

RELIABILITY ANALYSIS OF REINFORCED CONCRETE ELEMENTS WITH NORMAL CRACKS (ON RC BEAM EXAMPLE)

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Abstract: The purpose of research is development of methods for reliability analysis of structural reinforced concrete elements (on example of reinforced concrete beam) according to criterion of the normal cracks length in tensile zone of concrete. Cracks in tensile zone of concrete in beams leads to displacement of the beam's neutral axis towards the compressed part of the concrete, thereby increasing the compression stress in the concrete, to the increasing of the stress in the reinforcement bars on the crack width, to the growth of corrosion processes, etc. This leads to the decreasing of beams mechanical safety and reliability (as a quantitative measure of mechanical safety). The main parameters, which reduce the strength and stiffness of the beam, of the cracks is the length and width of the crack. The article describes the influence of the cracks length on the reliability of reinforced concrete elements. The article differs from existing approaches in that is it built on a limited statistical data of controlled parameters in mathematical models of limit state. The article also considers the influence of cracks in the structural element. The methods of reliability analysis allow to assess the mechanical safety of reinforced concrete elements and to make appropriate decisions about the safety, strengthening or replacement of the reinforced concrete element.

Keywords: reliability, normal cracks, beam, safety, failure probability, fuzzy variable, random set theory

РАСЧЕТ НАДЕЖНОСТИ ЖЕЛЕЗОБЕТОННЫХ ЭЛЕМЕНТОВ С НОРМАЛЬНЫМИ ТРЕЩИНАМИ НА ПРИМЕРЕ ЖЕЛЕЗОБЕТОННОЙ БАЛКИ

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Аннотация: Цель работы – разработка методов расчета надежности несущих железобетонных элементов на примере железобетонной балки по критерию длины серии нормальных трещин в растянутой зоне бетона балки. Трещины в растянутой зоне бетона балки приводит к смещению нейтральной оси балки в сторону сжатой части балки, повышая тем самым напряжения сжатия в бетоне, повышают деформации в арматуре на ширине раскрытия трещины, способствуют развитию коррозионных процессов и т.д. Это приводит к снижению механической безопасности балки, в качестве количественной меры которой используется надежность. Основными параметрами трещины, приводящими к снижению прочности и жесткости балки, является длина и ширина раскрытия трещины. Статья посвящена учету влияния длины серии нормальных трещин на надежность железобетонных элементов. Статья отличается от существующих подходов тем, что построена на ограниченном объеме статистической информации о контролируемых параметрах в математических моделях предельных состояний, а также учету влияния нескольких трещин в несущем элементе. Предложенные методы расчетов надежности позволяют оценить механическую безопасность несущих железобетонных элементов и принять адекватные решения о безопасности эксплуатации, усилению или замене балки.

Ключевые слова: надежность, нормальные трещины, балка, безопасность, вероятность отказа, нечеткая переменная, теория случайных множеств

1. INTRODUCTION

The Russian Federal Law No. 384 “Technical regulation for buildings and structures safety” came to force in 2010. Interstate Standard GOST 27751-2014 “Reliability of structures and foundations” came to force in 2015 on the basis of the Law No. 384. One of the main purposes of the Law is the provision of mechanical (structural) safety of buildings and structures during their operation. “Mechanical safety” is the state of structure in which there is no inadmissible risk of failure and harm to the life and health of citizens. Quantitative assessment of the mechanical safety may be the reliability (safety) of the structure or structural element. According to the European Standard Eurocode 0 “Basis of structural design”, the reliability – is an ability of a structure or a structural member to fulfill the specified requirements, including the design working life, for which it has been designed. Reliability is usually expressed in probabilistic terms. The measure of reliability is probability of failure or probability of non-failure. General principles of structural reliability are regulated by Eurocode 0 and International Standard ISO 2394:2015 «General principles on reliability of structures». Interstate Standard GOST 27751-2014 regulates the general principles of structural reliability in Russian Federation.

2. LITERATURE REVIEW

Probabilistic-statistical methods of reliability analysis were studied by many researches: [1, 2, 3 etc.]. Most of the work for the reliability analysis by probabilistic method is currently based on the hypothesis of normal distribution of random variables in mathematical models of limit state. The paper [4] presents the comparison between FORM (First Order Reliability Method) and ISM (The Importance Sampling Method) in reliability analysis of reinforced concrete buildings (including RC beams). Both methods are only used with a

large amount of statistical data, which in practice may cause some difficulties. The article [5] describes the reliability analysis methods for triangular and T-reinforced concrete beams. The method of reliability analysis also based on probabilistic methods. The article [6] presents reliability analysis of reinforced concrete beam exposed to fire using Monte Carlo simulation. The article [7] presents the reliability analysis of a corroded RC beam based on Bayesian updating of the corrosion model.

However, the vulnerability of probabilistic methods of reliability analysis appears in its practical use for individual structural elements in condition of limited time for inspection, the high cost of individual tests, inaccessibility or inability of the study to some parameters, and other reasons. In this regard, new methods of reliability analysis based on new mathematical theories: possibility theory [8], fuzzy set theory [9], theory of random sets [10, 11] etc., has been developed. The new methods allow to take into account the limited statistical information of controlled parameters in mathematical models of limit state in the reliability analysis, of course with the reducing the information content of the result. There is no theory to make up for the lack of statistical data in the reliability analysis. So, Dubois and Prad [8] notes that "the probability, on the one hand, and a pair of "possibility – necessity" correspond to the two extreme, and so the ideal, situations". It is also noted that a probability measure in a natural way synthesize database of accurate and differentiated knowledge, then as a measure of possibility is the essence of reflection is inaccurate, but coherent (i.e. corroborative) knowledge.

3. PROBLEM STATEMENT

A special problem occurs in assessing the safety level in reinforced concrete beams with normal cracks in the tensile zone of concrete, when statistical data of the controlled parameters is always individual and limited. In this case, the application of probabilistic and statistical

methods is not correct. The Figure 1 shows an example of a reinforced concrete beam with normal cracks. Without considering the reasons for the formation of cracks and their impact on the reduction of the RC beam strength and stiffness, let's consider the method of reliability analysis of RC beam with normal cracks on crack length criterion for subsequent decision-making about the possibility of further operation, strengthening or even replacing.

The relevance and novelty of the problem lies in the fact that in the standards of reinforced concrete elements design there are no requirements on the design RC elements according to the crack length criterion and guidelines for the reliability analysis consequently. At the same time, it is known [12] that the cracks in reinforced concrete beams reduces their reliability, and the critical crack length leads to it spontaneous self-growth and to the possibly failure. Piradov K.A. and Savitchkiy N.V. convincingly demonstrate in [13] the necessity for a transition to reinforced concrete structures design by fracture mechanics methods.

So, it is proposed to consider the methods of reliability analysis of reinforced concrete beams with normal cracks (Figure 1), with different types of statistical data about controlled parameters in the design mathematical models of limit state.

The reinforced concrete beam failure may occur according to various criteria of limit state (strength of concrete and rebar, the deflection, the cracks width, etc.). In this sense, the RC beam is a coherent mechanical system in terms of reliability theory. Cracks in RC beams are caused by tensile stresses and are characterized by width a_{crk} and length l_{crk} .

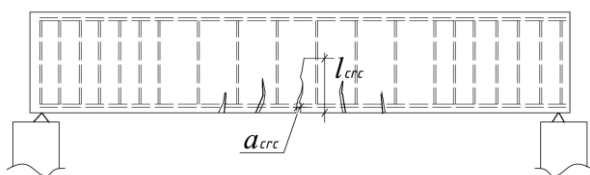


Figure 1. Reinforced concrete beam with normal cracks.

4. RELIABILITY ANALYSIS

The article [14] presents the method of reliability analysis of reinforced concrete beam with one crack by crack length with the mathematical model of limit state:

$$\tilde{l}_{crk} \leq l_{crk,ult} = 0,3h_0, \text{ or } X \leq k, \quad (1)$$

where $X = \tilde{l}_{crk}$ – length controlled (measured) crack the greatest length in the beam cross-section; $k = l_{crk,ult} = 0,3h_0$ – critical crack length (adopted as an option by [12]), which is deterministic value in this case; h_0 – distance from extreme compression fiber to centroid of longitudinal tension reinforcement.

There are other proposals about the critical length of cracks in reinforced concrete beams, for example $l_{crk,ult} = 0,5h$ [15]. Not discussing the problem of the size of the critical crack length, let's assume that it is known, and in the general case depends on the critical value of stress intensity factor K_{IC} . Evaluation of K_{IC} for reinforced concrete beams described in [16]. Every crack reduces the reliability of the beam. If to take the lack of interaction between cracks length (for simplicity), and the failure of the beam by the any of cracks leads to failure of the entire beam, then the beam with cracks can be seen as a coherent mechanical system in terms of reliability theory. In the reliability analysis by possibilistic methods for each criterion of limit state, and for the considered task for each normal crack, the possibility R and the necessity N non-failure of the RC beam as a coherent system [17] will be determined by the interval $[N_{i, \min}; R_{i, \min}]$.

Measurement of the crack length in reinforced concrete beams is challenging, since its length is determined not only by visible part, but by a part of loosened concrete at the crack tip [12]. It is proposed to use the method described in the patent [18] to measure the crack length (Figure 2).

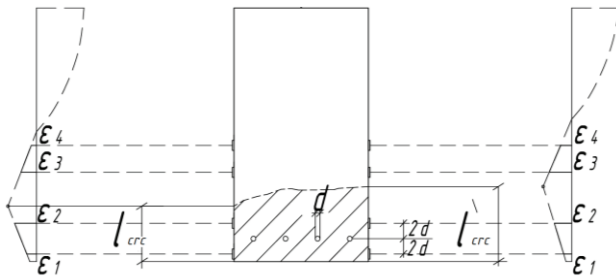


Figure 2. Method for measurement of crack length by [18].

As noted above, the reliability analysis of RC beam will be carried out on the basis of possibility theory [8, 9]. In this case, fuzzy variable described by the distribution function of possibilities $\pi_X(x)$. The most widely used in practice of reliability analysis of structural elements received a distribution function of possibilities with the analytical view:

$$\pi_X(x) = \exp \left[- \left(\frac{x - a_x}{b_x} \right)^2 \right], \quad (2)$$

where

$$a_x = 0,5 \cdot (X_{\max} + X_{\min})$$

– “conditional mean”;

$$b_x = 0,5(X_{\max} - X_{\min}) / \sqrt{-\ln \alpha}$$

– measure of “dispersion”, X_{\max} и X_{\min} - the maximum and minimum value in the set of values $\{x\}$ of fuzzy variable X ; $\alpha \in [0;1]$ - cut (risk) level, the value of which is set.

The choice of distribution function has an impact on the result of reliability analysis, including the width of the confidence interval $[N; R]$, which is the results of reliability analysis. There are no rules for functions choices and checking them such Pearson’ chi-squared test.

Cut level α can be considered as an indicator (measure) of uncertainty of the distribution

functions sets of random variables with their distribution in the shaded areas.

The reliability analysis of RC beam with one crack by possibilistic method carry out by the result of several measurements of crack length by mathematical model (1). We set α , and calculate the parameters a_x and b_x . If $a_x \leq k$ (that usually corresponds to the operational stage), then $R=1$. Possibility of failure Q calculated as

$$Q = \exp \left[- \left(\frac{k - a_x}{b_x} \right)^2 \right].$$

Reliability will be characterized by the interval $[N=1-Q; R]$.

Example 1. Let $X = \{17;20;23\}$ mm and $k = 24$ mm. Let’s find parameters of $\pi_X(x)$ with different values of α . By (2): $a_x = 0,5 \cdot (23 + 17) = 20$ mm and $b_x = 0,5(23 - 17) / \sqrt{-\ln \alpha}$ mm. As $a_x = 20 \leq k = 24$ mm, then $R=1$. Results of Q with different values of α shown in table 1. Figure 3 shows a graph of the dependence of possibility of failure Q and the cut level α :

$$Q(\alpha) = \exp \left[- \left(\frac{24 - 20}{0,5(23 - 17) / \sqrt{-\ln \alpha}} \right)^2 \right]$$

according to the Example 1.

Table 1. Given Data.

Cut level α	b_x , mm	Q
0,01	1,398	$2,78 \cdot 10^{-4}$
0,05	1,733	$4,865 \cdot 10^{-3}$
0,1	1,977	$1,677 \cdot 10^{-2}$
0,2	2,365	0,057
0,4	3,134	0,196

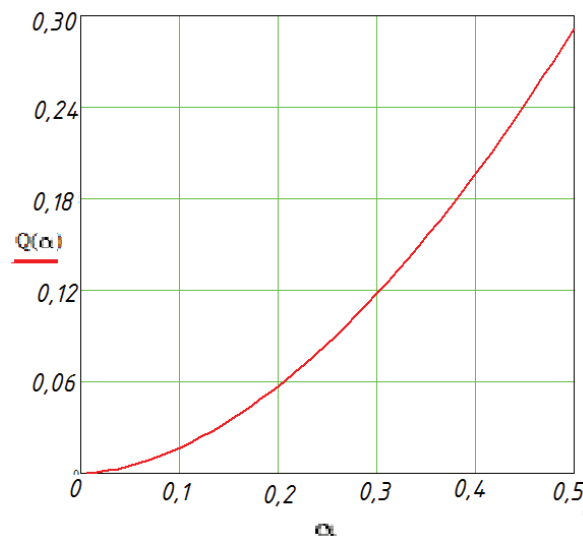


Figure 3. Dependence of possibility of failure Q and the cut level α .

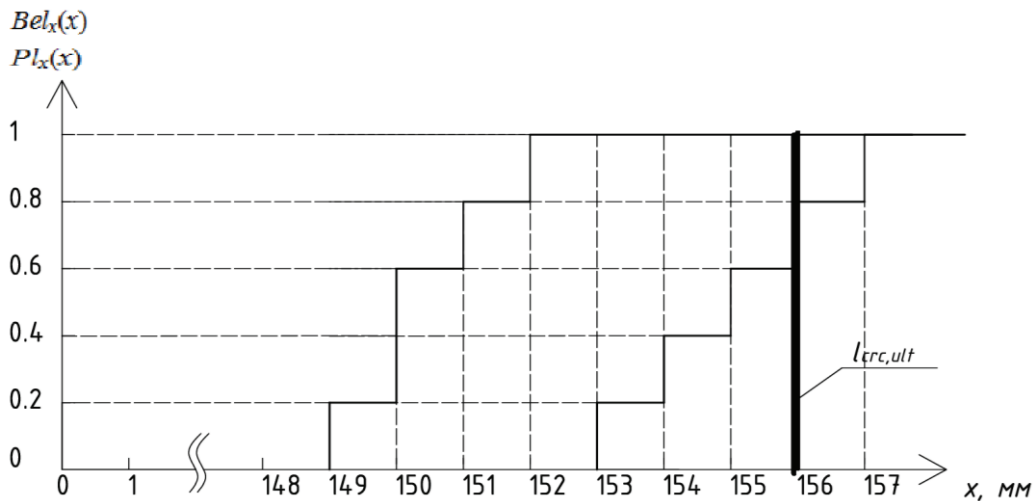


Figure 4. Graphs of belief function $Bel_x(x)$ and plausibility function $Pl_x(x)$ for $X = \tilde{l}_{crc}$.

From the above values of Q in the table 1 and from the graph (Figure 4) we can see, that the intensity of the impact of the cut (risk) level for the Q increases with growth α . At small values of $\alpha < 0,05$, the effect on Q is more subtle. Q increases dramatically with $\alpha > 0,1$. These results show the reduction of information content of the reliability result in the form of a confidence interval with increasing α values. That is a disadvantage of possibilistic method of reliability analysis for an individual object.

In this regard, the possibilistic method of reliability analysis is preferred in the comparative analysis of two or more objects by

reliability index. And value of α it is recommended to take in the interval $[0,01; 0,1]$, where α impact on the value of Q is relatively small. Provides information on the reliability analysis of reinforced concrete beams can be used to assess the level of safety of one beam and a series of such beams in order to compare them by reliability indexes.

5. RANDOM SET THEORY APPROACH

Let's consider the second approach to reliability analysis of reinforced concrete beam with normal

cracks on the basis of known theory of random sets (theory of evidence of Dempster-Shafer) [10, 11], in which there are no parameters and therefore their influence on the result of the reliability analysis. Reliability analysis of reinforced concrete beam we will still spend on the criterion of crack length for the mathematical model (1) by method based on the theory of random sets [19] with the data about X in the form of sub-intervals set of the measurement results of the controlled parameter X .

Example 2. Let $X = \tilde{l}_{cr,c}$ in interval form at different points in time during the operation process of reinforced concrete beam as: $X = \{[150; 154], [151; 155], [150; 156], [152; 157], [149; 153]\}$ mm and $l_{cr,c,ult} = k = 156$ mm. In accordance with the theory of random sets [19], Figure 4 shows graphs of belief function $Bel_x(x)$ and plausibility function $Pl_x(x)$ for $X = \tilde{l}_{cr,c}$, which does not contain parameters to set.

The upper \bar{P} and lower \underline{P} boundaries of the probability of non-failure interval is estimated by [19, 20] Fig. 4. with $l_{cr,c,ult} = k = 156$ mm as interval $[0,80; 1]$.

Example 3. Let's consider another feature of reliability analysis based on the theory of random sets. With the original data of the above example and with $k = 158$ mm, the interval of reliability is $[1; 1]$, i.e. the probability of failure is equal to 0, which is statistically incorrect. Thus, the reliability analysis of structural elements based on the theory of evidence (or theory of random sets) can be used in practice without the involvement of functions of random variables (and parameters definitions). However, the small amount of statistical data as sub-intervals sets and for large values of the non-failure probability of structural element this method may be inconclusive, since the probability of failure may be equal to one. There is another variant for the reliability analysis with a small amount of statistical data in form of intervals subset. So, it is proposed

[20] to use the advanced functions of belief and plausibility through the use of a imprecise Dirichlet model (IDM) as one of the types of robust models. In this case, the upper and lower bounds of probability of non-failure can be written as:

$$\underline{P}(A | c, s) = \frac{N \cdot Bel(A)}{N + s} = \chi Bel(A) \text{ and}$$

$$\bar{P}(A | c, s) = \frac{N \cdot Pl(A) + s}{N + s} = 1 - \chi [1 - Pl(A)], \quad (6)$$

where N – the number of tests (observations); s – the parameter characterizing the extent of «uncertainty», the value of which is set, where is the notation

$$\chi = (1 + s / N)^{-1}$$

and $\chi \in [0; 1]$. Let's consider the algorithm of calculation by this method with an example.

We use the statistical data in the first example, but with introducing the advanced belief and plausibility functions. Parameter $s=2$ as the most safe solution [21]. When $N=5$, extent of contamination is

$$\chi = (1 + s / N)^{-1} = (1 + 2 / 5)^{-1} = 0.714 .$$

Then

$$\underline{P}(A | c, s) = \chi \cdot 0.80 = 0.571$$

and

$$\bar{P}(A | c, s) = 1 - \chi [1 - 1] = 1 .$$

Reliability characterized by interval $[0.571; 1]$, which is wider then interval $[0.800; 1]$ in above example.

Figure 5 shows that the parameter s changes the lower bound of the non-failure probability within $[0,571; 0,800]$, in contrast to the parameter in possibilistic method, which causes the possibility of failure Q in the larger range $[0; 1]$.

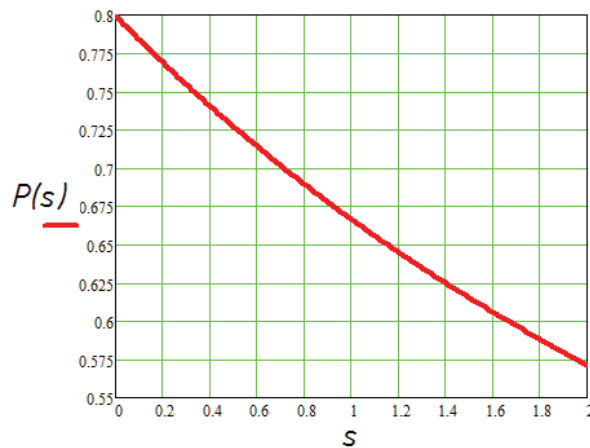


Figure 5. Probability of failure \underline{P} - parameter of «uncertainty» s graph.

Therefore, a volitional decision on the appointment of the parameter s to a lesser extent affects on the final result of reliability analysis. And this result can be considered as more credible.

Let's use the data from the example 3 (with $k = 158$ mm), but with the advanced functions of belief and plausibility. Let $s=1$, given the higher confidence in test results and greater the credibility of the expert values and thus a lower "uncertainty" of the statistical data. With $N=5$ we calculate:

$$\chi = (1 + s / N)^{-1} = (1 + 1 / 5)^{-1} = 0,833 .$$

Then

$$\underline{P}(A | c, s) = \chi \cdot 1 = 0,833$$

and

$$\overline{P}(A | c, s) = 1 - \chi[1 - 1] = 1 .$$

Reliability is characterized by the interval $[0,833; 1]$, which more correct then interval $[1; 1]$ in Example 3.

Thus, using of advanced functions of belief and plausibility using the imprecise Dirichlet model and random sets theory allow us to obtain more conclusive results of reliability analysis for a single crack in the concrete. But the advanced functions are parametric (including parameter s).

As a result of using the advanced functions of belief and plausibility, we can get the expanded boundaries of the mathematical expectation of a random variable X :

$$\begin{aligned} \underline{F}(x | c, s) &= \underline{P}(\{w \leq x\} | c, s) = \\ &= \begin{cases} (N + s)^{-1} \sum_{i: \sup A_i \leq x} c_i, & x < \Omega_* \\ 1, & x = \Omega_*; \end{cases} \\ \overline{F}(x | c, s) &= \overline{P}(\{w \leq x\} | c, s) = \\ &= \begin{cases} (N + s)^{-1} \left(s + \sum_{i: \sup A_i \leq x} c_i \right), & x > \Omega_* \\ 0, & x = \Omega_*. \end{cases} \end{aligned}$$

By [20], bounds of the mathematical expectation of a random variable X represented by intervals (such as probability of non-failure) can be found as:

$$\begin{aligned} \underline{E}X &= \int_{\Omega} \omega d\overline{F}(\omega | c, s) = \\ &= (N + s)^{-1} \left(s \cdot \Omega_* + \sum_{i=1}^n c_i \inf A_i \right); \\ \overline{E}X &= \int_{\Omega} \omega d\underline{F}(\omega | c, s) = \\ &= (N + s)^{-1} \left(s \cdot \Omega_*^* + \sum_{i=1}^n c_i \sup A_i \right). \end{aligned}$$

Let conventionally known intervals, which characterizing the reliability of a reinforced concrete beam according to the criterion of crack length: [0,995; 1], [0,997; 1], [0,994; 1], [0,995; 1]. It is known that the reliability of changes in the boundaries of the interval $[\Omega_*=0; \Omega^*=1]$. Take $s=0.4$. Then:

$$\begin{aligned} \underline{EX} &= (4 + 0,4)^{-1}(0,4 \cdot 0 + 0,995 + \\ &+ 0,997 + 0,994 + 0,995) = 0,9, \\ \bar{EX} &= (4 + 0,4)^{-1}(0,4 \cdot 1 + 1 + 1 + 1 + 1) = 1. \end{aligned}$$

Subject to uncertainty $s=0,4$ of source data, the mathematical expectation of the reliability of reinforced concrete beam according to the criterion of crack length is characterized by the interval [0.905; 1]. In the absence of "uncertainty" (when $s=0$):

$$\begin{aligned} \underline{EX} &= (4 + 0)^{-1}(0 \cdot 0 + 0,995 + \\ &+ 0,997 + 0,994 + 0,995) = 0,9953, \\ \bar{EX} &= (4 + 0)^{-1}(0 \cdot 1 + 1 + 1 + 1 + 1) = 1, \end{aligned}$$

statistical mathematical expectation of the reinforced concrete beam reliability according to the criterion of crack length is characterized by the interval [0,9953; 1].

Thus, advanced functions of belief and plausibility can be used in practice of reliability analysis, having the advantage over methods based on the theory of possibilities, that the s parameter is lesser affects on the results of reliability compared with the parameter (cut level α) in possibilistic method.

Returning to the cracks system in reinforced concrete beams and reliability analysis using the theory of random sets, with the conditions of independence in the interaction between cracks, the reliability analysis of reinforced concrete beam produced by each crack of the beam with the results by the lower and upper values of probabilities, and the reliability of the beam as system according to the criterion of crack length is characterized by [20] interval

$$\left[\prod_{i=1}^n \underline{P}_i; \prod_{i=1}^n \bar{P}_i \right], \quad i=1, \dots, n,$$

where n is the number of cracks in the beam.

Possibilistic method of reliability analysis may be recommended in case of a very small amount of statistical data in the form of individual values. In this case, it should carefully consider the purpose of the cut (risk) level α and take it on the recommendations and in the interval [0,01; 0,1], or other reasonable methods depending on the level of safety. Parameter "s" also is subject to additional research.

6. CONCLUSIONS

1. The article describes the method of reliability analysis of structural RC elements on example of RC beams with a series of normal cracks on the basis of the possibility theory;
2. Some recommendations are given for the appointment of the cut (risk) level in possibilistic methods of reliability analysis of structural elements;
3. The article presents the methods of reliability analysis of reinforced concrete beams with a series of normal cracks on the basis of the random set theory and on the advanced functions of belief and plausibility on the imprecise Dirichlet model;
4. The developed methods can be used in the reliability analysis of other structural elements for the requirements of standards on safety of structures and foundations.

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