + \delta^2 \chi_{LT,w}} = 0.113 - 0.128 \approx 0.12. \tag{12}$$

Thus, the standard deviations of buckling resistance moments may be treated as the same values for rolled and welded sections expressed as $\sigma M_{b,R} = \delta M_{b,R} \times M_{b,R_m}$.

3. Safety margin of buckling beams

The time-dependent safety margin of single (individual or component) steel beams (sections) may be defined as their performance process. According to Melchers (1999) and JCSS (2000), this safety margin may be expressed as a random process:

$$Z(t) = g[\theta, X(t)] = \theta_R M_{b,R} - \theta_g M_g - \theta_q M_{q_s}(t) - \theta_q M_{q_e}(t), \tag{13}$$

where M_g, $M_{q_s}(t)$ and $M_{q_e}(t)$ are the stochastically independent bending moments caused by permanent g, sustained q_s and extraordinary q_e loads (Fig. 2). The additional random variables (θ) represent the uncertainties of calculation models including uncertainties of their probability distributions. These variables may be modeled either by the density functions or simply as their means θ_{Rm}, θ_{gm}, θ_{qm} and standard deviations $\sigma\theta_R$, $\sigma\theta_g$, $\sigma\theta_q$ (see Section 5.1).

According to Ellingwood (1981), Raizer (1998) and EN 1990 (2002), EN 1994-1-1 (2004), the permanent bending moment M_g can be described by the normal

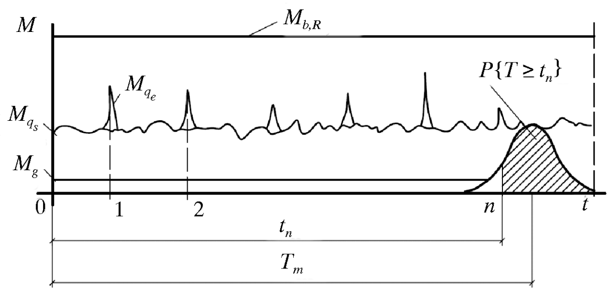

Fig. 2. Model for structural safety analysis of beams

distribution law. Therefore, for the sake of design simplifications, it is expedient to present the expression (13) in the form:

$$Z(t) = R_c - S_q(t), \quad (14)$$

where the component

$$R_c = \theta_R M_{b,R} - \theta_g M_g \quad (15)$$

may be treated as the conventional beam resistance which may be modeled by normal distribution irrespective of the fact that a distribution of the buckling resistance moment $M_{b,R}$ may only be close to this distribution (ISO 2394 1998; Kala *et al.* 2009)

$$S_q(t) = \theta_q M_{q_s}(t) + \theta_q M_{q_e}(t) \quad (16)$$

is the variable bending moment process induced by service live actions.

The means and variances of the probability distibutions of random functions R_c and S_q are:

$$R_{cm} = \theta_{Rm} M_{b,Rm} - \theta_{gm} M_{gm}, \quad (17)$$

$$\sigma^2 R_c = \theta_{Rm}^2 \sigma^2 M_{b,R} + M_{b,R}^2 \sigma^2 \theta_R + \theta_{gm}^2 \sigma^2 M_g + M_{gm}^2 \sigma^2 \theta_g, \quad (18)$$

$$S_{qm} = \theta_{qm}\left(M_{q_s m} + M_{q_e m}\right), \quad (19)$$

$$\sigma^2 S_q = \theta_{qm}^2\left(\sigma^2 M_{q_s} + \sigma^2 M_{q_e}\right) + \left(M_{q_s m} + M_{q_e m}\right)\sigma^2 \theta_q, \quad (20)$$

where $M_{b,Rm}$ is given by Eqs. (9) or (11),

$$\sigma^2 M_{b,R} = \left(\delta M_{b,R} \times M_{b,Rm}\right)^2 \quad (21)$$

when the coefficient of correlation, $\delta M_{b,R}$, is equal to 0.10 and 0.12 for rolled and welded sections, respectively.

The sustained and extraordinary components of service loads are modeled as time-variant stochastic processes. It is proposed to model the variable loads with a mean equal to $q_k/(1 + k_{0.95}\delta q)$, where $k_{0.95}$ is their characteristic fractile factor. The variance of bending variable moments is expressed as:

$$\sigma^2 M_q = \left(\delta M_q \times M_{qm}\right)^2. \quad (22)$$

In reality, the lognormal distribution may be used for sustained loads (JCSS 2000; EN 1990 2002; ISO 2394 1998). The sum of sustained and intermittent extraordinary load components may be assumed to be exponentially distributed (JCSS 2000; Vrouwenvelder 2002). The Type 1 (Gumbel) distribution may be also used. Besides, the Gumbel distribution law is quite appropriate for the probabilistic analysis of structures exposed to recurrent extreme action effects.

4. Survival probability and reliability index

When the variable action effect $S_q(t)$ by (16) may be treated as the recurrent extreme bending moment, the time-dependent safety margin (14) may be expressed as a random sequence:

$$Z_k = R_{ck} - S_k, \; k=1, 2,\ldots, n-1, n, \quad (23)$$

where R_{ck} by Eq. (15) and S_k by Eq. (16) are the conventional resistance and the variable extreme action effect at the sequence cut k the probability distributions of which are normal and Type 1, respectively; n is the number of extreme action effects during design working life t_n of the structures (Fig. 2) (Kudzys and Kliukas 2009).

When R_{ck} and S_k are independent, the instantaneous survival probability of members at any cut k of their safety margin sequences, assuming that they were safe at time less then t_k, may be calculated using formula:

$$P_k = P\{Z_k > 0\} = \int_0^\infty f_{R_{ck}}(x) F_{S_k}(x)\,dx, \quad (24)$$

where $f_{R_{ck}}(x)$ is the density function of conventional resistance R_{ck},

$$F_{S_k}(x) = \exp\left[-\exp\left(\frac{S_{km} - x}{0.7794\sigma S_k} - 0.5772\right)\right] \quad (25)$$

is the Gumbel distribution function of the recurrent action effect S_k the mean and standard deviation of which are S_{km} by Eq. (19) and σS_k from Eq. (20).

The time-dependent survival probability of members as series stochastic systems may be calculated using the numerical integration and Monte-Carlo simulation methods. However, it is more reasonable to use the unsophisticated analytical method of transformed conditional probabilities.

When the conventional resistance may be treated as a stationary process, the long-term survival probability of beams obtains the following form:

$$P\{Z > 0\} = P\{T \geq t_n\} = P_k^n\left[1 + \rho_{kl}^a\left(\frac{1}{P_k} - 1\right)\right]^{n-1}, \quad (26)$$

where $\rho_{kl} = \rho(Z_k, Z_l) = Cov(Z_k, Z_l)/(\sigma Z_k \times \sigma Z_l)$ is the coefficient of auto correlation of random safety margin sequence cuts; $a = \left[(4.5 + 4\rho_{kl})/(1 - 0.98\rho_{kl})\right]^{0.5} \approx \left[4.5/(1 - 0.98\rho_{kl})\right]^{0.5}$ is its bounded index; P_k is the instantaneous survival probability by Eq. (24).

The probabilistic analysis of structures subjected to two stochastically independent variable extreme actions is presented by Kvedaras and Kudzys (2005), Kudzys (2005). This analysis is based on the fact that a member failure may occur not only under joint action effects but also when the value of one out of two actions is extreme or when the conventional bivariate distribution of two extreme action processes exists.

When the variable action effect $S_q = M_q$ is distributed by the lognormal and exponential laws, the instantaneous probability (24) may be treated as the long-term survival probability and calculated by the analytical formulae:

$$P\{Z > 0\} = \int_0^\infty f_{R_c}(x)\left\{0.5\left[1 + erf\left(\frac{\ln x - M_{qm}}{\sigma M_q\sqrt{2}}\right)\right]\right\}dx\,, \quad (27)$$

$$P\{Z > 0\} = 1 - \Phi\left(-\frac{R_{cm}}{\sigma R_c}\right) - \exp\left(-\frac{R_{cm}}{M_{qm}} + 0.5\frac{\sigma^2 R_c}{\sigma^2 M_q}\right) \times \left[1 - \Phi\left(-\frac{R_c}{\sigma R_c} + \frac{\sigma R_c}{\sigma M_q}\right)\right], \quad (28)$$

where the mean and variance for R_c are calculated from Eqs. (17) and (18).

The survival probability of members may be introduced by the generalized reliability index

$$\beta = \Phi^{-1}(P\{Z > 0\}), \quad (29)$$

where $\Phi^{-1}(\bullet)$ is the cumulative distribution function of the standard normal distribution. This index helps us specify the degree of reliability of members according to the consequences of their failure.

According to Eurocode EN 1990 (2002), for an ultimate limit state design of structural members the minimum values for the reliability index during the 50-year reference period are: $\beta_{min} =$ 3.3, 3.8 and 4.3 when their reliability classes are RC1, RC2 and RC3, respectively. Beams and other particular members of the structure may be designated in the same reliability class as for the entire structure.

5. Numerical illustrations

5.1. Beams in engineering buildings of reliability class RC2

Let us consider, as the numerical example, the comparatively deep sections unrestrained between beam supports and exposed to storage silos weights and heavy variable central loads acting at the centroidal beam axis. The characteristic values of permanent, G_k, and leading variable, Q_k, loads are equal to 16.55 kN/m and 116.0 kN/m, respectively. Thus, the variable load is significantly larger than permanent one. The design values of load and bending moment are: F_d =1.35×16.55+1.5×116=196.3 kN/m and M_{Ed} = 825.6 kNm. The multiplication factor for actions $K_{F1} = 1.0$ (EN 1990 2002).

The length and the cross section depth of uniform beams HEA 550 of building silos (Fig. 3) are: $L = 5.80$ m and $h = 0.54$ m. The steel grade S275 belongs to Class 1 of Standard EN 10025-2 for hot rolled structural steel. Thus, the nominal (characteristic) value of its yield strength $f_y = 275$ MPa. The distance between shear centres of beam flanges $h_s = 0.516$ m; the second moments of area about minor (z–z) axis for rolled and welded cross sections $I_z = 1.082\times10^{-4}\,\text{m}^4$.

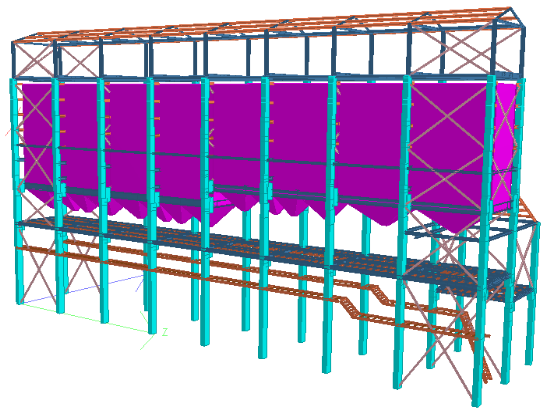

Fig. 3. Scheme of storage silos

Therefore, the warping constant of section is equal to

$$I_w = I_z \times h_s^2/4 = 1.082\times10^{-4}\times0.516^2/4 = 7.20\times10^{-6}\ \text{m}^6.$$

The plastic section modulus about major (y–y) axis $W_y = W_{pl,y} = 46.44\times10^{-4}\,\text{m}^3$.

The parameters

$$\alpha_1 = -0.042 + 0.2204\frac{t_w}{t_f} + 0.1355\frac{r}{t_f} - 0.0865\frac{rt_w}{t_f^2} - 0.0725\frac{t_w^2}{t_f^2} = -0.042 + 0.2204\frac{0.0125}{0.024} + 0.1355\frac{0.027}{0.024} - 0.0865\frac{0.027\cdot0.0125}{0.024^2} - 0.0725\frac{0.0125^2}{0.024^2} = 0.1548$$

and $$D_1 = (t_f + r)^2 + t_w\left[r + \left(\frac{t_w}{4}\right)\right]\Big/(2r + t_f) = \frac{(0.024 + 0.027)^2 + 0.0125\left[0.027 + \frac{0.0125}{4}\right]}{2\times0.027 + 0.024} = 0.0382\,\text{m}.$$

The torsion constant of section is equal to

$$I_t = \frac{2}{3}bt_f^{\,3} + \frac{1}{3}\left(h - 2t_f\right)t_w^{\,3} + 2\alpha_1 D_1^4 - 0.42t_f^{\,4} = \frac{2}{3}\times 0.30\times 0.024^3 + \frac{1}{3}\left(0.54 - 2\times 0.024\right)0.0125^3 + 2\times 0.1548\times 0.0382^4 - 0.42\times 0.024^4 = 0.03605\times 10^{-4}\ \text{m}^4 .$$

Thus, according to Eq. (6), the elastic critical moment for lateral-torsional buckling is

$$M_{cr} = 1.132\frac{\pi^2 210\times 10^6\times 108.2\times 10^{-6}}{(1.0\times 5.8)^2}\times \left\{\left[\left(\frac{1.0}{1.0}\right)^2\frac{7.202\times 10^{-6}}{108.2\times 10^{-6}} + \frac{(1.0\times 5.8)^2 81\times 10^6\times 3.61\times 10^{-6}}{\pi^2 210\times 10^6\times 108.2\times 10^{-6}} + \left[0.459\times 0,27\right]^2\right]^{0.5} - 0.459\times 0.27\right\} = 1740.9\,\text{kNm}.$$

According to Eq. (5), the non-dimentional slenderness for constant cross section is:

$$\bar{\lambda}_{LT} = \left(46.44\times 10^{-4}\times 275\times 10^3/1740.87\right)^{0.5} = 0.86 .$$

According to Eq. (4), the values of joint parameters for rolled and equivalent welded beams are:

$$\Phi_{LT,r} = 0.5\left[1 + 0.21(0.86 - 0.4) + 0.75\cdot 0.86^2\right] = 0.83 ,$$

and $\Phi_{LT,w} = 0.5\left[1 + 0.49(0.86 - 0.4) + 0.75\cdot 0.86^2\right] = 0.89$.

Thus, according to Eq. (3), the reduction factors for buckling resistance moments of rolled and equivalent welded beams are:

$$\chi_{LT,r} = \frac{1}{0.83 + \left(0.83^2 - 0.75\times 0.86^2\right)^{0.5}} = 0.84 ,$$

and $\chi_{LT,w} = \dfrac{1}{0.89 + \left(0.89^2 - 0.75\times 0.86^2\right)^{0.5}} = 0.73$.

According to Eq. (2), the design buckling resistance moments for rolled and welded beams are:

$M_{b,R_rd} = 0.84\times 46.44\times 10^{-4}\times 275\times 10^3/1.0 = 1072.8$ kNm $= M_{b,R_rk} >> M_{Ed}\times K_{FI} = 825.6\times 1.0 = 825.6$ kNm ,

$M_{b,R_wd} = 0.73\times 46.44\times 10^{-4}\times 275\times 10^3/1.0 = 932.3$ kNm$= M_{b,R_wk} > 825.6$ kNm. Thus, the welded sections are in perfect safety suitable for the retaining construction of silos.

The values of means and variances of buckling resistances of considered uniform sections are:

$$M_{b,R_rm} = 1072.8/(1 - 1.645\times 0.10) = 1284\,\text{kNm} ,$$

$$\sigma^2 M_{b,R_r} = (0.10\times 1284)^2 = 16487\,(\text{kNm})^2 ,$$

$$M_{b,R_wm} = 932.3/(1 - 1.645\times 0.12) = 1161.5\,\text{kNm} ,$$

$$\sigma^2 M_{b,R_w} = (0.12\times 1161.5)^2 = 19427\,(\text{kNm})^2 .$$

The values of means and variances of bending moments caused by permanent and sustained variable loads are:

$M_{gm} = 70.1$ kNm, $\sigma^2 M_g = (0.10\times 70.1)^2 = 49.1$ (kNm)^2;

$M_{qm} = 278.7$ kNm, $\sigma^2 M_q = (0.4\times 278.7)^2 = 12431$(kNm)^2

for the lognormal distribution of variable loads and $M_{qm} = 212.1$ kNm, $\sigma^2 M_q = (1.0\times 212.1)^2 = 44977$ (kNm)^2 for the exponential distribution of these loads.

The mean values and standard deviations of model additional variables are: $\theta_{Rm} \approx \theta_{gm} \approx \theta_{qm} \approx 1.0$, and $\sigma\theta_{R_r} \approx 0.05$, $\sigma\theta_{R_w} \approx 0.10$, $\sigma\theta_g \approx \sigma\theta_q \approx 0.10$ (Hong and Lind 1996, Holicky 2005). Then, according to Eqs. (17)–(20), the revised statistical parameters of conventional resistances and variable action effects of considered beams are:

$$R_{c,rm} = 1.0\times 1284 - 1.0\times 70.1 = 1213.9\,\text{kNm} ,$$

$$\sigma^2 R_{c,r} = 1.0\times 16486 + 1284^2\times 0.05^2 + 1.0\times 49.1 + 70.1^2\times 0.10^2 = 20706\,(\text{kNm})^2 ,$$

$$R_{c,wm} = 1.0\times 1161.5 - 1.0\times 70.1 = 1091.4\ \text{kNm} ,$$

$$\sigma^2 R_{c,w} = 1.0\times 19425 + 1161.5^2\times 0.10^2 + 1.0\times 49.1 + 70.1^2\times 0.10^2 = 33013\,(\text{kNm})^2 ,$$

$S_{qm} = 1.0\times 278.7 = 278.7$ kNm and $\sigma^2 S_q = 1.0\times 12431 + 278.7^2\times 0.10^2 = 13208$ (kNm)^2 for the lognormal distribution,

$S_{qm} = 1.0\times 212.1 = 212.1$ kNm and $\sigma^2 S_q = 1.0\times 212.1^2 + 212.1^2\times 0.10^2 = 45427$ (kNm)^2 for the exponential distribution.

According to Eqs. (27) and (28), the survival probabilities of rolled and welded beams are equal to 0.99989 and 0.99937 or 0.99588 and 0.99159 when the distributions of variable loads are lognormal or exponential, respectively. The reliability indices of beams are presented in Fig. 4.

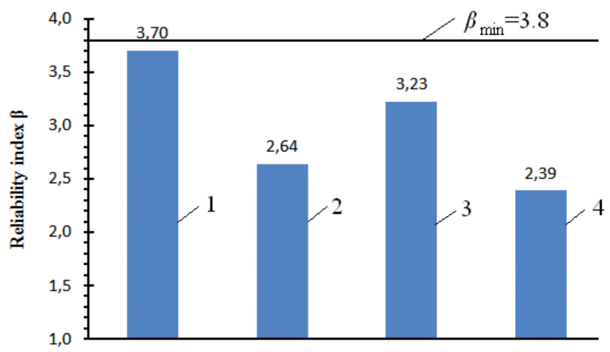

Fig. 4. Reliability indices of rolled (1, 2) and welded (3, 4) sections HEA 550 when the distributions of variable loads are lognormal (1, 3) and exponential (2, 4)

Fig. 4 shows that the structural safety of sections HEA 550 may be insufficient in spite of their perfect safety data calculated according to Eurocode 3 (EN 1993-1-1 2004) recommendations.

5.2. Beams in engineering works of reliability class RC1

The lenght and the cross-section depth of uniform beams HEA 500 are: $L = 5.80$ m and $h = 0.49$ m. The steel grade S275 belongs to Class 1 of Standard EN 10025-2 for hot rolled structural steel. The distance between shear centres of beam flanges $h_s = 0.467$ m; the second moments of area about minor (z–z) axis for rolled and welded cross-sections $I_z = 1.037 \times 10^{-4}$ m^4. Therefore, the warping constant of section is equal to $I_w = 5.654 \times 10^{-6}$ m^6.

The plastic section modulus of beams about major (y–y) axis $W_y = W_{pl,y} = 39.76 \times 10^{-4}$ m^3. When the parameters $\alpha_1 = 0.1594$ and $D_1 = 0.0371$m, the torsion constant of section is equal to $I_t = 0.03174 \times 10^{-4}$ m^4.

Thus, the elastic critical moment for lateral-torsional buckling is $M_{cr} = 1555.8$ *kN*m and the non-dimentional slenderness for constant cross-section $\bar{\lambda}_{LT} = 0.84$. The values of joint parameters for rolled and equivalent welded beams are: $\Phi_{LT,r} = 0.81$ and $\Phi_{LT,w} = 0.87$. According to Eq. (3), the reduction factors for buckling resistance moments of rolled and equivalent welded beams are: $\chi_{LT,r} = 0.86$ and $\chi_{LT,w} = 0.74$.

The multiplication factor for actions $K_{F1} = 0.9$ (EN 1990 2002). Therefore, the bending moment of beams is equal to 825.6 × 0.9 = 743 kNm. The design buckling resistance moments for rolled and welded beams, respectively, are: $M_{b,R_rd} = 940.3$ kNm $>> 743.0$ kNm, $M_{b,R_wd} = 809.1$ kNm > 743.0 kNm.

According to Eq. (2), the means and variances of buckling resistances of considered uniform sections are:

$$M_{b,R_rm} = 1125.5 \text{ kNm}, \ \sigma^2 M_{b,R_r} = 12666.7 (\text{kNm})^2;$$

$$M_{b,R_wm} = 1008.1 \text{ kNm}, \ \sigma^2 M_{b,R_w} = 14634.8 (\text{kNm})^2.$$

According to Eqs. (17)–(20), the revised statistical parameters of considered beams are:

$$R_{c,rm} = 1055.4 \text{ kNm}, \ \sigma^2 R_{c,r} = 15931.5 \text{ (kNm)}^2,$$

$$R_{c,wm} = 938.1 \text{ kNm}, \ \sigma^2 R_{c,w} = 24896 \text{ (kNm)}^2;$$

$S_{qm} = 278.7$ kNm and $\sigma^2 S_q = 13207.5 (\text{kNm})^2$ for the lognormal distribution of sustained loads,

$S_{qm} = 212.1$ kNm and $\sigma^2 S_q = 45427 (\text{kNm})^2$ for their exponential distribution.

According to Eqs. (27) and (28), the survival probabilities of rolled and welded beams are equal to 0.99962 and 0.99807 or 0.99176 and 0.98417 when the distributions of variable loads are lognormal or exponential, respectively. The reliability indices of beams are presented in Fig. 5.

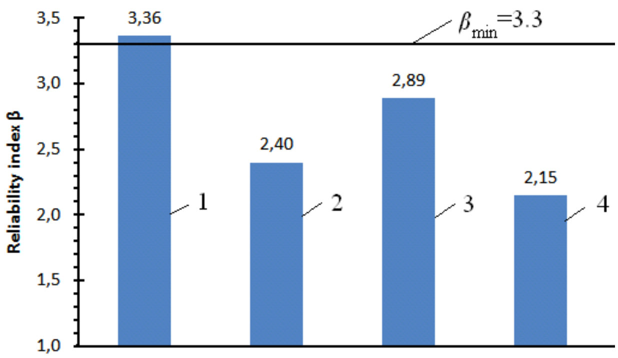

Fig. 5. Reliability indices of rolled (1, 2) and welded (3, 4) sections HEA 500 when the distributions of variable loads are lognormal (1, 3) and exponential (2,4)

The data presented in Fig. 5 corroborate the evidence that beams (especially formed from welded sections) designed by Eurocode 3 (EN 1993-1-1 2004) recommendations may be insufficiently safe if they are subjected to lateral-torsional buckling.

6. Conclusions

The relevant semi-probabilistic method of partial safety factor design helps us assess the effect of mechanical features of rolled and welded steel sections on their slendernesses and lateral-torsional buckling resistances. However, this method prevents us from estimating their reliability indices. The analysis data indicated that due to welding the design buckling resistance of beams decreases 13–14%. When the design values of buckling resistance moments of rolled sections exceed their design bending moments by 25–30% and their performance may be treated as perfectly sufficient, the structural safety of equivalent welded sections may be insufficient.

It is not complicated to predict the survival probability of beams subjected to lateral-torsional buckling and at the same time to ground their engineering decision by probability-based approaches presented in section 4 of this paper and its numerical illustrations. Therefore, instead of assessing the design values of buckling resistance and bending moments for rolled and welding beams, it is expedient to determine their reliability indices, compare them with specified values and select an objectively relevant structural decision.

The probability-based prediction of structural safety for crucial constructions in a simple and easily perceptible manner is acknowledged as the main task facing modern building and bridge engineers. Therefore, parallel with design code semi-probabilistic methods, the presented unsophisticated probability-based approaches may stimulate engineers to use probabilistic models in their design practice.

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ŠONINIU SUKIMU KLUPDOMŲ VALCUOTŲJŲ IR SUVIRINTŲJŲ SIJŲ KONSTRUKCINĖ SAUGA

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S a n t r a u k a

Aptariamas tikslingumas naudoti tikimybinius metodus skaičiuojant šoniniu sukimu klupdomas sijas. Analizuojamos valcuotųjų ir suvirintųjų I profiliuočių, kaip ypačiųjų elementų, klupdomųjų atsparių momentų vertės ir jų neapibrėžtys. Modeliuojama nuolatine ir laikinąja vertikalia apkrova klupdomų plieninių profiliuočių ribinė sauga. Nagrinėjama šoniniu sukimu klupdomų profiliuočių išlikties tikimybė ir patikimumo indeksas. Pateiktas ir skaitiniu pavyzdžiu iliustruotas pastatų ir inžinerinių statinių valcuotųjų ir suvirintųjų sijų tikimybinės saugos prognozavimas.

Reikšminiai žodžiai: plieniniai profiliuočiai, klupdymas šoniniu sukimu, ribinė sauga, išlikties tikimybė, suvirintosios sijos.

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