SOME GENERALIZATIONS AND REPRESENTATIONS OF THE POSSIBILITIES OF MULTI-PARAMETRIC MAPPING OF AFFIN SPACE IN AN APPENDIX TO THE CURVOLINEAR FORMING AND TRANSMISSION MOVEMENT

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Abstract: *The works of Prof. B. A. Perepelitsa from Kharkov Polytechnic Institute and his disciples to develop an applied methodology of multiparameter mappings in relation to the profiling and functioning of complex curvilinear objects and transmission mechanisms in mechanical engineering, mainly with examples of gears, are presented. The completed review is gratefully devoted to the 10th anniversary of Fiability & Durability Journal, which has repeatedly provided its platform to the authors to highlight this development of the wide Romanian, European and world scientific community.*

Keywords: multiparameter mappings, curved surfaces, variable curvature, complex shaping, gears and gearing.

1. INTRODUCTION TO REVIEW AND BEGINNINGS OF DEVELOPMENT

Historically, Ukraine has been traditionally noticeable in the global market for the development of the theory and practice of complex shaping, especially in the current challenges of the aerospace, nuclear power engineering, defense industry, including in terms of compact gears and CVTs for armored military and advanced civil equipment. So, in the world chronological list of works of scientists who made the most notable contribution to the theory of gears according to the modern version [1] (S. P. Radzevich, 2018), the first seven, followed by the representation of Italy (L. Da Vinci, 1493), Russia (L. Euler, 1754), England (R. Willis, 1841), France (T. Olivier, 1842) and the USA (F. Reuleaux, 1861, and J. W. Gibbs, 1863), closes H. I. Gofman's master degree work on mechanics [2]. It developed the analytical form of arbitrary gears and the basics of the modern analytical theory of gearing, created engineering methods for calculating and designing gearing, developed new types of gearing, the theory of kinematic pairs and chains, investigated the structure and derived the general equations of mechanisms, first classified them based on objective features which are determined by structure levels and movements.

The same prestigious list of works by 60 scientists includes Kharkov researcher A. I. Pavlov [3] and Hungarian researcher I. Dudas [4], who collaborates with the Kharkov scientific school in the applied development of the mathematical apparatus of multi-parameter mappings of space for gear wheels and gears [5]. The works [6-8] and some others in the list [1] also represent a number of ideologically close to development presented here on the mathematical apparatus and approaches used in it. In our experience, [9-14] also should be considered in this series of works.

At the same time, some well-known developments to enhance the adaptive capabilities of the mathematical apparatus of multiparameter mappings of affine space, for which the improvement of the theory and practice of gears is not an end in itself, but one of the directions of applied implementations, i. e. subordinate theme, are missing in the selected series of particularly significant works on the theory of gears [1]. Most likely, this is what explains the absence. Development of the Kharkov scientific school of Prof. B. A. Perepelitsa and his disciples at the Kharkov Polytechnic Institute (NTU "KhPI") [15-21] refers precisely to such works of a wide range of applications, including with respect to the description, improvement and synthesis of new gears and gearings. The history of this development is especially rich in fruitful cooperation with the Kiev Polytechnic Institute (P. R. Rodin, academic fundamentalist of the theory of instrumental production for complex cutting, and others at the NTUU "Igor Sikorsky KPI") [22, 23] and the Institute for Superhard Materials (A.V. Krivosheya, world class coryphaeus in the theory and practice of tooth processing, now the leading specialist in Ukraine in this object-oriented area of applied development of the general theory of multiparameter mappings, and others at the V. Bakul ISM of the NAS of Ukraine) [24-27]. The significant results of this collaboration are jointly summarized in [28] by the key participants mentioned above.

2. BRIEF EXCURSE TO GENERAL THEORY AND METHODOLOGY

The fundamentals of the new direction in the theory of shaping and designing of cutting tools and objects to be processed by them, based on the formalizing and generalizing capabilities of the mathematical apparatus of multiparameter mappings of affine space, were developed to the beginning of the 80-s of last century [15-17].

It was established that a general mathematical model of shaping during cutting can be a regulated multiparameter mapping. The general character of the model is that the workpieces with their edges are images, and the shaping movements of any complexity are one-parameter mappings. Consequently, it means to build one or another model of shaping by synthesize the corresponding mapping, that is, to completely determine its components. Initially, this requires completely, but in general terms, to determine the structure of the model, parameters and operators, the nature of the pre-image, operators, and functional connections between the parameters. Then it is necessary to assign or calculate the specific numerical content of all components of the model in accordance with the initial data. In the synthesis of the model, it is necessary to take into account the known regulatory conditions of contact and intersection, as well as the conditions for the completeness of shaping.

There are formulated the theoretical principles of modeling, which reflect different degrees of algorithmic generality: the principles of directional synthesis, coincidence, replacement, and inversion of mappings; principle of regulated intersection of images, etc. Based on these principles and the general model, systems of typical and specific models can be formed, each of which corresponds to a method or group of methods of real shaping, which potentially allows to predict and develop new processing methods.

Multiparameter mappings are used as a general methodological basis for geometric design and shaping. In both processes, the same theoretical principles of modeling and the general mathematical apparatus operate. This commonality allowed to develop an automated structural modeling system, which covers a wide range of seemingly different in nature modeling and control problems in the design and shaping.

The basis of the universality of the system are a multiparameter mapping and a structural method based on the unification of parameters, operators and functional relationships. It is enough to specify a mapping in order to fully define the corresponding part or tool as a geometric shape.

By specifying the structural models of the workpiece, the machining tool and the shaping, as well as the corresponding numerical parameter arrays, this information can be converted into a form convenient for technological realization, for example, for computer-controlled machines and systems equipped with microprocessors and mini-computers. At the same time, it is advisable to introduce unified operators into the structure of the forming surface at the design stage of the part, providing for the use of the same operators during processing. In other words, if necessary, it is possible to foresee and use an algorithmic generality between the design of the part and its shaping.

A technique has been developed for applying of mappings for the mathematical description and construction of parts and tools as limited multi-parameter images. For example, by successive one-parameter mappings with the help of two operators (rotation and parallel transfer), it is possible to synthesize a gear from a starting point as a multi-parameter area of space. Gear equations in vector, matrix and coordinate form can be obtained by substituting operators into the general equation of their mapping. The combination of these equations and parameter intervals fully describes the gear wheel as a set of its points (inside and on the surface). In the particular case when the operators of rotation and parallel translations are mapping, and the independent parameter is time, the one-parameter mapping represents motion. Consequently, the general mapping equation in this case is an equation of arbitrarily complex motion.

Schematically generalized kinematic scheme of shaping for gear links of various classes, types and types proposed by prof. B. A. Perepelitsa is represented in Fig. 1. The movements of the initial shape forming link in its own coordinate system are indicated in the first coordinate system, and movements of the first coordinate system with the moving initial shape forming link are indicated in the second coordinate system, etc.

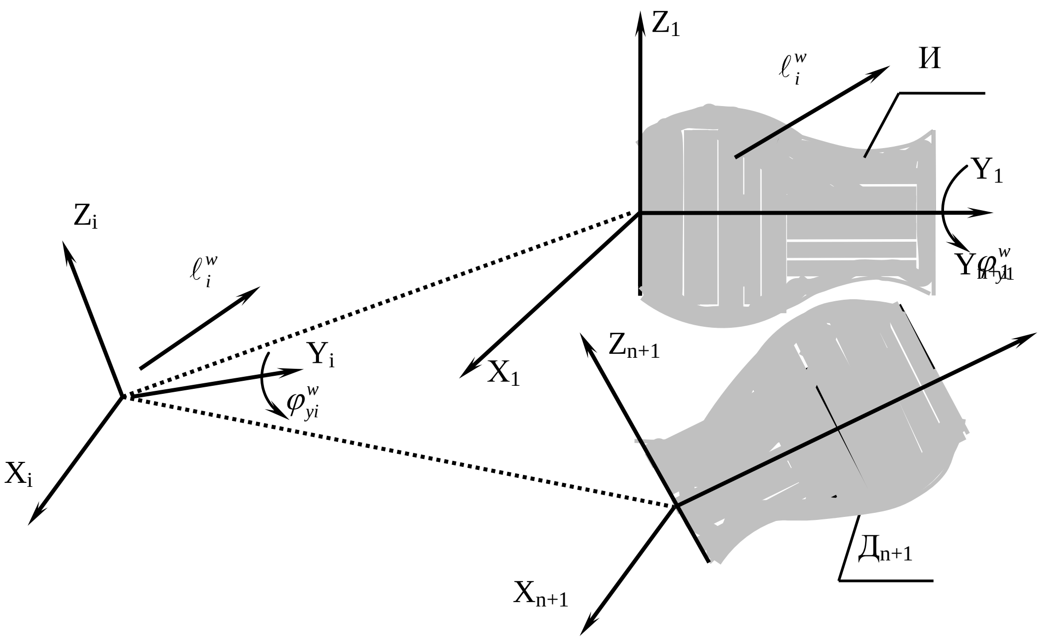

Fig. 1. The generalized kinematic scheme of shaping for gear links [17]:
$И_1$ and Дn+1 – forming and formable gear links, respectively

According to the generalized kinematic scheme of shaping (Fig. 1), its generalized unified mathematical model, using 4th order matrices, can be represented as (original identification [17] is preserved):

$$
\begin{cases}
m_{r\partial(n+1)И/Д} = m_{v_n^w c_n^w} m_{\varphi_n^w \ell_n^w} \ldots \\
m_{v_i^w c_i^w} m_{\varphi_i^w \ell_i^w} \ldots m_{v_1^w c_1^w} m_{\varphi_1^w \ell_1^w} m_{r_{u1}} \\
\varphi_{1A}^u < \varphi_1^u < \varphi_{1A}^u \\
\varphi_{1A}^v < \varphi_1^v < \varphi_{1B}^v \\
\varphi_{1A}^w < \varphi_1^w < \varphi_{1B}^w \\
\left.\begin{array}{l} \varphi_i = f_{\varphi_i}\left(\varphi_1^w\right) \\ \ell_i = f_{\ell_i}\left(\varphi_1^w\right) \end{array}\right\}
\end{cases}
. \qquad (1)
$$

In system (1), the last two equations are the equations of communication; φ_1^u, φ_1^v – independent parameters of the surface of the gear rims of the forming gear link; $\varphi_{1A}^u, \varphi_{1B}^u$ and $\varphi_{1A}^v, \varphi_{1B}^v$ – respectively, the initial and final values of the independent parameters of the surface of the gear rims of the forming gear link; φ_1^w – independent parameter of the movement of the forming gear link; $\varphi_{1A}^w, \varphi_{1B}^w$ – the initial and final value of the independent parameter of the movement of the forming gear link; $m_{r_{u1}}$ – matrix equation of the forming surface or line of the forming gear link in the first coordinate system; $m_{\varphi_i^w \ell_i^w}$ – motion matrix of the forming gear link in the coordinate system *i*; $m_{v_i^w c_i^w}$ – position matrix, i. e. matrix of coordinate transformations in the transition from the coordinate system *i* to the coordinate system *i* + 1; $m_{r_\partial(n+1)И/Д}$ – the matrix equation of motion of the forming link ($И_1$), specified in the 1st coordinate system relative to the fixed formable gear link (Д), written in the coordinate system *n* + 1, i. e. in the coordinate system of the taken form gear link.

3. ANNEX TO SURFACES OF COMPLEX FORM

The development of structural mathematical modeling in relation to the processes of shaping the cutting of parts of complex shape leads to the development of matrix formalization of geometric objects of processing, that allow, firstly, on a uniform algorithmic basis, to consider both the geometric design of surfaces, and their shaping, and with a significant reduction in the amount of processed information; secondly, not only to model, but also to investigate the geometric properties of structures, to predict and develop on the basis of this new surfaces (Fig. 2), tools and kinematic schemes of shaping; thirdly, to provide the necessary prerequisites for processing complex-shaped parts without a drawing, using structural mathematical models as an intermediate storage medium. Based on the previous national experience of the calculation and methodological support for the processing of complex non-linear surfaces on machine tools with numerical

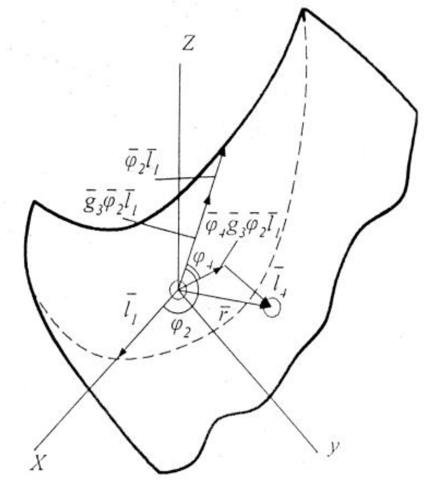

Fig. 2. *3D model of the turbine blade surface in the structure with mapping operators* [29].

control [30], a consistent development of a common method for calculating the trajectory and coordinates of instantaneous points of contact, applied to any shaped instrumental surface and based on special improvements and applied system development of the mathematical apparatus of multiparameter mappings of affine space, is carried out [18-20]. The addition of the existing methods of analytical description of discretely defined surfaces by the developed method of virtual modeling made it possible to expand the range of types and profiles of tool surfaces used on CNC machines.

4. ANNEX TO GEAR AND GEARING

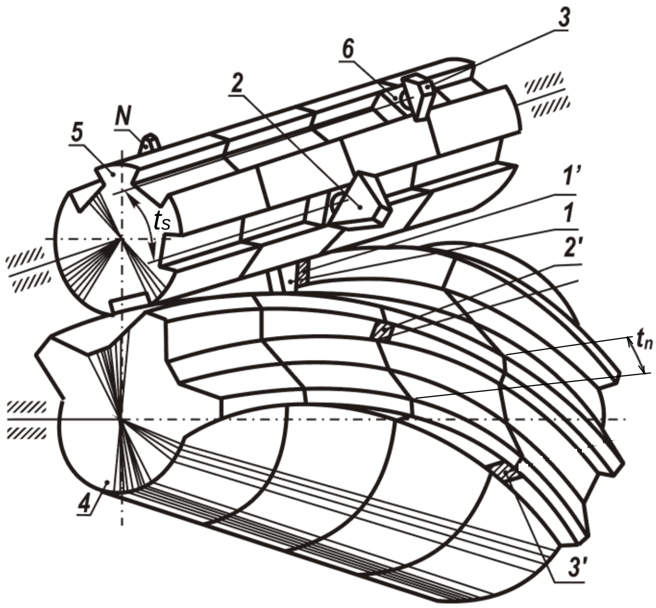

Fig. 2. *General schematics of double-link variator on the base of bevel gear with equidistant tooth lengthwise curves: 1, 2, 3, … N – movable tooth of spur gear; 1', 2 ', 3' – applicable contact zones; 4 – bevel gear with constant normal pitch; 5 – spur compound gear; 6 – sliding elements jointed with tooth* [31&21]

The relevance of improving the mathematical apparatus of analysis and synthesis of gearing, especially from the last quarter of the last century, respectively, and the development response, is fueled by the globalization of competition in the automotive industry, the demands of the aerospace industry and the military-industrial complex to create gears and variators of increased compactness and (or) supporting capacity, in particular two-parameter [31-35] (Fig. 3), with the spherical design of the teeth carrying elements [36, 37], etc.

5. DIRECTIONS OF INTEGRATION IN MECHANICAL ENGINEERING

Founded by Prof. B. A. Perepelitsa, with a brief overview of presented here related publications, working systematically develops applied principles and virtual concretization tools for a scientific theory of multiparameter mappings of affine space in the application to shaping of complex curvilinear surfaces, including gearing, that makes it possible to synthesize novel gears with the desired performance.

Ways of further integration of development in modern engineering, in accordance with its achieved theoretical level and applied assimilation, should be primarily associated with the triad of continuously relevant organizational, technical and technological tasks of increasing the competitiveness of enterprises of the engineering complex with mechanical and laser shaping of constructional parts and forming tools for aircraft , transport, energetics and others branches of the economy.

First, it is a virtual modeling of unitary objects and technical systems with new design solutions of increased functional efficiency, including using the developed structural approach to the mathematical description and visualization of kinematic curves [21, 38].

Secondly, it is the integration into the design of individual machining operations with varying degrees of certainty at the micro level kinematic-geometric schematics, including the prediction of the output microcutting parameters for optimizing the processing with abrasive tools at the macro level, by analogy with the developed adaptation of the general methodology that takes into account the functional specifics interference objects set forth in the works of the scientific school of prof. B. A. Perepelitsa [19, 23, 39].

Thirdly, is to use development in the creation of applied mathematical software for renovation algorithms and the original integrated design of highly efficient multi-operational production processes and complexes of such processes of an industrial enterprise, including in the framework of the implementation of the concept of simulation modeling in accordance with the experience of development of the NTU "KhPI" in this direction [26, 40].

6. CONCLUSION

The history of publication presentations of the development and adaptation of the mathematical apparatus of multiparameter mapping of affine space in application to curvilinear shaping and transfer motion in the journal Fiability & Durability [29, 34, 35, 38] goes back to its first issue [29]. The authors heartily thank the “Constantin Brancusi” University of Targu Jiu, which is the publisher, and its Fiability & Durability Editorial Board for the opportunities to cover this and other developments of the wide Romanian, European and world scientific community in the first decade of the journal and along with it.

REFERENCES

[1] **Radzevich, S. P.** *Theory of gearing: Kinematics, Geometry, and Synthesis, Second Edition.* Boca Raton, Florida, CRC Press, 2018, 898 p.

[2] **Gofman, H. I.** *The gear theory, generalized and developed by analysis*. Odessa, Novorossiysk University, 1886, 157 p. – In Russian.

[3] **Pavlov, A. I.** *Modern theory of gearing.* Kharkov, Kharkov National Automobile and Road University, 2005, 100 p. – In Russian.

[4] **Dudas, I.** *The theory and practice of worn gear drives*. London, Penton Press, 2000, 314 p.

[5] **Perepelitsa, B., I. Dudas, and N. V. Zubkova.** Unification of operators and elements of space mapping in 3D modeling of cutting tools. *Science and education. Collection of works for the 40th anniversary of the cooperation of the National Technical University "Kharkov Polytechnic Institute" and the University of Miskolc.* Kharkov, NTU “KPI”, 2004, pp. 169-174. – In Russian.

[6] **Baxter, M. L.** Basic Geometry and Tooth Contact of Hypoid Gears. *Industrial Mathematics*, 1961, Vol. 11, Part 2, pp. 19-42.

[7] **Lashnev, S. I.** *Generation of gear tooth flanks by means of rack-type and worm-type cutting tools.* Moscow, Mashinostroenie, 1971, 216 p. – In Russian.

[8] **Litvin, F. L.** *Theory of gearing*. 2nd edition. Moscow, Nauka, 1968, 584 p. – In Russian.

[9] **Lyukshin, V. S.** *Theory of screw surfaces in the design of cutting tools.* Moscow, Mashinostroenie, 1967, 372 p. – In Russian.

[10] **Grechishnikov, V. A., G. I. Kirsanov, and A. V. Kataev.** *Automated design of metal-cutting tools*. Moscow, Moscow Machine Tool Institute, 1984, 109 p. – In Russian.

[11] **Druzhinsky, I. A.** *Complex surfaces: Mathematical description and technological*

support. Leningrad**,** Mashinostroenie, 1985, 263 p. – In Russian.
[12] **Radzevich, S. P.** *Shaping complex surfaces on CNC machines*. Kiev, Vishcha shkola, 1991, 191 p. – In Russian.
[13] **Litvin, Faydor L.** *Development of Gear Technology and Theory of Gearing: US NASA Reference Publication 1406*. Cleveland, Ohio, Lewis Research Center, December 1997, 120 p.
[14] **Grechishnikov, V.A., A.R. Maslov, Yu.M. Solomentsev, and A.G. Shirtladze.** *Instrumental provision of automated production.* Moscow, Stankin, 2000, 204 p. – In Russian.
[15] **Perepelitsa, B. A.** On geometric shapes as affine varieties. *Applied geometry and engineering graphics.* Kiev, 1974, pp. 47-48. – In Russian.
[16] **Perepelitsa, B. A.** Cutters as affine manifolds. *Cutting and tooling*. Kharkov, Publisher at the Kharkov State University of the Vishcha Shkola Publishing Association, 1976, Iss. 16, pp. 36-40. – In Russian.
[17] **Perepelitsa, B. A.** *Mapping of affine space in theory of surface forming by cutting*, Kharkov, Vyshcha shkola, 1981, 152 p. – In Russian.
[18] **Perepelitsa, B. A.** *Automated profiling of cutting tools (theory and algorithms)*. Kharkov, KhPI, 1985, 107 p. – In Russian.
[19] *Development of the theory of three-dimensional (3D) geometric modeling of cutting tools and the processes of shaping surfaces based on multiparameter mappings of affine space*: *Research report (final).* Under the guidance of **B. Perepelitsa**. NTU “KhPI”, Theme / Report registration in Ukraine No. 0100U001083 / No. 0202U002517. Kharkov, 2001, 448 p. – In Russian.
[20] **Perepelitsa, B. A.** Systems of parameters of space mappings in the annex to 3D modeling. *Modern technologies in mechanical engineering.* Kharkov, NTU "KhPI", 2006, Vol. 1, pp. 402-408. – In Russian.
[21] *Creation of theory of unified multiparametric information database for CAD/CAM systems of gearing, tools and gear processing*: Research report (final). Under the guidance of **B. Perepelitsa** & **V. Dobroskok**. NTU “KhPI”, Theme / Report registration in Ukraine No.0108U001445/No.0210U001273. Kharkov, 2009, 452 p. – In Russian.
[22] **Rodin, P.R.** *Fundamentals of cutting tools design.* Kiev, Vishcha Shkola, 1990, 424 p.
[23] *3D-modeling tools, forming and allowance removal in cutting*. Ed. by **P. R. Rodin**. Kharkov, NTU "KhPI", 2001, 304 p. – In Russian.
[24] **Krivosheya, A. V.** Schematic representation of methods for synthesizing of shape-producing gearing systems. *Bulletin of KhSPU*. Kharkov, 2001, Iss. 6, pp. 129-139. – In Russian.
[25] **Krivosheya, A. V. et al.** The generalized classification of kinematic schemes for shaping gear wheels. *High technology in mechanical engineering.* Kharkov, NTU "KhPI", 2006, Iss. 2 (13), pp. 402-408. – In Russian.
[26] **Gutsalenko, Yu. G., A. V. Krivosheya, and A. V. Fadeev.** Working out means of CAD/CAM system infotainment for gearing design and production, *Physical and computer technologies, Proceedings of the 15th international scientific and technical conference, December 2-3, 2009, Kharkov*. KhNPK “FED”, 2009, pp. 234-240. – In Russian.
[27] **Krivosheya, A. V. et al.** On the problem of classification of kinematic schemes and mathematical models for the shaping of gears. *Visnyk NTU "KhPI".* 2014, No. 31 (1074), pp. 75-84. – In Russian.
[28] **Perepelitsa, B. A., P. R. Rodin, A. V. Krivosheya, and Yu. G. Gutsalenko.**

Multiparametric mapping of space in theory of tooth gear forming. *Cutting and tool in technological systems*. Kharkov, NTU "KhPI", Iss. 71 (2006), pp. 103-106. – In Russian.
[29] **Perepelitsa, B., Yu. Gutsalenko, and N. Krukova.** Methodology of data calculation for shaping of objects with curvilinear intricate profile. *Fiability & Durability*. Targy Jiu, Romania, Academica Brancusi, No. 1/2008, pp. 69-72.
[30] **Rodin, P. R., G. A. Linkin, and V. N. Tatarenko.** *Processing of shaped surfaces on machine tools with numerical program control.* Kiev, Technika, 1976, 198 p.
[31] **Кovalyukh, V. R.** Two-parameter gearing with equal-high-broad teeth and gashes. *Mechanical Engineering*, 1982, No.1, pp. 43-45. – In Russian.
[32] **Kovalyukh, V. R., and R. V. Kovalyukh.** A new method of controlling the speed of rotation of a gear drive and the synthesis of gear variators on its basis. *Theory of mechanisms and machines*. Kharkov, NTU "KhPI", 1988, Iss. 44, pp. 107-112. – In Russian.
[33] **Gutsalenko, Yu. G. et al.** Tooling design and development of shaping technology of bevel gears of double-link variators. *XXV microCAD International Scientific Conference 31 March – 1 April 2011. Section L: Production Engineering and Manufacturing Systems. University of Miskolc (Hungary)*. Miskolc, University of Miskolc, 2011, pp. 73-77.
[34] **Gutsalenko, Yu. G., A. L. Mironenko, and T. E. Tretyak.** Equidistant tooth generation on noncylindrical surfaces for two-parameter gearing. *Fiability & Durability*. Targy Jiu, Romania, Academica Brancusi, No. 2 (8)/2011, pp. 67-72.
[35] **Tretyak, T., Yu. Gutsalenko, and A. Mironenko.** Modeling of running cutters for shaping of improved noninvolute tooth gears. *Fiability & Durability*. Targy Jiu, Romania, Academica Brancusi, No. 1 (9)/2012, pp. 164-169.
[36] **Ternyuk, N. E., V. R. Kovaliukh, and Yu. V. Nasedkin.** Ways to machining of spherical gears. *Modern problems of cutting with tools from superhard materials: Abstracts of reports at the All-Union Scientific and Technical Conference*. Kharkov, 1981, pp. 46-49. – In Russian.
[37] **Kovaliukh, R. V. et al.** Kinematic schemes of forming of spherical gears with equidistant teeth lines. *Cutting and tooling in technological systems.* Kharkov, Kharkov State Politechnic University, 1996, Iss. 50, pp. 105-109. – In Russian.
[38] **Tretyak, T. et al.** Structural approach to the mathematical description and computer visualization of plane kinematic curves for the display of gears. *Fiability & Durability*. Targy Jiu, Romania, Academica Brancusi, No. 1 (21)/2018, pp. 7-11.
[39] **Perepelitsa, B. A., Yu. G. Gutsalenko, and N. V. Zubkova.** Simulation methodology of cutting process and calculation of output indices on the base of multiparametric mappings, *Visnyk NTU "KhPI"*, 2004, No. 44, pp. 155-160. – In Russian.
[40] **Shelkovoi, O. M., V. A. Fadeev, Yu. G. Gutsalenko, and E. A. Kostygov.** Experience of application of simulation modeling in solving the problem of modernization of the production units in the aviation industry. *Aerospace and technology*. Kharkov, National Aerospace University "Kharkiv Aviation Institute", 2008, Iss. 7, pp. 181-188. – In Ukrainian.