On Block Representations
in Image Processing Problems

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Abstract—Any orthogonal transformation of the digital grayscale image can be represented by a set of images to be an orthonormal basis. For such representation digital data scattering was considered that is important in applications, particularly for the robust watermarking techniques. We introduce a block matrix, elements of which are basis images. This matrix is found to be useful for representation of multi-dimensional arrays, that can describe a set of digital images. This representation has new features concerning the data scattering. A steganographic scheme for frequency domain watermarking based on this representation is considered.

I. INTRODUCTION

The article is devoted to one of the most intensively developing area of information protection — computer steganography. In this paper we continue our topic devoted to the applications of the basis images in digital watermarking [1], [2] and propose a new steganographic scheme.

Information is important for a human and plays an important role, as well as the problem of its protection. However, none of the great number of proposed methods of protecting information give full guarantee. Absence of universal methods is a source of permanent search of new solutions, among which is digital steganography using computer technologies [3]. Besides of that it is necessary to expand the application area of known mathematical methods of the digital image representation and its transforms.

An image given in a digital form can be processed using orthogonal transformations. The transformations are often used for extracting of the redundancy from the frequency representation of the digital image. The redundancy can be achieved by orthogonal transformations, which concentrate the energy, the square of the brightness of the pixel. As a result, the image in the frequency domain in a small neighborhood will be assembled with pixels containing almost the entire energy of the image. By setting the threshold, pixels with small energy can be deleted, and return to the spatial domain, however without a noticeable loss of quality. This scheme relies to the lossy compression [4].

Two-dimensional unitary transformation of the original digital image has different goals. Besides of the performing of the compression methods this transform is widely used for the coding of the image and extracting necessary characteristics of the image. Despite the fact that in the frequency domain the image acquires an unusual form, it can demonstrate its useful properties. In the paper [5] partial unitary transforms are described and a set of the basis images is obtained for such unitary transforms as WHT (Walsh-Hadamard Transform) and Haar transform. The size of given basis matrix is $N \times N$, and the paper considers an example for $N = 8$. The set of the basis images for the two-dimensional DCT (Discrete Cosine Transform) is illustrated in the monograph [6]. The basis images were considered as well for the DST (Discrete Sine Transform), WHT, Haar transform. Note that there are also exist nonstandard wavelet type transforms [7], [8] based on decomposition of the space of linear splines with irregular partition on closed segment.

The orthogonal transformations of the digital images with using block representations have various applications. The topic is not limited by image processing for compression methods, now it has also interdisciplinary applications, such as biomedicine, robotics, watermarking techniques in steganography. The papers [9], [10] rely to analysis of the bottom of the eyeball in the frequency domain for the glaucoma diagnosis purpose using DWT (Discrete Wavelet Transform) and DOST (Discrete Orthogonal Stockwell Transform).

In the robotics block matrices are used for the modeling kinematics of the arms of the manipulating robotic [11].

Orthogonal transformation is used for masking when the image is converted to a noise-like signal before transferring over communication link or wireless line. At the paper [12] quasi-orthogonal matrices for bilateral masking are used.

Images are stored in a compressed form, they can be edited by means of resizing, rotation without transforming into the spatial domain. It is possible to make a search using a representation in the frequency domain. For this purpose, a special DCT transform with blocking techniques is proposed. In the papers [13], [14] the modification sets of the DCT matrix blocks are obtained. Other orthogonal transforms also can be modified, for example, Walsh-Hadamard, Karhunen-Loeve and others.

The embedding into frequency domain using algorithms QIM (Quantization Index Modulation) is presented in [15]. The algorithms are based on the controlled requantizing. Initial data, which can be taken from the frequency domain are quantized with a step S. In a simple case the bit is embedded into a cell with size S by means of adding or extracting the value equal to $S/4$ from the coordinate of the center. A variety of the dots appears, the distances between them

are divisible by $S/2$. The using of the special correlation-stable functions, which reduce the correlation between bits of the digital watermarks and single pixels of the image, make the algorithms stable to the statistical attacks, but having low stability to the JPEG compression.

The watermarking schemes using wavelet orthogonal transform are also available. The technique uses two orthogonal transformations, for example, DCT-SVD [16]–[18]. SVD (Singular Value Decomposition) is conversion of a rectangular matrix into a block-diagonal matrix. Embedding can be performed using the coefficients of the orthogonal SVD transform, which operates frequency wavelet blocks *LL*, *LH*, *HL*, *HH*. DWT transform is not necessarily a one-level [19], [20]. A useful calculation tool is IWT (Integer Wavelet Transform) [21]. In this transform, the accuracy of calculations is limited, then the brightness values of the image pixels and transform coefficients are recorded in the same integer encoding. This eliminates the loss of rounding and creates reversibility at least in part of integer encoding. The method with two orthogonal transforms is stable to attacks such as JPEG compression, cropping, noise adding, Gaussian and median filtering. At the analysis, the stability measure was the degree of correlation between the initial and extracted watermarks [22].

II. BASIS IMAGES

Being a set of numbers the digital image can be represented using matrices to be an orthonormal basis.

A. Orthogonal tramsform

Let U be a square matrix with real elements and have the following properties

$$UU^T = U^TU = I,$$

where I is the identity matrix.

The columns u_m and the rows u_n^T of U form the orthonormal bases

$$\langle u_m, u_n \rangle = \delta_{mn},$$
$$\langle u_m^T, u_n^T \rangle = \delta_{mn},$$

where brackets denote scalar product, δ_{mn} is the Kronecker symbol. The matrix U is called *orthogonal* [23].

Any digital arrays can be transformed with the help of the orthogonal matrices. Such orthogonal transformation keeps metrics, particulary Euclidian distance known also as Root Mean Square Error. This transformation is reversible and conserves the Shannon information.

B. Scattering and concentrating

Considering the orthogonal transformation of digital data two properties may be made about data distributing. They are scattering and concentrating to be attractive for the image processing problems. The next example illustrates these features for a 1-D array.

Let $f := \{f_k\}$, $k = 1, \ldots, N$ be a vector and let $U := \{U_{ij}\}$, $i, j = 1, \ldots, N$ be an orthogonal matrix. Taking into account that $f = If = UU^Tf$, we find

$$f = Ug,$$
$$g = U^Tf,$$
(1)

where a new vector $g = \{g_k\}$, $k = 1, 2, \ldots N$ is called a *representation* of f. In literature g is often called the *frequency representation* of f. We use this definition for the data in the spatial domain. We introduce these terms to distinguish the arrays. Equations (1) can be written in the form

$$f_k = \sum_p U_{kp} g_p,$$

$$g_p = \sum_k U_{pk} f_k.$$

It results in data scattering. Consider an item f_k of the vector f. Assume it was obtained by embedding a bit of message by a steganographic technique. Orthogonal transformation distributes it in all items of g with its weight U_{kp}, where $p = 1, 2, \ldots, N$. By this way data scatters across the digital volume of frequency domain. To retrieve the f_k it needs to know all items of vector g. When all items in frequency domain transform into one it can be considered as concentrating. As result two above properties can be written in the form

$$f_k \leftrightarrows \{g_1, g_2, \ldots, g_N\}. \tag{2}$$

Indeed the presented observation concerns digital data however it has a simple physical analog. So in the optics a lens that can focus a light beam is described by a spatial Fourier transform. This is the orthogonal transform generalized to complex number.

Another example that illustrates scattering and concentration includes orthogonal transformation of a unit vector. Let f have only one nonzero item $f_k = \delta_{ka}$. Then its transform Uf is equal to a matrix U vector row $u_a^T = \{U_{ap}\}$, $p = 1, 2, \ldots, N$. In the case of 2-D array this example refers to basis images.

C. Representation over basis images

Let F be a $M \times N$ matrix, that describes a grayscale image. To get orthogonal transform of F it needs two orthogonal matrices U and V of size $N \times N$ and $M \times M$. Then taking into account that $F = UU^TFVV^T$ the orthogonal transform takes the form

$$F = UGV^T,$$
$$G = U^TFV.$$
(3)

where G is a rectangle $M \times N$ matrix to be representation of F. In accordance with the introduced terms the matrix G is assumed to be an image in the frequency domain and F is the image in the spatial domain. In the frequency domain image may look senseless. But the information is not lost and the orthogonal transform will allow us to return back to the original.

Using the matrix form of (3)

$$F_{xy} = \sum_{k,p} U_{xk} G_{kp} V_{py}^T = \sum_{k,p} (u_k \otimes v_p)_{xy} G_{kp},$$

we introduce a representation over the basis matrices to be a tensor product of matrices U and V, denoted by $\otimes$. Thus, if u_k is column-vector $u_k = (U_{1k}, U_{2k}, \ldots, U_{Nk})^T$ and $v_k = (V_{1k}, V_{2k}, \ldots, V_{Nk})^T$, then $u_k \otimes v_p := u_k v_p^T$ and $(u_k \otimes v_p)_{xy} = u_{xk} v_{yp}$.

We will consider a particular case $U = V$, when all matrices are $N \times N$ arrays. Then orthogonal transform (3) can be written as

$$F = \sum_{k,p} G_{kp}\, a_{kp},$$

$$G = \sum_{x,y} F_{xy}\, d_{xy},$$

where matrices a_{kp} and d_{xy} are tensor products of columns and rows of the orthogonal matrix U

$$\begin{aligned} a_{kp} &:= u_k \otimes u_p, \\ d_{xy} &:= u_x^T \otimes u_y^T. \end{aligned} \qquad (4)$$

These matrices are called the *basis images*. It follows from the definition that element (x, y) of matrix a_{kp} denoted by $a_{kp}(x, y)$ and element $d_{kp}(x, y)$ can be rewritten as

$$\begin{aligned} a_{kp}(x, y) &= U_{xk} U_{yp}, \\ d_{kp}(x, y) &= U_{kx} U_{py}. \end{aligned}$$

Each of two basis image sets has N^2 elements to be the $N \times N$ matrices.

D. Properties of the basis images

Two sets of basis images given by (4) are orthonormal bases. They have the similar properties. The principal properties are the following.

- The matrix product of two basis images is an another basis image

$$a_{kp}\, a_{mn} = a_{kn} \delta_{pm}. \qquad (5)$$

- Basis images are orthonormal items in the sense that

$$\langle a_{kp}, a_{mn} \rangle = \delta_{km} \delta_{pn}, \qquad (6)$$

where the scalar product of matrices is denoted by $\langle A, B \rangle = \sum_{m,n} A_{mn} B_{mn}$.

- The sum of the diagonal items is

$$\sum_k a_{kk} = I, \qquad (7)$$

$$\sum_k a_{kk}(x, y) = \delta_{xy},$$

$$\sum_x a_{kp}(x, x) = \delta_{kp}.$$

The presented properties result in the representation of any grayscale image in the form

$$F = \sum_{k,p} G_{kp}\, a_{kp}, \qquad (8)$$

where $G_{kp} = \langle F, a_{kp} \rangle$.

The are two ways how to create basis images. The first of them follows from the definition (4). It needs to know orthogonal matrix U. The second way requires the result of transform only. The reason is that calculations sometimes can be done by the algorithms that do not use U directly. The

well known example is DWT. All calculations are based on the filter function bank. Now we consider the second way.

Let $F = a_{ab}$ in the representation (8). Then according to property (6) we get $G_{kp} = \delta_{ak} \delta_{bp}$. It means that G has one nonzero pixel, which position is (a, b) and brightness is equal to 1. In another words, orthogonal transform of the basis image is a binary matrix of single brightness. We denote such matrix as

$$\mathbb{I}_{(ab)}(k, p) := \{\delta_{ka} \delta_{pb}\}.$$

We enumerate matrices $\mathbb{I}_{(ab)}$ by the lower subscript ab in brackets to avoid confusing with matrix elements.

The following equations are valid

$$a_{ab} = U \mathbb{I}_{(ab)} U^T, \qquad (9)$$

$$\mathbb{I}_{(ab)} = U^T a_{ab} U,$$

$$d_{ab} = U^T \mathbb{I}_{(ab)} U, \qquad (10)$$

$$\mathbb{I}_{(ab)} = U d_{ab} U^T.$$

These equations allow us to get the various wavelet basis images using the Matlab tools directly without referring to the orthogonal matrix U [2], [24].

E. Basis wavelet images

Some examples of the basis wavelet matrixes were discussed in the papers [2], [24]. However they have particular features thanks to the block structure of the orthogonal matrix U. For one level DWT, $j = 1$, it has two blocks L and H, known also as Low and High frequency bands. It results in block structure of the wavelet basis images, that can be obtained using (9).

Assume G is a frequency representation of a $N \times N$ grayscale image $F = U G U^T$. In what follows we will use the Matlab notations and write DWT and IDWT (inverse DWT) in the form

$$G = dwt(F) = \begin{bmatrix} cA & cH \\ cV & cD \end{bmatrix},$$

$$F = idwt(cA, cH, cV, cD).$$

where cA, cH, cV and cD are wavelet coefficients known as the approximation coefficient, the horizontal, the vertical and the diagonal detail blocks or LL, LH, HL and HH frequency bands. The matrix G can be considered as a 3d array G_{kpz}, where $k, p = 1, \ldots, N/2$ and item $z = 1, 2, 3, 4$ denotes block cA, cH, cV, or cD respectively. For example, $G_{kp4} = (cD)_{kp}$.

Assume $(k, p) \in cD$, then we find a set of the basis images

$$\mathbb{E}_{(kpD)} = idwt(O, O, O, \mathbb{I}_{kp}),$$

where O is a zero matrix $N \times N$. The subset $E_D = \{\mathbb{E}_{(kpD)}\}$ has $N^2/4$ basis images of dimension $N \times N$. By this way the following set can be achieved $\{\{E_A\}, \{E_H\}, \{E_V\}, \{E_D\}\}$. This set includes four blocks labeled by A, H, V, and D as approximation, horizontal, vertical and diagonal details. The images from the blocks are basis images of the a-type (9). Fig. 1 presents the basis images for $N = 6$, the wavelet 'sym6'. There are 9 items in each of blocks $\mathbb{E}_{(kpz)}$, $k, p = 1, 2, 3$, $z = 1, 2, 3, 4$. Indeed the images from the H, V and D blocks
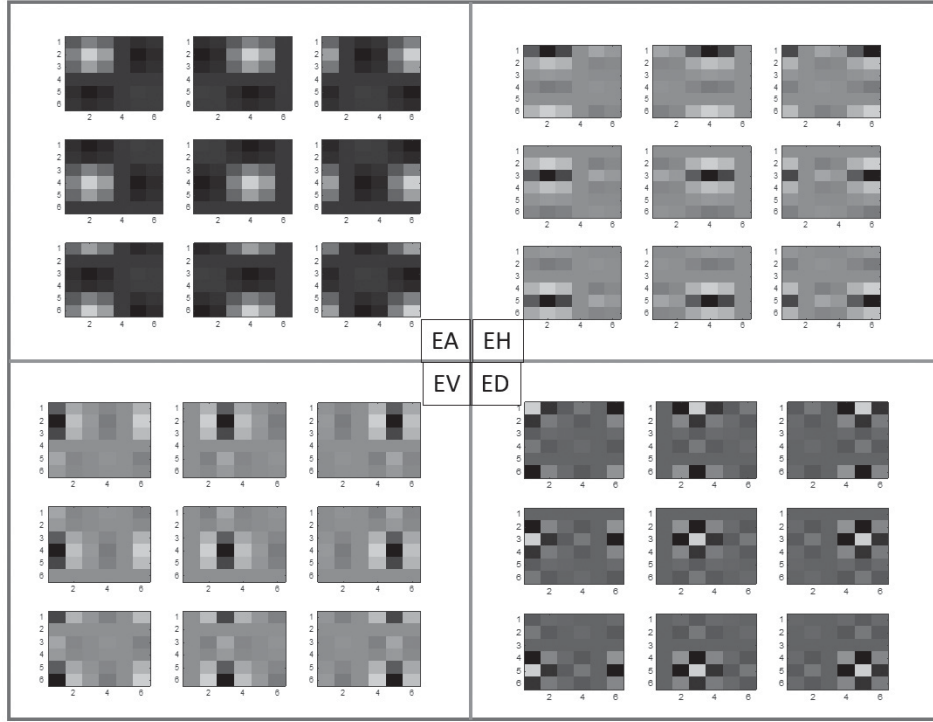
Fig. 1.   Basis images for $N = 6$, 'sym6'

have the characteristic lines in the appropriate direction. Also block A has its particular features.

Four basis images $\mathbb{E}_{(1,1,A)}$, $\mathbb{E}_{(1,1,H)}$, $\mathbb{E}_{(1,1,V)}$ and $\mathbb{E}_{(1,1,D)}$ are shown at Fig. 2. There are black pixels that illustrate the structure of the blocks.

The next basis of the d-type given by (10) can be obtained as

$$\mathbb{J}_{(xy)} = dwt(\mathbb{I}_{xy}) = \begin{bmatrix} \mathbb{J}_{(xyA)} & \mathbb{J}_{(xyH)} \\ \mathbb{J}_{(xyV)} & \mathbb{J}_{(xyD)} \end{bmatrix}.$$

In this set each of basis images has the block structure, so items consist of four blocks that refer to the A, H, V and D frequency bands. Fig. 3 shows basis images for $N = 6$, wavelet 'db6'. Each of the 36 items consists of four blocks as it is presented in $\mathbb{J}_{(55)}$.

III.   BLOCK REPRESENTATION

With the help of the basis images a block matrix can be introduced for the representation of images.

A. Block matrix

Consider a square $N \times N$ block matrix b (consisted of blocks). Let each block of b is a basis image $b_{mn} := a_{nm}$.

Being matrices elements of b does not commute and the properties of b are defined by the basis images. The introduced block matrix is a 4-D array of size $(N \times N) \times (N \times N)$, items of which are

$$K_{kpxy} = a_{kp}(x, y) = U_{xk}U_{yp}.$$

The following property is valid

$$bb = I. \tag{11}$$

Really from the definition and the properties (5) and (7) we find

$$(bb)_{mn} = \sum_k b_{mk}b_{kn} = \sum_k a_{km}a_{nk} =$$

$$\delta_{mn}\sum_k a_{kk} = \delta_{mn}.$$

It follows from (11) that

$$\langle b_m^T, b_n \rangle = \delta_{mn}.$$

Is the matrix b orthogonal? In the case of a block matrix this question is not trivial because it needs the transposing operation that is not well defined here. Nevertheless we can consider vectors consisting of rows and columns of the basis image. Introduce a block row r_k items of which are $b_{k1}, b_{k2}, \ldots, b_{kN}$ or basis images $a_{1k}, a_{2k}, \ldots, a_{Nk}$. By selecting a row x of all basis images we get a row r_{kx}. Similarly we can get a column. Then we find that such rows and columns are orthonormal vectors. This is a reason to call the block matrix b orthogonal.

B. The block vector representation

Consider a block vector f (consisted of blocks). Using (11) we have
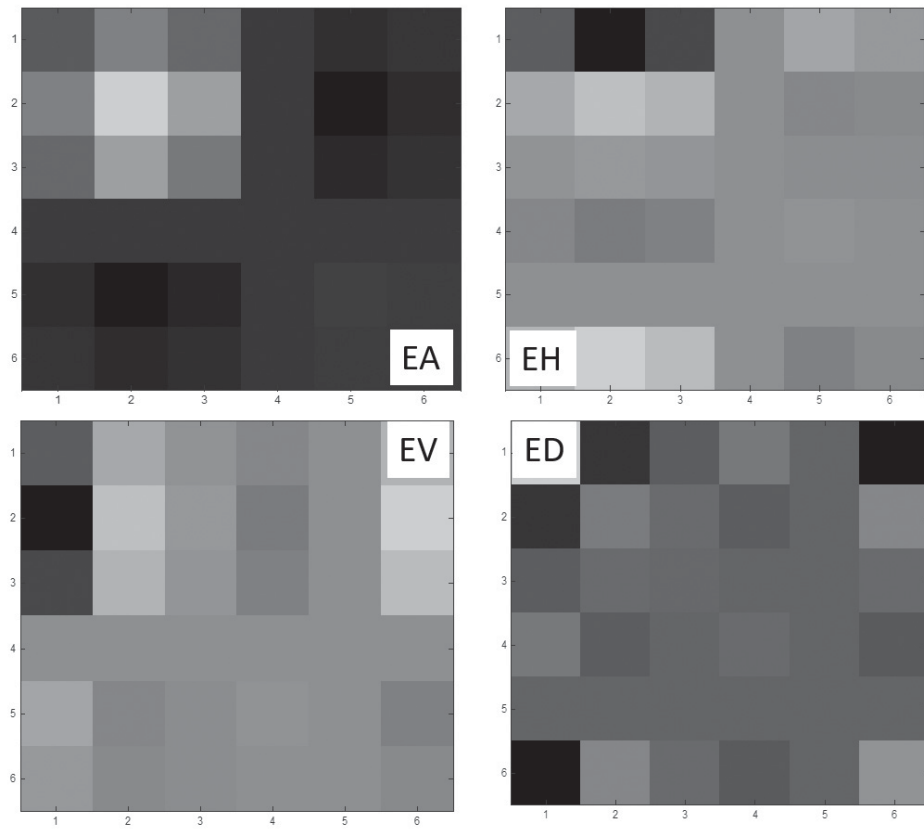
$$f = bbf = bg,$$
$$g = bf,$$

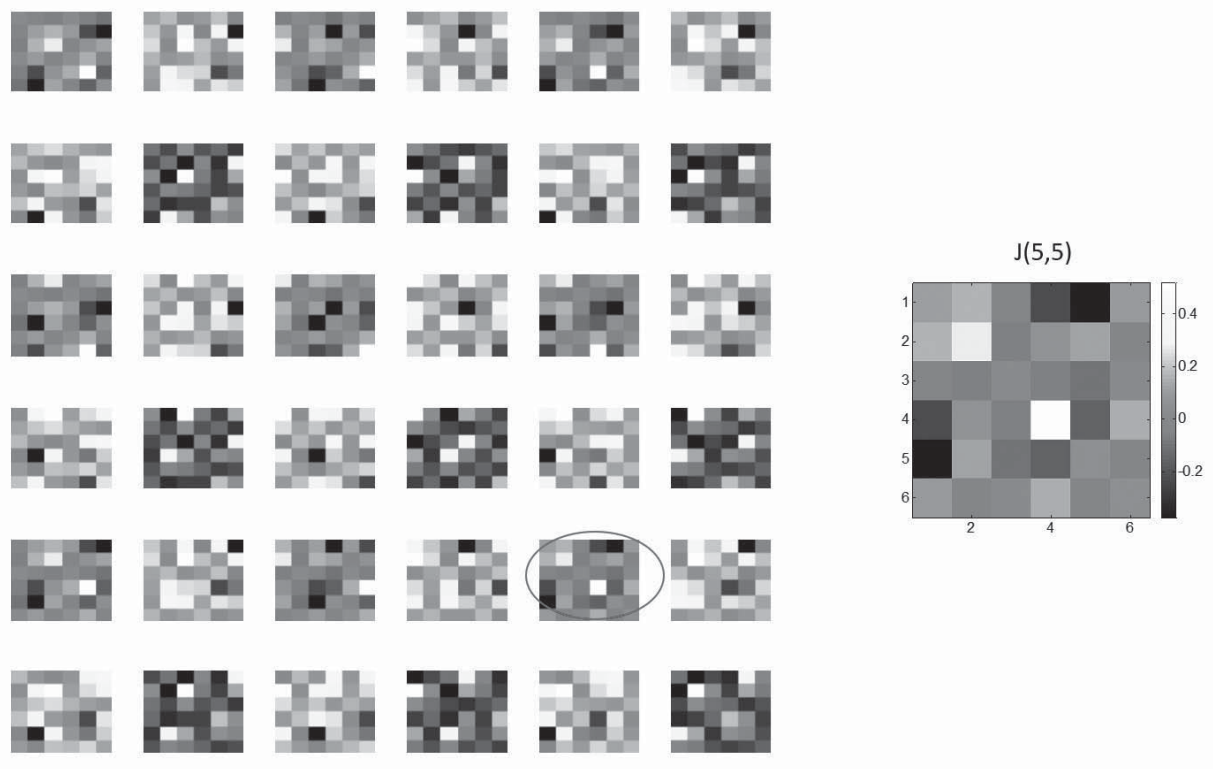Fig. 2.   Basis images for $\mathbb{E}_{(1,1,z)}, z = 1, 2, 3, 4$

Fig. 3.   Basis wavelet images for $N = 6$, 'db6'

where vector g is a representation of f.

Rewrite these equations in the matrix form taking into account that $b_{mn} = a_{nm}$, we have

$$f_k = \sum_k g_p \, a_{pk},$$
$$g_p = \sum_k f_k \, a_{kp}. \tag{12}$$

Note that the block matrix b results in the block vectors f and g, where $f = (f_1, f_2, \ldots, f_N)$ and $g = (g_1, g_2, \ldots, g_N)$, where each items f_k, g_p may be chosen as a vector, a matrix and so on.

The introduced block representation has the form of (2) however it involves more degree of freedom and it has some new properties as for data scattering.

Let each of items f_k and g_p be a 4-D array denoted by the four integers x, y, α, β. Such array f can be described by a $(N \times N)$-matrix, which elements are $(N \times N)$-matrices, i. e. the size of f is $(N \times N) \times (N \times N)$. Assume that f_k depends on α and β only

$$f_k = \delta_{xy} \psi_k(\alpha, \beta),$$
$$g_p = g_p(x, y, \alpha, \beta).$$

For this case equations (12) take the form

$$I \otimes \psi_k = \sum_p (a_{pk} \otimes I) g_p,$$
$$g_p = \sum_k a_{kp} \otimes \psi_k, \tag{13}$$

where $I \otimes \psi_k = f_k$.

C. Scattering

It follows from (13) that g_p has the form

$$g_p(x, y, \alpha, \beta) = a_{1p}(x, y)\psi_1(\alpha, \beta) + a_{2p}(x, y)\psi_2(\alpha, \beta) +$$
$$\cdots + a_{Np}(x, y)\psi_N(\alpha, \beta).$$

Instead of f_k this array is not factorized over variables x, y and α, β. From mathematical point of view this is a non separable array. It means that there are two degrees of freedom or two arrays to be correlated. Then by affecting one of the array we can change another one. So using the scalar product of basis images (6) we have

$$\sum_{x,y} a_{kp}(x, y) g_p(x, y, \alpha, \beta) = \psi_k(\alpha, \beta)$$

or

$$\langle a_{kp} \otimes I, g_p \rangle = \psi_k.$$

These equations illustrate a main feature of the block representation as for digital data scattering. It means that item ψ_k scatters into all g_p with its weight given by a basis image a_{kp}. However in contrast to (2) it can be established from any of g_p

$$\psi_k \to \{g_1, g_2, \ldots, g_N\},$$
$$\psi_k \leftarrow g_p. \tag{14}$$

This result can be useful for applications.

IV. WATERMARKING SCHEME

The block representation results in some new detection algorithms in frequency watermarking.

A. The scheme

Referring to image processing we will call the arrays f and g the data in the spatial and frequency domain representation. For frequency embedding it needs the following steps.

1) Transform the data into frequency domain $f \to g$ and embed a message M with the help of the chosen algorithm $g \to g_M = emb(g, M, K)$, where K is a parameter including possible secrete key.
2) Transform data into spatial domain and send it to a receiver.
3) Extract the embedded information by a chosen detection algorithm $f_M \to M = det(f_M, K)$.

All chain of transforms looks as follows

$$f \to g \to g_M \to f_M \to g_M \to M.$$

In the scheme the transform $g_M \to f_M$ scatters the embedded data into all spatial domain. It results in robustness of the watermark to some degradations and attacks when sending information. Indeed the sending may include some transform of the cover work f_M, particulary JPEG lossy compression when the image is stored in a graphical format. By scattering the degradation may be decreased.

This is a common feature of the frequency embedding scheme and the following conclusions may be maid about the block representation.

The introduced representation allows us to process a set of images or database as an input array. But what is more important it is the detection algorithm. In contrast to usual solutions the embedded information can be extracted more effective thanks to scattering features (14).

V. CONCLUSIONS

Orthogonal transformation has some attractive features for applications. It can be performed by various ways and it processes various type of digital data. We introduce a transformation based on basis images. It involves the multi-dimensional arrays, that can be presented for example by a set of images. Operating by such arrays results in a new features as for digital data scattering is important for robust watermarking schemes. Moreover, introduced block representation leads to effective algorithms of block parallelization.

VI. ACKNOWLEDGMENTS

REFERENCES

[1] A. Makarov, E. Yakovleva, "Comparative analysis of halftoning algorithms for digital watermarking", *18th Conference of Open Innovations Association and Seminar on Information Security and Protection of Information Technology (FRUCT-ISPIT)*, 2016, pp. 193–199.

[2] V. Gorbachev, E. Kaynarova, A. Makarov, E. Yakovleva, "Digital image watermarking using DWT basis matrices", *21th Conference of Open Innovations Association (FRUCT)*, 2017, pp. 127–133.

[3] E. M. Kaynarova, V. N. Gorbachev, A. N. Kulik, I. K. Metelev, "The methods of the digital steganography for the protection of the image information", *Journal proceedings of institutes of higher educations. Printing and publishing problems*, 2011, Is. 2., pp. 32–49.

[4] V. N. Gorbachev, V. N. Drozdov, E. M. Kaynarova. E. S. Yakovleva, *Elements of processing of digital images*, Saint-Petersburg, VVM, 2014, 75 p.

[5] W. K. Pratt, *Digital Image Processing*. Moscow: Mir, 1982, book 1.

[6] D. Salomon, *Data Compression Methods*. M.: Technosfera, 2004.

[7] A. A. Makarov, "Algorithms of Wavelet Compression of Linear Spline Spaces", *Vestnik St. Petersburg University: Mathematics*, 45:2, 2012, 82–92.

[8] A. A. Makarov, "On Two Algorithms of Wavelet Decomposition for Spaces of Linear Splines", *J. Math. Sci.*, 232:6, 2018, 926–937.

[9] A. Rajan, G. P. Ramesh, "Automated early detection of glaucoma in wavelet domain using optical coherence tomography images", *Biosciences Biotechnology Research Asia*, 2015, V. 12, Is. 3, pp. 2821–2828.

[10] K. Gokul Kannan, T. R. Ganeshbabu, "Glaucoma image classification using discrete orthogonal stockwell transform", *Int.J.Adv.Sig.Img.Sci*, 2017, V. 3, Is. 1, 6 p.

[11] A. G. Leskov, K. V. Bazhinova, S. D. Moroshkin, E. V. Feoktistova, "Modeling of robotic arms kinematics by means of block matrixes", *Engineering Journal: Science and Innovation*, 2013, V. 9, 12 p.

[12] Yu. N. Balonin, A. A. Vostrikov, E. A. Kapranova, A. M. Sergeev, O. I. Sinitsyna, S. A. Chernyshev, "Mersenne matrices for digital masking method and its special images", *Fundamental research, Modern problems of science and education*, 2017, Is. 4 (part 1), pp. 13–18.

[13] A. G. Shoberg, K. A. Shoberg, "Influence of execution of orthogonal block transform types and results of comparison", *Conf. Series: Journal of Physics*, 2018, 1015, 032130, 5 p.

[14] A. G. Shoberg, S. V. Sai, K. A. Shoberg, "The transform performing algorithm for frequency domain search", *IOP Conf. Series: Journal of Physics*, Conf. Series, 2017, 803, 012147, 4 p.

[15] V. A. Mitekin, V. A. Fedoseev, "New secure QIM-based information hiding algorithms", *Computer Optics*, 2018, V. 42, Is. 1, pp. 118–127.

[16] F. Huang, Z. H. Guan, "A hybrid SVD-DCT watermarking method based on LPSNR", *Pattern Recognition Letters*, 2004, pp. 1769–1775.

[17] S. J. Horng, D. Rosiyadi, P. Fan, X. Wang and M. K. Khan, "An adaptive watermarking scheme for e-government document images", *Multimedia Tools and Applications*, 2014, pp. 3085–3103.

[18] Ke Ji, Jianbiao Lin, Hui Li, Ao Wang, Tianjing Tang, "A DCT And SVD based Watermarking Technique To Identify Tag", arXiv:1502.02969, 5 p.

[19] V. S. Jabade, S. R. Gengaje, "Logo based image copyright protection using discrete wavelet transform and fuzzy inference system", International Journal of Computer Applications, 2012, (0975 - 8887), V. 58, Is. 10, pp. 22–28.

[20] M. Hamghalam, S. Mirzakuchaki, M. A. Akhaee, "Geometric modeling of the wavelet coefficients for image watermarking using optimum detector", *IET Image Processing*, 2014, V. 8, N. 3, pp. 162–172.

[21] Shreyash Shantam, "Robust lossless image watermarking in integer wavelet domain using SVD", *International Journal for Research in Applied Science & Engineering Technology (IJRASET)*, June 2017, V. 5, Is. VI, pp. 2462–2467.

[22] M. Jamali, S. Rafiee, S. M. Reza Soroushmehr, N. Karimi, S. Shirani, K. Najarian, S. Samavi, "Adaptive blind image watermarking using fuzzy inference system based on human visual perception", arXiv:1709.06536, 12 p.

[23] G. Streng, *Linear Algebra and its Applications*, Moscow, Mir, 1980, 456 p.

[24] E. S. Yakovleva, A. A. Makarov, V. N. Gorbachev, E. M. Kaynarova, "Wavelet Methods in Steganography", *Russian Forum of Young Scientists, KnE Engineering*, 2018, pp. 318-330.