Gaussian single-valued neutrosophic numbers and its application in multi-attribute decision making

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Abstract: The fuzzy set and intuitionistic fuzzy set are two useful mathematical tool for dealing with impression and uncertainty. However sometimes these theories may not suffice to model indeterminate and inconsistent information encountered in real world. To overcome this insufficiency, neutrosophic set theory and single-valued neutrosophic set (SVNS) theory which is useful in practical applications, were proposed. Many researchers have studied on single-valued triangular neutrosophic numbers and single-valued trapezoidal neutrosophic numbers. In this paper, concepts of Gaussian single-valued neutrosophic number (GSVNN), α-cut of a GSVNN and parametric form of a GSVNN are defined, and based on α-cuts of GSVNNs, arithmetic operations for GSVNNs are defined. Also, some results are obtained related to arithmetic operations of GSVNNs. Furthermore, a decision making algorithm is developed by using GSVNNs operations, and its an application in medical diagnosis is given.

Keywords: Neutrosophic set, Single-valued neutrosophic number, Gaussian single-valued neutrosophic number, $\alpha-$cut, decision making

1 Introduction

The concept of fuzzy set was defined by Zadeh [38] in 1965. A fuzzy set A on a fixed set X is character-ized by membership function denoted by μ_A such that $\mu_A : A \to [0, 1]$. In 1976, Sanchez [32] proposed a method to solve basic fuzzy relational equations, and in [33] he gave a method for medical diagnosis based on composition of fuzzy relations. In 2013, ¸Celik and Yamak [11] applied the fuzzy soft set theory to Sanchezs approach for medical diagnosis by using fuzzy arithmetic operations, and presented a hypothetical case study to illustrate process of proposed method. Concept of Gaussian fuzzy number and its $\alpha-$cuts were defined by Dutta and Ali [13]. Garg and Singh [15] suggested the numerical solution for fuzzy system of equations by using the Gaussian membership function to the fuzzy numbers considering in its parametric form. In 2017, Dutta and Limboo [14] introduced a new concept called Bell-shaped fuzzy soft set, and gave some applications of this set in medical diagnosis based on C¸ elik and Yamak's work [11].

The concept of neutrosophic set, which is a generalization of fuzzy sets [38], intuitionistic fuzzy sets [1], was introduced by Smarandache [34] to overcome problems including indeterminate and inconsistent information. A neutrosophic set is characterized by three functions called truth-membership function $(T(x))$, indeterminacy-membership function $(I(x))$ and falsity membership function $(F(x))$. These functions are real standard or nonstandard subsets of $]^{-}0, 1^{+}[$. In some areas such as engineering and real scientific fields, modeling of some problems is difficult with real standard or nonstandard subsets of $]^{-}0, 1^{+}[$. To make a success of this difficulties, concepts of single-valued neutrosophic set (SVNS) and interval neutrosophic set (INS) were in-troduced by Wang et al. in [35] and [36]. Recently, many researchers have studied on concept of single-valued neutrosophic numbers, which are a special case of SVNS, and is very important tool for multi criteria decision making problems. For example, Liu et al. [19] proposed some new aggregation operators and presented some new operational laws for neutrosophic numbers (NNs) based on Hamacher operations and studied their prop-erties. Then, they proposed the generalized neutrosophic number Hamacher weighted averaging (GNNHWA) operator, generalized neutrosophic number Hamacher ordered weighted averaging (GNNHOWA) operator, and generalized neutrosophic number Hamacher hybrid averaging (GNNHHA) operator, and explored some properties of these operators and analyzed some special cases of them. Biswas et al. [4] studied on trape-zoidal fuzzy neutrosophic numbers and its application in multi-attribute decision making Deli and Subaş[16] defined single-valued triangular neutrosophic numbers (SVTrNN) and proposed some new geometric operators for SVTrNNs. They also gave MCDM under SVTrN information based on geometric operators of SVTrNN. In [17], Deli and Subaş defined $\alpha-$cut of SVNNs to apply the single-valued trapezoidal neutrosophic num-bers (SVTNNs) and SVTrNNs, then they used these new concepts to solve a MCDM problem. Also, many researchers studied on applications in decision making and group decision making of neutrosophic sets and their some extensions and subclasses, based on similarity measures [37, 23, 24, 25, 21, 27], TOPSIS method [26, 5, 7, 10, 39], grey relational analysis [2, 12], distance measure [8], entropy [28], correlation coefficient [18] and special problem in real life [3, 6, 9, 30, 31, 20, 22].

The SVTrNNs and SVTNNs are useful tool indeterminate and inconsistent information. However, in some cases obtained data may not be SVTrN or SVTN. Therefore, in this study, a new kind of SVNNs called Gaussian single-valued neutrosophic numbers (GSVNNs) is introduced. Also, $\alpha-$cut, parametric form of GSVNNs, and arithmetic operations of GSVNNs by using $\alpha-$cuts of GSVNNs are defined, and some results are obtained related to $\alpha-$cut of GSVNNs. Furthermore, based on Çelik and Yamak's work in [11] and Dutta and Limboo's work in [14], a decision making method is proposed for medical diagnosis problem. Finally, a hypothetical case study is given to illustrate processing of the proposed method.

2 Preliminaries

2.1 Single-valued neutrosophic sets

A neutrosophic set $\tilde{a}$ on the universe of discourse X is defined as follows:

$$\tilde{a} = \left\{\langle x, a_t(x), a_i(x), a_f(x))\rangle : x \in X\right\}$$

where a_t, a_i, a_f: $X \to]^{-}0, 1^{+}[$ and $^{-}0 \leq a_t(x) + a_i(x) + a_f(x) \leq 3^{+}$ [34]. From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^{-}0, 1^{+}[$. In some real life applications, modeling of problems by using real standard or nonstandard subsets of $]^{-}0, 1^{+}[$ may not be easy sometimes. Therefore concept of single valued neutrosophic set (SVN-set) was defined by Wang et al. [36] as follow:

Let $X \neq \emptyset$, with a generic element in X denoted by x. A single-valued neutrosophic set ($SVNS$) $\tilde{a}$ is characterized by three functions called truth- membership function $a_t(x)$, indeterminacy-membership function $a_i(x)$ and falsity-membership function $a_f(x)$ such that $a_t(x)$, $a_i(x)$, $a_f(x) \in [0, 1]$ for all $x \in X$.

If X is continuous, a $SVNS$ $\tilde{a}$ can be written as follows:

$$\tilde{a} = \int_X \langle a_t(x), a_i(x), a_f(x)\rangle /x, \quad \text{for all } x \in X.$$

If X is crisp set, a $SVNS$ $\tilde{a}$ can be written as follows:

$$\tilde{a} = \sum_x \langle a_t(x), a_i(x), a_f(x)\rangle /x, \quad \text{for all } x \in X.$$

Here $0 \leq a_t(x) + a_i(x) + a_f(x) \leq 3$ for all $x \in X$. For convenience, a SVNN is denoted by $\tilde{a} = \langle a_t, a_i, a_f\rangle$.

Definition 2.1. (Gaussian fuzzy number) A fuzzy number is said to be Gaussian fuzzy number $GFN(\mu, \sigma)$ whose membership function is given as follows:

$$f(x) = \exp(-\frac{1}{2}(\frac{x-\overline{\mu}}{\sigma})^2), -\infty < x < \infty,$$

where $\overline{\mu}$ denotes the mean and σ denotes standard deviations of the distribution.

Definition 2.2. α-cut of Gaussian fuzzy number: Let membership function for Gaussian fuzzy number is given as follows:

$$f(x) = \exp(-\frac{1}{2}(\frac{x-\overline{\mu}}{\sigma})^2).$$

Then, α-cut is given $A_\alpha = \left[\overline{\mu} - \sigma\sqrt{-2\log\alpha}, \overline{\mu} + \sigma\sqrt{-2\log\alpha}\right]$

3 Gaussian SVN-number:

Definition 3.1. A SVN-number is said to be Gaussian SVN-number $GSVNN\Big((\overline{\mu}_t, \sigma_t), (\overline{\mu}_i, \sigma_i), (\overline{\mu}_f, \sigma_f)\Big)$ whose truth-membership function, indeterminacy-membership function and falsity-membership function are given as follows:

$$\varphi(x_t) = \exp\left(-\frac{1}{2}(\frac{x_t-\overline{\mu}_t}{\sigma_t})^2\right),$$

$$\varphi(x_i) = 1 - \left(\exp(-\frac{1}{2}(\frac{x_i-\overline{\mu}_i}{\sigma_i})^2\right),$$

$$\varphi(x_f) = 1 - \left(\exp(-\frac{1}{2}(\frac{x_f-\overline{\mu}_f}{\sigma_f})^2\right),$$

respectively. Here $\overline{\mu}_t$ ($\overline{\mu}_i, \overline{\mu}_f$) denotes mean of truth-membership (indeterminacy-membership, falsity-membership) value. σ_t (σ_i, σ_f) denotes standard deviation of the distribution of truth-membership (indeterminacy-membership, falsity-membership) value.

Example 3.2. Let $\tilde{A} = GSVNN\Big((0.4, 0.2), (0.6, 0.3), (0.3, 0.1)\Big)$ be Gaussian SVN-number. Then graphics of truth-membership function, indeterminacy-membership function and falsity-membership function of GSVNN $\tilde{A}$ are depicted in Fig 1.

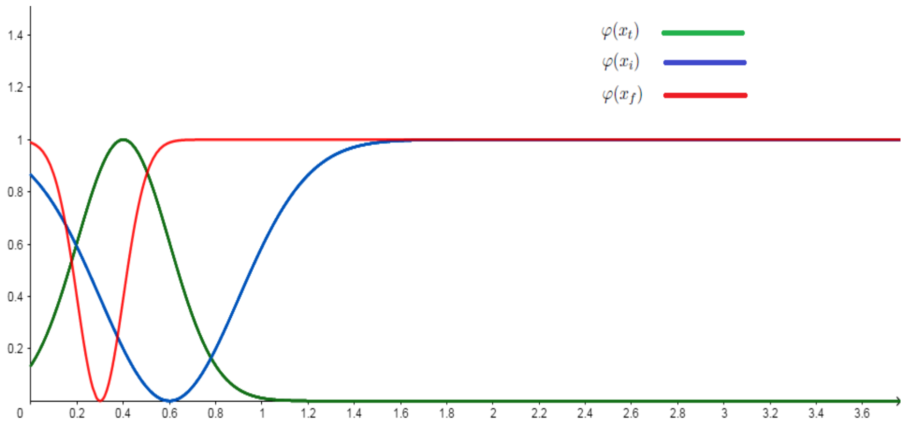

Figure 1: GSVNN $\tilde{A}$

Definition 3.3. **α-cut of Gaussian SVN-number:** Truth-membership function, indeterminacy-membership function and falsity-membership function for Gaussian SVN-number $\tilde{A}$ are given as follows:

$$\varphi(x_t) = \exp(-\frac{1}{2}(\frac{x_t - \overline{\mu}_t}{\sigma_t})^2),$$

$$\varphi(x_i) = 1 - \exp(-\frac{1}{2}(\frac{x_i - \overline{\mu}_i}{\sigma_i})^2),$$

$$\varphi(x_f) = 1 - \exp(-\frac{1}{2}(\frac{x_f - \overline{\mu}_f}{\sigma_f})^2)$$

respectively.

Then α-cuts of them are as follows:

$$A_{t_\alpha} = \Big[\overline{\mu}_t - (\sigma_t\sqrt{-2\log\alpha}), \overline{\mu}_t + (\sigma_t\sqrt{-2\log\alpha})\Big],$$

$$A_{i_\alpha} = \Big[\overline{\mu}_i - (\sigma_i\sqrt{-2\log(1-\alpha)}), \overline{\mu}_i + (\sigma_i\sqrt{-2\log(1-\alpha)})\Big],$$

$$A_{f_\alpha} = \Big[\overline{\mu}_f - (\sigma_f\sqrt{-2\log(1-\alpha)}), \overline{\mu}_f + (\sigma_f\sqrt{-2\log(1-\alpha)})\Big],$$

respectively.

3.1 Arithmetic operations of Gaussian SVN-numbers

Let $\tilde{A} = GSVNN\Big((\overline{\mu}_{A_t}, \sigma_{A_t}), (\overline{\mu}_{A_i}, \sigma_{A_i}), (\overline{\mu}_{A_f}, \sigma_{A_f})\Big)$ and $\tilde{B} = GSVNN\Big((\overline{\mu}_{B_t}, \sigma_{B_t}), (\overline{\mu}_{B_i}, \sigma_{B_i}), (\overline{\mu}_{B_f}, \sigma_{B_f})\Big)$ be two Gaussian SVN-numbers. Then their α-cuts $(0 < \alpha < 1)$ of these numbers are as follows:

$$A_{t_\alpha} = \Big[\overline{\mu}_{A_t} - (\sigma_{A_t}\sqrt{-2\log\alpha}), \overline{\mu}_{A_t} + (\sigma_{A_t}\sqrt{-2\log\alpha})\Big],$$

$$A_{i_\alpha} = \Big[\overline{\mu}_{A_i} - (\sigma_{A_i}\sqrt{-2\log(1-\alpha)}), \overline{\mu}_{A_i} + (\sigma_{A_i}\sqrt{-2\log(1-\alpha)})\Big],$$

$$A_{f_\alpha} = \Big[\overline{\mu}_{A_f} - (\sigma_{A_f}\sqrt{-2\log(1-\alpha)}), \overline{\mu}_{A_f} + (\sigma_{A_f}\sqrt{-2\log(1-\alpha)})\Big]$$

and

$$B_{t_\alpha} = \Big[\overline{\mu}_{B_t} - (\sigma_{B_t}\sqrt{-2\log\alpha}), \overline{\mu}_{B_t} + (\sigma_{B_t}\sqrt{-2\log\alpha})\Big],$$

$$B_{i_\alpha} = \Big[\overline{\mu}_{B_i} - (\sigma_{B_i}\sqrt{-2\log(1-\alpha)}), \overline{\mu}_{B_i} + (\sigma_{B_i}\sqrt{-2\log(1-\alpha)})\Big],$$

$$B_{f_\alpha} = \Big[\overline{\mu}_{B_f} - (\sigma_{B_f}\sqrt{-2\log(1-\alpha)}), \overline{\mu}_{B_f} + (\sigma_{B_f}\sqrt{-2\log(1-\alpha)})\Big],$$

respectively.

Based on α-cuts of $\tilde{A}$ and $\tilde{B}$, arithmetic operations between GSVNN $\tilde{A}$ and GSVNN $\tilde{B}$ are defined as follows:

1. Addition:
   $A_{t_\alpha} + B_{t_\alpha} = \Big[(\overline{\mu}_{A_t} + \overline{\mu}_{B_t}) - (\sigma_{A_t} + \sigma_{B_t})\sqrt{-2\log\alpha}, \overline{\mu}_{A_t} + (\sigma_{A_t} + \sigma_{B_t})\sqrt{-2\log\alpha}\Big]$
   $A_{i_\alpha} + B_{i_\alpha} = \Big[(\overline{\mu}_{A_i} + \overline{\mu}_{B_i}) - (\sigma_{A_i} + \sigma_{B_i})\sqrt{-2\log(1-\alpha)}, (\mu_{A_i} + \mu_{B_i}) + (\sigma_{A_i} + \sigma_{B_i})\sqrt{-2\log(1-\alpha)}\Big],$
   $A_{f_\alpha} + B_{f_\alpha} = \Big[(\overline{\mu}_{A_f} + \overline{\mu}_{B_f}) - (\sigma_{A_f} + \sigma_{B_f})\sqrt{-2\log(1-\alpha)}, (\mu_{A_f} + \mu_{B_f}) + (\sigma_{A_f} + \sigma_{B_f})\sqrt{-2\log(1-\alpha)}\Big],$
   Truth-membership function, indeterminacy-membership function and falsity-membership function of addition of GSVNNs $\tilde{A}$ and $\tilde{B}$ are as follows:

   $$\varphi_{(A+B)}(x_t) = \exp\Big(-\frac{1}{2}(\frac{x_t - (\overline{\mu}_{A_t} + \overline{\mu}_{B_t})}{\sigma_{A_t} + \sigma_{B_t}})^2\Big),$$

   $$\varphi_{(A+B)}(x_i) = 1 - \exp\Big(-\frac{1}{2}(\frac{x_i - (\overline{\mu}_{A_i} + \overline{\mu}_{B_i})}{\sigma_{A_i} + \sigma_{B_i}})^2\Big),$$

   and

   $$\varphi_{(A+B)}(x_f) = 1 - \exp\Big(-\frac{1}{2}(\frac{x_f - (\overline{\mu}_{A_f} + \overline{\mu}_{B_f})}{\sigma_{A_f} + \sigma_{B_f}})^2\Big),$$

   respectively.

2. Substraction:
   $A_{t_\alpha} - B_{t_\alpha} = \Big[(\overline{\mu}_{A_t} - \overline{\mu}_{B_t}) - (\sigma_{A_t} + \sigma_{B_t})\sqrt{-2\log\alpha}, \overline{\mu}_{A_t} - (\sigma_{A_t} + \sigma_{B_t})\sqrt{-2\log\alpha}\Big]$
   $A_{i_\alpha} - B_{i_\alpha} = \Big[(\overline{\mu}_{A_i} - \overline{\mu}_{B_i}) - (\sigma_{A_i} + \sigma_{B_i})\sqrt{-2\log(1-\alpha)}, (\mu_{A_i} - \mu_{B_i}) + (\sigma_{A_i} + \sigma_{B_i})\sqrt{-2\log(1-\alpha)}\Big],$
   $A_{f_\alpha} - B_{f_\alpha} = \Big[(\overline{\mu}_{A_f} - \overline{\mu}_{B_f}) - (\sigma_{A_f} + \sigma_{B_f})\sqrt{-2\log(1-\alpha)}, (\mu_{A_f} - \sigma_{B_f}) + (\sigma_{A_f} + \sigma_{B_f})\sqrt{-2\log(1-\alpha)}\Big],$
   Truth-membership function, indeterminacy-membership function and falsity-membership function of substraction of GSVNNs $\tilde{A}$ and $\tilde{B}$ are as follows:

   $$\varphi_{(A-B)}(x_t) = \exp\Big(-\frac{1}{2}(\frac{x_t - (\overline{\mu}_{A_t} - \overline{\mu}_{B_t})}{\sigma_{A_t} + \sigma_{B_t}})^2\Big),$$

   $$\varphi_{(A-B)}(x_i) = 1 - \exp\Big(-\frac{1}{2}(\frac{x_i - (\overline{\mu}_{A_i} - \overline{\mu}_{B_i})}{\sigma_{A_i} + \sigma_{B_i}})^2\Big),$$

and

$$\varphi_{(A-B)}(x_f) = 1 - \exp\Big(-\frac{1}{2}(\frac{x_f - (\overline{\mu}_{A_f} - \overline{\mu}_{B_f})}{\sigma_{A_f} + \sigma_{B_f}})^2\Big),$$

respectively.

3. Multiplication:
$A_{t_\alpha}.B_{t_\alpha} = \Big[(\overline{\mu}_{A_t} - \sigma_{A_t}\sqrt{-2\log\alpha})\cdot(\overline{\mu}_{B_t} - \sigma_{B_t}\sqrt{-2\log\alpha}), (\overline{\mu}_{A_t} + \sigma_{A_t}\sqrt{-2\log\alpha})\cdot(\overline{\mu}_{B_t} + \sigma_{B_t}\sqrt{-2\log\alpha})\Big]$
$A_{i_\alpha}.B_{i_\alpha} = \Big[(\overline{\mu}_{A_i} - \sigma_{A_i}\sqrt{-2\log(1-\alpha)})\cdot(\overline{\mu}_{B_i} - \sigma_{B_i}\sqrt{-2\log(1-\alpha)}), (\overline{\mu}_{A_i} + \sigma_{A_i}\sqrt{-2\log(1-\alpha)})\cdot$
$(\overline{\mu}_{B_i} + \sigma_{B_i}\sqrt{-2\log(1-\alpha)})\Big]$
$A_{f_\alpha}.B_{f_\alpha} = \Big[(\overline{\mu}_{A_f} - \sigma_{A_f}\sqrt{-2\log(1-\alpha)})\cdot(\overline{\mu}_{B_f} - \sigma_{B_f}\sqrt{-2\log(1-\alpha)}), (\overline{\mu}_{A_f} + \sigma_{A_f}\sqrt{-2\log(1-\alpha)})\cdot$
$(\overline{\mu}_{B_f} + \sigma_{B_f}\sqrt{-2\log(1-\alpha)})\Big]$

4. Division:
$\frac{A_{t_\alpha}}{B_{t_\alpha}} = \Big[\frac{\overline{\mu}_{A_t} - (\sigma_{A_t}\sqrt{-2\log\alpha})}{\overline{\mu}_{B_t} + (\sigma_{B_t}\sqrt{-2\log\alpha})}, \frac{\overline{\mu}_{A_t} + (\sigma_{A_t}\sqrt{-2\log\alpha})}{\overline{\mu}_{B_t} - (\sigma_{B_t}\sqrt{-2\log\alpha})}\Big],$

$\frac{A_{i_\alpha}}{B_{i_\alpha}} = \Big[\frac{\overline{\mu}_{A_i} - (\sigma_{A_i}\sqrt{-2\log(1-\alpha)})}{\overline{\mu}_{B_i} + (\sigma_{B_i}\sqrt{-2\log(1-\alpha)})}, \frac{\overline{\mu}_{A_i} + (\sigma_{A_i}\sqrt{-2\log(1-\alpha)})}{\overline{\mu}_{B_i} - (\sigma_{B_i}\sqrt{-2\log(1-\alpha)})}\Big],$

$\frac{A_{f_\alpha}}{B_{f_\alpha}} = \Big[\frac{\overline{\mu}_{A_f} - (\sigma_{A_f}\sqrt{-2\log(1-\alpha)})}{\overline{\mu}_{B_f} + (\sigma_{B_f}\sqrt{-2\log(1-\alpha)})}, \frac{\overline{\mu}_{A_f} + (\sigma_{A_f}\sqrt{-2\log(1-\alpha)})}{\overline{\mu}_{B_f} - (\sigma_{B_f}\sqrt{-2\log(1-\alpha)})}\Big].$

3.2 Parametric Form of SVN-numbers

A SVN $\tilde{n}$ in parametric form is a triple of pairs $((\underline{n}_t(x), \overline{n}_t(x)), (\underline{n}_i(x), \overline{n}_i(x)), (\underline{n}_f(x), \overline{n}_f(x)))$ of the functions $\underline{n}_t(x), \overline{n}_t(x), \underline{n}_i(x), \overline{n}_i(x), \underline{n}_f(x)$ and $\overline{n}_f(x)$ for $0 \leq x \leq 1$ which satisfies the following conditions.

1. (a) $\underline{n}_t(x)$ is bounded and monotonic increasing left continuous function,
   (b) $\overline{n}_t(x)$ is bounded and monotonic decreasing right continuous function,
   (c) $\underline{n}_t(x) \leq \overline{n}_t(x)$ for $0 \leq x \leq 1$.
2. (a) $\underline{n}_i(x)$ is bounded and monotonic decreasing left continuous function,
   (b) $\overline{n}_i(x)$ is bounded and monotonic increasing right continuous function,
   (c) $\underline{n}_i(x) \leq \overline{n}_i(x)$ for $0 \leq x \leq 1$.
3. (a) $\underline{n}_f(x)$ is bounded and monotonic decreasing left continuous function,
   (b) $\overline{n}_f(x)$ is bounded and monotonic increasing right continuous function,
   (c) $\underline{n}_f(x) \leq \overline{n}_f(x)$ for $0 \leq x \leq 1$.

A SVN-number $\tilde{\alpha} = \langle \alpha_t, \alpha_i, \alpha_f \rangle$ is simply represented by $\underline{n}_t(x) = \overline{n}_t(x) = \alpha_t$, $\underline{n}_i(x) = \overline{n}_i(x) = \alpha_i$ and $\underline{n}_f(x) = \overline{n}_f(x) = \alpha_f$, $0 \leq x \leq 1$. For $\tilde{n} = ((\underline{n}_t(x), \overline{n}_t(x)), (\underline{n}_i(x), \overline{n}_i(x)), (\underline{n}_f(x), \overline{n}_f(x)))$ and $\tilde{m} =$

$((\underline{m}_t(x), \overline{m}_t(x)), (\underline{m}_i(x), \overline{m}_i(x)), (\underline{m}_f(x), \overline{m}_f(x)))$, we may define addition and scalar multiplication as

$$\underline{(n+m)}_t(x) = \underline{(n)}_t(x) + \underline{(m)}_t(x), \quad \underline{(n+m)}_i(x) = \underline{(n)}_i(x) + \underline{(m)}_i(x), \quad \underline{(n+m)}_f(x) = \underline{(n)}_f(x) + \underline{(m)}_f(x)$$

$$\overline{(n+m)}_t(x) = \overline{(n)}_t(x) + \overline{(m)}_t(x), \quad \overline{(n+m)}_i(x) = \overline{(n)}_i(x) + \overline{(m)}_i(x), \quad \overline{(n+m)}_f(x) = \overline{(n)}_f(x) + \overline{(m)}_f(x)$$

and

$$\underline{(cn)}_t(x) = c\underline{n}_i(x), \underline{(cn)}_i(x) = c\underline{n}_i(x), \underline{(cn)}_f(x) = c\underline{n}_f(x), c \neq 0$$
$$\overline{(cn)}_t(x) = c\overline{n}_i(x), \overline{(cn)}_i(x) = c\overline{n}_i(x), \overline{(cn)}_f(x) = c\overline{n}_f(x), c \neq 0,$$
$$\underline{(cn)}_t(x) = c\overline{n}_i(x), \underline{(cn)}_i(x) = c\overline{n}_i(x), \underline{(cn)}_f(x) = c\overline{n}_f(x), c \leq 0$$
$$\overline{(cn)}_t(x) = c\underline{n}_i(x), \overline{(cn)}_i(x) = c\underline{n}_i(x), \overline{(cn)}_f(x) = c\underline{n}_f(x), c \leq 0.$$

If $c = \langle c_t, c_i, c_f \rangle$ is a SVN-value, then $c\tilde{n}$ is defined as follows:

$$\underline{(cn)}_t(x) = c_t\underline{n}_t(x), \underline{(cn)}_i(x) = c_i\underline{n}_i(x), \underline{(cn)}_f(x) = c_f\underline{n}_f(x),$$
$$\overline{(cn)}_t(x) = c_t\overline{n}_t(x), \overline{(cn)}_i(x) = c_i\overline{n}_i(x), \overline{(cn)}_f(x) = c_f\overline{n}_f(x).$$

Let $\tilde{A}$ be a GSVNN as $\varphi_A(x_t) = \exp(-\frac{1}{2}(\frac{x_t - \overline{\mu}_t}{\sigma_t})^2), \varphi_A(x_i) = -\exp(-\frac{1}{2}(\frac{x_i - \overline{\mu}_i}{\sigma_i})^2) + 1$ and $\varphi_A(x_f) = -\exp(-\frac{1}{2}(\frac{x_f - \overline{\mu}_f}{\sigma_f})^2) + 1$. Then, parametric form of GSVNN $\tilde{A}$ can be transformed as

$$\begin{aligned}((\underline{n}_t(x), \overline{n}_t(x)), (\underline{n}_i(x), \overline{n}_i(x)), (\underline{n}_f(x), \overline{n}_f(x))) \; = \; & \Big((\overline{\mu}_t - \sigma_t\sqrt{-2\log\alpha}), \overline{\mu}_t + \sigma_t\sqrt{-2\log\alpha}),\\ & (\overline{\mu}_i - \sigma_i\sqrt{-2\log(1-\alpha)}, \overline{\mu}_i + \sigma_i\sqrt{-2\log(1-\alpha)}),\\ & (\overline{\mu}_f - \sigma_f\sqrt{-2\log(1-\alpha)}, \overline{\mu}_f + \sigma_f\sqrt{-2\log(1-\alpha)})\Big)\end{aligned}$$

Example 3.4. Let us consider $(0.5, 0.2, 0.8) = GSVNN((0.5, 0.02), (0.2, 0.05), (0.8, 0.01))$ to be a SVN-number with Gaussian membership functions. Then, its parametric form is as follows:

$$\Big((0.5 - 0.02\sqrt{-2ln(\alpha)}, 0.5 + 0.02\sqrt{-2ln(\alpha)}), (0.2 - 0.05\sqrt{-2ln(1-\alpha)}, 0.2 + 0.05\sqrt{-2ln(1-\alpha)}),$$
$$(0.8 - 0.01\sqrt{-2ln(1-\alpha)}, 0.8 + 0.01\sqrt{-2ln(1-\alpha)})\Big)$$

Proposition 3.5. *Addition of two GSVNNs is a GSVNN. Namely;*

$$(\tilde{A} + \tilde{B})(x_t) = \Big(\mu_A(x_t) + \mu_B(x_t) - (\sigma_{A_t} + \sigma_{B_t})\sqrt{-2\ln(\alpha)}, \mu_A(x_t) + \mu_B(x_t) + (\sigma_{A_t} + \sigma_{B_t})\sqrt{-2\ln(\alpha)}\Big),$$
$$(\tilde{A} + \tilde{B})(x_i) = \Big(\mu_A(x_i) + \mu_B(x_i) - (\sigma_{A_i} + \sigma_{B_i})\sqrt{-2\ln(1-\alpha)}, \mu_A(x_i) + \mu_B(x_i) + (\sigma_{A_i} + \sigma_{B_i})\sqrt{-2\ln(1-\alpha)}\Big),$$
$$(\tilde{A} + \tilde{B})(x_f) = \Big(\mu_A(x_f) + \mu_B(x_f) - (\sigma_{A_f} + \sigma_{B_f})\sqrt{-2\ln(1-\alpha)}, \mu_A(x_f) + \mu_B(x_f) + (\sigma_{A_f} + \sigma_{B_f})\sqrt{-2\ln(1-\alpha)}\Big).$$

Proof. The proof is obvious from definition. □

Let us consider GSVNNs $A = GSVNN((0.3, 0.5), (0.4, 0.2), (0.7, 0.1))$ and $B = GSVNN((0.6, 0.2), (0.5, 0.1), (0.4, 0.3))$. Their graphics are shown in Figs (2) and (3)

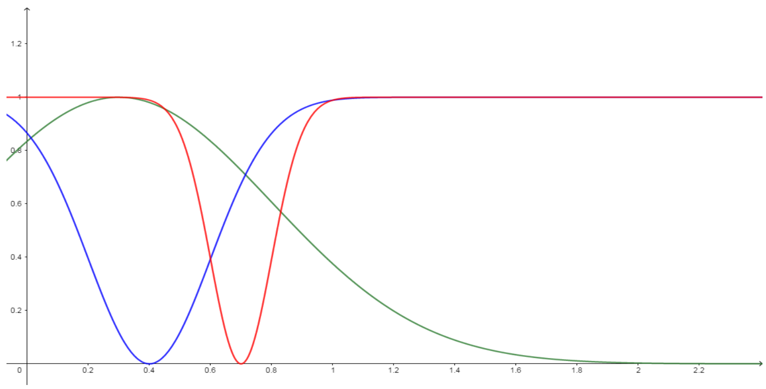

Figure 2: GSVNN $\tilde{A}$

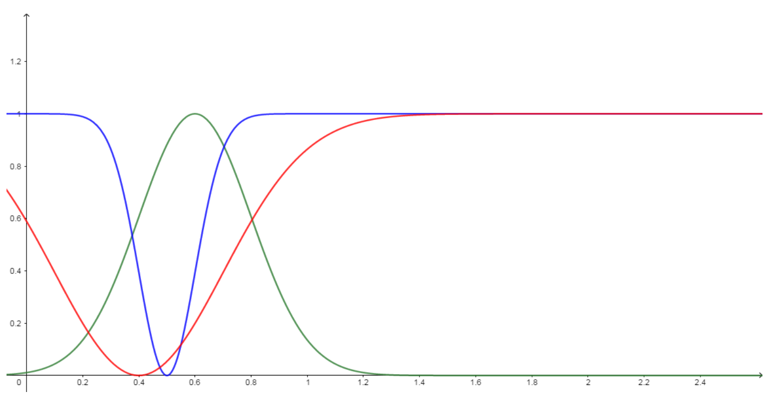

Figure 3: GSVNN $\tilde{B}$

Truth-membership, indeterminacy-membership and falsity-membership function of GSVNN $A + B$ and $A - B$ are as follows:

$$\varphi_{(A+B)}(x_t) = \exp\Big(-\frac{1}{2}(\frac{x-0.9}{0.7})^2\Big),$$

$$\varphi_{(A+B)}(x_i) = 1 - \exp\Big(-\frac{1}{2}(\frac{x-0.9}{0.3})^2\Big),$$

$$\varphi_{(A+B)}(x_f) = 1 - \exp\Big(-\frac{1}{2}(\frac{x-1.1}{0.4})^2\Big),$$

and

$$\varphi_{(A-B)}(x_t) = \exp\Big(-\frac{1}{2}(\frac{x-0.3}{0.7})^2\Big),$$

$$\varphi_{(A-B)}(x_i) = 1 - \exp\Big(-\frac{1}{2}(\frac{x-0.1}{0.3})^2\Big),$$

$$\varphi_{(A-B)}(x_f) = 1 - \exp\Big(-\frac{1}{2}(\frac{x-0.3}{0.4})^2\Big).$$

Then, figures of GSVNNs $\tilde{A} + \tilde{B}$ and $\tilde{A} - \tilde{B}$ are as in Figs (4) and (5).

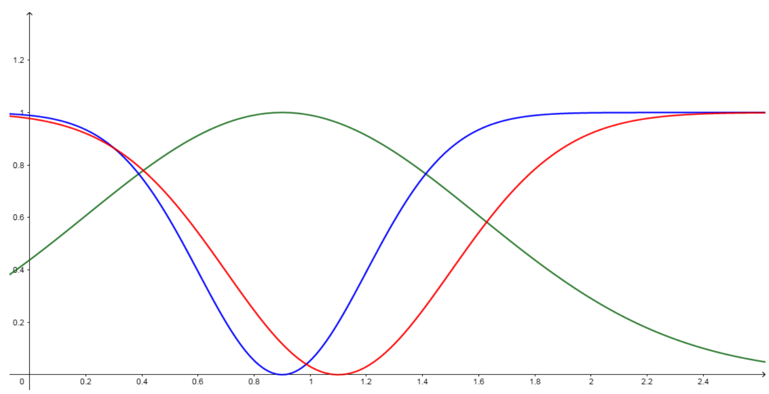

Figure 4: GSVNN $\tilde{A} + \tilde{B}$

4 Application of Gaussian SVN-numbers in Medical Diagnosis

Let us consider the decision-making problem adapted from [11].

4.1 Method and Algorithm

Let $P = \{p_1, p_2, ..., p_p\}$ be a set of patients, $S = \{s_1, s_2, ..., s_s\}$ be set of symptoms and $D = \{d_1, d_2, ..., d_d\}$ be a set of diseases. Patients $p_i (i = 1, 2, ..., p)$ are evaluated by experts by using Table 1 for each symptom $s_j (j = 1, 2, ..., s)$, and patient-symptom (PS) matrix is given as follows:

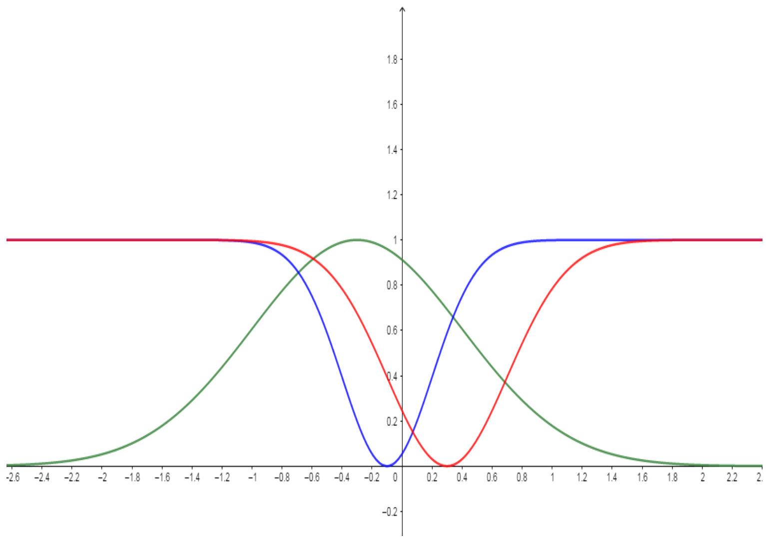

Figure 5: GSVNN $\tilde{A} - \tilde{B}$

$$PS = \begin{pmatrix} \tilde{m}_{11} & \tilde{m}_{12} & \cdots & \tilde{m}_{1s} \\ \tilde{m}_{21} & \tilde{m}_{22} & \cdots & \tilde{m}_{2s} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{m}_{p1} & \tilde{m}_{p2} & \cdots & \tilde{m}_{ps} \end{pmatrix}$$

Here $\tilde{m}_{ij} = \langle m_{t_{ij}}, m_{i_{ij}}, m_{f_{ij}} \rangle$ denotes SVN-value of patient p_i related to symptom s_j.

Table 1: SVN-numbers for linguistic terms

Linguistic terms	Linguistic values of SVN-numbers
Absolutely low(AL)	$\langle 0.05, 0.95, 0.95 \rangle$
Low(L)	$\langle 0.20, 0.75, 0.80 \rangle$
Fairly low(FL)	$\langle 0.35, 0.60, 0.65 \rangle$
Medium(M)	$\langle 0.50, 0.50, 0.50 \rangle$
Fairly high(FH)	$\langle 0.65, 0.40, 0.35 \rangle$
High(H)	$\langle 0.80, 0.25, 0.20 \rangle$
Absolutely high(AH)	$\langle 0.95, 0.10, 0.05 \rangle$

Symptoms $s_i (i = 1, 2, ..., s)$ are evaluated with Gaussian SVN-numbers for each disease $d_k (j = 1, 2, ..., d)$, and symptoms-disease (SD) matrix is given as follows:

$$SD = \begin{pmatrix} \tilde{n}_{11} & \tilde{n}_{12} & \cdots & \tilde{n}_{1d} \\ \tilde{n}_{21} & \tilde{n}_{22} & \cdots & \tilde{n}_{2d} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{n}_{s1} & \tilde{n}_{s2} & \cdots & \tilde{n}_{sd} \end{pmatrix}.$$

Here $\tilde{n}_{jk} = GSVNN\langle (n_{t_{jk}}, \sigma_t), (n_{i_{jk}}, \sigma_i), (n_{f_{jk}}, \sigma_f) \rangle$ denotes Gaussian SVN-value of symptom s_j related to disease d_k $(k = 1, 2, ..., d)$.

Decision matrix (PD) is defined by using composition of matrices PS and SD as follows:

$$PD = \begin{pmatrix} \tilde{q}_{11} & \tilde{q}_{12} & \cdots & \tilde{q}_{1d} \\ \tilde{q}_{21} & \tilde{q}_{22} & \cdots & \tilde{q}_{2d} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{q}_{p1} & \tilde{q}_{s2} & \cdots & \tilde{q}_{pd} \end{pmatrix} = \begin{pmatrix} \tilde{m}_{11} & \tilde{m}_{12} & \cdots & \tilde{m}_{1s} \\ \tilde{m}_{21} & \tilde{m}_{22} & \cdots & \tilde{m}_{2s} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{m}_{p1} & \tilde{m}_{p2} & \cdots & \tilde{m}_{ps} \end{pmatrix} \circ \begin{pmatrix} \tilde{n}_{11} & \tilde{n}_{12} & \cdots & \tilde{n}_{1d} \\ \tilde{n}_{21} & \tilde{n}_{22} & \cdots & \tilde{n}_{2d} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{n}_{s1} & \tilde{n}_{s2} & \cdots & \tilde{n}_{sd} \end{pmatrix}.$$

Here $\tilde{q}_{ik}$ $(i = 1, 2, ..., p; k = 1, 2, ..., d)$ is calculated by

$$(\langle \tilde{u}_t, \tilde{u}_i, \tilde{u}_f \rangle GSVNN\langle (n_{t_{jk}}, \sigma_t), (n_{i_{jk}}, \sigma_i), (n_{f_{jk}}, \sigma_f)\rangle)(x) =$$

$$\begin{aligned} &\langle (u_t n_{t_{jk}} - u_t\sigma_t\sqrt{-2ln(\alpha)}, u_t n_{t_{jk}} + u_t\sigma_t\sqrt{-2ln(\alpha)}), \\ &(u_i n_{i_{jk}} - u_i\sigma_i\sqrt{-2ln(1-\alpha)}, u_i n_{i_{jk}} + u_i\sigma_i\sqrt{-2ln(1-\alpha)}), \\ &(u_f n_{f_{jk}} - u_f\sigma_f\sqrt{-2ln(1-\alpha)}, u_f n_{f_{jk}} + u_f\sigma_f\sqrt{-2ln(1-\alpha)})\rangle. \end{aligned}$$

For the sake of shortness, $(\langle \tilde{u}_t, \tilde{u}_i, \tilde{u}_f \rangle GSVNN\langle (n_{t_{jk}}, \sigma_t), (n_{i_{jk}}, \sigma_i), (n_{f_{jk}}, \sigma_f)\rangle)(x)$ will be denoted by $\langle (\underline{a}, \overline{a}), (\underline{b}, \overline{b}), (\underline{c}, \overline{c})\rangle$.

For obtained parametric forms of Gaussian SVN-numbers, score functions are defined as follows:

$$S_{q_{ik}} = \frac{4 - (\underline{a} - \underline{b} - \underline{c}) + (\overline{a} - \overline{b} - \overline{c})}{6} \tag{4.1}$$

If $\max S_{q_{ik}} = S_{q_{it}}$ for $1 \leq t \leq k$, then it is said that patient p_i suffers from disease d_t. In case $\max S_{q_{ik}}$ occurs for more than one value, for $1 \leq t \leq k$, then symptoms can be reassessed.

Algorithm 1

Input: The matrix PS (patient-symptom) obtained according to opinion of expert (decision maker)
Output: Diagnosis of disease

algorithmic

1. Construct matrix PS according to opinions of experts by using Table 1.
2. Construct matrix SD by using GSVNNs.
3. Calculate decision matrix PD.
4. Compute score values of elements of decision matrix PD.
5. Find t for which $\max S_{q_{ik}} = S_{q_{it}}$ for $1 \leq t \leq k$

5 Hypothetical case study

In this section, a hypothetical case study is given to illustrate processing of the proposed method.

There are three patients p_1, p_2, p_3, p_4 and p_5 who it is considered that they suffer from d_1 =viral fever, d_2 =tuberculosis, d_3 =typhoid, d_4 =throat disease or d_5 =malaria. In these diseases, common symptoms are s_1 =temperature, s_2 =cough, s_3 =throat pain, s_4 =headache, s_5 =body pain.
Step 1: In the results of observation made by an expert, suppose that matrix PS is as follows:

$$PS = \begin{array}{cc} & \begin{array}{ccccc} s_1 & s_2 & s_3 & s_4 & s_5 \end{array} \\ \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{array} & \begin{pmatrix} \langle 0.95, 0.10, 0.05\rangle & \langle 0.65, 0.40, 0.35\rangle & \langle 0.20, 0.75, 0.80\rangle & \langle 0.20, 0.75, 0.80\rangle & \langle 0.80, 0.25, 0.20\rangle \\ \langle 0.35, 0.60, 0.65\rangle & \langle 0.65, 0.40, 0.35\rangle & \langle 0.50, 0.50, 0.50\rangle & \langle 0.80, 0.25, 0.20\rangle & \langle 0.95, 0.10, 0.05\rangle \\ \langle 0.65, 0.40, 0.35\rangle & \langle 0.50, 0.50, 0.50\rangle & \langle 0.80, 0.25, 0.20\rangle & \langle 0.20, 0.75, 0.80\rangle & \langle 0.80, 0.25, 0.20\rangle \\ \langle 0.20, 0.75, 0.80\rangle & \langle 0.80, 0.25, 0.20\rangle & \langle 0.95, 0.10, 0.05\rangle & \langle 0.35, 0.60, 0.65\rangle & \langle 0.20, 0.75, 0.80\rangle \\ \langle 0.80, 0.25, 0.20\rangle & \langle 0.35, 0.65, 0.65\rangle & \langle 0.05, 0.10, 0.95\rangle & \langle 0.80, 0.25, 0.20\rangle & \langle 0.20, 0.250.80\rangle \end{pmatrix} \end{array}$$

Step 2: Suppose that matrix SD is as follows:

$$SD = \begin{array}{cc} & \begin{array}{ccccc} d_1 & d_2 & d_3 & d_4 & d_5 \end{array} \\ \begin{array}{c} s_1 \\ \\ s_2 \\ \\ s_3 \\ \\ s_4 \\ \\ s_5 \end{array} & \left(\begin{array}{ccccc}
\begin{array}{c}\langle(.30, .10),\\ (.50, .01),\\ (.20, .03)\rangle\end{array} & \begin{array}{c}\langle(.25, .03),\\ (.70, .02),\\ (.50, .01)\rangle\end{array} & \begin{array}{c}\langle(.25, .15),\\ (.25, .015),\\ (.50, .02)\rangle\end{array} & \begin{array}{c}\langle(.60, .03),\\ (.40, .18),\\ (.20, .1)\rangle\end{array} & \begin{array}{c}\langle(.71, .04),\\ (.52, .05),\\ (.45, .02)\rangle\end{array} \\ \hline
\begin{array}{c}\langle(.65, .12),\\ (.48, .02),\\ (.25, .04)\rangle\end{array} & \begin{array}{c}\langle(.25, .03),\\ (.32, .02),\\ (.60, .01)\rangle\end{array} & \begin{array}{c}\langle(.60, .10),\\ (.64, .06),\\ (.18, .018)\rangle\end{array} & \begin{array}{c}\langle(.60, .09),\\ (.80, .001),\\ (.40, .13)\rangle\end{array} & \begin{array}{c}\langle(.60, .05),\\ (.30, .05),\\ (.20, .02)\rangle\end{array} \\ \hline
\begin{array}{c}\langle(.40, .1),\\ (.23, .08),\\ (.50, .01)\rangle\end{array} & \begin{array}{c}\langle(.50, .06),\\ (.35, .02),\\ (.32, .03)\rangle\end{array} & \begin{array}{c}\langle(.45, .12),\\ (.40, .15),\\ (.56, .03)\rangle\end{array} & \begin{array}{c}\langle(.45, .10),\\ (.90, .015),\\ (.50, .18)\rangle\end{array} & \begin{array}{c}\langle(.88, .02),\\ (.60, .07),\\ (.40, .02)\end{array} \\ \hline
\begin{array}{c}\langle(.90, .35),\\ (.43, .05),\\ (.80, .07)\rangle\end{array} & \begin{array}{c}\langle(.25, .01),\\ (.12, .09),\\ (.44, .04)\rangle\end{array} & \begin{array}{c}\langle(.60, .22),\\ (.30, .19),\\ (.13, .022)\rangle\end{array} & \begin{array}{c}\langle(.65, .14),\\ (.23, .012),\\ (.41, .20)\rangle\end{array} & \begin{array}{c}\langle(.90, .06),\\ (.30, .20),\\ (.65, .01)\rangle\end{array} \\ \hline
\begin{array}{c}\langle(.50, .09),\\ (.32, .021),\\ (.44, .06)\rangle\end{array} & \begin{array}{c}\langle(.33, .02),\\ (.70, .08),\\ (.60, .07)\rangle\end{array} & \begin{array}{c}\langle(.75, .03),\\ (.50, .11),\\ (.25, .02)\rangle\end{array} & \begin{array}{c}\langle(.48, .12),\\ (.43, .02),\\ (.41, .04)\rangle\end{array} & \begin{array}{c}\langle(.28, .02),\\ (.63, .20),\\ (.50, .08)\rangle\end{array}
\end{array}\right) \end{array}$$

Step 3: Elements of decision matrix $PD = PS \circ SD$ are obtained as follows:
For the sake of shortness, some annotations are adapted as follows:

$x = \sqrt{-2ln(\alpha)})$ and $y = \sqrt{-2ln(1-\alpha)})$

$$\begin{aligned}
\tilde{q}_{11} &= \Big((0.285 - 0.095x, 0.285 + 0.095x), (0.050 - 0.001y, 0.050 \\
&+ 0.001y), (0.010 - 0.002y, 0.010 + 0.002y)\Big) + \Big((0.423 \\
&- 0.078x, 0.423 + 0.078x), (0.192 - 0.008y, 0.192 \\
&+ 0.008y), (0.088 - 0.014y, 0.088 + 0.014y)\Big) \\
&+ \Big((0.08 - 0.02x, 0.08 + 0.02x), (0.173 - 0.06y, 0.173 \\
&+ 0.06y), (0.40 - 0.008y, 0.40 + 0.008y)\Big) \\
&+ \Big((0.18 - 0.07x, 0.18 + 0.07x), (0.323 - 0.038y, 0.323 \\
&+ 0.038y), (0.64 - 0.056y, 0.64 + 0.056y)\Big) \\
&+ \Big((0.40 - 0.072x, 0.40 + 0.072x), (0.080 - 0.005y, 0.080 \\
&+ 0.005y), (0.088 - 0.012y, 0.088 + 0.012y)\Big) \\
&= \Big((1.368 - 0.335x, 1.368 + 0.335x), (0.817 - 0.112y, \\
&\quad 0.817 + 0.112y), (1.226 - 0.092y, 1.226 + 0.092y)\Big)
\end{aligned}$$

By similar way, we have $\tilde{q}_{12} = \Big((0.814 - 0.078x, 0.814 + 0.078x), (0.726 - 0.113y, 0.726 + 0.113y), (0.963 - 0.074y, 0.963 + 0.074y)\Big)$

$\tilde{q}_{13} = \Big((1.438 - 0.300x, 1.438 + 0.300x), (0.931 - 0.308y, 0.931 + 0.308y), (0.690 - 0.053y, 0.690 + 0.053y)\Big)$

$\tilde{q}_{14} = \Big((1.564 - 0.231x, 1.564 + 0.231x), (1.314 - 0.044y, 1.314 + 0.044y), (0.960 - 0.363y, 0.960 + 0.363y)\Big)$

$\tilde{q}_{15} = \Big((1.645 - 0.103x, 1.645 + 0.103x), (1.005 - 0.278y, 1.005 + 0.278y), (1.033 - 0.048y, 1.033 + 0.048y)\Big)$

$\tilde{q}_{21} = \Big((1,923 - 0.529x, 1.923 + 0.529x), (0.747 - 0.069y, 0.747 + 0.069y), (0.650 - 0,056y, 0.650 + 0.056y)\Big)$

$\tilde{q}_{22} = \Big((1.014 - 0.087x, 1.014 + 0.087x), (0.823 - 0.061y, 0.823 + 0.061y), (0.813 - 0.037y, 0.813 + 0.037y)\Big)$

$\tilde{q}_{23} = \Big((1.895 - 0.382x, 1.895 + 0.382x), (0.731 - 0.167y, 0.731 + 0.167y), (0.707 - 0.040y, 0.707 + 0.040y)\Big)$

$\tilde{q}_{24} = \Big((1.801 - 0.345x, 1.801 + 0.345x), (1.111 - 0.121y, 1.111 + 0.121y), (0.623 - 0.243y, 0.623 + 0.243y)\Big)$

$\tilde{q}_{25} = \Big((2.065 - 0.124x, 2.065 + 0.124x), (0.870 - 0.155y, 0.870 + 0.155y), (0.718 - 0.036y, 0.718 + 0.036y)\Big)$

$\tilde{q}_{31} = \Big((1,420 - 0.347x, 1.429 + 0.347x), (0.900 - 0.077y, 0.900 + 0.077y), (1.023 - 0,101y, 1.023 + 0.101y)\Big)$

$\tilde{q}_{32} = \Big((1.002 - 0.101x, 1.002 + 0.101x), (0.793 - 0.111y, 0.793 + 0.111y), (1.011 - 0.061y, 1.011 + 0.061y)\Big)$

$\tilde{q}_{33} = \Big((1.543 - 0.312x, 1.543 + 0.312x), (0.870 - 0.244y, 0.870 + 0.244y), (0.531 - 0.044y, 0.531 + 0.044y)\Big)$

$\tilde{q}_{34} = \Big((1.564 - 0.269x, 1.564 + 0.269x), (1.065 - 0.090y, 1.065 + 0.090y), (0.780 - 0.304y, 0.780 + 0.304y)\Big)$

$$\tilde{q}_{35} = \Big((1.870 - 0.095x, 1.870 + 0.095x), (0.891 - 0.263y, 0.891 + 0.263y), (0.958 - 0.045y, 0.958 + 0.045y)\Big)$$

$$\tilde{q}_{41} = \Big((1.375 - 0.352x, 1.375 + 0.352x), (1.016 - 0.066y, 1.016 + 0.066y), (1.107 - 0, 126y, 1.107 + 0.126y)\Big)$$
$$\tilde{q}_{42} = \Big((0.879 - 0.095x, 0.879 + 0.095x), (1.237 - 0.136y, 1.237 + 0.136y), (1.302 - 0.094y, 1.302 + 0.094y)\Big)$$
$$\tilde{q}_{43} = \Big((1.318 - 0.307x, 1.318 + 0.307x), (0.943 - 0.238y, 0.943 + 0.238y), (0.749 - 0.051y, 0.749 + 0.051y)\Big)$$
$$\tilde{q}_{44} = \Big((1.423 - 0.264x, 1.423 + 0.264x), (0.986 - 0.156y, 0.986 + 0.156y), (0.798 - 0.271y, 0.798 + 0.271y)\Big)$$
$$\tilde{q}_{45} = \Big((1.829 - 0.092x, 1.829 + 0.092x), (1.178 - 0.327y, 1.178 + 0.327y), (1.243 - 0.092y, 1.243 + 0.092y)\Big)$$

$$\tilde{q}_{51} = \Big((1.308 - 0.425x, 1.308 + 0.425x), (0.648 - 0.041y, 0.648 + 0.041y), (1.190 - 0.104y, 1.190 + 0.104y)\Big)$$
$$\tilde{q}_{52} = \Big((0.579 - 0.050x, 0.579 + 0.050x), (0.623 - 0.063y, 0.623 + 0.063y), (1.362 - 0.101y, 1.362 + 0.101y)\Big)$$
$$\tilde{q}_{53} = \Big((1.063 - 0.343x, 1.063 + 0.343x), (0.719 - 0.133y, 0.719 + 0.133y), (0.975 - 0.065y, 0.975 + 0.065y)\Big)$$
$$\tilde{q}_{54} = \Big((1.329 - 0.197x, 1.329 + 0.197x), (0.875 - 0.055y, 0.875 + 0.055y), (1.185 - 0.348y, 1.185 + 0.348y)\Big)$$
$$\tilde{q}_{55} = \Big((1.589 - 0.103x, 0.509 + 0.103x), (0.618 - 0.152y, 0.618 + 0.152y), (1.130 - 0.102y, 1.130 + 0.102y)\Big)$$

Step 4: By using Eq. (4.1), scores of elements of decision matrix PD are obtained as follows:

$$SM = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{array} \begin{array}{c} \begin{array}{ccccc} d_1 & d_2 & d_3 & d_4 & d_5 \end{array} \\ \begin{pmatrix} 0.442 & 0.375 & \mathbf{0.606} & 0.430 & 0.536 \\ \mathbf{0.842} & 0.459 & 0.819 & 0.689 & 0.826 \\ 0.499 & 0.399 & \mathbf{0.714} & 0.573 & 0.674 \\ 0.417 & 0.113 & 0.542 & \mathbf{0.546} & 0.470 \\ 0.490 & 0.198 & 0.456 & 0.423 & \mathbf{0.617} \end{pmatrix} \end{array}$$

Step 5: According to score matrix SM, we say that patient 1 suffer from typhoid, patient 2 suffer from viral fever, patient 3 suffer from typhoid, patient 4 suffer from threat disease and patient 5 suffer from malaria.

6 Conclusion

In this paper, some new concepts and operations was defined such as GSVNNs, $\alpha-$cuts of GSVNNs, parametric forms of GSVNNs and arithmetic operations of GSVNNs. Also, based on operations between parametric forms of GSVNNs and composition of matrices, a decision making method was proposed and presented an application in medical diagnosis based on hypothetical data. In future, Cauchy single-valued neutrosophic numbers may be defined and its properties can be investigated. Also, this study can be extended for other distributions in mathematical statistics. Furthermore, decision making methods can be developed for proposed new SVNNs.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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