Is the Effect of Risk on Stock Returns Different in Up and Down Markets? A Multi-Country Study

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Received: 24.11.2015 Accepted: 02.09.2016 Published: 30.09.2016
doi:10.33818/ier.278045

ABSTRACT

Several empirical studies in finance have examined whether or not the risk associated with any stock market responds differently in two different states of the stock market, especially in bull and bear markets. This paper studies this problem through a model where (i) the conditional mean specification considers the threshold autoregressive model for two market situations characterized as up and down markets, (ii) the conditional variance specification is asymmetric in the sense of capturing leverage effect, and (iii) the conditional variance directly affects the conditional mean through the risk premium term in the risk-return relationship. Using daily returns on stock indices of eight countries, comprising four developed countries - the USA, the UK, Hong Kong, and Japan - and four important emerging economies, called the BRIC group, we have found that the nature of risk-return relationship is different in up and down markets. Furthermore, the risk aversion parameter is positive in the down markets and negative in the up markets. This finding supports the hypothesis that investors require a premium for taking downside risk and pay a premium for upside variation. Finally, the findings suggest that the nature of risk-return relationship is same for the two groups of countries.

Key words: *Asymmetric Risk Aversion, Leverage Effect, Up and Down Markets, Threshold Regression, Exponential GARCH-M*
JEL Classifications: C51, C58, G12, G15

1. INTRODUCTION

Modelling stock market volatility has been the subject of vast theoretical and empirical investigations during the last three decades by academics and practitioners alike. Initially, risk, or more generally volatility, was assumed to be constant. However, this assumption of constant risk, particularly in the context of financial time series, was very restrictive. Subsequently, with the introduction of ARCH (GARCH) model (Engle, 1982 and Bollerslev, 1986), along with its various extensions and generalisations, are used to capture time-varying risk. In the latter category, the exponential GARCH (EGARCH; Nelson, 1991) and the threshold GARCH (TGARCH; Glosten et al., 1993) models are two of the most popular models capturing asymmetry in volatility.

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The authors are thankful to two anonymous referees for insightful comments. Errors, if any, are the sole responsibility of the authors.

In contrast, conditional mean models from similar consideration of asymmetric responses are rather limited. Whether or not risk or beta of the capital asset pricing model (CAPM) responds asymmetrically to good and bad news, as measured by positive and negative returns, respectively, has long been investigated in the context of financial literature. Many studies (see, for instance, Kim and Zumwalt, 1979; Bhardwaj and Brooks, 1993; Pettengil et al., 1995; Howton and Peterson, 1998; Crombez and Vennet, 2000 and Faff, 2001) have examined, more generally, the validity of several asset pricing models, especially the CAPM, taking into account the market movements, defined as 'up' and 'down' markets. To classify 'up' and 'down' markets, various definitions have been used. For instance, Kim and Zumwalt (1979) and Chen (1982) have used three threshold levels viz., average monthly market return, average risk free rate, and zero. When the realized market return is above (below) the threshold level, the market is considered to be in the up (down) market state. The overwhelming empirical evidence in studies with various states of market condition implies that the CAPM cannot perform, under constant risk, in different market conditions. Levy (1974) and Fabozzi and Francis (1977, 1978) suggested that there was a need to have separate betas for bull and bear markets. By defining the bull and bear markets using a threshold model, Kim and Zumwalt (1979) found no evidence to support the beta instability, but concluded that investors like to receive a positive premium for accepting downside risk, associating a negative premium with the up-market beta. Using daily returns data, Granger and Silvapulle (2002) investigated the asymmetric response of beta to different market conditions by modelling the mean and the volatility of CAPM as nonlinear threshold models with three regimes. The results are in accordance with the widely-held view that the portfolio beta increases (decreases) when the market is bearish (bullish). Furthermore, Galagedera and Faff (2005) investigated whether the risk-return relationship varies depending on changing market volatility and up-down market conditions.

To capture such 'asymmetries' or differential effects on returns, as understood by different market conditions, threshold autoregressive (TAR) models, originally used by Tong (1978) are used. This class of models is characterized by a regime switching mechanism. In time series literature, several models with regime switching mechanisms, considering both the conditional mean and conditional variance have been proposed (see, for instance, Li and Li, 1996; Hagerud, 1997; Gonzalez-Rivera, 1998; Lundbergh and Terasvirta, 1998 and Brannas and DeGoojer, 2004). However, risk-return relationships with an asymmetric risk aversion coefficient, characterized by market conditions like bull and bear markets, along with asymmetry in conditional variance due to leverage effect, are very limited.

In 1987 Engel et al. (1987) introduced the ARCH-in-mean (ARCH-M) model, which incorporates risk premium by introducing volatility directly into the conditional mean equation of returns. As a result, risk, inter alia, could affect returns directly. Although such a risk-return relationship is expected to be positive in nature, since an increase in risk represented through conditional variance is likely to lead to a rise in expected returns, the empirical evidence is somewhat mixed. In respect to correlation behaviour of stock returns and subsequent volatility, French et al. (1987) and Campbell and Hentschel (1992) found the relative risk aversion parameter to be positive, while Turner et al. (1989) found it to be negative. As noted by Bekaert and Wu (2000), sometimes this coefficient is statistically insignificant as well. If the relation between market returns and conditional volatility is not found to be positive, then the validity of time-varying risk premium is in doubt. However, such findings may also be due to the fact that in this theory the relative risk aversion parameter is taken to be constant, which may not be true in some situations. This assumption may lead to misleading inferences on other parameters as well.

Thus, by giving more consideration to different market movements[1] in conditional mean and also to asymmetry in conditional variance in terms of behaviour of return shocks in the GARCH-in-mean (GARCH-M) modelling framework, we can expect to get a better understanding of the 'risk-return' relationship. This paper makes an attempt in this direction and aims to find if the associated risk responds differently to different states of a stock market. In fact, this enables us to test the conclusion of Kim and Zumwalt (1979): that investors would like to receive a positive (negative) premium for accepting down (up) market risk.

Though, most studies on this topic have considered data on developed countries, significant work on emerging economies exists as well (see, for instance, Harvey, 1995; De Santis, 1997; Bailey, 1994; Bhar and Nikolova, 2009). While the paper viz., Bhar and Nikolova (2009) worked with data from BRIC countries; the other three use data from China, India, and some other developing countries.

In this paper, our aim is to study whether or not the two risk aversion parameters corresponding to the two market conditions exist in the stock market. If so, then what are the natures of these risks to determining future returns in different regimes? In the GARCH-in-mean framework, however, there is no such model that explicitly considers the conditional mean specification for up and down markets. The main contribution of the paper is the introduction of such models, where two different risk aversion parameters for two different regimes - to be henceforth called as up and down markets -- are considered (see, for details, Chen, 1982 and Chen, 2009). The conditional variance model is taken to be the EGARCH[2] model in order to capture the leverage effect. Further, one of the proposed models considers smooth transition mechanism in the conditional mean process, proposed by Chan and Tong (1986) and Terasvirta (1994), which allows for transition from one market condition to the other continuously.

The models proposed in this paper along with two other existing models viz., the AR-GARCH-M and AR-EGARCH-M, which are taken as benchmark models, have been fitted to individual time series of stock returns for eight countries, comprising four developed economies - the USA, the UK, Hong Kong, and Japan, and four important emerging economies - called the BRIC economies, comprising Brazil, Russia, India, and China. These two groups of countries provide a favourable sample for this study; despite the high economic potential and growing importance of the BRIC[3], these countries are still much behind the developed countries considered here, and hence the nature of the risk-return behaviour of investors in these two groups of countries may differ substantially. The benchmark models have been considered in order to find to what extent the performances of the proposed models improve with the introduction of (i) up and down movements of the stock market, and (ii) two different risk aversion parameters for these two states of the market. The main findings of this study are: (i) the risk-return relationship is different in the up and down markets, (ii) the risk aversion parameter is positive in the down market and negative in the up market, and (iii) the pattern of risk-return relationship is same for the two groups of countries.

[1] There are some studies where asymmetry in mean in terms of return shocks, which is also called the asymmetric reverting behaviour in return dynamics, have been considered while specifying the model for conditional mean (see, for details, Nam et al., 2001; Nam, 2003 and Kulp-Tag, 2007).

[2] This model is considered to find if the (asymmetric) EGARCH, rather than the (symmetric) GARCH, is the appropriate model for volatility for the return series considered in this study.

[3] Some details on the background, stages of development and importance of this group in international portfolio management, which indeed influenced our choice of this group, are given in Section 3.

The organization of the paper is as follows: Section 2 introduces the proposed models, Section 3 discusses the empirical results, and Section 4 ends with some concluding remarks.

2. THE PROPOSED MODEL

All three proposed models for return, r_t, have the following conditional mean specification[4] as discussed in the preceding section:

$$r_t = \begin{cases} \mu_1 + \phi_1 r_{t-1} + \delta_1 \sqrt{h_t} + \varepsilon_t & \text{if } \bar{r}_t^k \le 0 \\ \mu_2 + \phi_2 r_{t-1} + \delta_2 \sqrt{h_t} + \varepsilon_t & \text{if } \bar{r}_t^k > 0 \end{cases} \quad (2.1)$$

where $\varepsilon_t = v_t \sqrt{h_t}$ with $\varepsilon_t \mid \Psi_{t-1} \sim N(0, h_t)$, v_t is independently and identically distributed $N(0,1)$, Ψ_{t-1} is the information or conditioning set up to time $t-1$, h_t is the conditional variance at time t, and $\bar{r}_t^k$ is as defined in the next paragraph.

In the literature on the threshold autoregressive (TAR) model, the choice of the threshold variable is often taken as a lagged value of its own. In our case the two regimes refer to low and high market situations and the return series is at daily level; the threshold variable is taken to be $\bar{r}_t^k$, where $\bar{r}_t^k$ is the average of past k returns i.e., $\bar{r}_t^k = (\sum_{i=1}^{k} r_{t-i})/k$, as suggested by Chen (2009). Since the data are at a high frequency, averaging, which adjusts for fluctuations, is expected to identify the two market situations better. Obviously, an appropriate choice of k is a relevant issue. Therefore, we used several choices for k and then adopted the one for which the underlying log-likelihood value for our model is maximum.

Combining the two conditional mean specifications stated in equation (2.1), we can write the model compactly as:

$$r_t = (\mu_1 + \phi_1 r_{t-1} + \delta_1 \sqrt{h_t}) D(\bar{r}_t^k \le 0) + (\mu_2 + \phi_2 r_{t-1} + \delta_2 \sqrt{h_t})(1 - D(\bar{r}_t^k \le 0)) + \varepsilon_t \quad (2.2)$$

where $D(\cdot)$ is an indicator function taking value 1 if $\bar{r}_t^k \le 0$ and 0 otherwise.

Rigorous derivations of the conditions for stationarity of nonlinear time series models are not as available. Chan and Tong (1985) have shown that a sufficient condition for stationarity of the TAR model is $\max(|\phi_1|, |\phi_2|) < 1$), (see also, Chan et al., 1985 and Franses and van Dijk, 2000; for a 'rough and ready' check for nonstationarity of nonlinear time series models).

The first proposed model, called the TAR-GARCH-M, assumes h_t to have the GARCH(1,1) specification[5] i.e.,

$$h_t = \omega + \alpha\, \varepsilon_{t-1}^2 + \beta\, h_{t-1} \quad (2.3)$$

where $\omega > 0$, $\alpha \ge 0$, $\beta \ge 0$, and $\alpha + \beta \le 1$. In case of the second model, designated as the TAR-EGARCH-M, h_t is taken to be the EGARCH(1,1) model, which is given by:

$$\ln h_t = \omega + \alpha \left(\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right) + \lambda \left[\frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}} - E\left(\frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}} \right) \right] + \beta \ln h_{t-1} \quad (2.4)$$

Unlike the (symmetric) GARCH model, imposing any restrictions on the parameters of EGARCH model are not necessary to ensure that h_t is positive. Note that if $0 < \lambda < 1$, the process

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[4] The model has been specified with one lag only since this was found to be adequate for all the return series. Further, in specifying the risk premium term, $\sqrt{h_t}$ has been considered although other functional forms like h_t and log h_t can also be taken.

[5] The order pair (1,1) for the GARCH/EGARCH model has been found to be adequate for all the return series, and hence the model has been specified for this order only.

generates volatility clustering. This condition, along with $\alpha < 0$, delivers leverage effect since under these restrictions a negative shock has a larger effect on the conditional variance than a positive shock of the same size.

This model assumes that the border between the two regimes signifying change in mean is given by a specific value of the threshold variable, which is zero for our model. A more gradual transition between different regimes can be obtained by replacing the indicator function $D(\bar{r}_t^k \leq 0)$ in equation (2) by a continuous function $G(\bar{r}_t^k, \gamma)$, which changes smoothly from 0 to 1 as $\bar{r}_t^k$ increases. The resulting model, which allows for an abrupt transition as well, is called the smooth transition autoregressive (STAR) model (Terasvirta, 1994). While this model has been extensively used in capturing regime switching behaviour (see also, in this context, Silvernnonien and Throp, 2013), it is recently being applied to incorporate asymmetry in conditional variance also (see Nam et al., 2001 and Nam, 2003; for instance).

Hence, the STAR with EGARCH-in-mean (STAR-EGARCH-M) model has been considered as another competing model, and this is represented as:

$$r_t = (\mu_1 + \phi_1 r_{t-1} + \delta_1 \sqrt{h_t})\,(1 - G(\bar{r}_t^k;\gamma)) + (\mu_2 + \phi_2 r_{t-1} + \delta_2 \sqrt{h_t}) G(\bar{r}_t^k;\gamma) + \varepsilon_t \qquad (2.5)$$

and the specification of h_t is as given in equation (2.4). While other choices like the exponential function exist, we have taken the popular choice of the logistic function for $G(\bar{r}_t^k;\gamma)$ i.e., $G(\cdot) = 1/(1+\exp(-\gamma\,\bar{r}_t^k)$. This transition function takes values between 0 and 1 depending on the values of $\bar{r}_t^k$ i.e., $0 < G(\bar{r}_t^k;\gamma) < 0.5$ for $\bar{r}_t^k$, $G(\bar{r}_t^k;\gamma) = 0.5$ for $\bar{r}_t^k = 0$ and $0.5 < G(\bar{r}_t^k;\gamma) < 1$ for $\bar{r}_t^k > 0$. Hence, the logistic transition function incorporates the monotonically changing market conditions from down to up. The two regimes are associated with the very small and large values of the transition variable, $\bar{r}_t^k$, compared to the threshold value of 0. If γ approaches 0 yielding $G(\bar{r}_t^k;\gamma) \simeq 0.5$, the STAR-EGARCH-M model reduces to a simple AR-EGARCH-M model. On the other hand, when γ approaches infinity, the transition function becomes 1 for $\bar{r}_t^k > 0$ and 0 for $\bar{r}_t^k \leq 0$, hence reducing to the indicator function.

3. EMPIRICAL RESULTS

In this section we discuss the results of estimation of all the models considered in this paper. We begin by describing the data sets and the results of some standard statistical tests that have been carried out to determine important characteristics of these time series.

3.1. Data and Summary Statistics

This study considers stock index data at daily frequency for eight countries - four from developed countries and four from important emerging economics. Specifically, the countries in these two groups are the USA, the UK, Hong Kong, and Japan from the developed economics, and Brazil, Russia, India, and China representing important emerging economies. Most empirical studies on such models have been carried out with returns on stock indices of developed economics like the USA, the UK, and Japan. Despite being a highly developed economy and important economic power[6] globally, there are not many studies using such nonlinear models with returns on the Hong Kong stock market index. We have included Hong Kong in our study, for this particular reason.

[6] This stock market is now the second and sixth largest stock market in Asia and in the world, respectively.

Relatively fewer studies exist with returns from developing economies compared to developed economies; hence, we have decided to include some important members from developing countries in our study. Accordingly, the BRIC group of economies have been chosen[7]. Following Kundu and Sarkar (2016), we now present some facts about these four countries to indicate their growing importance and justify their selection for this study. The BRIC group of countries, consisting of Brazil, Russia, India, and China, have common features like large land area, huge populations, and rapid economic growth. For instance, in 2010, these countries together accounted for over a quarter of the world's land area and more than 40% of the world's population. The BRIC markets have also become attractive destinations for FDI.

These four countries have been accepted as the fastest growing "emerging markets" since the early 2000s. In 2000, the share of these four developing countries in global GDP, in terms of purchasing power parity (PPP), was 16.4%, but in 2010 this figure rose to 25%. China and India were the main contributors to the rise, with shares increasing from 7.2% to 13.3% and 3.6% to 5.3%, respectively, in the above-mentioned decade. The share of this group in world trade has also improved significantly during the last two decades - from 3.6% to over 15%. Although the largest increase in terms of value has been in case of China - from less than 2% to over 9% - others too have made significant progress. Brazil's share has risen from 0.8% to 1.2%, while those of Russia and India rose from 1.5% to 2.3% and 0.5% to 1.8%, respectively. According to an estimate by Goldman Sachs, the four original BRIC countries are expected to represent 47 per cent of global GDP by 2050, which would dramatically change the list of the world's 10 largest economies.

Regarding the performance of the stock markets, there have been marked and significant improvements during the first decade of the existence of BRIC countries. A significant rise in equity indices was also observed between the years 2000 and 2008. During this period, the price-earnings ratio as an indicator of capital markets has been relatively stable. The strength of the stock markets in BRIC economies, measured in terms of market capitalization to GDP, deepened progressively over the years. Combined external financing of capital markets in BRIC from bonds, equities, and loans (in absolute term) also increased significantly during this period.

In all these countries, more than one index on stock prices is available. We have, however, taken only one index for each country. The index taken is the one that is considered to be the most important for the stock market of the country concerned. These indices have also been used in most existing studies for these countries. Accordingly, the stock indices considered in this study are: BOVESPA (for Brazil), MICEX (for Russia), SENSEX (for India), SSE COMPOSITE (for China), S&P 500 (for the US), FTSE ALL (for the UK), HANG SENG (for Hong Kong), and NIKKEI 225 (for Japan). All of the time series (at daily frequency) have been downloaded from the official website of Yahoo Finance[8] (http://finance.yahoo. com/). The sample time period for all the series is 01 January 2000 to 31 December 2012. The starting time point chosen coincides with the formation of this group. The total numbers of observations are not exactly the same for all the eight series because of varying numbers of holidays in different countries when stock markets remain closed. Thus, SSE COMPOSITE of China has the highest number of observations, i.e. 3329, while the NIKKEI 225 of Japan yields the lowest number of observations, i.e. 3191. These indices (in level) are plotted in Figure 3.1. These plots suggest

[7] Since the joining of South Africa in this group in April 2011, this group is called the BRICS. However, we confined ourselves to the original BRIC group since South Africa was not a member of this group for almost the entire time period considered in this study.

[8] Though we have used the data from yahoo finance, there are also other reliable database sources such as Datastream and Bloomberg.

that all the series are nonstationary, and that there are wide variations in stock prices in each series. Returns (in percentage), r_t, defined as $r_t = (\ln(p_t) - \ln(p_{t-1})) \times 100$, where p_t is the closing stock price index of a country on the t^{th} day, are plotted in Figure 3.2. These plots indicate that all of the returns series are stationary and that volatility is present in each series, as expected. Further, these plots do not give indication of any structural break in any of the eight return series.

Figure 3.1 Plots of the stock indices

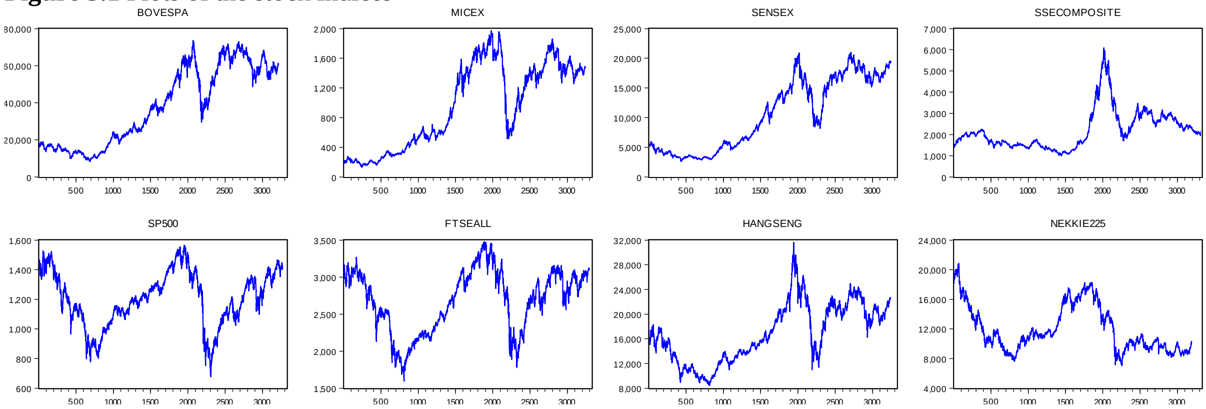

Figure 3.2 Plots of returns on all stock indices

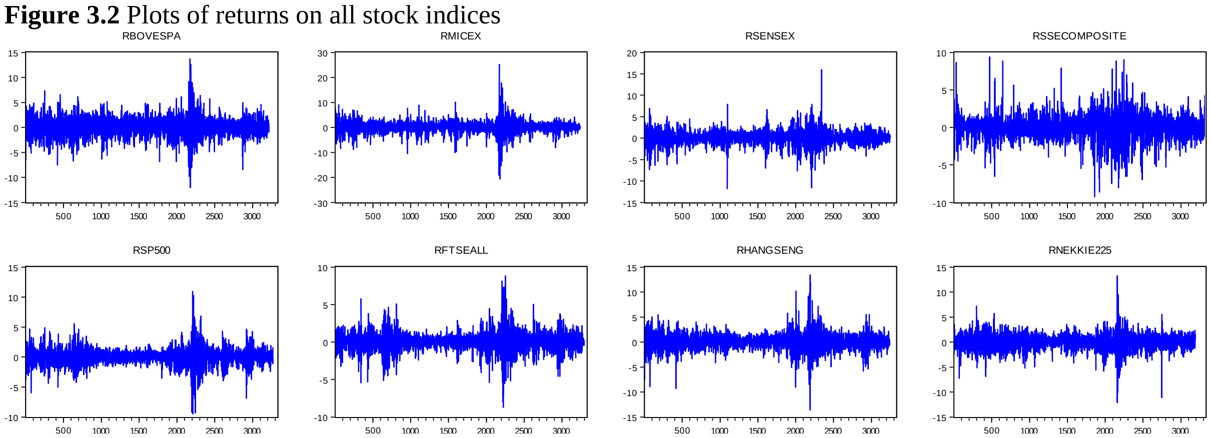

Table 3.1 presents the summary statistics of returns on all the eight time series. Mean values of returns for all BRIC countries are positive and negative for all developed economies, except for Hong Kong. Although small in magnitude, the skewness coefficients for all the return series have negative values, indicating that all the return distributions are skewed to the left. Furthermore, Japan has the maximum asymmetry for returns distribution. All the kurtosis values are higher than 3 with the maximum being 15.4304 for MICEX of Russia. Consequently, the resulting J-B test statistic values reject the assumption of normality strongly for all the series.

All the series are found to have unit roots at their level values by the augmented Dickey Fuller (ADF) test. However, the ADF test on returns confirmed that all of the returns series are stationary. Also, all the return series have significant (linear) autocorrelations as well as squared autocorrelations, as exhibited by the values of $Q(5)$, $Q(10)$, $Q^2(5)$, and $Q^2(10)$ test statistics.

Country / Summary	Brazil	Russia	India	China	The US	The UK	Hong Kong	Japan
Mean	0.0399	0.0661	0.0396	0.0152	-0.0006	-0.0004	0.0082	-0.0189
Median	0.0937	0.1526	0.1117	0.0000	0.0488	0.0402	0.0286	0.0051
Maximum	13.6766	25.2261	15.9899	9.4007	10.9572	8.8107	13.4068	13.2345

Minimum	-12.0961	20.6571	-11.8092	-9.2561	-9.4695	-8.7099	-13.5820	-12.1110
Std. dev.	1.9048	2.3290	1.6517	1.5882	1.3508	1.2385	1.6158	1.5667
Skewness	-0.0953	-0.1960	-0.1777	-0.0831	-0.1584	-0.1767	-0.0657	-0.3933
Kurtosis	6.6994	15.4304	9.3485	7.5141	10.3268	8.7030	10.5496	9.6856
J-B	1837.04	2086.92	5466.37	2829.46	7323.49	4479.70	7703.89	6023.19
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
ADF	-55.9043	-54.5671	-53.1190	-57.4533	-44.8785	-29.3910	-57.9338	-58.0035
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$Q(5)$	13.8192	12.1271	21.5550	10.2975	42.0542	48.4091	7.5071	7.0895
	(0.02)	(0.03)	(0.00)	(0.07)	(0.00)	(0.00)	(0.19)	(0.21)
$Q(10)$	23.1694	15.7780	38.5307	18.9498	49.1160	64.2120	17.7793	14.1138
	(0.01)	(0.11)	(0.00)	(0.04)	(0.00)	(0.00)	(0.06)	(0.17)
$Q^2(5)$	943.6102	560.6082	522.6123	250.7641	1340.2018	1302.9052	1351.6053	1617.4151
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$Q^2(10)$	1903.1032	835.0211	845.1342	454.6938	2609.1054	2122.8019	2026.8029	2589.5043
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
D_{max}	6.82	14.37*	7.62	11.46	14.66*	14.68*	8.56	5.31
WD_{max}	9.07	16.91*	11.02	13.47*	14.66*	17.69*	9.13	8.81
No. of obs.	3214	3242	3246	3329	3269	3294	3244	3191

Table 3.1 Summary statistics of daily stock returns on the eight stock markets.

Notes: c_0 Figures in parentheses indicate p−values. J-B stands for the Jarque-Bera normality test. $Q(\bullet)$ and $Q^2(\bullet)$ represents the Ljung-Box test statistics of returns and squared returns respectively. Unit root test is based on augmented Dickey-Fuller (ADF) test with linear trend and intercept terms. D_{max}, and WD_{max} are the tests for structural stability (Bai and Perron, 1998, 2003). * indicates significance at 5% level (critical value at 5% level is 11.70 for D_{max} and 12.81 for WD_{max}).

Finally, the structural stability of each of the return series during the sample period has been examined by carrying out the Bai-Perron test (1998, 2003). Since the data period is long enough spanning 13 years, testing for stability of the parameters is necessary, especially because of the Global Financial Crisis in 2007 - 08. Since this test can be done with both stationary and nonstationary data, we have applied the test at both levels. This test, which is used widely for testing the presence of multiple structural breaks, essentially requires computing the test statistics, which Bai and Perron (1998, 2003) denoted as D_{max}, WD_{max} and $Sup\ F(l+1|\,l)$, $l \geq 1$.

As stated in Kundu and Sarkar (2016), the results of the Bai-Perron test with all eight stock indices (all of which are nonstationary), show that there are structural breaks in most of the series, and this is quite expected. Regarding the test results on the returns series, each of which has been found to be stationary, we note that the WD_{max} test statistic values are greater than the critical value of 12.81 at 5% level of significance for returns on Russia, China, the US, and the UK stock indices. This leads to the conclusion that the null hypothesis of 'no structural break' is rejected for these four series, and hence the return series are structurally unstable for half of the countries considered. For the other four returns series the statistic values are not significant, although at 10% level of significance.

3.2. Findings on the Models

In this study, we have considered, in total five models and obtained the ML estimates of all the parameters involved. Besides the three proposed models - TAR-GARCH-M, TAR-EGARCH-M, and STAR-EGARCH-M - the two others are AR-GARCH-M and AR-EGARCH-M. The latter two models are two simple models in 'volatility-in-mean' framework, which have been taken as the benchmark models. These two benchmark models are utilized to determine the extent to which up and down market movements in conditional mean and/or asymmetry in conditional variance, captured through the EGARCH model, lead to improvements in explaining returns dynamics.

Under the assumption of normal distribution for ε_t, i.e. $\varepsilon_t\,|\,\psi_{t-1} \sim (0,h_t)$, the likelihood function is obviously highly nonlinear. Imposing stationarity conditions, the ML estimates[9] thus obtained for all the models are reported and discussed below.

Country / Parameter	Brazil	Russia	India	China	The US	The UK	Hong Kong	Japan
μ	-0.1506	0.1290	0.0941	-0.1119	-0.0078	-0.0011	0.0129	0.0536
	(0.23)	(0.20)	(0.17)	(0.16)	(0.87)	(0.97)	(0.83)	(0.47)
ϕ	0.0112	0.0238	0.0797	0.0036	-0.0586	-0.0450	0.0149	-0.0044
	(0.54)	(0.20)	(0.00)	(0.84)	(0.00)	(0.01)	(0.41)	(0.81)
δ	0.1421	-0.0018	0.0048	0.1030	0.0597	0.0568	0.0346	-0.0106
	(0.06)	(0.97)	(0.93)	(0.09)	(0.22)	(0.21)	(0.50)	(0.86)
ω	0.0655	0.0902	0.0551	0.0288	0.0151	0.0132	0.0142	0.0417
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
α	0.0720	0.1056	0.1291	0.0625	0.0864	0.1141	0.0671	0.1052
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
β	0.9086	0.8767	0.8527	0.9267	0.9043	0.8799	0.9271	0.8794
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
MLLV	-6312.9611	-6706.2816	-5684.6883	-5895.7102	-4862.3414	-4679.8743	-5529.8469	-5527.3343

Table 3.2 Summary statistics of daily stock returns on the eight stock markets.
Notes: In Table 3.2, we specify the following models for r_t and h_t:

$$r_t = (\mu + \phi\, r_{t-1} + \delta\sqrt{h_t}) + \varepsilon_t\,, \qquad \varepsilon_t\,|\,\psi_{t-1} \sim (0,h_t),$$
$$h_t = \omega + \alpha\,\varepsilon^2_{t-1} + \beta\, h_{t-1}$$

The entries in the parentheses are the p–values; MLLV is the maximized log-likelihood value.

We first report the estimates of the AR-GARCH-M model for all of the eight returns series in Table 3.2. In the context of our study, this is the simplest model since it considers neither different market movements like the up and down markets for conditional mean nor the leverage effect for conditional variance. The intercept μ is statistically not significant for all the series. The first-order autocorrelation coefficient ϕ is significant only for three returns series viz., SENSEX for India, S&P 500 for the USA, and FTSE ALL for the UK. The coefficients of GARCH model are significant at 1% level for all returns series, indicating the presence of strong volatility in all. For examining the risk-return relationship, the risk aversion parameter, δ, is the most important one. The findings on δ are somewhat unexpected since for each of the eight return series, δ is found to be statistically insignificant. This finding implies that time-varying risk does not directly influence returns, irrespective of whether the stock markets refer to the developed or the BRIC group of emerging economies. This has been observed by others as well. For instance, Bekaert and Wu (2000) have stated the coefficient linking (symmetric) volatility to returns is statistically insignificant. These findings on returns for all eight stock markets raise the issue of considering asymmetry in volatility, especially because leverage effect is so common and prevalent in stock markets.

The estimation results for the AR-EGARCH-M model, where instead of GARCH conditional variance is taken to be the EGARCH model, are presented in Table 3.3. As evident in this table, the first-order autocorrelation coefficient is significant only for three series i.e., SENSEX, S&P 500, and FTSE ALL, which are, as expected, the same as in case of the AR-GARCH-M model. From the estimates of the parameters of the EGARCH model, all four parameters i.e., ω, α, λ, and β are significant for all of the eight series. The parameter α is negative for all, except for returns on SSE COMPOSITE of China and HANG SENG of Hong Kong. For these two countries, $\hat{\alpha}$ equals 0.0192 and 0.0628, respectively. Thus, while asymmetry in volatility in returns of all the eight developed and emerging economies has been empirically found, it may be worthwhile to note that the nature of this asymmetry is somewhat mixed, as also noted by Bekaert and Wu (2000). To be more specific, in six out of eight, the relation between current volatility and past returns is negative, which is indeed often the case (see, for instance; Turner

[9] The parameters of these models have been obtained using program written in GAUSS.

et al., 1989; Glosten et al., 1993 and Nelson, 1991), whereas for the remaining two it is positive. While the explanations for observed negative correlations are provided in terms of leverage effect and volatility feedback, the observed positive correlation cannot, as such, be justified from theories of economics and finance. The explanation lies in statistical reasoning. Specifically, the positive correlation may be due to the improper specification of the conditional mean model. In our final model where the two market conditions have been taken into account, asymmetric coefficient in the conditional variance is negative for all the series.

Country Parameter	Brazil	Russia	India	China	The US	The UK	Hong Kong	Japan
μ	-0.0947	0.0152	0.0867	-0.0365	0.0409	0.0077	0.0733	0.0866
	(0.42)	(0.09)	(0.18)	(0.63)	(0.30)	(0.83)	(0.21)	(0.23)
ϕ	0.0263	0.0161	0.0907	0.0047	-0.0541	-0.0292	0.0235	-0.0085
	(0.15)	(0.38)	(0.00)	(0.79)	(0.00)	(0.10)	(0.19)	(0.68)
δ	0.0771	-0.0419	-0.0318	0.0562	-0.0434	-0.0001	-0.0432	0.0770
	(0.28)	(0.43)	(0.54)	(0.34)	(0.34)	(0.99)	(0.39)	(0.19)
ω	0.0312	0.0438	0.0306	0.0180	0.0046	-0.0005	0.0107	0.0218
	(0.00)	(0.00)	(0.00)	(0.00)	(0.07)	(0.84)	(0.00)	(0.00)
α	-0.0819	-0.0457	-0.0997	0.0192	-0.1274	-0.1247	0.0628	-0.0980
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
λ	0.1284	0.2246	0.2431	0.1207	0.1067	0.1224	0.1301	0.1867
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
β	0.9724	0.9733	0.9631	0.9873	0.9832	0.9830	1.9869	0.9697
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
MLLV	–6284.1864	–6712.2342	–5660.9226	–5893.7096	–4791.0282	–4606.7713	–5497.6119	–5490.2236

Table 3.3 Estimates of the parameters of AR(1)-EGARCH(1,1)-M model.
Notes: In Table 3.3, we specify the following models for r_t and h_t:

$$r_t = (\mu + \phi\, r_{t-1} + \delta \sqrt{h_t}) + \varepsilon_t\,, \qquad \varepsilon_t \,|\, \psi_{t-1} \sim (0, h_t),$$

$$\ln h_t = \omega + \alpha\left(\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right) + \lambda\left[\frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}} - E\left(\frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}}\right)\right] + \beta \ln h_{t-1}$$

The entries in the parentheses are the *p*–values; MLLV is the maximized log-likelihood value.

Insofar as the risk aversion parameter δ is concerned, the findings are exactly the same as in the case of the AR-GARCH-M model viz.; this parameter is insignificant for all of the eight return series. Thus, the conclusion, based on these two benchmark models, is that there is no direct effect of risk on expected returns in all of the eight series.

We now consider the first proposed model viz., the TAR-GARCH-M model, where two regimes based on positive and non-positive past average returns are considered for the conditional mean model. As mentioned in Section 2, the two regimes - up and down- have been chosen based on the value of $\bar{r}_t^k$, the average of k past returns, being > 0 and ≤ 0, respectively. For the choice of k, we considered several values, especially because the data are at daily frequency. Thus, starting with $k = 5$ the values have been considered with small gaps viz., $k = 7, 10, 15, 20, 25, 30$, then with big jumps like $k = 50, 75, 100$, and finally with $k = 150$. For each of these choices, the model has been estimated and the particular value of k has been chosen for which the log-likelihood value was the maximum. The values of k thus obtained are: 20 for returns on BOVESPA, SENSEX, SSE COMPOSITE, and S&P 500, 15 for returns on MICEX, 10 for returns on FTSE ALL, and 5 for returns on HANG SENG and NIKKEI 225.

Country Parameter	Brazil	Russia	India	China	The US	The UK	Hong Kong	Japan
μ_1	-0.2722	-0.2486	0.1304	-0.2708	-0.0669	-0.0649	-0.0812	-0.0999
	(0.13)	(0.11)	(0.22)	(0.01)	(0.40)	(0.31)	(0.37)	(0.41)
	-0.0098	0.0045	-0.0152	-0.0096	-0.0867	-0.0311	-0.0155	-0.0177

ϕ_1	(0.71)	(0.87)	(0.76)	(0.70)	(0.00)	(0.24)	(0.56)	(0.52)
δ_1	0.1920	0.1655	-0.0910	0.1838	0.1116	0.1433	0.0928	0.1066
	(0.07)	(0.04)	(0.37)	(0.03)	(0.13)	(0.03)	(0.22)	(0.25)
μ_2	-0.0767	0.3477	0.1528	0.0506	0.0370	0.0566	0.0895	0.1759
	(0.66)	(0.01)	(0.10)	(0.66)	(0.53)	(0.26)	(0.27)	(0.09)
ϕ_2	0.0239	0.0329	0.1712	0.0022	-0.0299	-0.0478	0.0430	0.0248
	(0.36)	(0.21)	(0.00)	(0.93)	(0.26)	(0.09)	(0.14)	(0.40)
δ_2	0.1054	-0.1246	-0.1059	0.0187	0.0012	-0.0256	-0.0343	-0.1261
	(0.35)	(0.13)	(0.22)	(0.83)	(0.98)	(0.69)	(0.63)	(0.14)
ω	0.0677	0.0947	0.0570	0.0293	0.0154	0.0136	0.0148	0.0448
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
α	0.0727	0.1068	0.1314	0.0626	0.0868	0.1154	0.0670	0.1072
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
β	0.9070	0.8739	0.8496	0.9259	0.9035	0.8784	0.9266	0.8759
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
MLLV	–6312.0557	–6701.4757	–5680.5637	–5885.9288	–4861.2393	–4679.0630	–5299.0078	–5527.0409

Table 3.4 Estimates of the parameters of TAR(1)-GARCH(1,1)-M model.
Notes: In Table 3.4, we specify the following models for r_t and h_t:

$$r_t=\begin{cases}\mu_1+\phi_1 r_{t-1}+\delta_1\sqrt{h_t}+\varepsilon_t \text{ for down market}\\ \mu_2+\phi_2 r_{t-1}+\delta_2\sqrt{h_t}+\varepsilon_t \text{ for up market}\end{cases},\quad \varepsilon_t\,|\,\psi_{t-1}\sim(0,h_t),$$

$$h_t=\omega+\alpha\,\varepsilon_{t-1}^2+\beta\,h_{t-1}$$

The entries in the parentheses are the p–values; MLLV is the maximized log-likelihood value.

From the estimates of this model, presented in Table 3.4, with the introduction of two states for stock markets and specification of the model for conditional mean accordingly, the findings are now somewhat different. The autocorrelation coefficient for the down state, ϕ_1, is significant only for S&P 500 returns, while for the up state, ϕ_2, is significant for India and the UK. The parameters of the GARCH model are statistically significant for all the returns, as expected. As for the two risk aversion parameters, δ_1 is significant for stock returns on Brazil, Russia, China, and the UK, while δ_2 is insignificant for all eight countries. Thus, with the introduction of regimes characterized by up and down market conditions and time-varying risk represented by GARCH, the expected return is significantly and directly influenced by (symmetric) volatility, unlike in the first two models. There is also some improvement compared to the AR-GARCH-M model in terms of the maximized log-likelihood value for most of the stock indices, attesting to the importance of considering two market conditions in the conditional mean model for determining returns.

We now consider the other two proposed models - the TAR-EGARCH-M and the STAR-EGARCH-M models. The conditional mean specifications of these are given in equations (2.2) and (2.5), respectively, and the conditional variance is given by the EGARCH model (cf. equation 2.4) for both. Empirical results for the two models are presented in Tables 3.5 and 3.6, respectively. As evident in Table 3.5 all three parameters of the EGARCH model viz., α, λ and β, are significant as in the case of the AR-EGARCH-M model. The estimate of α, however, is now negative for all eight returns series, establishing thereby the presence of leverage effect in returns of all of the eight stock markets considered in this study. The most significant observation, however, is that either of the two risk aversion parameters for the two market movements considered, δ_1 and δ_2, is significant for all the return series. Furthermore, $\hat{\delta}_1$ is positive and the coefficient is significant for stock returns on Brazil, Russia, China, the USA, and the UK, while $\hat{\delta}_2$ is negative and the coefficient is significant for four return series viz., those of Russia, India, Hong Kong, and Japan. In terms of the two regimes characterized by two different market conditions, these findings suggest that response of risk to these two market conditions is asymmetric. This finding of asymmetry in risk aversion suggests that a single regime model where the risk aversion coefficient is fixed, is unable to guide the investor to differentiate between downside risk or loss and upside risk or gain. In a dual risk aversion model, where up and down market conditions are considered when prices are falling and consequently average return is negative, an investor will take risk only when an extra premium is associated with the market risk. In an up market, however, when the return is in general positive, the investor may pay a premium such that the risk aversion coefficient becomes negative in the up market situation.

Country Parameter	Brazil	Russia	India	China	The US	The UK	Hong Kong	Japan
μ_1	-0.6719	-0.3300	0.0712	-0.3025	-0.3081	-0.1548	-0.1012	-0.1062
	(0.00)	(0.02)	(0.49)	(0.00)	(0.00)	(0.02)	(0.28)	(0.38)
ϕ_1	0.0041	-0.0035	-0.0107	-0.0096	-0.0821	-0.0248	-0.0254	-0.0328
	(0.88)	(0.90)	(0.84)	(0.70)	(0.00)	(0.36)	(0.35)	(0.29)
δ_1	0.3242	0.1694	-0.0916	0.1810	0.1940	0.1134	0.0551	0.0558
	(0.00)	(0.01)	(0.36)	(0.02)	(0.00)	(0.07)	(0.45)	(0.53)
μ_2	0.0338	0.3551	0.2209	0.1087	0.0793	0.0420	0.1898	0.2001
	(0.83)	(0.00)	(0.01)	(0.31)	(0.10)	(0.35)	(0.01)	(0.04)
ϕ_2	0.0423	0.0233	0.1926	-0.0023	-0.0270	-0.0448	0.0514	0.0292
	(0.08)	(0.32)	(0.00)	(0.93)	(0.27)	(0.07)	(0.06)	(0.30)
δ_2	0.0232	-0.1586	-0.2142	-0.0155	-0.0711	-0.0120	-0.1372	-0.1976
	(0.82)	(0.02)	(0.01)	(0.85)	(0.24)	(0.84)	(0.06)	(0.02)
ω	0.0336	0.0450	0.0300	0.0188	0.0032	-0.0012	0.0099	0.0223
	(0.00)	(0.00)	(0.00)	(0.00)	(0.25)	(0.66)	(0.00)	(0.00)
α	-0.1020	-0.0502	-0.1045	-0.0268	-0.1446	-0.1365	-0.0672	-0.1023
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
λ	0.1126	0.2209	0.2422	0.1262	0.0789	0.1145	0.1251	0.1830
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
β	0.9698	0.9719	0.9629	0.9863	0.9817	0.9814	0.9870	0.9690
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
MLLV	– 6274.5898	– 6704.5386	– 5655.5614	– 5877.2182	– 4776.9058	– 4603.5382	– 5493.2968	– 5488.4212

Table 3.5 Estimates of the parameters of TAR(1)-EGARCH(1,1)-M model.
Notes: In Table 3.5, we specify the following models for r_t and h_t:

$$r_t=\begin{cases}\mu_1+\phi_1 r_{t-1}+\delta_1\sqrt{h_t}+\varepsilon_t & \text{for down market}\\ \mu_2+\phi_2 r_{t-1}+\delta_2\sqrt{h_t}+\varepsilon_t & \text{for up market}\end{cases}, \quad \varepsilon_t\,|\,\psi_{t-1}\sim(0,h_t),$$

$$\ln h_t=\omega+\alpha\left(\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right)+\lambda\left[\frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}}-E\left(\frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}}\right)\right]+\beta\ln h_{t-1}$$

The entries in the parentheses are the *p*–values; MLLV is the maximized log-likelihood value.

Furthermore, we briefly point out the findings, which are different in these two groups of countries. Among the developed markets, the US and the UK had a positive risk return relation in the down market condition and an insignificant risk aversion coefficient in the up market, while Japan and Hong Kong faced a negative relationship in the up market. On the other hand, among the developing countries considered, all but India have downside risk. These findings demonstrate that there is no substantial difference in the nature of risk-return behaviour of investors belonging to developed and BRIC economies in up and down market situations.

Table 3.6 presents the estimates of the last model considered viz., the STAR-EGARCH-M model. This model is different from the TAR-EGARCH-M model in that now there is a smooth transition mechanism from one market condition to the other through the assumption of logistic function. The two models are otherwise the same. As evident from the entries in this table, the empirical results are almost the same as those for the TAR-EGARCH-M model for each of the eight stock returns. This is so because the estimate of γ, the parameter of smoothness in the logistic transition function, is very high for all of the stock-returns except for returns on SENSEX of India and SSE COMPOSITE of China. As already stated in Section 2, under this condition, the transition is almost instantaneous at $\bar{r}_t^k=0$, and hence the STAR model for conditional mean reduces to the TAR model (see also, Silvennoinen and Thorp, 2013, for similar evidence). Accordingly, no further discussions on the findings of this model are given.

Country / Parameter	Brazil	Russia	India	China	The US	The UK	Hong Kong	Japan
μ_1	-0.6769 (0.00)	-0.3281 (0.02)	-0.0402 (0.84)	-0.3761 (0.00)	-0.3079 (0.00)	-0.1747 (0.01)	-0.1152 (0.18)	-0.1090 (0.36)
ϕ_1	0.0035 (0.90)	-0.0028 (0.92)	-0.0587 (0.49)	-0.0132 (0.60)	-0.0830 (0.01)	-0.0232 (0.41)	-0.0264 (0.36)	-0.0329 (0.28)
δ_1	0.3241 (0.00)	0.1677 (0.01)	-0.0743 (0.58)	0.2183 (0.01)	0.1910 (0.00)	0.1249 (0.05)	0.0647 (0.35)	0.0580 (0.51)
μ_2	0.0184 (0.91)	0.3467 (0.00)	0.3913 (0.06)	0.1505 (0.21)	0.0778 (0.10)	0.0513 (0.26)	0.2063 (0.01)	0.1971 (0.04)
ϕ_2	0.0436 (0.09)	0.0240 (0.30)	0.1409 (0.07)	0.0036 (0.88)	-0.0261 (0.30)	-0.0454 (0.07)	0.0582 (0.05)	0.0279 (0.33)
δ_2	0.0350 (0.73)	-0.2426 (0.03)	-0.2426 (0.04)	-0.0278 (0.75)	-0.0688 (0.25)	-0.0234 (0.70)	0.1538 (0.03)	-0.1939 (0.02)
ω	0.0339 (0.00)	0.0450 (0.00)	0.0302 (0.00)	0.0188 (0.00)	0.0032 (0.20)	-0.0011 (0.68)	0.0098 (0.00)	0.0223 (0.00)
α	-0.1030 (0.00)	-0.0503 (0.00)	-0.1048 (0.00)	-0.0289 (0.00)	-0.1453 (0.00)	-0.1374 (0.00)	-0.0675 (0.00)	-0.1023 (0.00)
λ	0.1126 (0.00)	0.2208 (0.00)	0.2427 (0.00)	0.1255 (0.00)	0.0785 (0.00)	0.1130 (0.00)	0.1248 (0.00)	0.1830 (0.00)
β	0.9694 (0.00)	0.9719 (0.00)	0.9629 (0.00)	0.9862 (0.00)	0.9817 (0.00)	0.9815 (0.00)	0.9871 (0.00)	0.9689 (0.00)
γ	18.85 (0.42)	341.57 (0.47)	1.50 (0.13)	0.71 (0.12)	45.13 (0.38)	26.36 (0.37)	13.70 (0.36)	500.01 (0.49)
MLLV	-6274.4172	-6704.7575	-5654.4256	-5875.6370	-4776.6193	-4602.7198	-5492.6637	-5488.5261

Table 3.6 Estimates of parameters of STAR(1)-EGARCH(1,1)-M model.

Notes: In Table 3.6, we specify the following models for r_t and h_t:

$$r_t=(\mu_1+\phi_1 r_{t-1}+\delta_1\sqrt{h_t})\,(1-G(r_t^k;\gamma,c))+(\mu_2+\phi_2 r_{t-1}+\delta_2\sqrt{h_t})G(r_t^k;\gamma,c)+\varepsilon_t\,,\quad \varepsilon_t\,|\,\psi_{t-1}\sim N(0,h_t),$$

$$G(r_t^k;\gamma,c)=\frac{1}{1+\exp(-\gamma[r_t^k-c])}$$

$$\ln h_t=\omega+\alpha\left(\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right)+\lambda\left[\frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}}-E\left(\frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}}\right)\right]+\beta\ln h_{t-1}$$

The entries in the parentheses are the p–values; MLLV is the maximized log-likelihood value.

In order to find if the chosen lag values of the models for both the conditional mean and conditional variance are adequate, we have carried out the Ljung-Box test, $Q(\cdot)$, with both standardized residuals and squared standardized residuals for all five models. The values of this test statistic are provided in Table 3.7 for the proposed TAR-EGARCH-M model only[10]. As evident in Table 3.7 the chosen lag value of unity for the conditional mean is adequate for all eight return series since the test statistic values suggest that the null hypothesis of 'no autocorrelation in standardized errors' cannot be rejected. As for the adequacy of the order pair (1,1) for the conditional variance model of EGARCH, the values of $Q^2(\cdot)$ indicate, at 1% level of significance, that the null hypothesis of 'no autocorrelation in squared standardized errors' is not rejected for returns on stock indices of Brazil, China, India, the UK, and Japan. However, this null hypothesis is rejected for the remaining three series viz., for returns on Russia, the US, and Hong Kong.

Country / Statistic	Brazil	Russia	India	China	The US	The UK	Hong Kong	Japan
$Q(5)$	4.4053	5.1346	8.1960	6.4387	9.1276	6.0071	8.3223	0.8842
	(0.49)	(0.40)	(0.15)	(0.26)	(0.10)	(0.31)	(0.14)	(0.89)
$Q(10)$	8.3952	10.3242	16.4307	22.7153	13.9194	8.6823	11.0464	7.6421

[10] The test statistic values are almost the same for the STAR-EGARCH-M model and hence these are not being reported. As for the other models, the conclusions are, by and large, the same.

	(0.59)	(0.41)	(0.09)	(0.01)	(0.18)	(0.56)	(0.35)	(0.66)
$Q^2(5)$	12.5082	19.4176	4.0754	3.0411	26.0054	1.6951	19.4491	7.1519
	(0.03)	(0.00)	(0.53)	(0.69)	(0.00)	(0.89)	(0.00)	(0.21)
$Q^2(10)$	19.9892	21.2706	8.3980	5.0272	42.2025	9.4082	29.3843	9.9140
	(0.03)	(0.01)	(0.59)	(0.89)	(0.00)	(0.49)	(0.00)	(0.45)

Table 3.7 $Q(\cdot)$ and $Q^2(\cdot)$ test statistic values for the residuals of the TAR-EGARCH-M model.
Notes: The entries in the parentheses are the *p*–values.

Summing up the empirical findings so far, we can state that the twin issues of different stock market conditions and asymmetry in conditional variance are very important in risk-return relationships where time-varying risk is assumed to directly affect the conditional mean. As noted in Tables 3.2 through 3.6, substantial gains in terms of maximized log likelihood values are made in the model hierarchy considered in this paper. Starting with the AR-GARCH-M model where there is no regime in conditional mean as well as no asymmetry in volatility, the modelling performance improves when asymmetry in conditional variance is considered. The performance of the TAR-EGARCH-M model shows that asymmetry in conditional variance is present in all of the return series. The two proposed models i.e., the TAR-EGARCH-M and the STAR-EGARCH-M models, perform almost the same, suggesting thereby that there is no smooth transition from one market state to the other. Finally, Table 3.8 demonstrates that the proposed TAR-EGARCH-M model (also the STAR-EGARCH-M model) has strong statistical support compared to the benchmark AR-EGARCH-M model in terms of the likelihood ratio (LR) test for all countries other than Japan.

Country	Brazil	Russia	India	China	The USA	The UK	Hong-Kong	Japan
H_1 / H_0	TAR-EHARCH-M							
AR-EGARCH-M	19.1932 (0.00)	15.4224 (0.00)	10.7052 (0.01)	33.0038 (0.00)	28.2269 (0.00)	6.4523 (0.09)	8.6237 (0.03)	3.5717 (0.31)

Table 3.8 Likelihood ratio test statistic values.
Notes: The entries in the parentheses are the *p*–values.

3.3. Results of the Wald Test

The most important hypothesis of interest for the proposed models with EGARCH volatility specification is whether risk responds to returns differently in the up and down market movements. In terms of the parameters, the null and alternative hypotheses are $H_0 : \delta_1 = \delta_2$ and $H_1 : \delta_1 \neq \delta_2$, respectively. This null hypothesis has been tested by using the Wald test; Table 3.9 provides the statistic values for the TAR-EGARCH-M model[11]. Before testing this null hypothesis, we first tested the null hypothesis $\mu_1 = \mu_2$, $\phi_1 = \phi_2$, $\delta_1 = \delta_2$ in order to infer whether introduction of two states of market is statistically tenable or not. The Wald test statistic values for the eight return series corresponding to this null hypothesis are also presented in Table 3.9. Based on results presented in Table 3.9, the null hypothesis is rejected for all of the eight return series, thus empirically supporting the introduction of two market states for the conditional mean model. To be more specific, this finding suggests that the two market situations indeed require two different conditional mean models for the two regimes. In order to assess if rejection of this null hypothesis is due to differences in autocorrelations only, we also tested the null hypothesis given as $\phi_1 = \phi_2$. We found that this hypothesis is 'not rejected' in all but two return series; the two exceptions are the returns on SENSEX of India and HANG SENG of Hong Kong. It may be worth recalling that ϕ_1 and /or ϕ_2 have been found to be statistically significant only in case of few stock returns.

[11] The test statistic values in case of the STAR-EGARCH-M model are almost the same and hence these are not reported separately.

Finally, the results of the Wald test for $H_0 : \delta_1 = \delta_2$ show that this null hypothesis is rejected in all cases except for returns on SENSEX and FTSE ALL, thus, implying that except for these two stock indices, the relative risk aversion parameter differs in the two market conditions for the remaining six return series viz., BOVESPA, MICEX, SSE COMPOSITE, S&P 500, HANG SENG and NIKKEI 225. This empirically establishes the fact that investors' reactions to returns in response to risk are different in up and down states of the stock market, and that this is so regardless of whether the stock market belongs to any of the developed or BRIC economies.

H_0 \ Country	Brazil	Russia	India	China	The US	The UK	Hong Kong	Japan
$\mu_1 = \mu_2$, $\phi_1 = \phi_2$ and $\delta_1 = \delta_2$	27.1837 (0.00)	17.3195 (0.00)	12.0445 (0.01)	22.0121 (0.00)	48.9482 (0.00)	12.4868 (0.01)	11.9175 (0.01)	7.3422 (0.06)
$\phi_1 = \phi_2$	0.9026 (0.34)	0.4838 (0.48)	8.9617 (0.00)	0.0432 (0.83)	2.0035 (0.15)	0.2758 (0.59)	3.4002 (0.07)	2.1367 (0.14)
$\delta_1 = \delta_2$	5.7236 (0.02)	11.6848 (0.00)	0.9487 (0.33)	3.3650 (0.07)	14.3499 (0.00)	2.2395 (0.13)	3.7520 (0.06)	4.1912 (0.04)

Table 3.9 Results of the Wald test for the TAR-EGARCH-M model.
Notes: The entries in the parentheses are the *p*–values.

4. CONCLUSIONS

This paper proposes models for studying the risk-return relationship in a framework, where (i) risk directly affects return, as in the GARCH-in-mean model, (ii) two market situations, called the up and down markets (which are based on average of past returns), are considered, and (iii) the risk aversion parameter is assumed to be different. The last assumption enables us to find if risk responds differently in the two market situations. The specification of conditional variance has been taken to be the EGARCH model in order to incorporate the asymmetric behaviour of return shocks on conditional variance. The two proposed models capturing these features, designated as the TAR-EGARCH-M and the STAR-EGARCH-M models, differ only in that the logistic transition function is considered for smooth transition from one regime to the other in case of the latter model. We have taken daily level data on stock indices for a group of eight countries, of which four are developed and four are important emerging economies, represented by the acronym BRIC. The stock indices considered at daily frequency are: S&P 500 (the US), FTSE ALL (the UK), Hang Seng (Hong Kong), NIKKEI 225 (Japan), BOVESPA (Brazil), MICEX (Russia), BSE SENSEX (India), and SSE COMPOSITE (China). Returns from these stock indices have been used to estimate the proposed models along with two benchmark models where no market conditions have been considered.

The empirical findings are overwhelmingly in favour of the TAR-EGARCH-M model. The two mean regimes referring to up and down markets are statistically valid for all of the eight return series. More importantly, according to the results, risk in terms of time-varying conditional variance responds differently in the two market conditions, in the sense that the risk aversion parameter is positive in the down market and negative in the up market. These empirical findings lend support to the observations made by Fabozzi and Francis (1977) and Kim and Zumwalt (1979), which stated that investors require a premium for taking downside risk and pay a premium for upside variation. Finally, it is also found that the nature of risk-return relationship is basically the same for the two groups of countries.

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