Horizontal Flight Dynamics Simulations using a Simplified Airplane Model and Considering Wind Perturbation

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Abstract: *An in-house method of Newton-Euler inverse dynamics guidance based on a simplified airplane model in horizontal flight was proposed in a previous paper presented at NMAS 2018 workshop [1]. The goal was to guide the airplane between two locations situated at 100 km distance in the horizontal plane, considering some simplifying assumptions: • the airplane was considered a material point (no motion equations involving torques are considered here); • the thrust was constant in magnitude during the entire motion; • the airplane is inclined at time t with the rolling angle $\varphi(t)$; • the main parameter for controlling the flight path was considered to be the sideslip angle β (angle between the thrust vector and the velocity vector); • the lift force balanced the weight, the centrifugal force and the wind perturbation lateral force; • the wind perturbation was considered linear by pieces of 10 km distance. So, the horizontal flight guidance parameter is the sideslip angle β, while the rolling angle φ is determined from the condition that the flight remains in the horizontal plane, which has to be permanently fulfilled. This paper presents several simulations validating the proposed inverse dynamics guidance tool for airplane horizontal flight. Various wind perturbation possibilities have been tested, considering this wind perturbation as linear by pieces during the horizontal flight. In conclusion, this guidance method worked well for the simplified horizontal flight case study.*

Key Words: *guidance method, horizontal flight, inverse dynamics, simplified airplane motion, rolling angle, sideslip angle*

1. PROBLEM

Since airplanes are omnipresent in human life for more than a century, elaborate aircraft dynamics complete 3D models can be found in the literature [2,3]. Interesting studies have been carried out also concerning optimal flight paths and speeds [4]. Even if the airplane flight in horizontal plane represents a very simplified motion, it is always useful to dispose of an in-house simulation tool for this simplified horizontal flight, considering wind perturbation. For this purpose, a simplified model for flight dynamics in horizontal plane with wind perturbation was considered, as proposed by several authors [5,6]. The simulations performed so far validate the proposed numerical guidance algorithm based on this simplified model [1]. Figure 1 shows a vertical view of the horizontal flight and airplane model, illustrating two of the three flight angles used by the simplified model: the yaw angle ψ_{abs} (angle between aircraft absolute velocity vector $\mathbf{V}_{abs}$ and **x** axis) and the sideslip angle β_{abs} (angle between the thrust vector **T** and the absolute velocity vector $\mathbf{V}_{abs}$). The other flight angles involved in this simplified flight dynamics model are: the flight path angle $\gamma = 0$ (for the horizontal flight) and the rolling angle φ, i.e., angle between the vertical plane and the plane of symmetry of the airplane.

Let us remark that the absolute velocity vector $\mathbf{V}_{abs}$ designates the velocity relative to the ground, i.e., with respect to the inertial frame (O; **x**,**y**), being written as the vector sum of the velocity of the airplane relative to the atmosphere **V** and the velocity of the atmosphere relative to the ground **w** (wind velocity):

$$\mathbf{V}_{\text{abs}} = \mathbf{V} + \mathbf{w} \tag{1}$$

The flight dynamics equations below involve the velocity of the airplane relative to the atmosphere **V**, also the sideslip angle β and the yaw angle ψ are the ones relative to the atmosphere.

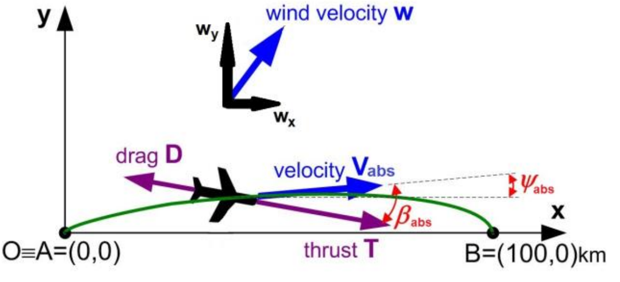

Fig. 1 – Vertical view of the airplane flight [1]

The goal of the airplane flight is to move from point A(0,0) to point B(100 km, 0), starting from the initial speed $V(t_0 = 0) = 500\ \text{km/h}$.

The airplane is subject to wind blowing with velocity **w**, defined in the inertial frame (O; **x**,**y**): $\mathbf{w} = w_x\vec{\imath} + w_y\vec{\jmath}$.

2. SIMPLIFIED FLIGHT DYNAMICS MODEL FOR AIRPLANE HORIZONTAL FLIGHT, CONSIDERING WIND PERTURBATION

To model the aircraft dynamics in the horizontal plane, we have considered the simplifying assumption that the aircraft mass m is constant, considering a relatively short segment of the flight path (100 km flight distance on **x** axis) [5].

The condition that the flight remains permanently in the horizontal plane is the flight path angle γ being null: $\gamma = 0$.

In this particular case, the motion of the aircraft is characterized by the following 6 kinematic parameters: the x and y position coordinates in the horizontal plane, the aircraft velocity V relative to the atmosphere, the relative yaw angle ψ, the rolling angle φ and the relative sideslip angle β. Then, the horizontal flight equations are [5,6]:

$$\begin{cases} \dfrac{dx}{dt} = V\cos\psi + w_x \\ \dfrac{dy}{dt} = V\sin\psi + w_y \\ \dfrac{dV}{dt} = \dfrac{1}{m}(T\cos\beta - D) - (\dot{w}_x\cos\psi + \dot{w}_y\sin\psi) \\ \dfrac{d\psi}{dt} = \dfrac{1}{mV}(L + T\sin\beta)\sin\varphi + \dfrac{1}{V}(\dot{w}_x\sin\psi - \dot{w}_y\cos\psi) \\ \dfrac{d\gamma}{dt} = \mathbf{0} = \dfrac{1}{mV}[(L + T\sin\beta)\cos\varphi - G] \end{cases} \tag{2}$$

where [6]:

$$\begin{aligned} \dot{w}_x &= \frac{\partial w_x}{\partial x}(V\cos\psi + w_x) + \frac{\partial w_x}{\partial y}\left(V\sin\psi + w_y\right) \\ \dot{w}_y &= \frac{\partial w_y}{\partial x}(V\cos\psi + w_x) + \frac{\partial w_y}{\partial y}\left(V\sin\psi + w_y\right) \end{aligned} \tag{3}$$

Let us recall here that V is the value of the velocity of the airplane relative to the atmosphere, given by (1): $\mathbf{V} = \mathbf{V}_{\text{abs}} - \mathbf{w}$. The thrust T is considered constant during the flight, while drag force D and lift force L depend on the relative speed V, following the classical expressions below:

$$D = \frac{1}{2}\rho V^2 S C_D \tag{4}$$

$$L = \frac{1}{2}\rho V^2 S C_L \tag{5}$$

3. NUMERICAL CASE STUDY

The aircraft of mass $m = 10\text{ t} = 10000\text{ kg}$ flies in an horizontal plane from point A(0,0) to point B(100km, 0), starting from the initial speed $V(t_0 = 0) = 500\text{ km/h}$. The complete initial conditions are as follows:

$$\begin{cases} x_0 = x(t_0 = 0) = 0, \quad y_0 = x(0) = 0, \quad V_0 = V(0) = 500\,\dfrac{\text{km}}{\text{h}}, \\ \psi_0 = \psi(t_0 = 0) = 0, \quad \varphi_0 = \varphi(0) = 0, \quad \beta_0 = \beta(0) = 0. \end{cases} \tag{6}$$

In the absence of wind, the aircraft will normally fly from point A(0,0) to point B(100 km, 0), in $t_F = \frac{100\text{ km}}{500\text{ km/h}} = 0.2\text{ h} = 12\text{ min} = 720\text{ s}$.

But there is wind in the flight horizontal plane, more precisely $\mathbf{w} = w_x\vec{\imath} + w_y\vec{\jmath}$, thus the aircraft will be deviated. In this case study, the wind speed components w_x and w_y are considered to vary linearly by pieces of 10 km on the **x**-axis abscissa, as shown in the results section below.

The following values have been considered here for the coefficients and constants involved in expressions (4) and (5) of the drag and lift forces: $\rho = 1.22\text{ kg/m}^3$ (air), wing area $S = 54.5\text{ m}^2$, drag coefficient $C_D = 0.0418$, lift coefficient $C_L = 0.239$. The value of the constant thrust during the flight is calculated for our numerical case study as: $T = 17.15$ kN. Let us recall the unknowns of the flight equations system (1): x, y, V, ψ, φ and β.

4. GUIDANCE ALGORITHM

The guidance algorithm used to attend the desired final position B(100 km, 0) is a simplistic inverse dynamics guidance [1]. The flight dynamics equations (2) are integrated in time, from $t_0 = 0$ until the estimated $t_F^{\text{estimate}} = 720$ s (or updated value). A constant time step $\Delta t = 0.1$ s is used here.

The estimated flight time t_F^{estimate} is computed for a constant aircraft speed V_0, but in fact the speed will slightly vary during the flight. The estimated number of constant time step integrations is $N_{\text{estimate}} = \frac{t_F^{\text{estimate}}}{\Delta t} = 7200$.

So, the simplified horizontal flight dynamics system comprises:

- 6 unknown parameters: x, y, V, ψ, φ and β;
- 5 scalar dynamics equations (2), from which the first 4 are differential equations, while the last one is transformed in an algebraic equation by imposing the horizontal flight condition $\gamma = 0 \Rightarrow \frac{d\gamma}{dt} = 0$.
- it results that there will be 6–5=1 free variables, i.e., the flight control parameter. Naturally, this flight control parameter will be either the sideslip angle β, or the rolling angle φ, or still a combination of β and φ.

The simplistic idea of our (inverse dynamics) guidance iterative algorithm is as follows:

Step 1) we start from the initial conditions (6);

Step 2) for each $t_i \in [0, t_F^{\text{estimate}}]$, with $i = 0, \dots, N_{\text{estimate}}$, the yaw angle is ψ_i, but we want it to move towards $\psi_i^{\text{desired}} = \psi_i^{\text{guidance}}$ computed by the following simplistic/rudimentary guidance algorithm:

- *Let us consider the first two equations from the flight dynamics system (2) at intermediary time t_i, by roughly approximating $\frac{dx}{dt} = \frac{\Delta x}{\Delta t}$ and $\frac{dy}{dt} = \frac{\Delta y}{\Delta t}$:*

$$\begin{cases} \left(\frac{\Delta x}{\Delta t}\right)_i = V_i \cos\psi_i + v_{x,i} \\ \left(\frac{\Delta y}{\Delta t}\right)_i = V_i \sin\psi_i + v_{y,i} \end{cases} \tag{7}$$

- *Let us use the following rough approximations:*

$$\begin{cases} \left(\frac{\Delta x}{\Delta t}\right)_i = \frac{x_{\text{final}} - x_i}{t_F^{\text{estimate}} - t_i} = \frac{100\text{km} - x_i}{t_F^{\text{estimate}} - t_i} \\ \left(\frac{\Delta y}{\Delta t}\right)_i = \frac{y_{\text{final}} - y_i}{t_F^{\text{estimate}} - t_i} = \frac{0 - y_i}{t_F^{\text{estimate}} - t_i} \end{cases} \tag{8}$$

- *By dividing the first and the second equation (7) and replacing the rough guidance approximations (8), one obtains:*

$$\tan\psi_i^{\text{guidance}} = \frac{100\text{km} - x_i}{-y_i} \Rightarrow \psi_i^{\text{guidance}} = \text{atan}\left(\frac{x_i - 100\text{km}}{y_i}\right) \tag{9a}$$

As usual, the inverse tangent function must be carefully numerically handled. There is also possible to avoid using the inverse tangent function, by means of the following alternative expression:

$$\begin{aligned}\sin\psi_i^{\text{guidance}} &= \frac{1}{V_i}\left(\frac{-y_i}{t_F^{\text{estimate}}-t_i}-v_{y,i}\right) \Rightarrow \psi_i^{\text{guidance}}\\ &= \operatorname{asin}\left[\frac{-1}{V_i}\left(\frac{y_i}{t_F^{\text{estimate}}-t_i}+v_{y,i}\right)\right]\end{aligned} \tag{9b}$$

Step 3) Still at the intermediary time t_i, let us compute the necessary variation of ψ_i so that to change (to be guided) from ψ_i to the desired ψ_i^{guidance}:

$$\frac{\Delta\psi_i}{\Delta t}=\frac{\psi_i^{\text{guidance}}-\psi_i}{\Delta t_i^{\text{guidance}}} \tag{10}$$

where ψ_i^{guidance} is given by (9) and the guidance maneuver time can be considered either the time until the final position $\Delta t_i^{\text{guidance}}=t_F^{\text{estimate}}-t_i$, or less.

Taking into account the guidance formula (10), the last two scalar equations from expressions (2) become, at t_i:

$$\begin{cases}(L_i+T_i\sin\beta_{i+1})\sin\varphi_{i+1}=mV_i\dfrac{\Delta\psi_i}{\Delta t}-m(\dot w_{x,i}\sin\psi_i-\dot w_{y,i}\cos\psi_i)\\ (L_i+T_i\sin\beta_{i+1})\cos\varphi_{i+1}=G\end{cases} \tag{11}$$

with $\dot w_{x,i}$ and $\dot w_{y,i}$ given by (3).

From equations (11), one easily obtains the explicit expressions of the rolling angle and the sideslip angle at t_i:

$$\begin{cases}\beta_{i+1}=\operatorname{asin}\left\{\dfrac{1}{T_i}\left[\sqrt{G^2+\left(mV_i\dfrac{\psi_i^{\text{guidance}}-\psi_i}{\Delta t_i^{\text{guidance}}}-m\left(\dot w_{x,i}\sin\psi_i-\dot w_{y,i}\cos\psi_i\right)\right)^2}-L_i\right]\right\}\\ \varphi_{i+1}=\operatorname{asin}\dfrac{mV_i\dfrac{\psi_i^{\text{guidance}}-\psi_i}{\Delta t_i^{\text{guidance}}}-m(\dot w_{x,i}\sin\psi_i-\dot w_{y,i}\cos\psi_i)}{L_i+T_i\sin\beta_{i+1}}\end{cases} \tag{12}$$

Once computed β_{i+1} and φ_{i+1}, the first 3 scalar differential equations in (2) are easily integrated using finite differences integration, computing x_{i+1}, y_{i+1} and V_{i+1}.

Steps 2) and 3) of this guidance algorithm are iteratively applied until $x_i=x_{\text{final}}=100$ km.

Let us recall that the value of thrust T is maintained constant during the flight, while the sideslip angle β is the main variable control parameter and the rolling angle φ is imposed by the condition of flight in the horizontal plane.

5. RESULTS VALIDATING THE HORIZONTAL FLIGHT DYNAMICS SIMULATION TOOL

For the case study presented above, simulations results are obtained using the in-house horizontal flight dynamics simulation tool based on the simplistic guidance algorithm from previous section. Three tests are presented, all succeeding to bring the airplane close to the final desired destination point B(100km, 0). As mentioned, the wind velocity $\mathbf{w} = w_x\vec{\imath} + w_y\vec{\jmath}$ was considered in the horizontal plane, deviating the aircraft from the initial straight-line trajectory. The wind velocity components w_x and w_y are considered here to vary linearly by pieces of 10 km on the **x**-axis abscissa, as shown in the three case studies below. These wind profiles were chosen randomly, for testing purposes. Figure 2 shows the **x** and **y**-axis components of the wind velocity for the first case study. The two components of the wind velocity vary linearly by pieces of 10 km on the **x**-axis abscissa, their intermediate values being randomly chosen in order to test as much as possible the reliability of the proposed guidance algorithm.

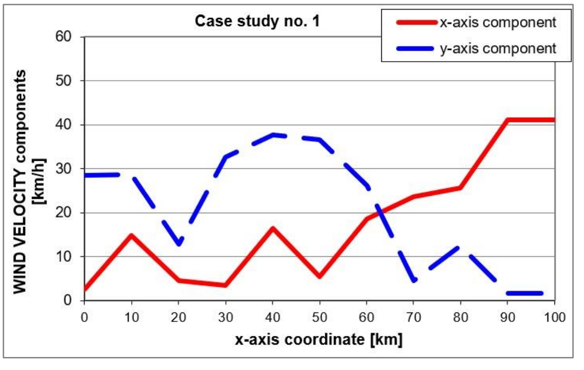

Fig. 2 – **x** and **y**-axis components of the wind velocity for the **first case study**

For this first case study corresponding to the wind velocity shown in Figure 2, the flight path in the **x-y** plane from point A(0,0) to point B(100 km, 0), obtained using the simplistic guidance algorithm proposed in previous section, is presented in Figure 3. The variation of flight angles for the flight path from Figure 3 is shown in Figure 4, the main flight control parameter being the sideslip angle β, while the yaw angle ψ is imposed by our simplistic guidance algorithm and the rolling angle φ is imposed by the condition of flight in the horizontal plane.

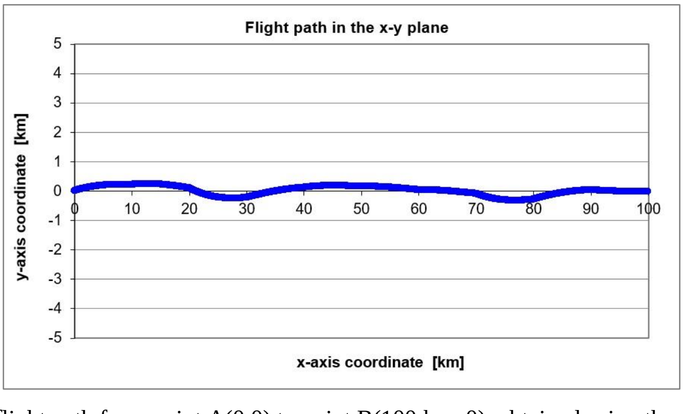

Fig. 3 – **x**-**y** plane flight path from point A(0,0) to point B(100 km, 0), obtained using the proposed simplistic guidance algorithm, for the **first case study**, i.e., for the wind velocity evolution shown in Figure 2

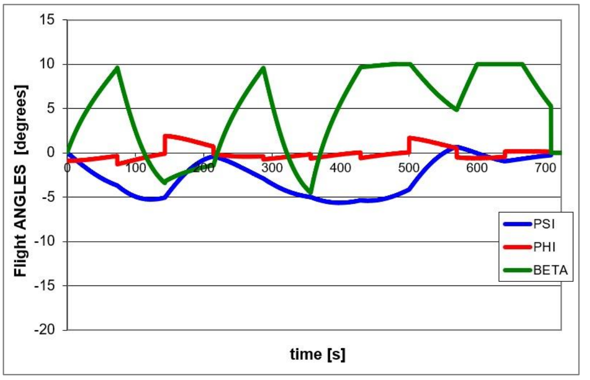

Fig. 4 – Variation of the flight angles (yaw angle ψ, sideslip angle β and rolling angle φ) for the flight path from Figure 3 (**first case study**), where β is the main flight control parameter

As can be observed on Figures 4, 7 and 10, limitations have been considered in what concerns the control angle, more precisely the absolute value of the sideslip angle was imposed to be less than 10° ($|\beta| < 10°$). The second case study corresponds to the **x** and **y**-axis components of the wind velocity which vary linearly by pieces of 10 km as shown in Figure 5.

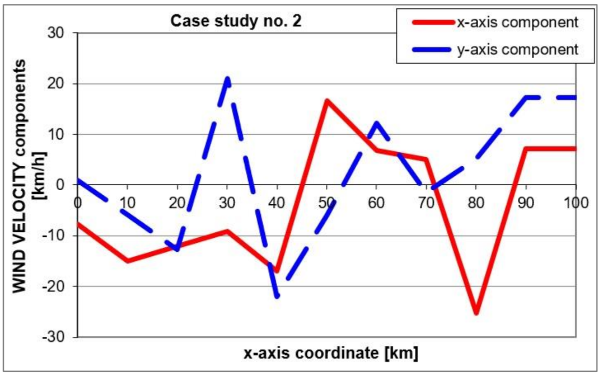

Fig. 5 – **x** and **y**-axis components of the wind velocity for the **second case study**

For this second case study corresponding to the wind velocity shown in Figure 5, the flight path in the **x-y** plane from A(0,0) to B(100 km, 0), obtained using the simplistic guidance algorithm proposed in previous section, is presented in Figure 6. Figure 7 shows the corresponding variation of flight angles ψ, β and φ.

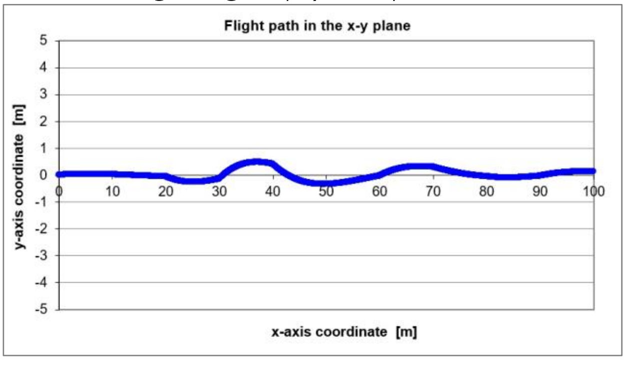

Fig. 6 – **x-y** plane flight path from point A(0,0) to point B(100 km, 0), obtained using the proposed simplistic guidance algorithm, for the **second case study**, i.e., for the wind velocity evolution shown in Figure 5

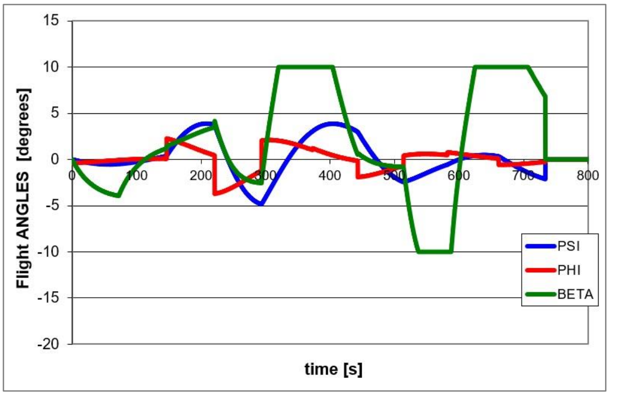

Fig. 7 – Variation of the flight angles (yaw angle ψ, sideslip angle β and rolling angle φ) for the flight path from Figure 6 (**second case study**), where β is the main flight control parameter

The third case study corresponds to the **x** and **y-**axis components of the wind velocity which vary linearly by pieces of 10 km as shown in Figure 8.

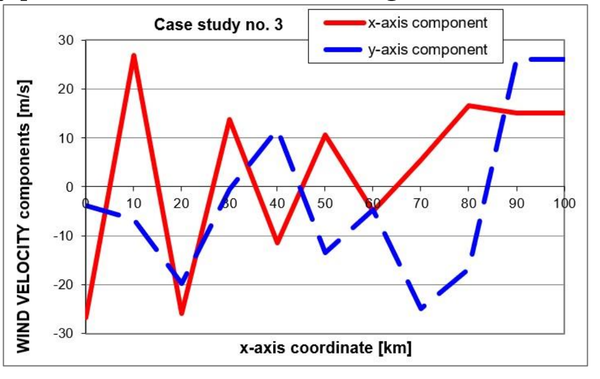

Fig. 8 – **x** and **y**-axis components of the wind velocity for the **third case study**

For this third case study corresponding to the wind velocity shown in Figure 8, the flight path in the **x-y** plane from point A(0,0) to point B(100 km, 0), obtained using the simplistic guidance algorithm proposed in previous section, is presented in Figure 9. Figure 10 shows the corresponding variation of flight angles β and φ, while the evolution of the yaw angle ψ is provided by our simplistic guidance algorithm.

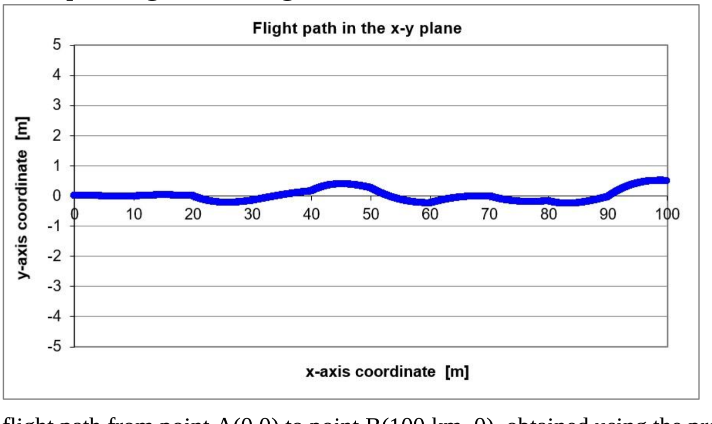

Fig. 9 – **x**-**y** plane flight path from point A(0,0) to point B(100 km, 0), obtained using the proposed simplistic guidance algorithm, for the **third case study**, i.e., for the wind velocity evolution shown in Figure 8

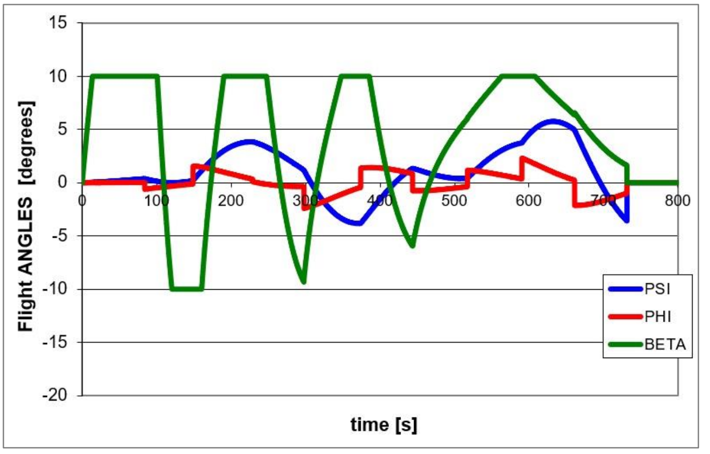

Fig. 10 – Variation of the flight angles (yaw angle ψ, sideslip angle β and rolling angle φ) for the flight path from Figure 9 (**third case study**), where β is the main flight control parameter

6. CONCLUSIONS AND FURTHER WORK

The three case studies show a good performance of the proposed guidance algorithm based on some simplistic approximations. The airplane arrives in all three cases close to the final desired destination point B(100 km, 0). Some appropriate adjustments must be performed to improve the precision of the guidance algorithm in the final part of the flight path; a more appropriate guidance strategy must be applied to cope with the wind deviation on the last 10 km flight path piece. Further work will try to solve this issue, searching for a more elaborated guidance and control algorithm in order to reduce the error in attaining the final position in horizontal flight and trying to obtain a trajectory closer to the straight-line one. Further simulations will also consider more realistic wind profiles.

Another more ambitious future work will be to extend the current horizontal flight dynamics simplified model to a simplified model of 3D flight dynamics, with a similar goal of guiding the airplane for an initial to a final position, considering wind perturbation.

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