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Synthesis of Petri Nets with Localities

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Abstract

Automated synthesis from behavioural specifications is an attractive way of constructing computational systems. In this paper, we look at a specific instance of this approach which aims at constructing GALS (globally asynchronous locally synchronous) systems. GALS systems are represented by Petri nets with localities, each locality defining a set of co-located actions, and specifications are given in terms of transition systems with arcs labelled by steps of executed actions. The proposed synthesis procedures are based on the regions of transition systems, and work without knowing which actions are to be co-located.

We consider two basic classes of Petri nets, viz. Elementary Net System with Localities (ENL-system) and Place/Transition nets with localities (PTL-nets). In particular, we discuss ENL-systems where there is no conflict between events coming from different localities. In such a case, the synthesis problem reduces to checking just one co-location relation. This result is then extended to PTL-nets.

Keywords: concurrency, Petri nets, localities, GALS, net synthesis, step sequence semantics, transition systems, theory of regions, conflict.

1 Introduction

Many computational systems exhibit behaviour adhering to the ‘globally asynchronous locally synchronous (GALS)’ paradigm. Examples can be found in hardware design, where a VLSI chip may contain multiple clocks

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responsible for synchronising different subsets of gates [6], and in biologically inspired membrane systems representing cells within which biochemical reactions happen in synchronised pulses [16]. To formally capture GALS systems, the paper [9] introduced *Place/Transition nets with localities* (PTL-nets), where each locality identifies a distinct set of actions which must be executed synchronously, i.e., in a maximally concurrent manner. Intuitively, this way of executing actions can be thought of as *local maximal concurrency.*

An attractive way of constructing complex computational systems is their automated synthesis from a range of behavioural specifications, e.g., given in terms of suitable transition systems. In such a case, synthesis procedures are often based on the regions of transition systems [1, 2, 3, 7, 14, 15, 17].

In the paper [10], we used localities together with local maximal concurrency for the case of Elementary Net Systems (EN-systems) — a fundamental class of safe Petri nets [15] — leading to EN-systems with localities (ENL-systems). We aimed there at finding a characterisation of all transition systems generated by such nets, and in so doing provide a solution to the corresponding synthesis problem (from transition systems to ENL-systems). Later we extended our approach to cover EN-systems with context arcs and localities [11]. The papers [10, 11] adapted the classical theory of regions [2] to cope with local maximal concurrency and this work was later generalised to other classes of Petri nets, including PTL-nets, in [5]. However, in all these papers it was assumed that localities are given at the outset rather than 'discovered' during runs of a synthesis algorithm. In this paper, we again consider ENL-systems and PTL-systems, but this time we aim at efficient synthesis procedures in the case that localities are not given in advance.

To explain the basic idea behind Petri nets with localities, let us consider the ENL-system in Figure 1 modelling two co-located consumers and one producer residing in a remote location. (Note that in the diagrams co-located events or transitions are shaded in the same way.) In the initial state, the net can execute the singleton step $\{c_4\}$. Another enabled step is $\{p_1\}$ which removes the token from b_1 and inserts tokens into b and b_2. After that, there are three enabled steps, viz. $\{p_2\}$, $\{c_1, c_4\}$ and $\{p_2, c_1, c_4\}$. The last one, $\{p_2, c_1, c_4\}$, corresponds to what is usually called *maximal concurrency* as no more activities can be added to it without violating the constraints imposed by the available resources (represented by tokens). However, the

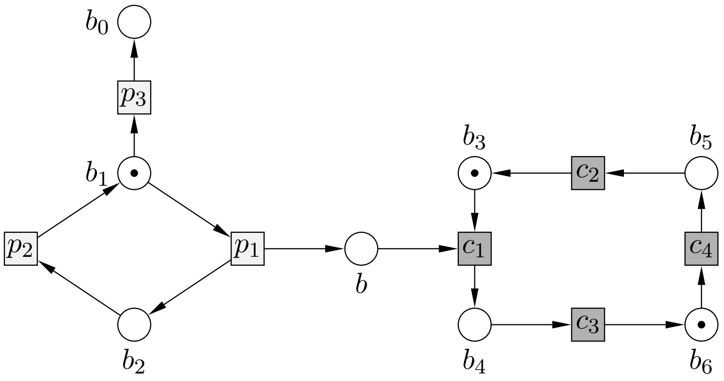

Figure 1: A one-producer/two-consumers system where boxes representing co-located events are shaded in the same way.

previously enabled step $\{c_4\}$ which is still *resource enabled* is disallowed by the control mechanism of ENL-systems. It rejects $\{c_4\}$ since we can add to it c_1 which is enabled and co-located with c_4, obtaining a resource enabled step $\{c_1, c_4\}$. It can therefore be said that the step execution mechanism employed by ENL-systems (and PTL-nets) is *local maximal concurrency.*

The synthesis procedure of [10] assumed that the events of the ENL-systems to be constructed come with a given co-location relation. Such an assumption may be difficult to fulfil in practice, and in this paper we drop it. After doing so, we discover that for the class of ENL-systems with localised conflicts, there exists just one co-location relation which needs to be investigated by the synthesis procedure. In the second part of the paper, we basically repeat, for PTL-nets, the discussion carried out for ENL-systems. (Note that a preliminary version of this part was presented at PNSE'09 [12].) In particular, we show that the idea of localised conflicts works also in this case.

2 Preliminaries for ENL-systems

Let E be a fixed finite non-empty set of *events.* A *co-location relation on E* is any equivalence relation $\simeq$ on the set of events. Moreover, for an event e and a non-empty set of events U (called a *step*), we will denote $e \simeq U$ whenever there is at least one event $f \in U$ satisfying $e \simeq f$.

A *step transition system on E* is a triple $\mathfrak{ts} \overset{\mathrm{df}}{=} (Q, A, q_0)$ where Q is a non-empty finite set of *states*, $A \subseteq Q \times (2^E \setminus \{\varnothing\}) \times Q$ is a finite set of *transitions (arcs)*, and $q_0 \in Q$ is the *initial state.* We will write $q \xrightarrow{U} q'$ (or simply $q \xrightarrow{U}$) whenever (q, U, q') is a transition. Moreover, for every state q of the step transition system $\mathfrak{ts}$, we assume that:

- $\mathit{allSteps}_q$ is the set of all steps labelling arcs outgoing from q.
- $\mathit{minSteps}_q$ is the set of all minimal steps (w.r.t. set inclusion) belonging to $\mathit{allSteps}_q$.
- E_q is the union of all the steps labelling arcs outgoing from q.
- $\simeq_q$ is the restriction of a co-location relation $\simeq$ to $E_q \times E_q$.

To ease the presentation, we will assume that each event of E occurs in at least one of the steps labelling the transitions of $\mathfrak{ts}$.

2.1 ENL-systems and their step transition systems

An *elementary net system with localities* (ENL-system) is a tuple

$$\mathfrak{enl} \overset{\mathrm{df}}{=} (B, E, F, \simeq, c_0)$$

such that B is a finite set of *conditions* disjoint from the events, $F \subseteq (B \times E) \cup (E \times B)$ is the *flow relation*, $\simeq$ is a co-location relation on E, and $c_0 \subseteq B$ is the *initial configuration* (in general, any subset of B is a *configuration*).

In diagrams, conditions (local states) are represented by circles, events (actions) by boxes, the flow relation by directed arcs, and each configuration (global state) by tokens (small black dots) placed inside those conditions which belong to this configuration. Moreover, as we already mentioned, boxes representing co-located events are shaded in the same way (see Figure 1).

For every event e, its *pre-conditions* and *post-conditions* are given respectively by ${}^\bullet e \overset{\mathrm{df}}{=} \{b \mid (b, e) \in F\}$ and $e^\bullet \overset{\mathrm{df}}{=} \{b \mid (e, b) \in F\}$ (both sets are assumed to be non-empty and disjoint). Two events are *in conflict* (or conflicting) if they share a pre-condition, or share a post-condition. The dot-notation extends to sets of events in the usual way, e.g., ${}^\bullet U \overset{\mathrm{df}}{=} \bigcup\{{}^\bullet e \mid e \in U\}$.

The semantics of $\mathfrak{enl}$ is based on steps of simultaneously executed events. We first define *potential steps of* $\mathfrak{enl}$ as all non-empty sets of non-conflicting events. A potential step U is then *resource enabled* at a configuration c if ${}^\bullet U \subseteq c$ and $U^\bullet \cap c = \varnothing$, and *control enabled* if, in addition, there is no event $e \notin U$ such that $e \doteq U$ and the step $U \cup \{e\}$ is resource enabled at c. A control enabled step U can be *executed* leading from c to the configuration $c' = (c \setminus {}^\bullet U) \cup U^\bullet$. We denote this by $c[U\rangle c'$ (or $c[U\rangle$). It is easy to see that the following hold:

Fact 1 *If a step U is resource enabled at a configuration c then there is a step W which is control enabled at c such that $U \subseteq W$ and $e \doteq U$, for every $e \in W \setminus U$.*

Fact 2 *Two events, e and f, resource enabled at a configuration c are in conflict* iff *there is no step resource enabled at c to which they both belong. Note: an event is resource enabled at a configuration if the singleton step containing this event is resource enabled.*

The *step transition system* of $\mathfrak{enl}$ is given by:

$$\mathfrak{ts}_{\mathfrak{enl}} \overset{\text{df}}{=} \big(C\,,\, \{(c,U,c') \in 2^B \times 2^E \times 2^B \mid c \in C \wedge c[U\rangle c'\}\ , c_0\big)\ ,$$

where C — the set of *reachable* configurations — is the least set of configurations containing c_0 and closed w.r.t. the step execution relation. To ease the presentation, we will assume that $\mathfrak{enl}$ does not have *dead events*, i.e., each event occurs in at least one of the steps labelling the arcs of the step transition system $\mathfrak{ts}_{\mathfrak{enl}}$. Figure 2 shows three different ENL-systems generating the same step transition system.

To link the nodes (global states) of a step transition system $\mathfrak{ts}$ with the conditions (local states) of the hypothetical ENL-system corresponding to it, we use the notion of a *region* defined as a triple

$$\mathfrak{r} \overset{\text{df}}{=} (in, r, out) \in 2^E \times 2^Q \times 2^E$$

such that, for every transition $q \xrightarrow{U} q'$, the following hold:

R1 If $q \in r$ and $q' \notin r$ then $|U \cap in| = 0$ and $|U \cap out| = 1$.

R2 If $q \notin r$ and $q' \in r$ then $|U \cap in| = 1$ and $|U \cap out| = 0$.

R3 If $U \cap out \neq \varnothing$ then $q \in r$ and $q' \notin r$.

R4 If $U \cap in \neq \varnothing$ then $q \notin r$ and $q' \in r$.

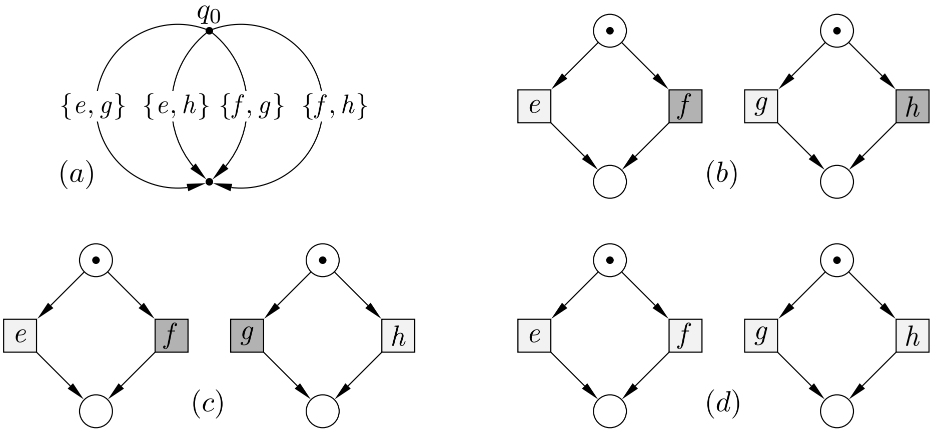

Figure 2: A step transition system (a); and three ENL-systems with different co-location relations generating it (b, c, d).

There are exactly two *trivial* regions satisfying $r = \varnothing$ or $r = Q$, viz. $(\varnothing, \varnothing, \varnothing)$ and $(\varnothing, Q, \varnothing)$. Moreover, (in, r, out) is a region iff so is its *complement* $(out, Q \setminus r, in)$. In general, a region cannot be identified only by its set of states r; in other words, *in* and *out* may not be recoverable from r. For example, the step transition system of Figure 3(a) has two different regions, $\mathfrak{r}_1 = (\varnothing, \{q_0\}, \{e\})$ and $\mathfrak{r}_3 = (\varnothing, \{q_0\}, \{f\})$, which have the same set of states.

The set of all non-trivial regions will be denoted by $\mathfrak{R}_{\mathfrak{ts}}$ and, for every

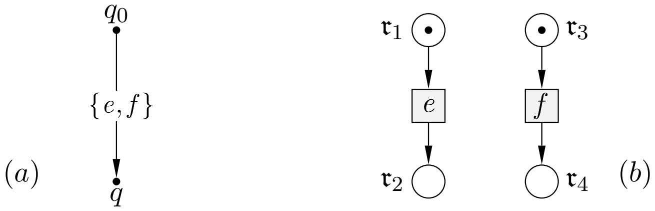

Figure 3: An ENL-transition system with co-located events e and f (a), and the ENL-system resulting from the synthesis (b). Note that the non-trivial regions in this case are: $\mathfrak{r}_1 = (\varnothing, \{q_0\}, \{e\})$, $\mathfrak{r}_2 = (\{e\}, \{q\}, \varnothing\})$, $\mathfrak{r}_3 = (\varnothing, \{q_0\}, \{f\})$ and $\mathfrak{r}_4 = (\{f\}, \{q\}, \varnothing)$.

state q, $\mathfrak{R}_q$ is the set of all non-trivial regions (in, r, out) containing q, i.e., $q \in r$. The sets of *pre-regions*, ${}^\circ e$, and *post-regions*, e°, of an event e comprise all the non-trivial regions (in, r, out) respectively satisfying $e \in out$ and $e \in in$. This extends in the usual way to sets of events, e.g., ${}^\circ U \overset{\mathrm{df}}{=} \bigcup\{{}^\circ e \mid e \in U\}$.

To characterise step transition systems of ENL-systems, we need two more notions. The set of *potential steps of* $\mathfrak{ts}$ comprises all non-empty sets U of events such that ${}^\circ e \cap {}^\circ f = e^\circ \cap f^\circ = \varnothing$, for each pair of distinct events $e, f \in U$. A potential step U is then *region enabled at state* q if ${}^\circ U \subseteq \mathfrak{R}_q$ and $U^\circ \cap \mathfrak{R}_q = \varnothing$.

A step transition system $\mathfrak{ts} = (Q, A, q_0)$ is an *ENL-transition system* w.r.t. a co-location relation $\bumpeq$ if the following hold:

A1 Each state is reachable from the initial state.

A2 For every event e, both ${}^\circ e$ and e° are non-empty.

A3 For all distinct states q and q', $\mathfrak{R}_q \neq \mathfrak{R}_{q'}$.

A4 For every state q and step U, we have that $q \xrightarrow{U}$ *iff* U is region enabled at q and there is no event $e \notin U$ such that $e \bumpeq U$ and the step $U \cup \{e\}$ is region enabled at q.

One can show (see [10]) that the step transition system of an ENL-system with the co-location relation $\bumpeq$ is an ENL-transition system w.r.t. $\bumpeq$. Moreover, the following hold:

Fact 3 *If a step U is region enabled at a state q then there is a step W such that $q \xrightarrow{W}$ and $U \subseteq W$ and $e \bumpeq U$, for every $e \in W \setminus U$.*

Fact 4 *If $q \xrightarrow{U} q'$ is one of the transitions of $\mathfrak{ts}$, then the step U is region enabled at q and the following are satisfied:*

$$\mathfrak{R}_q \setminus \mathfrak{R}_{q'} = {}^\circ U \quad \textit{and} \quad \mathfrak{R}_{q'} \setminus \mathfrak{R}_q = U^\circ \quad \textit{and} \quad {}^\circ U = \biguplus_{e \in U} {}^\circ e \quad \textit{and} \quad U^\circ = \biguplus_{e \in U} e^\circ \ .$$

Note that the problem of checking whether a step transition system is an ENL-transition system w.r.t. a given co-location relation is NP-complete (this is the case even if no two events are co-located).

2.2 Synthesis of ENL-systems with fixed localities

ENL-systems generate ENL-transition systems. The converse also is true, and the translation from ENL-transition systems to the corresponding ENL-systems is based on the regions of step transition systems.

Let $\mathfrak{ts} = (Q, A, q_0)$ be an ENL-transition system w.r.t. a (given) co-location relation $\stackrel{\frown}{-}$. Then the net system *associated* with $\mathfrak{ts}$ is defined as:

$$\mathfrak{enl}_{\mathfrak{ts}}^{\stackrel{\frown}{-}} \stackrel{\mathrm{df}}{=} (\mathfrak{R}_{\mathfrak{ts}}\,,\ E, F_{\mathfrak{ts}}\,,\ \stackrel{\frown}{-}\,,\ \mathfrak{R}_{q_0})$$

where $F_{\mathfrak{ts}} \stackrel{\mathrm{df}}{=} \{(\mathfrak{r}, e) \in \mathfrak{R}_{\mathfrak{ts}} \times E \mid \mathfrak{r} \in {}^{\circ}e\} \cup \{(e, \mathfrak{r}) \in E \times \mathfrak{R}_{\mathfrak{ts}} \mid \mathfrak{r} \in e^{\circ}\}$. It turns out that such a construction always produces an ENL-system which generates a transition system isomorphic to $\mathfrak{ts}$. (Note that one does not have to use all the regions in $\mathfrak{R}_{\mathfrak{ts}}$ to construct a desired ENL-system, and a method to reduce their number is described in [13].)

Theorem 1 ([10]) *Let $\mathfrak{ts}$ be an ENL-transition system w.r.t. a co-location relation $\stackrel{\frown}{-}$. Then $\mathfrak{enl}_{\mathfrak{ts}}^{\stackrel{\frown}{-}}$ is an ENL-system and its step transition system is isomorphic to $\mathfrak{ts}$. Moreover, the isomorphism ψ between $\mathfrak{ts}$ and the step transition system of $\mathfrak{enl}_{\mathfrak{ts}}^{\stackrel{\frown}{-}}$ is given by $\psi(q) \stackrel{\mathrm{df}}{=} \mathfrak{R}_q$, for every state q of $\mathfrak{ts}$.*

The above result assumes that a suitable co-location relation is known in advance, but we might want to weaken the synthesis problem by considering only a step transition system and finding a co-location relation during the synthesis procedure. Usually, there will be many co-location relations that would make a particular step transition system synthesisable, see Figure 2. They form a pool of relations from which one might select an ‘optimal’ one. However, this pool of suitable relations is naturally restricted by a given step transition system as seen below.

Proposition 1 *Consider an ENL-transition system w.r.t. some co-location relation and its state q.*

1. *If $U \uplus W \in allSteps_q$ and $U \in allSteps_q$ then there are no co-located events $e \in U$ and $f \in W$.*

2. *If $U \in minSteps_q$ and for each pair of events, one in U and the other in $E_q \setminus U$, there is a step in $allSteps_q$ to which the events belong, then all the events in U are co-located.*

3. *If two events in E_q are such that, for every $U \in allSteps_q$, they either both belong to U or both do not belong to U, then the events are co-located.*

Proof: Similar to proofs of Propositions 6.1, 6.2 and 6.3 in [11]. □

For example, if we consider the step transition system in Figure 3(a) then it follows from Proposition 1(3) that e and f must be co-located, whereas e and f in Figure 4(a) must belong to different locations (see Proposition 1(1)).

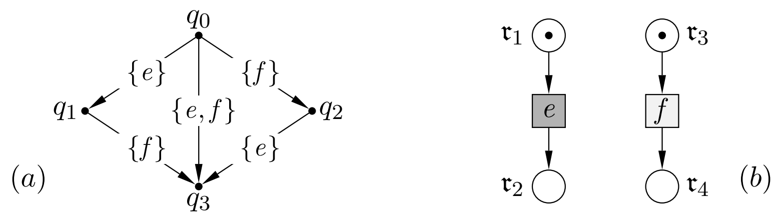

Figure 4: An ENL-transition system with non co-located events e and f (a), and the ENL-system resulting from the synthesis (b). The non-trivial regions in this case are: $\mathfrak{r}_1 = (\varnothing, \{q_0, q_2\}, \{e\})$, $\mathfrak{r}_2 = (\{e\}, \{q_1, q_3\}, \varnothing)$, $\mathfrak{r}_3 = (\varnothing, \{q_0, q_1\}, \{f\})$ and $\mathfrak{r}_4 = (\{f\}, \{q_2, q_3\}, \varnothing)$.

3 ENL-systems with localised conflicts

As mentioned above, the axioms (A1-A4) and the synthesis algorithm are formulated w.r.t. a known co-location relation. However, one could convincingly argue that the distribution of events into separate subsystems should be part of a realistic synthesis procedure. Given that the number of co-location relations is finite for a given finite set of events, one might, of course, enumerate them all and check the axioms (A1-A4) for each and every one. This, however, would be both wasteful (as many potential relations could be totally inappropriate) and impractical (since the total number of co-location relations for n different events is the n-th number in the fast-growing sequence of *Bell numbers*). To address this problem, we will now attempt to make the number of relevant co-location relation as low as possible.

Proposition 1 narrows down the range of co-location relations worth considering. Another result aimed at the same reduction is presented next.

Proposition 2 *Let* $\mathfrak{ts}$ *be a step transition system. Moreover, let* $\doteq$ *and* $\doteq'$ *be two* state consistent *co-location relations, i.e.,* $\doteq_q$ *and* $\doteq'_q$ *coincide for every state* q*. Then* $\mathfrak{ts}$ *is an ENL-transition system w.r.t.* $\doteq$ iff *it is an ENL-transition system w.r.t.* $\doteq'$*.*

Proof: Clearly, $\mathfrak{ts}$ satisfies (A1-A3) w.r.t. $\doteq$ iff the same holds w.r.t. $\doteq'$. For (A4), Fact 3 implies that events belonging to steps which are region enabled at a given state q always belong to E_q, and so we can take advantage of the fact that $\doteq$ and $\doteq'$ are state consistent (note that being a region enabled step does not depend on the co-location relation). □

Though the above result is straightforward, it is potentially very useful. Basically, what it says is that, when checking the axioms (A1-A4), what really matters are the restrictions of the co-location relations to sets of events enabled at each of the states of the step transition system. Hence it suffices to check the axioms w.r.t. just one relation for any equivalence class of state consistent co-location relations. In the extreme case, the synthesis problem can be reduced to checking the axioms (A1-A4) for just *one* co-location relation.

An ENL-system has *localised conflicts* (or is ENL/LC-system) if no conflicting non-co-located events are resource enabled at any reachable configuration. A key property of such ENL-systems is captured by the next result.

Proposition 3 *Let* $\mathfrak{ts}$ *be the step transition system of an ENL/LC-system, and* q *be one of its states.*

1. *If* $U \in \mathit{allSteps}_q$ *and* W *is a non-empty maximal subset of* U *containing only co-located events, then* $W \in \mathit{minSteps}_q$*.*

2. *If* $U \in \mathit{minSteps}_q$*, then all the events in* U *are co-located.*

Proof: (1) Let $\doteq$ be the co-location relation of an ENL/LC-system $\mathfrak{enl}$ generating $\mathfrak{ts}$. In particular, this means that below we may treat the states of $\mathfrak{ts}$ as if they were reachable configurations of $\mathfrak{enl}$.

Suppose that $U \in \mathit{allSteps}_q$ and $W \notin \mathit{allSteps}_q$. Then (since W is resource enabled at q) there is event $e \notin W$ such that $e \doteq W$ (and so $e \notin U \setminus W$ as W is a maximal subset of co-located events) and the step $W \cup \{e\}$ is resource enabled at q. Since e is resource enabled at q and $U \in \mathit{allSteps}_q$ and $e \doteq U$ and $e \notin U$, it must be the case that $U \cup \{e\}$ is not a potential step. Hence, since U and $W \cup \{e\}$ are potential steps (as both are resource enabled at q), there is $f \in U \setminus W$ such that f and e are in conflict, producing a contradiction with $\mathfrak{enl}$ having localised conflicts. Hence $W \in \mathit{allSteps}_q$ and so, since all the events in W are co-located, $W \in \mathit{minSteps}_q$.

(2) follows immediately from (1). □

ENL/LC-systems are interesting because in this case we are able to characterise all possible co-location relations rather precisely.

Theorem 2 *Let $\mathfrak{ts}$ be the step transition system of an ENL/LC-system, and q be one of its states. Then two distinct events in E_q are co-located* iff *either there is no step in $\mathit{allSteps}_q$ to which the two events belong, or there is a step in $\mathit{minSteps}_q$ to which the two events belong.*

Proof: Let e and f be distinct events in E_q.

($\Longrightarrow$) Suppose that e and f are co-located and there is $U \in \mathit{allSteps}_q$ containing both e and f. Let W be the (non-empty) set of all events in U co-located with e and f. Then, by Proposition 3(1), we have $W \in \mathit{minSteps}_q$.

($\Longleftarrow$) If there is no $U \in \mathit{allSteps}_q$ comprising e and f, then e and f must be in conflict, and so they must be co-located as the ENL-system generating $\mathfrak{ts}$ has localised conflicts. If there is $U \in \mathit{minSteps}_q$ comprising e and f, we apply Proposition 3(2). □

Corollary 1 *Let $\mathfrak{ts}$ be the step transition system of an ENL/LC-system with the co-location relation $\simeq$. Then, for every state q of $\mathfrak{ts}$ we have that $\simeq_q$ is equal to $\simeq^{\mathfrak{ts},q}$, where:*

$$\simeq^{\mathfrak{ts},q} \overset{\mathrm{df}}{=} \bigcup_{U \in \mathit{minSteps}_q} U \times U \quad \cup \quad \Big((E_q \times E_q) \setminus \bigcup_{U \in \mathit{allSteps}_q} U \times U \Big) . \tag{1}$$

Proof: Follows from Theorem 2 and an observation that, by Proposition 3(1), for every event $e \in E_q$, there is $U \in \mathit{minSteps}_q$ such that $e \in U$. Hence $\simeq^{\mathfrak{ts},q}$ is reflexive. □

Proposition 4 *Let $\mathfrak{enl}$ and $\mathfrak{enl}'$ be two ENL-systems with the same sets of conditions and events, and the co-location relations $\simeq$ and $\simeq'$, respectively. If they generate the same step transition system $\mathfrak{ts}$ and $\simeq_q$ is equal to $\simeq'_q$, for every state q of $\mathfrak{ts}$, then $\mathfrak{enl}$ has localised conflicts* iff *$\mathfrak{enl}'$ has localised conflicts.*

Proof: Suppose that $\mathfrak{enl}$ is not an ENL/LC-system and $\mathfrak{enl}'$ is. Then there is a reachable configuration c of $\mathfrak{enl}$ and two distinct events $e \not\simeq f$ which are conflicting in $\mathfrak{enl}$ and resource enabled at c. Clearly, c is then a state of $\mathfrak{ts}$. From Facts 1 and 2 it follows that $e, f \in E_c$ and there is no step in $allSteps_c$ to which e and f both belong. Now, by $e, f \in E_c$ and the assumed consistency of $\simeq$ and $\simeq'$ at c, we have that $e \not\simeq' f$. On the other hand, by Theorem 2, $e \simeq' f$, a contradiction. □

Proposition 5 *Let* $\mathfrak{ts}$ *be the step transition system of an ENL-transition system* $\mathfrak{enl}$ *with the co-location relation* $\simeq$. *Then* $\mathfrak{enl}$ *is an ENL/LC-system* iff *there is no state* q *of* $\mathfrak{ts}$ *and two distinct events* $e \not\simeq_q f$ *in* E_q *which do not belong to at least one step in* $allSteps_q$.

Proof: The ($\Longrightarrow$) implication follows from Theorem 2, and ($\Longleftarrow$) from Fact 2 and the definition of an ENL/LC-system. □

4 Synthesis of ENL/LC-systems

We are interested in solving the following synthesis problem:

Problem 1 *Given a step transition system* $\mathfrak{ts}$ *find as efficient as possible a way of checking whether it is isomorphic to the step transition system of an ENL/LC-system, and if so construct such a system.*

We can approach this problem in stages. First, for every state q of $\mathfrak{ts}$, we construct $\simeq^{\mathfrak{ts},q}$ as in Corollary 1, and then form the co-location relation:

$$\simeq^{\mathfrak{ts}}_{min} \stackrel{\mathrm{df}}{=} \Big(\bigcup_{q \in Q} \simeq^{\mathfrak{ts},q} \Big)^* .$$

Next, we check whether $\simeq^{\mathfrak{ts},q}$ is equal to $\simeq^{\mathfrak{ts}}_{min} |_{E_q \times E_q}$, for every state q. If this is not the case, we know that Problem 1 is not feasible. Otherwise, in view of Corollary 1, $\simeq^{\mathfrak{ts}}_{min}$ is the finest (w.r.t. the number of equivalence classes) possible co-location relation for $\mathfrak{ts}$ although we still do not know whether it provides a positive answer to the synthesis problem. To establish this, we proceed to check whether the axioms (A1-A4) are satisfied for the co-location relation $\simeq^{\mathfrak{ts}}_{min}$. If so, $\mathfrak{ts}$ is an ENL-transition system, and we can use the procedure from Section 2.2 to obtain the synthesised ENL-system

$\mathfrak{enl}_{\mathfrak{ts}}^{\bumpeq^{\mathfrak{ts}}_{min}}$. What is more, one can easily check that, for all the states q of the step transition system $\mathfrak{ts}$, we have:

$$(E_q \times E_q) \setminus \bumpeq^{\mathfrak{ts}}_{min} \quad \subseteq \bigcup_{U \in allSteps_q} U \times U \ .$$

Hence, by Proposition 5, $\mathfrak{enl}_{\mathfrak{ts}}^{\bumpeq^{\mathfrak{ts}}_{min}}$ is an ENL/LC-system solving Problem 1.

The above outlines a procedure which takes advantage of the structural (and local) properties of the original step transition system. If it succeeds, we obtain an ENL/LC-system which solves the synthesis problem. Moreover, one can easily characterise all other ENL/LC-systems with this property using Propositions 2 and 4.

Let $G^{\mathfrak{ts}}$ be an undirected graph whose vertices are the equivalence classes of the co-location relation $\bumpeq^{\mathfrak{ts}}_{min}$, and there is an edge between vertices V and V' if there is a state q of $\mathfrak{ts}$ and two events, $e \in V$ and $f \in V'$, such that $e, f \in E_q$ and $e \not\bumpeq^{\mathfrak{ts}}_{min} f$. Then it follows from Propositions 2 and 4 that all other co-location relations which also provide a solution are given through the solutions of the vertex colouring problem for the graph $G^{\mathfrak{ts}}$. More precisely, for each valid colouring (i.e., one which uses different colours for vertices joined by an edge), we join into clusters of co-located events all equivalence classes of $\bumpeq^{\mathfrak{ts}}_{min}$ labelled with the same colour.

5 Preliminaries for PTL-nets

In the second part of this paper we consider PTL-nets rather than ENL-systems. Since these two classes of Petri nets differ in a number of subtle ways and, moreover, they employ different notation and terminology (for example, conditions are called places in PTL-nets and, crucially, can carry more than one token), we start by providing a fresh set of definitions to avoid any confusion.

Let T be a fixed finite non-empty set of *net transitions*. A *co-location relation on T* is any equivalence relation $\bumpeq$ on the set of net transitions. For a net transition t and a multiset of net transitions α (i.e., an element of $\mathbb{N}^T$), called subsequently a *step*, we will denote $t \bumpeq \alpha$ whenever there is at least one $u \in \alpha$ satisfying $t \bumpeq u$. Moreover, $\alpha|_t$ is α after deleting all the net transitions which are not co-located with t.

Below mappings like $f : T \to \mathbb{N}$ or $g : X \times T \to \mathbb{N}$, where X is a set, can accept steps α instead of single net transitions, in the following way:

$$f(\alpha) \overset{\text{df}}{=} \sum_{t \in T} \alpha(t) \cdot f(t) \quad \text{and} \quad g(x, \alpha) \overset{\text{df}}{=} \sum_{t \in T} \alpha(t) \cdot g(x, t) \ .$$

A *step transition system on* T is a triple $\mathfrak{ts} \overset{\text{df}}{=} (Q, A, q_0)$ where Q is a non-empty finite set of *states*, $A \subseteq Q \times \mathbb{N}^T \times Q$ is a finite set of *transitions (arcs)*, and $q_0 \in Q$ is the *initial state*. We will write $q \xrightarrow{\alpha} q'$ whenever (q, α, q') is an arc. Moreover, for every state q of the step transition system $\mathfrak{ts}$, we assume that:

- $\mathit{allSteps}_q$ is the set of all steps labelling arcs outgoing from q.
- $\mathit{minSteps}_q$ is the set of all non-empty steps $\alpha \in \mathit{allSteps}_q$ for which there is no non-empty $\beta \in \mathit{allSteps}_q$ strictly included in α.
- T_q is the set of all net transitions occurring in the steps of $\mathit{allSteps}_q$.
- $\frown\!\!\!\!-_q$ is the restriction of a co-location relation $\frown\!\!\!\!-$ to $T_q \times T_q$.

Similarly as before, we assume that each net transition of T occurs in at least one of the steps labelling the arcs of $\mathfrak{ts}$, and that each state is reachable from the initial state.

5.1 PTL-nets and their step transition systems

A *Place/Transition net with localities* (PTL-net) is a tuple

$$\mathfrak{ptl} \overset{\text{df}}{=} (P, T, W^+, W^-, \frown\!\!\!\!-, M_0) \ ,$$

such that P is a finite set of *places* disjoint from net transitions, $W^+, W^- : P \times T \to \mathbb{N}$ define directed *flow arcs* with non-negative integer weights, $\frown\!\!\!\!-$ is a co-location relation, and $M_0 : P \to \mathbb{N}$ is an *initial marking* (in general, any multiset of places is a marking). In diagrams, places (local states) are represented by circles, net transitions (actions) by boxes, the flow relation by directed arcs with the weights $W^+(p, t)$ (from t to p) and $W^-(p, t)$ (from p to t), and a marking (global state) by tokens inside the places. Note that zero weight arcs as well as unitary arc weight are omitted, see Figure 5.

A step α of net transitions is *resource enabled* at a marking M if, for every place $p \in P$, $M(p) \geq W^-(p, \alpha)$. Such a step is then *control enabled* if there is no net transition t such that $t \frown\!\!\!\!- \alpha$ and the step $t + \alpha$ is resource

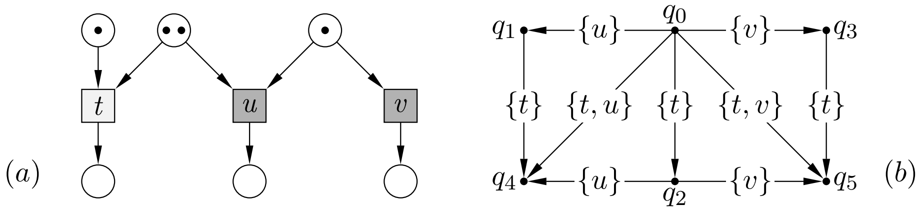

Figure 5: A PTL/LC-net (a); and its step transition system (b).

enabled at M. A control enabled step α can be *executed* leading to the marking M', for every $p \in P$ given by:

$$M'(p) \overset{\mathrm{df}}{=} M(p) - W^{-}(p,\alpha) + W^{+}(p,\alpha) \ .$$

We denote this by $M[\alpha\rangle M'$. We also assume that for each net transition t there is a place p such that $W^{-}(p,t) > 0$ (otherwise t would never occur in a control enabled step since, for every resource enabled step α containing t, the strictly greater step $\alpha{+}t$ would also be resource enabled). As a consequence, we have the following:

Fact 5 *For every step α which is resource enabled at a marking M there is a step containing α which is control enabled at M.*

The *step transition system of* $\mathfrak{ptl}$ is given by:

$$\mathfrak{ts}_{\mathfrak{ptl}} \overset{\mathrm{df}}{=} \big(\mathcal{M}\,,\ \{(M,\alpha,M') \in \mathbb{N}^P \times \mathbb{N}^T \times \mathbb{N}^P \mid M \in \mathcal{M} \wedge M[\alpha\rangle M'\}\,,\ M_0\big)\ ,$$

where $\mathcal{M}$ — the set of *reachable* markings — is the least set of markings containing M_0 and closed w.r.t. the step execution relation. Similarly as before, we will assume that each net transition occurs in at least one of the steps labelling the arcs of the step transition system $\mathfrak{ts}_{\mathfrak{ptl}}$.

5.2 Synthesis of PTL-systems with fixed localities

Let us consider the following net synthesis problem:

Problem 2 *Given a finite step transition system $\mathfrak{ts}$ and a co-location relation $\simeq$ on T, construct a PTL-net $\mathfrak{ptl}$ such that $\mathfrak{ts}$ is isomorphic to $\mathfrak{ts}_{\mathfrak{ptl}}$.*

It was shown in [5] that synthesis problems like Problem 2 can be solved using techniques coming from the theory of regions of step transition systems (see, e.g., [2, 8, 14]). In this particular case, a *region* of the step transition system $\mathfrak{ts}$ is a triple of mappings to non-negative integers

$$\big(\sigma : Q \to \mathbb{N}\,,\, \eta^+ : T \to \mathbb{N}\,,\, \eta^- : T \to \mathbb{N}\big)$$

such that, for every transition $q \xrightarrow{\alpha} q'$ of $\mathfrak{ts}$, we have:

$$\sigma(q) \geq \eta^-(\alpha) \;\text{ and }\; \sigma(q') = \sigma(q) - \eta^-(\alpha) + \eta^+(\alpha)\ .$$

Regions of this kind are used both to check the feasibility of Problem 2 and to construct a target PTL-net. At the centre of the synthesis procedure outlined below is checking of two required properties of $\mathfrak{ts}$, called *state separation* and *forward closure*.

Assume that $Q = \{q_0, \ldots, q_m\}$ and $T = \{t_1, \ldots, t_n\}$. We use three vectors of non-negative variables: $\mathbf{x} = x_0 \ldots x_m$, $\mathbf{y} = y_1 \ldots y_n$ and $\mathbf{z} = z_1 \ldots z_n$. We also denote $\mathbf{p} = \mathbf{xyz}$ and define a homogeneous linear system, where $\alpha \cdot \mathbf{z}$ denotes $\alpha(t_1) \cdot z_1 + \cdots + \alpha(t_n) \cdot z_n$, etc.:

$$\mathcal{P}\ :\ \begin{cases} x_i \geq \alpha \cdot \mathbf{z} \\ x_j = x_i + \alpha \cdot (\mathbf{y} - \mathbf{z}) \end{cases} \qquad \text{for all } q_i \xrightarrow{\alpha} q_j \text{ in } \mathfrak{ts}$$

The regions of $\mathfrak{ts}$ are determined by the integer solutions $\mathbf{p}$ of $\mathcal{P}$ assuming that $\sigma(q_i) = x_i$ (for $0 \leq i \leq m$) as well as $\eta^+(t_j) = y_j$ and $\eta^-(t_j) = z_j$ (for $1 \leq j \leq n$).

Remark 1 *Let α_i be the sum of the sequence of steps labelling arcs along the path from q_0 to q_i in a fixed spanning tree Tree of $\mathfrak{ts}$. One can eliminate each x_i with $i \geq 1$ through a substitution $x_i = x_0 + \alpha_i \cdot (\mathbf{y} - \mathbf{z})$, resulting in a system equivalent to $\mathcal{P}$ (note that as $\varnothing \in allSteps_{q_i}$ we do not need the inequality $x_0 + \alpha_i \cdot (\mathbf{y} - \mathbf{z}) \geq 0$):*

$$\mathcal{P}' : \begin{cases} x_0 + \alpha_i \cdot (\mathbf{y} - \mathbf{z}) \geq \alpha \cdot \mathbf{z} & \textit{for all } q_i \textit{ and } \alpha \in \textit{allSteps}_{q_i} \\ (\alpha_j - \alpha_i - \alpha) \cdot (\mathbf{y} - \mathbf{z}) = \mathbf{0} & \textit{for all } q_i \xrightarrow{\alpha} q_j \textit{ in } \mathfrak{ts} \textit{ but not in Tree} \end{cases}$$

The set of rational solutions of $\mathcal{P}$ forms a polyhedral cone in $\mathbb{Q}^{m+2n+1}$ (while that of $\mathcal{P}'$ forms a polyhedral cone in $\mathbb{Q}^{2n+1}$). Following [4], one can compute (in time polynomial in the size of $\mathfrak{ts}$) finitely many integer generating rays $\mathbf{p}^1, \ldots, \mathbf{p}^k$ of this cone such that each rational solution $\mathbf{p}$ of

$\mathcal{P}$ can be expressed as their linear combination with non-negative rational coefficients:

$$\mathbf{p} = \sum_{l=1}^{k} r_l \cdot \mathbf{p}^l \ .$$

Such rays are fixed and turned into net places if Problem 2 is feasible. More precisely, the initial marking of each $\mathbf{p}^l$ is given by x_0^l and, for every t_i, we have $W^-(\mathbf{p}^l, t_i) \overset{\mathrm{df}}{=} z_i^l$ and $W^+(\mathbf{p}^l, t_i) \overset{\mathrm{df}}{=} y_i^l$.

Checking state separation is carried out for each pair of distinct states, q_i and q_j, and amounts to deciding whether there exists an integer solution $\mathbf{p}$ of $\mathcal{P}$ with coefficients $r_1, \ldots, r_k$ such that $x_i \neq x_j$. Since the latter is equivalent to

$$\sum_{l=1}^{k} r_l \cdot x_i^l \neq \sum_{l=1}^{k} r_l \cdot x_j^l \ ,$$

one simply checks whether there exists at least one l such that $x_i^l \neq x_j^l$.

Checking forward closure is carried out for each state q_i, and starts by calculating the *region enabled* steps $\mathit{regSteps}_{q_i}$. One only needs to consider steps α with $|\alpha| \leq \mathit{max}$ where max is maximum size of steps labelling arcs in $\mathfrak{ts}$ since, as one can easily see,

$$\mathbf{p} = \underbrace{\mathit{max} \ \ldots \ \mathit{max}}_{m+1 \ \mathit{times}} \ \underbrace{1 \ \ldots \ 1}_{n \ \mathit{times}} \ \underbrace{1 \ \ldots \ 1}_{n \ \mathit{times}}$$

is an integer solution of $\mathcal{P}$. Such a step does not belong to $\mathit{regSteps}_{q_i}$ *iff* for some integer solution $\mathbf{p}$ of $\mathcal{P}$ with coefficients $r_1, \ldots, r_k$ we have $x_i < \alpha \cdot \mathbf{z}$. Since the latter is equivalent to

$$\sum_{l=1}^{k} r_l \cdot (x_i^l - \alpha \cdot \mathbf{z}^l) < 0 \ ,$$

one simply checks whether there exists at least one l such that $x_i^l - \alpha \cdot \mathbf{z}^l < 0$. Finally, one checks whether $\mathit{allSteps}_{q_i}$ is the set of all $\alpha \in \mathit{regSteps}_{q_i}$ for which there is no $t \in T$ such that $\alpha + t \in \mathit{regSteps}_{q_i}$ and $t \doteq \alpha$.

6 PTL-nets with partially localised conflicts

The synthesis procedure outlined above works when a co-location relation is given in advance. Suppose now that this is not he case. Then, as for

ENL-systems, a useful observation is that the synthesis procedure succeeds for $\simeq$ *iff* it succeeds for any co-location relation $\simeq'$ with which it is state consistent (i.e., the restrictions $\simeq_q$ and $\simeq'_q$ are the same, for every state q). Moreover, for a special subclass of PTL-nets — similar to ENL/LC-systems — all one needs to consider is just *one* co-location relation.

A PTL-net has *partially localised conflicts* (or is PTL/LC-net) if the following holds:

Assumption 1 *For all reachable markings M and steps α which are resource enabled at M, if t is a net transition resource enabled at M but the step $\alpha + t$ is not resource enabled at M, then $\alpha|_t + t$ is also not resource enabled at M.*

The intuition behind a PTL/LC-net is that all the actual (dynamic) conflicts for resources (tokens) in reachable markings involve only local conflicts. All conflicts between net transitions that are not co-located are only static (or structural). To see this, consider Assumption 1 and observe that $(\alpha-\alpha|_t)+t$ is resource enabled at M for any PTL/LC-net. Indeed, otherwise we could take Assumption 1 with the same t and $(\alpha - \alpha|_t)$ instead of α reaching the conclusion that $(\alpha-\alpha|_t)|_t+t$ is not resource enabled. But this would mean that t is not resource enabled (as $(\alpha - \alpha|_t)|_t + t = \varnothing + t = t$), contrary to what has been assumed.

Figure 5 shows an example of a PTL/LC-net which exhibits a dynamic conflict between (co-located) net transitions u and v, but the conflict between (not co-located) net transitions t and u is static.

Let $\mathfrak{ts}$ be the step transition system of a PTL/LC-net $\mathfrak{ptl}$ with the co-location relation $\simeq$. Moreover, let q be one of its states and

$$max_t^q \overset{\text{df}}{=} \max\{\alpha(t) \mid \alpha \in allSteps_q\} \ ,$$

for every net transition t in T_q. Below we treat q as a marking of $\mathfrak{ptl}$.

Proposition 6 *If $t \in \alpha \in allSteps_q$ then $\alpha|_t \in minSteps_q$.*

Proof: Suppose that $\alpha|_t \notin allSteps_q$. Then (since $\alpha|_t$ is resource enabled at q) there is a net transition u such that $u \simeq \alpha|_t$ and $\alpha|_t + u$ is resource enabled at q. By Assumption 1, $\alpha+u$ is resource enabled at q, contradicting $\alpha \in allSteps_q$. As a result, $\alpha|_t \in allSteps_q$ and so, since all the net transitions in $\alpha|_t$ are co-located, $\alpha|_t \in minSteps_q$. □

Corollary 2 *If $\alpha \in \mathit{minSteps}_q$, then all the net transitions in α are co-located.*

The next result shows that, for a step transition system of a PTL/LC-net, the local information at a state q about the steps enabled there will determine the co-location relation of any two net transitions that are involved in all these steps. More precisely, two net transitions will be co-located if either there is no step enabled at q where they both have the maximal number of occurrences, or there is a minimal step at q to which they both belong.

Theorem 3 *Two distinct net transitions $t, u \in T_q$ are co-located* iff *either there is no step $\alpha \in \mathit{allSteps}_q$ such that $\mathit{max}^q_t + \mathit{max}^q_u = \alpha(t) + \alpha(u)$, or there is a step in $\mathit{minSteps}_q$ to which the two net transitions belong.*

Proof: $(\Longrightarrow)$ Suppose that $\alpha \in \mathit{allSteps}_q$ is such that

$$\mathit{max}^q_t + \mathit{max}^q_u = \alpha(t) + \alpha(u) ,$$

and so $\alpha(t) \geq 1$ and $\alpha(u) \geq 1$. Then $t, u \in \alpha|_t$ and by Proposition 2, we obtain that $\alpha|_t \in \mathit{minSteps}_q$.

$(\Longleftarrow)$ Suppose that $t \neq u$ and there is no step $\alpha \in \mathit{allSteps}_q$ such that

$$\mathit{max}^q_t + \mathit{max}^q_u = \alpha(t) + \alpha(u) .$$

Let β be a step in $\mathit{allSteps}_q$ such that $\beta(t) + \beta(u)$ is maximal. Since

$$\beta(t) + \beta(u) < \mathit{max}^q_t + \mathit{max}^q_u ,$$

we assume, without loss of generality, that $\beta(t) < \mathit{max}^q_t$. Since $\beta \in \mathit{allSteps}_q$ we have that the step $\gamma = \beta(t) \cdot t + \beta(u) \cdot u$ is resource enabled at q. On the other hand, $\gamma + t$ is not resource enabled as otherwise there would have been a step in $\mathit{allSteps}_q$ containing it (see Fact 5), contradicting the choice of β. Hence, by $t \neq u$ and $t \in T_q$ and Assumption 1,

$$\gamma|_t + t = \beta(t) \cdot t + t$$

is not resource enabled at q. But this contradicts the definition of max^q_t and $\beta(t) + 1 \leq \mathit{max}^q_t$.

If $t, u \in \alpha \in \mathit{minSteps}_q$ then we apply Corollary 2. □

It follows from Theorem 3 that if we can synthesise a PTL/LC-net then the projections $\bumpeq_q$ of all suitable co-location relations are unique and can be computed locally for each state q from the steps in $\mathit{allSteps}_q$ and $\mathit{minSteps}_q$.

For the step transition system in Figure 5(b), we can show that the choice of a co-location relation as in Figure 5(a) was actually the only choice to make this step transition system synthesisable to a PTL-net. To see this, let us apply Theorem 6 to the states of the step transition system in Figure 5(b). For the initial state, we have $T_{q_0} = \{u, t, v\}$ and

$$\begin{array}{lcl} \mathit{allSteps}_{q_0} & = & \{\varnothing, \{u\}, \{t\}, \{t, u\}, \{v\}, \{t, v\}\} \\ \mathit{minSteps}_{q_0} & = & \{\{u\}, \{t\}, \{v\}\} \ . \end{array}$$

Consequently, $t \not\bumpeq_{q_0} u$ as there is no step in $\mathit{minSteps}_{q_0}$ which contains both t and u, and there is a step $\alpha = \{t, u\} \in \mathit{allSteps}_{q_0}$ such that

$$\alpha(t) + \alpha(u) = 2 = \mathit{max}_t^{q_0} + \mathit{max}_u^{q_0} \ .$$

Similarly, one can show that $t \not\bumpeq_{q_0} v$. For the last pair of net transitions, u and v, we obtain that $u \bumpeq_{q_0} v$ as there is no step $\alpha \in \mathit{allSteps}_{q_0}$ such that

$$\alpha(u) + \alpha(v) = 2 = \mathit{max}_u^{q_0} + \mathit{max}_v^{q_0} \ .$$

For the remaining states we have

$$T_{q_1} = \{t\} \qquad T_{q_2} = \{u, v\} \qquad T_{q_3} = \{t\} \qquad T_{q_4} = T_{q_5} = \varnothing \ ,$$

and so we only need to check whether $u \bumpeq_{q_2} v$. The answer is positive since

$$\begin{array}{lcl} \mathit{allSteps}_{q_2} & = & \{\varnothing, \{u\}, \{v\}\} \\ \mathit{minSteps}_{q_2} & = & \{\{u\}, \{v\}\} \end{array}$$

and so there is no step $\alpha \in \mathit{allSteps}_{q_2}$ such that

$$\alpha(u) + \alpha(v) = 2 = \mathit{max}_u^{q_2} + \mathit{max}_v^{q_2} \ .$$

After computing the projections $\bumpeq_q$, for all $q \in Q$, we form the transitive closure $\bumpeq_{\mathfrak{ts}}$ of their union and proceed as follows (note that in the case of our example, $\bumpeq_{\mathfrak{ts}} = \{(u, v), (v, u), (u, u), (v, v), (t, t)\}$).

First we check whether $\bumpeq_q$ is equal to $\bumpeq_{\mathfrak{ts}} |_{T_q \times T_q}$, for every state q. If this is not the case, we know that the synthesis problem to PTL/LC-nets is not feasible. Otherwise, we proceed with the procedure outlined in the previous section with a given co-location relation $\bumpeq_{\mathfrak{ts}}$, and its outcome determines the outcome of the whole synthesis process. Moreover, if the synthesis procedure succeeds, then any other good co-location relation can be obtained similarly as in the case of ENL/LC-systems.

7 Concluding remarks

In this paper, we discussed how one could synthesise GALS systems represented by Petri nets from their behavioural specifications given in terms of step transition systems without assuming anything about the co-location of actions. In particular, we investigated how this problem might be solved without considering all potential co-location relations. This has led to the identification of two net classed, ENL/LC-systems and PTL/LC-nets, characterised by the lack of dynamic conflicts between co-located actions, and for which it suffices to consider only *one* co-location relation. It is worth pointing out that nets of this kind have *practical* importance. Consider, for example, a distributed system of computing nodes, where each node executes actions in a synchronous manner, and the nodes themselves communicate by asynchronous message passing of signals. Then, provided that each computing node can be represented by a finite Place/Transition net, the overall network can be modelled by a PTL/LC-net.

In our future work we plan to consider more relaxed versions of the synthesis problem. For example, one can assume that a step transition system gives an upper bound on the desirable behaviour of the synthesised net, and the goal is to retain as much as possible of its behaviour in the constructed Petri net. Another direction for future work is net synthesis from behavioural specifications expressed in a temporal logic, such as that described in [18].

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