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Research Article

Bending Response of Nanobeams Resting on Elastic Foundation

Çiğdem Demir[1], Kadir Mercan[1], Hayri Metin Numanoglu[1], Ömer Civalek[1]

[1] Department of Civil Engineering, Mechanical Division, Akdeniz University
Antalya, TURKIYE

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Corresponding author: Ömer Civalek, ocivalek@akdeniz.edu.tr

Abstract. In the present study, the finite element method is developed for the static analysis of nano-beams under the Winkler foundation and the uniform load. The small scale effect along with Eringen's nonlocal elasticity theory is taken into account. The governing equations are derived based on the minimum potential energy principle. Galerkin weighted residual method is used to obtain the finite element equations. The validity and novelty of the results for bending are tested and comparative results are presented. Deflections according to different Winkler foundation parameters and small scale parameters are tabulated and plotted. As it can be seen clearly from figures and tables, for simply-supported boundary conditions, the effect of small scale parameter is very high when the Winkler foundation parameter is smaller. On the other hand, for clamped-clamped boundary conditions, the effect of small scale parameter is higher when the Winkler foundation parameter is high. Although the effect of the small scale parameter is adverse on deflection for simply-supported and clamped-clamped boundary conditions.

Keywords: Nonlocal elasticity theory, Static analysis, Weighted residual method, Winkler foundation, Euler-Bernoulli beam theory.

1. Introduction

The investigation of the mechanical properties of nano-structures has a great importance because of the increasing demands for the application of the nano-structures. Nanoscience and nanotechnology have greatly contributed to the design of small-scale structures and devices. Some of the potential applications of nanotechnology are medicine, electronics, food, fuel cells, solar cells, modern batteries, cleaner water, and chemical sensors. Besides, recent applications of nano-beams in engineering structures include nano-sensors and actuators for sensing and energy harvesting, micro- and nano-electro-mechanical systems (MEMS/NEMS). Some application cases of NEMS are nonvalite random access memory (NVRAM), nano-tweezers, tunable oscillator, rotational motors, nano-relays, and feedback-controlled nano-cantilevers. This paper aims to simulate the bending behavior of nano-structures including the size effect such as Carbon nanotube, Silicon carbide nanotube, Boron nitrite nanotube, etc. in a more precise and realistic method than classical continuum models.

Due to the size dependence on micro/nano materials, it is not possible to investigate the mechanical behavior of nano-structures in the classical continuum theory. Therefore, higher order continuum theories such as the nonlocal elasticity theory, the strain gradient elasticity theory, the modified couple stress theory, and the surface elasticity theory have been developed to take the size effect of micro/nano materials into account. The most widely known theory is Eringen's nonlocal elasticity theory [1, 2]. The superiority of the nonlocal elasticity theory to other size effect theories comes from the easiness of its application and from the number of small scale parameters of which the theory consists. The nonlocal elasticity theory includes atomic forces and the internal length scale of the micro/nano structure with only one parameter (e_0a).

The most important factor in the application of the nonlocal elasticity theory is the correct determination of small scale parameters of materials. It is necessary to calibrate e_0 by taking advantage of experimental or molecular dynamic simulation results. Eringen [2] equated the frequency expression of the Born- Kármán model for lattice dynamics to the nonlocal theory for plane waves to determine e_0. It was emphasized that the equality was perfect when ka = π and the value of e_0 in this case was 0.39. Wang and Hu [3] investigated the dispersion relations of nonlocal elasticity in the nonlocal elasticity of Timoshenko beam theory. They also worked on the molecular dynamic simulations for the flexural wave propagation in an armchair (5, 5) and for a wide range of wave numbers in the single-walled carbon nanotube for another armchair (10, 10). The interatomic interactions were defined by Tersoff-Brenner potential and the value of e_0a was obtained as 0.0355 nm. Duan et al. [4] presented a calibration of e_0 in the nonlocal Timoshenko beam theory using molecular dynamic simulation results at room temperature to use in the free vibration analysis of single-walled carbon nanotubes. It was determined that calibrated e_0 values are not constant values. The value of e_0 depended on the length/diameter ratio, boundary conditions, and mode shapes ranging from 0 to 19. Aydoğdu [5] studied longitudinal wave propagation in nano-rods using the nonlocal elasticity theory. The previous theories additionally used the combined rod theory which includes lateral inertia, shear and surface tension effects. The nonlocal parameter was calibrated using lattice dynamics and the nonlocal parameters were determined to be dependent on the material and geometry. By adopting the pseudo-differential operator and Padé's approximation, Wang et al. [6] calibrated a small scale coefficient for an initially stressed vibrating nonlocal Euler Bernoulli Beam with an analytical solution. It was shown that the lowest value of e_0 (0.289) reached the critical value of the comprehensive axial stress. It was found that when the axial stress was equal to zero, the value was equal to 0.408 and this value increased with the axial tensile stress. The calibration of e_0a studies are usually related to carbon nanotubes. Differently using nonlocal elasticity theory, Gao and Lei [7], who first examined the mechanical behavior of protein microtubules, showed that small-scale effects of MTs play an important role in the microtubules buckling. In another study, Gao and Lei [7] used the small scale effect value of $0 \leq e_0a \leq 70$ nm. However, since the e_0 parameter relates to the internal microstructures of the nanomaterials, a considerable effort must be made to achieve this value for each nanomaterial.

The first study on the application of nonlocal elasticity theory to nanotechnology was conducted by Peddieson et al. [8]. The static analysis of beams under concentrated and distributed load was examined in their study. Other researchers have worked on bending analysis of nanostructures along with various size dependence theories [9-11]. Wang and Liew [12] used the Euler-Bernoulli and Timoshenko beam theory for the static analysis with nonlocal continuum mechanics under the point load of micro- and nano-structures. The first detailed research on the nonlocal elasticity theory have been made by Reddy and Pang [13] and Reddy [14]. More recently, FanandZhao [15] have developed mechanics approach to analyze the micro- /nano-bridge test by using the nonlocal elasticity theory. De Rosa and Franciosi [16] have introduced a simple approach to analyze Euler-Bernoulli and Timoshenko nano-beams in a static case in order to detect the effect of the nonlocal parameter. Janghorban [17] have taken the thermal effect on micro-beams in the static analysis into consideration and also used two different differential quadrature methods along with the nonlocal elasticity theory.

On the other hand, many research about the vibration and buckling analysis of micro- /nano-beams and plates with the nonlocal effect have been conducted [18]. Farajpour et al. [19] have investigated the buckling behavior of an orthotropic single layered graphene sheet for various boundary conditions by using the nonlocal elasticity theory. The buckling behavior of nano-plates along with their post-buckling behavior have also been investigated by taking the size effect and the piezoelectric effect into consideration by Liu et al. [20] using the nonlocal Mindlin plate model and von Kármán geometric nonlinearity. Moreover, the same theory and method have been used to calculate the nonlinear vibration of piezoelectric of nano-plates by Liu et al. [21]. On the other hand, Asemi et al. [22] have studied the nonlinear stability of simply supported orthotropic single-layered graphene sheets by using the nonlocal elasticity theory. Besides, a vibration analysis has been made by Malekzadeh and Farajpour [23] in case of axisymmetric free and the forced case for the initially stressed circular nano-plate which is embedded in an elastic medium. Gürses et al. [24] investigated free vibration of the nano annular sector plate based on the nonl ocal thin plate model while they employed an eight-node discrete singular convolution transformation.

A beam on an elastic foundation has been used to describe a lot of engineering problems, with various applications in geo-technics, road, railroad, and marine engineering and biomechanics [25]. The polymer matrix was modelled via an elastic foundation in micro- /nano-engineering. The mechanical behavior of micro- /nano-structures with the effect of the elastic foundation have been studied widely while carbon nanotubes have comprised a huge amount of investigations [26]. Murmu and Pradhan [27] have investigated the thermal vibration of single-walled carbon nanotubes (SWCNTs) surrounded by an elastic matrix based on the nonlocal elasticity theory. Murmu and Pradhan [28] have investigated the stability of single-walled carbon nanotubes embedded in a two-parameter elastic foundation. The beam was modeled according to the Timoshenko beam theory. They used the differential quadrature method to solve both analyses. Pradhan and Reddy [29] have studied the buckling behavior of single walled carbon nanotubes (SWCNT) on the Winkler foundation under various boundary conditions via the differential transformation method (DTM). YoonandRu [30] have presented the vibration analysis of multi-walled carbon nanotubes along with the effect of Winkler foundation. More recently, some other kind of nanotubes have been investigated such as silicon carbide [31, 32] and boron nitrite [33, 34] nanotubes. Demir [35] has presented the vibration analysis of micro- /nano-structures on an elastic matrix along with the effect of the nonlocal elasticity by using the differential transform method (DTM).

Despite the application of the finite element formulation in the nonlocal elasticity theory, it has been investigated much less than other methods [36-43]. Previously, the finite element model was reported for the static analysis of nano-beams by Alshorbagy et al. [44]. The existing literature indicates that the number of studies which evaluate surrounding elastic medium for nano- /micro-beam bending is limited.

In this paper, the bending analysis of nano-beams, based on Euler–Bernoulli beam theory and Eringen's nonlocal elasticity theory, is investigated. The nano-beam is also subjected to Winkler elastic foundation and the governing equation is derived by using the principle of minimum potential energy. Finite Element Method (FEM) is employed to study the small scale effects on bending of nano-beams on the elastic foundation in simply-supported and clamped-clamped boundary conditions. Stiffness matrices and load vector are obtained via Galerkin weighted residual method. Moreover, Euler-Bernoulli beam theory is considered for the modelling of nano-beams.

2. Nonlocal Euler-Bernoulli Beam on Winkler Foundation

2.1. Governing Equation

The total potential energy ($\prod$) of an elastic body is defined as:

$$\prod = U - W \tag{1}$$

where *'U'* and *'W'* denote total strain energy and the work of external forces, respectively. Based on the classical Euler-Bernoulli beam theory, the displacement field can be written as:

$$u = -z\frac{dw}{dx}, \quad v = 0, \quad w = w(x) \tag{2}$$

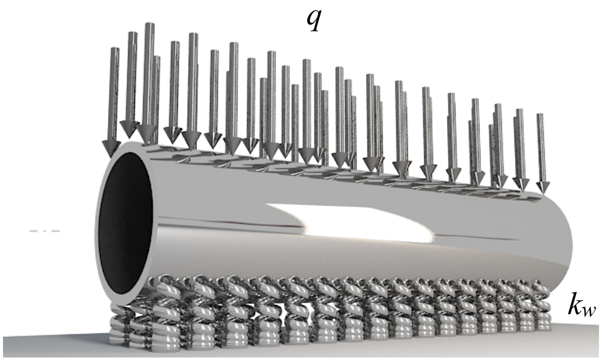

Fig. 1. Nano-beam subjected to uniformly distributed load and resting on Winkler foundation

where '*w*' is the transverse displacement of the beam. According to Euler-Bernoulli beam theory, the strain (ε_{xx})-displacement, the stress (σ_{xx})-displacement relationship, and general expression of the bending moment (M) can be written as:

$$\varepsilon_{xx} = \frac{du}{dx} = -z\frac{d^2w}{dx^2}, \quad \sigma_{xx} = -Ez\frac{d^2w}{dx^2}, \quad M = \int_A z\,\sigma_{xx}\,dA \tag{3}$$

In the above-mentioned equation, '*A*' is used to denote the cross section area. Total strain energy *(U)* and the work of external forces *(W)* for Euler-Bernoulli nano-beams on the Winkler foundation (*k*) are defined as [25]:

$$U = \frac{1}{2}\int_0^L\int_A \sigma_{xx}\,\varepsilon_{xx}\,dA\,dx + \frac{1}{2}\int_0^L kw^2dx, \quad W = \int_0^L qw\,dx \tag{4}$$

where '*q*' denotes the uniform load. Substitution of Eq. (4) into Eq. (1) yields:

$$\prod = \frac{1}{2}\int_0^L\int_A \sigma_{xx}\,\varepsilon_{xx}\,dA\,dx + \frac{1}{2}\int_0^L kw^2dx - \int_0^L qw\,dx \tag{5}$$

and when necessary arrangements are made according to Eq. (3), the first variation of the total potential energy *δΠ* is expressed as:

$$\int_0^L\int_A \sigma_{xx}\left(-z\frac{d^2\delta w}{dx^2}\right)dA\,dx + \int_0^L kw\,\delta w dx - \int_0^L\left(q\,\delta w\right)dx = 0 \tag{6}$$

The simplification of Eq. (6) gives:

$$\int_0^L\left(-M\frac{d^2\delta w}{dx^2}\right)dx + \int_0^L kw\,\delta w dx - \int_0^L\left(q\,\delta w\right)dx = 0 \tag{7}$$

If the integration is done by a part of Eq. (7), it yields:

$$\frac{d^2M}{dx^2} = -q + kw \tag{8}$$

Special form of the nonlocal constitutive relations for beams is as follows [13]:

$$\sigma_{xx} - (e_0a)^2\frac{d^2\sigma_{xx}}{dx^2} = E\,\varepsilon_{xx} \tag{9}$$

If both sides of Eq. (9) are multiplied by *z* and integrated with respect to the field, the moment expression according to Euler-Bernoulli beam theory [13] is as follows:

$$M-(e_0a)^2\frac{d^2M}{dx^2}=-EI\frac{d^2w}{dx^2} \tag{10}$$

where 'e_0a ' is the small scale parameter. When Eq. (8) is substituted in Eq. (10), the nonlocal moment expression is:

$$M=(e_0a)^2(kw-q)-EI\frac{d^2w}{dx^2} \tag{11}$$

Finally, the nonlocal bending equation is obtained by substituting Eq. (11) in Eq. (8) as:

$$EI\frac{d^4w}{dx^4}-(e_0a)^2\frac{d^2}{dx^2}(kw-q)+kw-q=0 \tag{12}$$

2.2. Galerkin Weighted Residual Method

In cases where the variational principle is known, the finite element equations can be obtained by the minimum potential energy method, whereas, in some areas outside solid mechanics the variational principle may not exist. In this case there are only differential equations and boundary conditions where the finite element equations can be obtained by the weighted residual method. The finite element method is based on the definition of functions representing the exact solution. The exact solution for displacement is that the right-hand side of the differential equation must be exactly equal to zero. In the approximate solution for the transverse displacement, the result is not equal to zero. In this case, Eq. (12) defines the residual as follows:

$$EI\frac{d^4w}{dx^4}-(e_0a)^2\frac{d^2}{dx^2}(kw-q)+kw-q=residual \tag{13}$$

Eq. (13) is multiplied by a weighting function *(h)* to define the weighted residual. The aim is to equate the mean weighted residual to zero. When the weighted residual is integrated over the length as:

$$\int_0^L h\left(EI\frac{d^4w}{dx^4}-(e_0a)^2\frac{d^2}{dx^2}(kw-q)+kw-q\right)dx=0 \tag{14}$$

Eq. (14) can be integrated by parts. According to the chain rule, the weak form of differential equation is:

$$\int_0^L\left(EI\frac{d^2h}{dx^2}\frac{d^2w}{dx^2}+(e_0a)^2k\frac{dh}{dx}\frac{dw}{dx}+(e_0a)^2q\frac{d^2h}{dx^2}+khw-qh\right)dx=0 \tag{15}$$

The approximate displacement function *(w),* which is substituted into Eq. (15) and manipulated by the finite element technique is actually the sum of *i* functions.

$$w=\sum_1^i H_id_i \quad \text{or} \quad w=H^Td \tag{16}$$

The vector *H* contains dimensionless functions of *x* and *d* is a vector of unknown displacements. Galerkin's method takes the weighting functions (*h*) equal to the dimensionless function (*H*) which is used to determine the approximate displacement function.

$$h=H \tag{17}$$

Eqs. (16)- (17) are substituted in Eq. (15) as:

$$\int_0^L\left(EI\frac{d^2H}{dx^2}\frac{d^2H^T}{dx^2}+(e_0a)^2k\frac{dH}{dx}\frac{dH^T}{dx}+kHH^T\right)\{d\}dx+\int_0^L\left((e_0a)^2q\frac{d^2H}{dx^2}-qH\right)dx=0 \tag{18}$$

The dimensionless functions *(H)* are interpolating (shape) functions; based on the influence of the nodal displacement, these functions are used to predict the displacement anywhere in the computational domain. The shape function (*H*) for the Euler-Bernoulli beam can be defined as:

$$H=\begin{Bmatrix}1-3\xi^2+2\xi^3\\ L(\xi-2\xi^2+\xi^3)\\ 3\xi^2-2\xi^3\\ L(-\xi^2+\xi^3)\end{Bmatrix} \tag{19}$$

Where $\xi = x / L$ represents the dimensionless local coordinate. If Eq. (19) is substituted in Eq. (18), the stiffness matrices are:

$$K_1 = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{20}$$

$$K_2 = \frac{k}{420}\begin{bmatrix} 156L & 22L^2 & 54L & -13L^2 \\ 22L^2 & 4L^3 & 13L^2 & -3L^3 \\ 54L & 13L^2 & 156L & -22L^2 \\ -13L^2 & -3L^3 & -22L^2 & 4L^3 \end{bmatrix} \tag{21}$$

$$K_3 = \frac{(e_0a)^2 k}{30L}\begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 4L^2 \end{bmatrix} \tag{22}$$

Total stiffness matrix *($\bar{K}$)* is defined as:

$$\bar{K} = K_1 + K_2 + K_3 \tag{23}$$

And the load vector *(F)* [37] is as follows:

$$F_1 = q\begin{bmatrix} L/2 \\ L^2/12 \\ L/2 \\ -L^2/12 \end{bmatrix} \quad \text{and} \quad F_2 = -(e_0a)^2 q\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \tag{24}$$

$$F = F_1 + F_2 \tag{25}$$

Displacements can be obtained when the following equilibrium equation of the system is solved.

$$\left[\bar{K}\right]\{d\} = \{F\} \tag{26}$$

3. Numerical Examples

In this study, in order to discuss the influence of Winkler foundation parameters, small scale parameters, and various boundary conditions on bending behavior of the nano-beam resting on the elastic foundation, a detailed parametric study is performed (Fig.1). Moreover, the non-dimensional parameter for the Euler-Bernoulli nano-beam is calculated according to the following relation:

$$K = \frac{kL^4}{EI}, Q = \frac{qL^4}{EI} \tag{27}$$

In the existing literature, studies on nano-beams resting on the Winkler elastic foundation for static analysis are quite limited. However, the study of Reddy and Pang [13] includes the analytical solution of the Euler-Bernoulli nano-beam for bending analysis. In addition, the maximum displacement equations for simply-supported and clamped-clamped boundary conditions are given in this study as follows, respectively.

$$w_{\max} = \frac{qL^4}{384EI}\left[5 + 48\left(\frac{e_0a}{L}\right)^2\right], w_{\max} = \frac{qL^4}{384EI} \tag{28}$$

A comparative study is performed to check the validity of the finite element method. To this end, the maximum deflection of the nano-beam under the uniform distributed load for simply-supported and clamped-clamped boundary conditions are compared with the above-mentioned equations (Eq.(28)) as proposed by Reddy and Pang [13] and Thai [45]. It is obvious in Tables 1 and 2 that there is a good harmony between the results. It is notable that the results of e_0a/L=0 indicate the classical non-dimensional deflection. As it can be clearly seen in Table 1, non-dimensional deflections are constant for different values of the slenderness ratio (*h/L*) in case of Euler-Bernoulli Beam Theory (EBBT) for various e_0a/L. On the other hand, it must be noted that EBBT gives accurate results when the slenderness ratio (*h/L*) is greater than 20, while in case that the slenderness ratio is lower than 20, Timoshenko Beam Theory (TBT) must be used. It is also found in Table 1 that an increase in the small scale parameter leads to increase in the non-dimensional maximum deflection for simply-supported boundary conditions. Moreover, in Table 2, it can be seen that the non-dimensional maximum deflection of the clamped-clamped nano-beam is not affected by the small scale parameter.

Table 1. Non-dimensional maximum center deflection of simply-supported nano-beam under uniform distributed load (K=0)

	e_0a/L	EBBT Ref. [13]	EBBT Ref. [45]	TBT Ref. [45]	EBBT Present
h/L=5	0	0.013021	0.013021	0.014321	0.013021
	0.05	0.013333	-	-	0.013333
	0.10	0.014271	0.014271	0.015674	0.014271
	0.15	0.015833	-	-	0.015833
	0.20	0.018021	0.018021	0.019734	0.018021
	0.25	0.020833	-	-	0.020833
h/L =10	0	0.013021	0.013021	0.013346	0.013021
	0.05	0.013333	-	-	0.013333
	0.10	0.014271	0.014271	0.014622	0.014271
	0.15	0.015833	-	-	0.015833
	0.20	0.018021	0.018021	0.018449	0.018021
	0.25	0.020833	-		0.020833
h/L =20	0	0.013021	0.013021	0.013102	0.013021
	0.05	0.013333	-	-	0.013333
	0.10	0.014271	0.014271	0.014359	0.014271
	0.15	0.015833	-	-	0.015833
	0.20	0.018021	0.018021	0.018128	0.018021
	0.25	0.020833	-	-	0.020833
h/L =100	0	0.013021	0.013021	0.013024	0.013021
	0.05	0.013333	-	-	0.013333
	0.10	0.014271	0.014271	0.014274	0.014271
	0.15	0.015833	-	-	0.015833
	0.20	0.018021	0.018021	0.018025	0.018021
	0.25	0.020833	-	-	0.020833

Table 2. Non-dimensional maximum center deflection of clamped-clamped nano-beam under uniform distributed load (K=0)

e_0a/L	Ref. [13]	Present
0	0.002604	0.002604
0.05	0.002604	0.002604
0.10	0.002604	0.002604
0.15	0.002604	0.002604
0.20	0.002604	0.002604
0.25	0.002604	0.002604

Variation of the non-dimensional center deflection in case of the simply-supported nano-beam for various Winkler foundation and small scale parameters are illustrated in Table 3 and Figs. 2 and 3. The effects of Winkler foundation parameter of nan-obeams on the non-dimensional deflection are plotted in Fig. 2. In Table 3 and Fig. 2, it can be seen that the maximum deflection corresponding to various Winkler foundation parameters is found at smaller values of foundation parameters as expected. It also can be seen in Table 3 that the increase in the small scale parameter increases the deflection. To examine the effect of high Winkler foundation parameter and small scale parameter, Fig. 4 is plotted. As it can be clearly seen in Fig. 4, in case of high elastic foundation parameter, the effect of small scale parameter is almost zero while this effect is clear in case of low Winkler foundation parameter values. As can easily be seen in Fig. 3, it can be emphasized that the increase in non-dimensional deflection is more evident regarding the small Winkler foundation parameter.

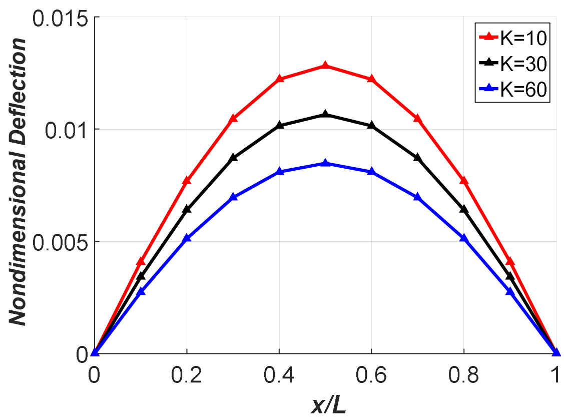

Fig. 2. Non-dimensional deflection of simply-supported nano-beam with e_0a/L=0.1

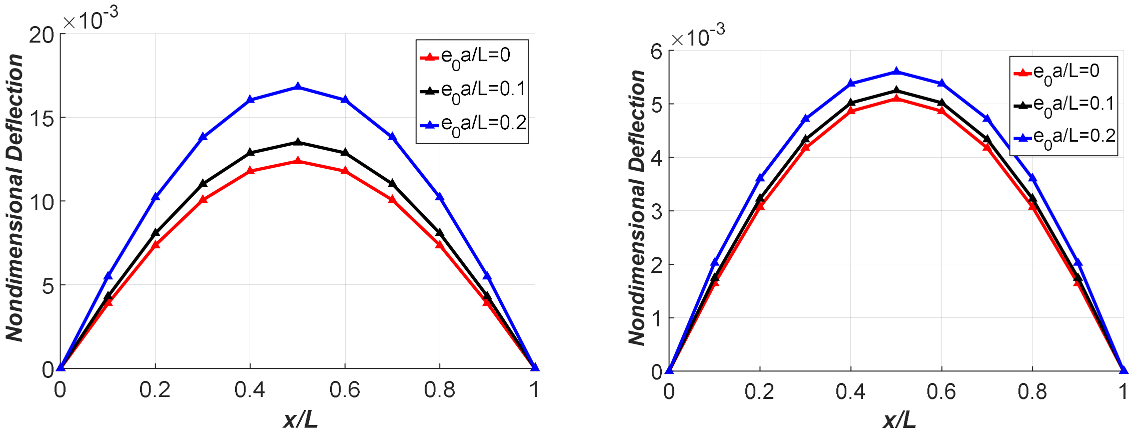

Fig. 3. Non-dimensional deflection of simply-supported beam for a) K=5 and b) K=150

Table 3. Non-dimensional maximum center deflection of simply-supported nano-beam under uniform distributed load and resting on Winkler foundation

e_0a/L	K				
	1	10	25	50	100
0	0.01289	0.01181	0.01037	0.00862	0.00643
0.05	0.01320	0.01207	0.01056	0.00874	0.00650
0.10	0.01411	0.01283	0.01113	0.00912	0.00669
0.15	0.01564	0.01406	0.01204	0.00970	0.00697
0.20	0.01777	0.01575	0.01324	0.01044	0.00732
0.25	0.02049	0.01784	0.01466	0.01128	0.00767

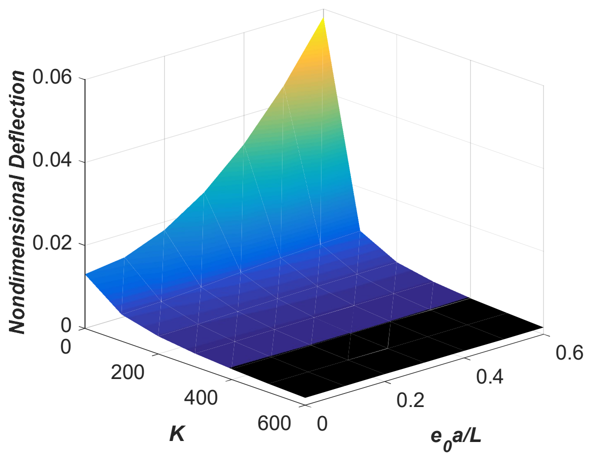

Fig. 4. Non-dimensional maximum deflection of simply-supported beam for various small scale parameter and elastic foundation parameter

Table 4. Non-dimensional maximum center deflection (Q_{max}) of clamped-clamped nano-beam under uniform distributed load and resting on Winkler foundation

e_0a/L	K				
	1	10	25	50	100
0	0.00260	0.00255	0.00248	0.00237	0.00218
0.05	0.00260	0.00255	0.00248	0.00237	0.00217
0.10	0.00260	0.00255	0.00247	0.00235	0.00214
0.15	0.00260	0.00254	0.00245	0.00232	0.00208
0.20	0.00260	0.00253	0.00243	0.00227	0.00201
0.25	0.00260	0.00252	0.00239	0.00222	0.00192

Table 4 and Figs. 5 and 6 similarly depict the effect of the Winkler foundation and small scale parameters on the non-dimensional deflection in case of clamped-clamped boundary conditions. It is notable that in this case, when the Winkler foundation parameter is equal to zero, the small scale parameter does not affect the deflection. Contrary to Fig. 4, in Fig. 7, the effect of small scale parameter in case of high elastic foundation parameter values is clear while it has no effect in case of low elastic foundation parameter values. Similar to the simply-supported boundary condition, the increase of Winkler foundation parameters reduces the deflection. It can be interpreted that the effect of Winkler foundation parameter on the non-dimensional deflection in case of simply-supported nano-beams are more prominent than in case of clamped-clamped supported nano-beams. Moreover, the increase of the small scale parameter decreases the deflection. In addition, when the two boundary conditions are compared, it can be seen that the small scale effect adversely affects the deflections of the nano-beam resting on the Winkler foundation. In the clamped-clamped boundary condition, the effect of the small scale parameter is more evident in the large values of the Winkler foundation parameter. In general, compared to two boundary conditions, nano-beam deflections with a clamped-clamped boundary condition are smaller than those with a simply-supported boundary condition.

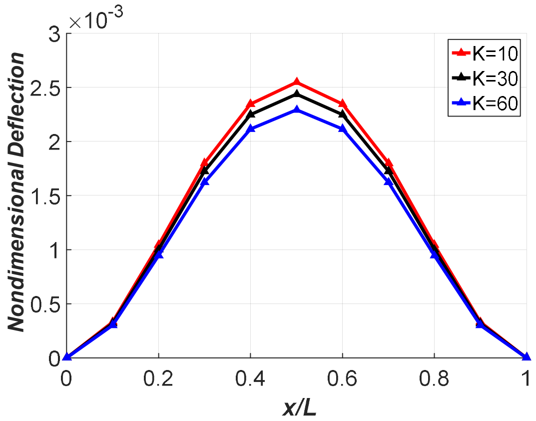

Fig. 5. Non-dimensional deflection of clamped-clamped beam e_0a/L=0.1

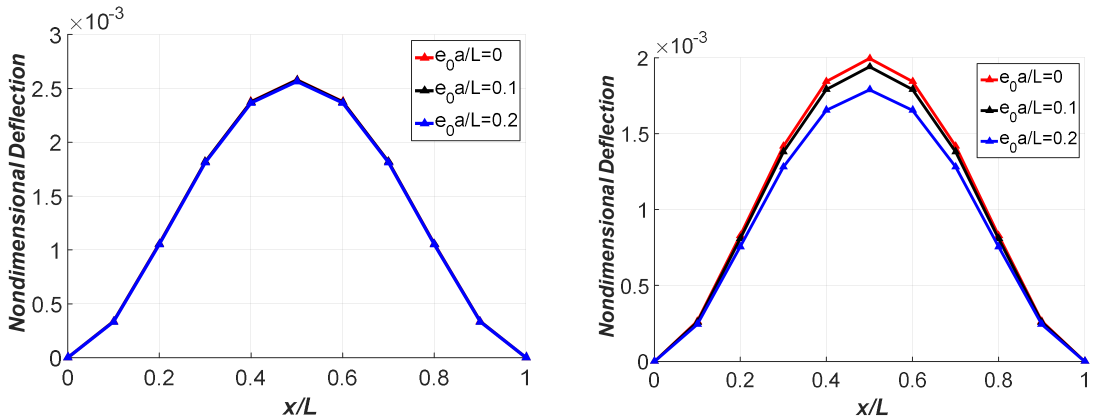

Fig. 6. Non-dimensional deflection of clamped-clamped beam with a) K=5 and b) K=150

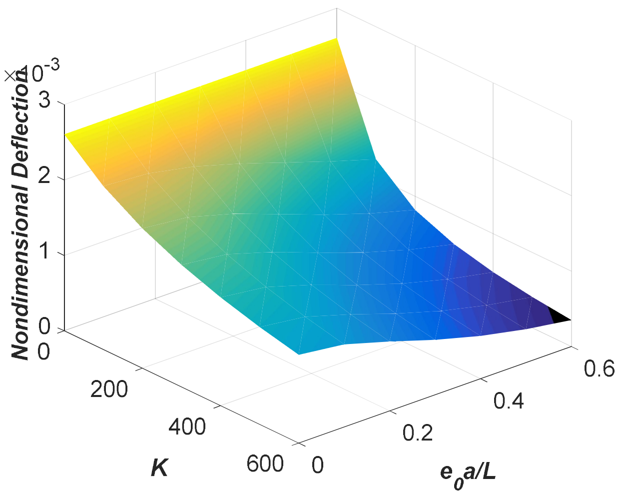

Fig. 7. Non-dimensional maximum deflection of clamped-clamped beam for various small scale parameter and elastic foundation parameter

4. Conclusions

The static analysis of the nano-beam resting on the elastic foundation is investigated based on the nonlocal Euler-Bernoulli beam theory. The deflection of a simply-supported and clamped-clamped beam is considered under a uniform load using the finite element method. The governing equations are derived by using the minimum potential energy principle. The Galerkin weighted residual method is used to solve the governing differential equation of the nano-beam. The stiffness matrix and the load vector are derived. The decrease in the deflection of nano-beams in both boundary conditions is observed by increasing the Winkler foundation parameter. Another consequence is that, in contrast to the simply-supported boundary condition, where the Winkler foundation parameter is higher, the small scale parameter is more important in case of clamped-clamped nano-beams. It should also be noted that while the nano-beam resting on the Winkler elastic foundation, the increase of the small scale parameter causes the deflection to decrease in the clamped-clamped boundary condition and to increase in the simply-supported boundary condition.

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