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THE CONTRIBUTION OF OPTIMAL TURBO FAN TRANSPORT AIRCRAFT CLIMB SCHEDULE TO AIR COMPANY ECONOMY

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Abstract. Today airlines are challenging two opposite goals: minimization of flight fuel consumption and minimization of elapsed flight time. A well-known cost structure and cost generators represent significant pre-conditions for defining cost optimization strategy in an airline company. Airline company management has a limited set of tools for cost managing, which include the following documents: Performance Engineers' Manual and Aircraft Flight Manual. In this particular paper review we discuss the problem of vertical flight path of turbo-fan aircraft, where we point out the impact of the choice of climb technique on the overall *en route* flight profile costs. In temporary aircraft flight preparation process, there is no stressing out the significance of the climb phase in minimizing costs of this particular flight phase. In the paper we show the procedure of defining optimal climbing resulting minimum costs, but also optimal function operational adjustment to the climb schedule. This way of the approximation of optimal function and its adjustment to the operational use enables the application of minimal cost climbing technique in operational use of transportation aircraft. On short-haul flights, climb phase can reduce cruise flight length up to 60% of total range. In the paper, we show the impact of climb regime on flight profile of turbo-fan aircraft considering the usage of time, fuel and costs. The impact is shown according to the data taken from the Performance Engineers Manual. The impact stresses the importance of minimum costs climbing regime to cutting down total flight costs. It also shows the conditions which need to be fulfilled in order to apply minimum costs climbing technique. We identify the scope of *CAS* speed during climb and *TOC* for flight's minimum total expenses by using minimum costs climbing technique. Conditions for achieving minimum costs climbing technique are the results of the logarithmic differential. In order to achieve optimal numerical results we used Newton-Raphson formula.

Keywords: air company, economy, transport aircraft, climb, performance, optimization, cost.

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1. Introduction

The world as we know it is changing in a rapid way due to the crisis of global economics which directly affects air transport. Concerns over fuel consumption are not just focused on road vehicles and electrical power plants, but now on air carriers and general aviation as well. All of these issues need to be addressed to future air traffic systems, and new technology, from basic aircraft configuration to the aircraft engines and subsystems, and the airspace in which they operate. All of this must be accomplished while maintaining or improving flight safety. All kinds of transport use combustion of fossil fuel as main propellant. One kind of traffic is air traffic with participation of 2–3% in global fossil fuel consumption. To compare, the whole transportation sector currently accounts for 20–25% of all fossil fuel consumption. Thus, the aviation sector consumes 13% of the fossil fuel used in transportation; it is the second biggest sector after road transportation, which consumes 80% (IPCC 2001). The average increase of global air transport is predicted around 5.3% per year, till 2025 (Rogers *et al.* 2002).

This paper develops a strategy how to satisfy two contradictor aims: minimization of flight fuel consumption and minimization of elapsed flight time. This strategy is based on the calculation of direct operating costs which consist of consumed fuel cost and elapsed flight time cost. Two major groups of direct operating costs reduction can be separated.

The first case are actions, which save fuel and flight time but need investment such as new generation engine, new generation combustion chamber, winglet installation, re-engine, better software for FMC improvement, etc. All have the same nature, investment cash or credit, which give additional ballast on air carrier finance with long-term and sometimes unexpected results (Arljukova 2008). The second group are the cases where air carrier is in search for fuel and time saving use assets which are available, without additional investment in aircraft. This paper is tribute to the second group of methods for fuel and time saving. The place where one can find improvements in fuel savings are the aircraft performances.

The aircraft performance plays major role in the cost of fuel consumption and cost of flight time reduction of today's commercial airlines. In all kinds of transport papers with the aim to determine reduction fuel consumption and time such as (Mickūnaitis *et al.* 2007) were published. Also, interesting investigation is done in the area application of alternative fuels: Lingaitis, Pukalskas (2008), Al-Hasan, Al-Momany (2008) and Boychenko *et al.* (2008).

2. The flight time and fuel consumption of short-haul flights

In order to make comparison of fuel consumption and elapsed flight time we establish comparison on short-haul flight profile (EUROCONTROL 2007) published document for the use of different cost-benefit analysis, in which average air route with distance of 826 km (445 Nm) for IFR (Instrumental Flight Rules) type of flights is recommended. The document analysis (Single European Sky II 2008) confirmed that the average flight in European Union is less than 500 Nm. Besides, short-haul in air traffic includes flight which is under 1.5 Fhr.

Most used aircraft in EU is B737300 in different series. B737300 aircrafts make up to 90 percent of low-cost airlines fleet, which in 2007 covered 40 percent of EU air traffic market. According to (EUROCONTROL 2007) data, in order to make cost-benefit analysis, statistics of the use of different types of aircrafts in European airlines fleet were made. According to the statistics, B737 makes up to 10.1 percent (1422 aircrafts) of total aircraft number, which makes it most widely used type of carrier on singular basis.

3. Fuel consumption and elapsed time in climb phase

The previous work published in papers had the focus on cruise flight phase analysis as most dominant. The aim of this paper is to develop the importance of climb flight phase and introduce impact of flight time and consumed fuel of climb phase.

Contrary to other analyses, this paper fixed cruise and descent phase and investigates possibilities of cost minimization. The analysis shows fuel and time consumption. On short haul flights, where climb phase reduces range in cruise flight phase, even higher direct operating costs are generated in climb compared to cruise flight phase.

The paper provides comparison between minimum fuel consumptions cost and minimum flight time cost as a result of transport aircraft climb phase, a part of *en route* flight. From the comparison which was executed for B737300 with engines CFM56-3-B1, we can find difference of climb fuel and time cost.

Aircraft climb schedule comparison is based on the real aircraft data extracted from (The Boeing Company 1981) Performance Engineers Manual. The combination of high speed drag polar and installed engine data offer possibilities to achieve the result which can be applied in real aircraft exploitation. The flight *DOC* (Direct Operations Costs in USD) consists of time dependent costs and fuel dependent costs. The time dependent cost is usually expressed in USD and is the function of time consumed and time price in USD/Fhr. The fuel cost is usually expressed in USD and is the function of consumed fuel and fuel price in USD/kg. The results of climb optimization are optimal *CAS* (Calibrated Air Speed) speed distribution with climb altitude, which result in minimum direct operating costs. The condition for optimal *CAS* speed is obtained by logarithmic differential method, which is numerically solved by Newton- Ramphson method. The final result is adopted for the application in on board FMC (Flight Management Computer), where climb schedule from minimum direct operating costs can be entered.

4. Aircraft climb and en route flight

The presented research is based on *en route* flight phases which include climb, acceleration, cruise and descent flight phases. All examined flight phases are segmented in small increments which is a usual method of step by step calculation in point-mass performance model as shown in (Jenkinson *et al.* 1999). *En-route* phase starts with climbing to 1500ft QNH to *TOC* (Top of Climb Altitude in ft) based on optimal climb law, for minimum direct operating costs, acceleration phase on cruising height to cruising speed $M_{cr} = 0.74$, cruising with constant height. From *TOD* start descent flight phase, with constant descent law 074/250 kt to 3000 ft QNH. QNH is a pressure setting used by pilots, ATC to refer to the barometric altimeter setting which will cause the altimeter to read altitude, AMSL (Above Mean Sea Level) within a certain defined region.

This cruise and descent regime was taken from aircraft's FPPM (Flight Planning and Performance Manual). Such cruising and descending phase model was chosen in order to display the usefulness of minimum direct operating costs climb.

For climbing maximum climb thrust setting is used, whereas in cruising, program of continual thrust setting decrease is used as cruise progresses, as the cruise is done under const h (pressure altitude in ft) and const M_{cr} (Mach number in cruising).

In descending, regime of minimum power is used (low idle thrust). A special care is applied on the climb flight phase. In order to set climb parameters, we applied basic climb equals, in which height altitude of 1500 ft QNH to *TOC* climb altitude was divided to b-th ($b = 1, ..., s$) segments.

Limitations on whose basis we calculate climb, cruise and descent flight phases are taken from (Houghton, Brock 1970) and (Jenkinson *et al.* 1999):

- $T_{\max c}$, maximum climb thrust in N

$$T_{\max c} = Tn_c \quad \forall h_b, b = 1, \ldots, s, \tag{1}$$

- Tn_c, available climb thrust (N),
- F_f (fuel flow in kg/sec)is function of h, M and climb thrust,
- climb can be considered up to height which represents operative top of the flight, and which is defined by max $ROC = 2.54$ *m/s*,
- flight is straight, without turns or change of flight direction,
- the change of γ, climb angle (in degree), is small, $\dot{\gamma} = 0$,
- the equations which describe flight during climb in each segment of climb are calculated for accepted assumption of small climb angle Houghton and Brock (1970), $\gamma < 13$, which results in $\cos\gamma \approx 1$, $\sin\gamma \approx \gamma$,
- Center of Gravity position, do not have influence on drag value obtained from high speed polar,
- *CAS=IAS* or we neglected installation factor of pito-static tube,
- ISA (International Standard Atmosphere) condition.

For each climb segment b, we calculated used time t_{cb}, fuel g_{cb} and range $X_{c\,b}$. Equations which describe climb, cruise and descent flight phase are taken from (Hull 2007). For basic segment b of climb, it is possible to define time in sec, fuel needed in kg, in order to climb from height h_{b-1} to height h_b:

$$t_{cb} = \int_{h_{b-1}}^{h_b} \frac{dh}{ROC_b}, \quad (b = 1, \ldots, s), \tag{2}$$

- fuel needed to climb from height h_{b-1} to height h_b

$$g_{cb} = \int_{h_{b-1}}^{h_b} \frac{F_{f\,b}}{ROC_b} dh, \quad (b = 1, \ldots, s), \tag{3}$$

- horizontal range covered during climbing X_{cb} from height h_{b-1} to height h_b

$$X_{cb} = t_{cb}\left(\cos\gamma_b\right)M_b\ a_{sl}\sqrt{\theta}_b,\ \left(b = 1,\ldots,s\right). \qquad (4)$$

Climbing parameters, from $b = 1$, which fit to the segment of height at the beginning of climbing to $b = s$, which fit to the segment of height *TOC* are:

– total amount of fuel consumed during climbing

$$g_c = \sum_{b=1}^{s} g_{cb}, \qquad (5)$$

– total time spent during climbing

$$t_c = \sum_{b=1}^{s} t_{cb}, \qquad (6)$$

– total path distance covered during climbing

$$R_c = \frac{\sum_{b=1}^{s} X_{cb}}{1000}. \qquad (7)$$

After aircraft reaches *TOC*, we calculate fuel, time and distance needed to speed up to Mach number in cruise, M_{cr}. After cruise flight phase we determined parameters in descent flight phase. As a final result we can obtain total *en route* flight fuel g_t and time t_t.

5. Optimization model

The time dependent cost *COt* is usually expressed in USD and is the function of time consumed and *ct* (time price in USD/Fhr). Today fuel cost *COf* is usually expressed in USD and is the function of consumed fuel and *cf* (fuel price in USD/kg). The elementary cost function *C* (total *en route* flight cost in USD) relates to only cost of time and cost of fuel spent during each flight phase.

$$C = COt + COf. \qquad (8)$$

For each segment of climbing, it is possible to define it by the method of logarithmic differential, optimal CAS_b speed for minimum climb costs. Total operation climb costs on b-th climb segment C_{cb} (USD), consist of climb fuel cost and climb time cost. We can develop the function of climb costs on b-th climb segment, as shown in equation.

$$C_{cb} = \left(ct \cdot t_{cb}\right)/3600 + cf \cdot g_{cb}. \qquad (9)$$

Minimizing climb cost C_{cb} is possible if we minimize sum of costs of fuel spent and elapsed time on b-th climb segment.

$$C_{cb} = \left(ct \cdot \frac{dh}{ROC_b}\right)/3600 + cf \cdot \frac{F_{f\,b}}{ROC_b}\ dh. \qquad (10)$$

From C_{cb} we can develop total climb cost C_c as a sum of elementary costs on climb b-th segments, where ($b = 1, \ldots, s$). From determined *en route* flight fuel g_t (kg) and flight time t_t (Fhr), we can develop total *en route* cost C (USD) as a sum of climb, cruise and descent direct operating costs. After we arrange this redefined condition of minimum climb costs on g-th climb segment, we can calculate the definition of costs change with altitude $RDOC_b\left(\frac{USD}{m}\right)$ (*Rate of Direct Operating Costs*).

$$RDOC_b = \frac{ct}{3600}\frac{1}{ROC_b} + cf\,\frac{F_{fb}}{ROC_b}\,. \tag{11}$$

The aim of optimizing is minimizing of $RDOC_b$. The aim of optimizing can be achieved if we define optimal distribution of CAS speed with climb altitude. The technique used to find the condition of optimal CAS_b for each b-th climb segment is logarithmic differential. Let's define logarithmic differential of $RDOC_b$ on b-th climb segment. The condition for minimizing $RDOC_b$ is achieved by logarithmic differential $RDOC_b$ by CAS_b. The result of differential is, then, equalized with 0 and solved by CAS_b. The condition for minimum climbs costs, in case we optimize climb by CAS_b, is shown in equation.

$$dRDOC_b = \left[\frac{\partial \ln RDOC_b}{\partial \ln CAS_b}\right] d\ln CAS_b = 0, \tag{12}$$

$$1 = \frac{\partial \ln K_b}{\partial \ln CAS_b} - \frac{\partial \ln(Tn_{cb} - Rx_b)}{\partial \ln CAS_b} + \frac{\partial \ln F_{fb}}{\partial \ln CAS_b} + \frac{\partial \ln\left(\frac{1}{3600}\frac{1}{F_{fb}}\frac{ct}{cf}+1\right)}{\partial \ln CAS_b}\,. \tag{13}$$

As we can see from equation (13) the dominant parameter is the ratio of unit time and unit fuel cost, ct/cf non-dimensional parameter.

The solutions of the condition shown in equation (13), for different values of ct/cf, was shown on Fig. 1. In airline industry common operational economic parameter is *Cost Index* or *CI* (McLean 2006), which can be now introduced for easier application of condition in equation (13).

$$\frac{ct}{cf \cdot 100} = CI\,. \tag{14}$$

From equation (13) we can obtain conditions for CAS_b where ($b = 1, \ldots, s$) speed distribution with altitude, which minimize C_c. The optimal CAS_b distribution with altitude h is shown on Fig. 1 for different values of ct/cf.

6. Climb schedule determination

Once distribution of optimal CAS speed (for all b-th flight increments), with altitude h is defined, for minimum costs climb technique, it is necessary to enable operative use of achieved result, so that it can be applicable in FMC. That presents new optimization problem that is: adjustment of theoretic results to practical use in the way that optimization results are approximated in the form of constant speed CAS and constant M number (climb schedule) so that to be entered before take-off in FMC. Post-optimal results adjustment to operative use requires solving a new optimization task. The aim of post-optimal adjustment for application in FMC:

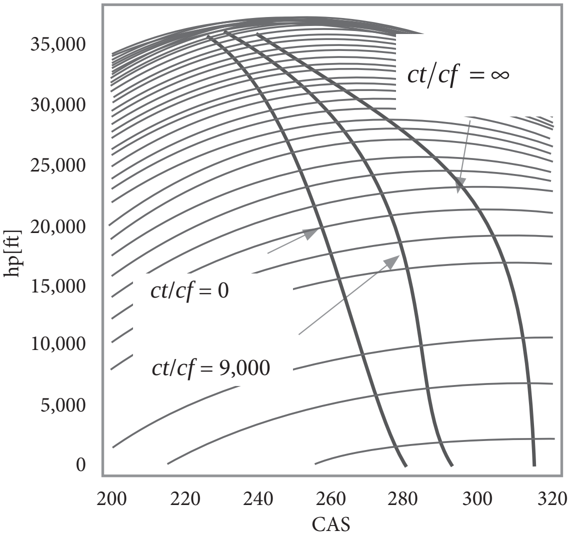

Fig. 1. The contours of constant *RDOC* and function of optimal *CAS* speed distribution for minimum *RDOC*, with h, for minimal direct operating costs, for B737300 with CFM56 engines, for $ct/cf = 0, 9000, \infty$

- to define constant speed *CAS*,
- to define h_c (cross over pressure altitude in ft)$_{o,}$ up to which constant *CAS* is applied and from what we begin to apply climb with const M number,
- to find *M* number that is constant during climb,
- constant *CAS* and constant *M* represent optimum combination (climb schedule).

It should take optimal result to get as close as possible to operative use, which means ATC restrictions must be obeyed.

The transport aircraft in flight operations use climb schedule in form of *IAS*/*M* as stated in (McLean 2006). Sub-optimal schedule of climbing has minimum exception margin from the optimal speed distribution in climbing as shown on Fig. 2. Sub-optimal climb schedule consists of *CAS* speed and *M* number, which makes it useful in operative exploitation obeying system limitations. Speed limits in all flight phases are in form of maximum speed CAS_{MO} (Calibrated Air Speed, Max Operating) and maximum M_{MO} (Mach number, Max Operating).

7. Software and test facility for validation

For the shown model of flight profile in which standard cruise and standard descend is presented, the influence of different climb technique has been analyzed. Climb techniques which are taken into consideration are: climb for minimum time cost, climb for minimum fuel cost and climb for minimum direct operating costs.

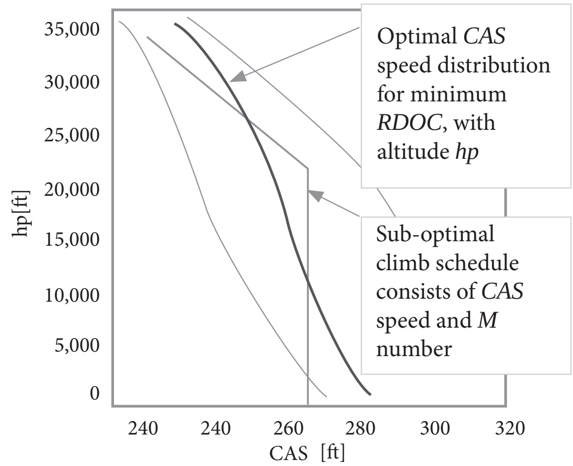

Fig. 2. Adjustment optimal *CAS* distribution with altitude *h* with climb schedule: const *CAS*, const *M* and h_{co}

There have been several experiments on presented model, in order to notice advantages and disadvantages of the use of each of the two climb techniques during *en route* flight profile. Aircraft flight model and presented model of aerodynamic and engine characteristics is used for the development of software *OPT-CL* on platform *Mathematica*®*5.1*.

Results which are acquired by software are *en route* flight profile with special climb flight profile presentation, diagram of change of *CAS* speed with altitude and tabular presentation of the result for the whole flight profile for the variation of climb technique.

The results obtained from software *OPT-CL*, which are prepared for operational use, were then validate at Flight Simulator at The Faculty of Traffic and Transport Engineering shown on Fig. 3. The Flight Simulator was developed by the authors of this article.

Fig. 3. The flight simulator at the faculty of traffic and transport engineering, University of Belgrade

8. Analyzing of influence of climb technique

There have been several experiments on the presented model in order to notice advantages and disadvantages of the use of each of the three climb ways during *en route* flight profile.

In order to research influence of the climb technique we took into consideration the following three factors that influence time, fuel and costs of total flight: *CI* (from

5 to 90), aircraft mass at the beginning of climb m_{c1} and altitude *TOC* (from 20,000 ft to 30,000 ft).

We analyzed the influence of mass change from m_{cl1} = 61,000 kg, $m_{c\ 1}$ = 51,000 kg and $m_{c\ 1}$ = 41,000 kg. For cruise, we analyzed M_{cr} = 0.74.

All analyses are done for the range *R* = 840 km. Results are presented in diagrams and show the range of altitude and value *CI* on which climb technique for minimum costs is applicable, because total costs of *en route* flight profile are minimum.

9. The analysis of results

Analysis of results that are achieved for the given variation of entering data enabled us to determine the range of use of climb technique for minimum costs in order to get minimum total costs of *en route* flight.

Let us first define the minimum of costs two values: total *en route* costs min($C_{\min time}$, $C_{\min fuel}$), for the application of climb technique for minimum amount of fuel ($CI \to 0$), $C_{\min fuel}$ (USD) and application of climb technique for the minimum amount of time ($CI \to \infty$), $C_{\min time}$ (USD). Where $C_{\min time}$ denotes climb cost for minimum time climb in USD, $C_{\min time}$ denotes to climb cost for minimum fuel climb in USD and $C_{\min time}$ denotes to climb cost for minimum cost climb in USD.

We can, then, define the savings, *St* (cost savings in %) that presents relation between min($C_{\min time}$, $C_{\min fuel}$) and total *en route* costs for application of climb technique for minimum costs, $C_{\min cost}$ (USD).

$$St = \frac{\min\left(C_{\min time}, C_{\min\ fuel}\right) - C_{\min \cos t}}{\min\left(C_{\min time}, C_{\min\ fuel}\right)} \cdot 100 . \tag{15}$$

The results of the first group of analysis, shown on Fig. 4 and Fig. 7 calculated for *TOC* on 20,000 ft, 22,000 ft, 25,000 ft, 27,000 ft and 30,000 ft and aircraft mass at the beginning of *en route* flight profile $m_{c\ 1}$ = 51,000 kg.

For such testing conditions, the interval *CI* has been defined, in which climb technique for minimum climb cost should be applied, in order to achieve minimum costs during *en route* flight profile. For *TOC* = 30,000 ft it is the range *CI* from 20 to 70, whereas the saving compared to climb technique with minimum time and climb technique with minimum fuel is equal (*break even cost*).

In case of *CI* = 50 as shown in Fig. 4 we can make savings up to 0.0399% of total costs of *en route* flight, if we apply climb technique for minimum costs. For *CI* less than 20, it should be necessary to apply minimum fuel climb technique.

For *CI* with value above 70 it should be necessary to apply minimum time climb technique in order to achieve minimum costs of *en route* flight profile. For the same conditions, if we only change *TOC* to 25,000 ft as shown in Fig. 4, we get that the range is *CI*, which require use of minimum costs climb technique is from 5 to 95. The saving for this conditions in time and fuel is equal for *CI* = 50 and is 0,053% of total costs of *en route* flight profile.

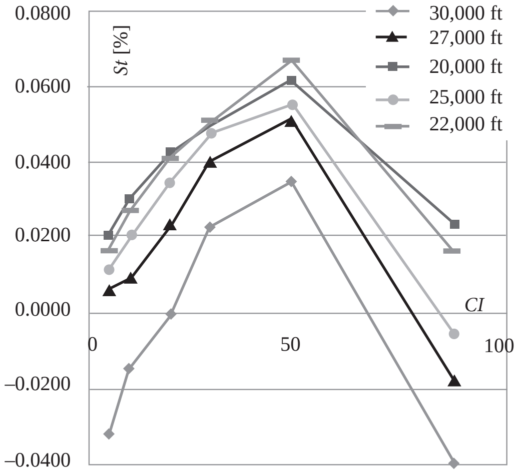

Fig. 4. Percentage of saving for *en route* flight profile total costs

Last case that has been analyzed is *TOC* = 20,000 ft, where the range of application of minimum costs climb technique *CI* from 5 to 90. The savings compared to techniques, climb techniques for minimum time and minimum fuel for *CI* = 50 and which is 0.05% total costs of *en route* flight profile.

For application *TOC* = 30,000 ft we can clearly identify

- zone of application climb technique for minimum fuel use from *CI* 0 to 20,
- zone of application of climb technique for minimum costs from *CI* 20 to 70 and
- zone of application of climb technique for minimum time from *CI* 70 to 90.

In *CI* zone from 20 to 70, we can define *CI* for which the maximum saving as a result of applying of minimum costs climb technique is *CI* = 50, for which we achieve saving of 0.0399% of total costs of *en route* flight profile.

For variation of *TOC* from 20,000 ft to 27,000 ft, we can conclude that only application of minimum costs climb technique can result in minimum of total costs of *en route* flight profile for *CI* from 5 to 80. Maximum of savings are achieved on *CI* = 50 and for *TOC* = 22,000 ft and is 0.0674% of total costs of *en route* flight profile. For the same case, if we analyze only climb phase, Fig. 5, we can achieve maximum savings which vary from 1.2% to 1.4% of climb phase costs.

Results of second group of analysis, shown on Fig. 6, 7 and 8 was calculated for *TOC* on 20,000 ft, 22,000 ft, 25,000 ft, 27,000 ft i 30,000 ft and aircraft mass at the beginning of e*n route* flight profile $m_{c\,1}$ = 41,000 kg, Fig. 6, $m_{c\,1}$ = 51,000 kg Fig. 7 and $m_{c\,1}$ = 61,000 kg, Fig. 8.

We can clearly define the change and movement of range *CI* in which it is necessary to apply minimum costs climb technique. In case $m_{c\,1}$ = 41,000 kg the range needed is *CI* from 5 to 20, whereas in case $m_{c\,1}$ = 61,000 kg the range needed is *CI* from 50 to 85.

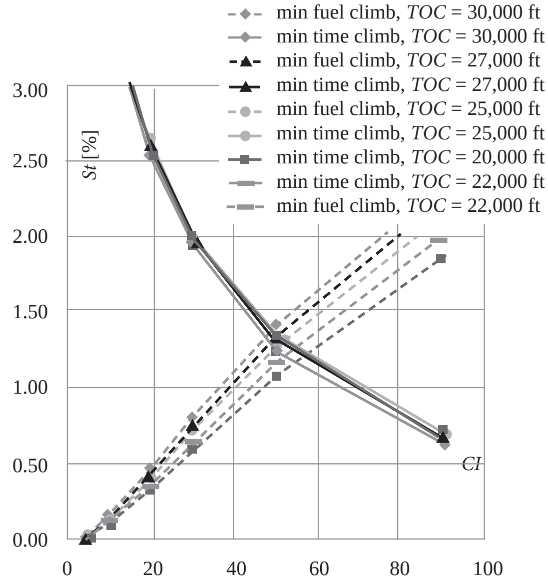

Fig. 5. Percentage of saving for climb phase. Costs

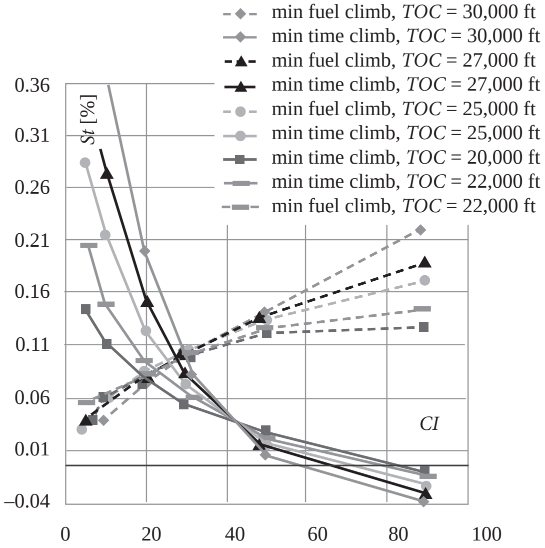

Fig. 6. Determination of the range of use of minimum costs climb technique, $m_{c\,1}$ = 41,000 kg

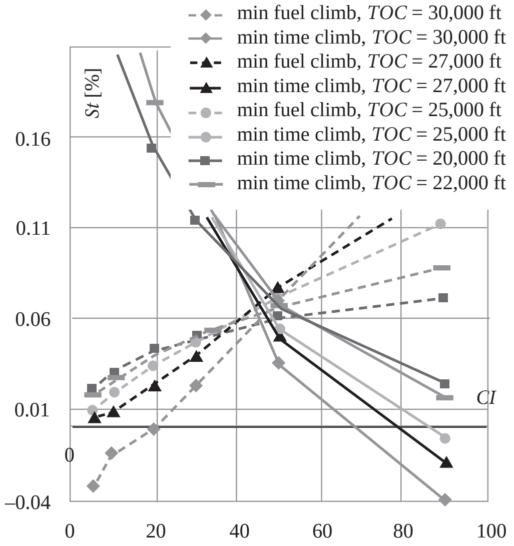

Fig. 7. Determination of the range of use of minimum costs climb technique, $m_{c\,1}$ = 51,000 kg

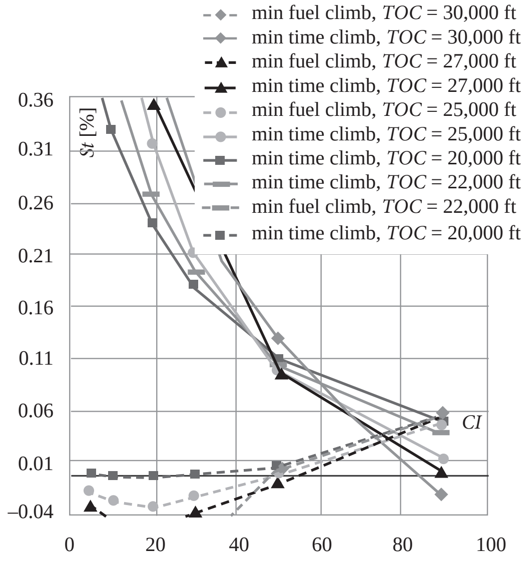

Fig. 8. Determination of the range of use of minimum costs climb technique, $m_{cl\,1}$ = 61,000 kg

The subject of the third group of analysis points out ranges *TOC* for the application of minimum costs climb technique, if we analyze total costs of *en route* flight for given *CI*, generates minimum costs.

The third group of analysis was shown on Fig. 9, 10 and 11. Flight analysis shows that for given starting conditions and $m_{c\,1}$ = 51,000 kg, for *CI* = 90 we achieve minimum total costs at the range of *TOC* from 20,000 ft to 25,000 ft, if we apply minimum cost climb technique. After *TOC* = 25,000 ft minimum of total costs is generated by minimum time climb technique, as shown on Fig. 9.

On Fig. 10 the same case of flight is shown, except *CI* = 30, where we can notice that minimum costs of total *en route* flight profile are achieved for range of *TOC* from 20,000 ft to 30 000 ft, if we apply minimum costs climb technique.

On Fig. 11 same flight scenario is shown, except *CI* = 5, where we can notice that minimum costs of the whole *en route flight* profile are achieved by using minimum costs climb technique or minimum fuel climb technique, for total *TOC* range which is use for this analysis.

The final, forth group of analysis was detected optimal *FL* for *TOC* for different values of *CI* which minimize *en route* costs *C*. Then, we apply different climb techniques for different *CI* and compare generated costs *C* for optimal *TOC* to other *TOC*, available for this aircraft.

There can be determined the measure of *en route* cost deviation, ΔC_D (%) of *en route* costs, generated with the application of minimum climb cost technique $C_{\mathrm{min}cost}$ (USD), as shown in equation (16). The referent *en route* cost can be developed as minimum from en route costs form flight with application of climb for minimum of fuel $C_{\mathrm{min}fuel}$ (USD) and climb for minimum time $C_{\mathrm{min}time}$ (USD), for all available *TOC*. That deviation ΔC_D (*en route* cost deviation in %), must be minimized, which is criterion for defining optimal *TOC* for is given aircraft mass, range and *CI*.

$$\Delta C_D = \frac{\min\left[\min\left(C_{\mathrm{min}fuel}, C_{\mathrm{min}time}\right)\forall TOC{=}20\,000\mathrm{ft},\ldots,30\,000\mathrm{ft}\right] - C_{\mathrm{min}cost}}{\min\left[\min\left(C_{\mathrm{min}fuel}, C_{\mathrm{min}time}\right)\forall TOC{=}20\,000\mathrm{ft},\ldots,30\,000\mathrm{ft}\right]}\cdot 100, \qquad (16)$$

$$\Delta C_D \underset{TOC}{\rightarrow} 0. \qquad (17)$$

In case of climb with $m_{c\,1}$ = 41,000 kg, Fig. 12, we found TOC_{opt} = 30,000 ft for range of *CI* from 5 to 50. For *CI* value from 50 to 90 there can be determined the decrease of TOC_{opt} (Optimal Top of Climb in ft) from 30,000 ft to 26,000 ft.

In case of climb with $m_{c\,1}$ = 51,000 kg, Fig. 13, we found TOC_{opt} form 28,000 ft to 30,000 ft for range of *CI* from 5 to 50. For *CI* value from 50 to 90 it can be determined decrease of TOC_{opt} from 28,000 ft to 22,000 ft. In case of climb with $m_{c\,1}$ = 61,000 kg, Fig. 14, we found TOC_{opt} form 27,000 ft to 24,000 ft for range of CI from 5 to 30. For CI value from 30 to 90 it can be determined decrease of TOC_{opt} from 24,000 ft to 20,000 ft.

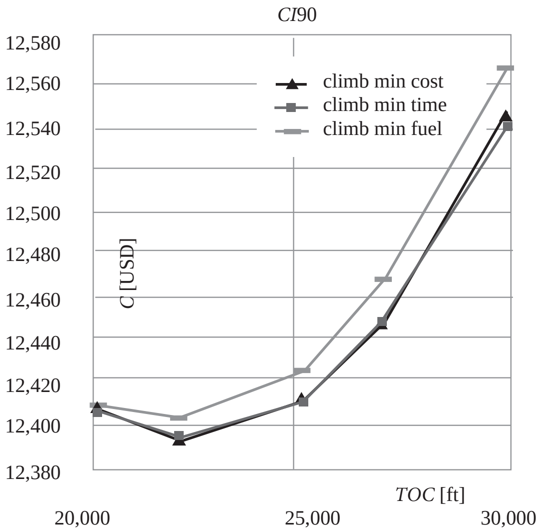

Fig. 9. Comparison of *en route* flight total costs for three types of climb techniques, *CI* = 90

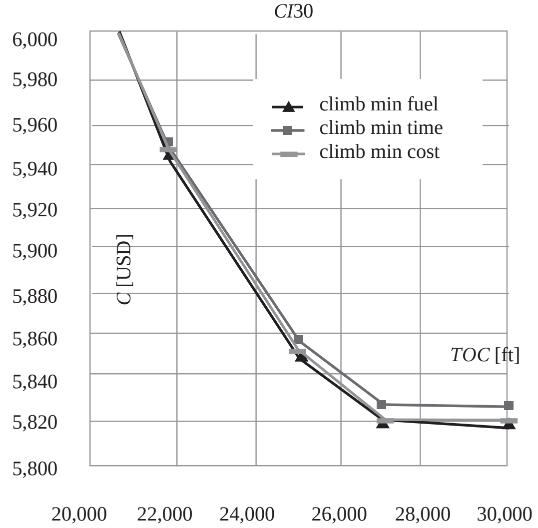

Fig. 10. Comparison of *en route* flight total costs for three types of climb techniques, *CI* = 30

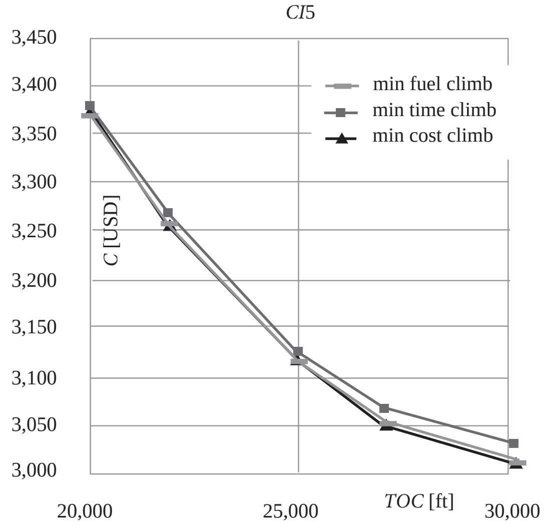

Fig. 11. Comparison of *en route* flight total costs for three types of climb techniques, $CI = 5$

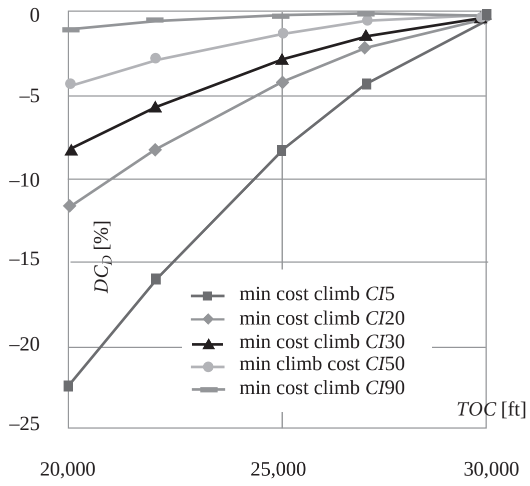

Fig. 12. The change of ΔC_D with *TOC*, for minimum cost climb technique, $m_{c\,1} = 41{,}000$ kg

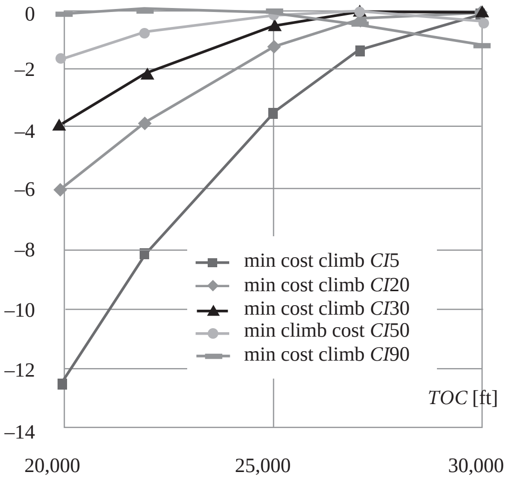

Fig. 13. The change of ΔC_D with *TOC*, for minimum cost climb technique, $m_{c\,1}$ = 51,000 kg

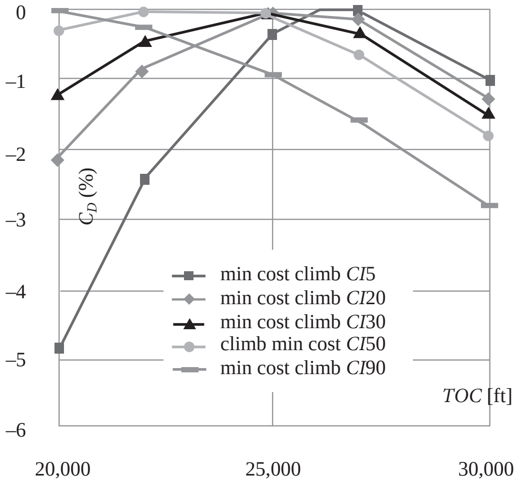

Fig. 14. The change of ΔC_D with *TOC*, for minimum cost climb technique, $m_{c\,1}$ = 61,000 kg

10. Conclusions

1. This paper connects direct operating cost of en route flight to minimum fuel consumption, minimum flight time and minimum costs for existing turbo fan transport aircraft. Also, there was examined the influence of different climb technique on total en route

direct operating costs. As support in economical flight planning process, we develop conditions for direct operating cost determination and minimization, during climb phase.

2. In the paper, new approach for defining conditions for minimum cost climb technique was defined, and we also defined new parameter for achieving optimization goal, minimization of *Rate of Direct Operating Cost*.
3. The optimal solutions for climb was then standardized in the form of climb schedule and prepare for operational use, for all kinds turbo fan transport aircraft.
4. As an extension of climb flight phase, then was examined total *en route* flight and there was determined optimal *FL* for *TOC*, for minimum direct operating cost. Only full understanding and wide approach will allow minimization of total turbo fan aircraft direct operating costs of *en route* flight. The management of air company derive strategy of minimization cost, presented by *CI*. The flight operations department, according to air company policy, with application of presented optimization model, can prepare operative parameters of flights: optimal climb schedule and TOC_{opt}.
5. Recalling assumptions of constant cruise Mach number and constant descent schedule, which isolated climb influence on total *en route* costs, it is obvious existence of further development of cost optimization in cruise and descent phase.
6. In the process of defining optimal *TOC*, it was shown influence of choice of TOC_{opt} on *en route* total operating costs. This results show the importance in both climb technique and choice of *TOC* under different costs represented by *CI*.
7. The paper defines conditions of avoiding trap which can arise, if we apply on *en route* flight, all the time, climb technique for minimum direct operating costs. The paper defines border of application different climb technique, which guarantee minimum *en route* direct operating flight costs.
8. In dramatic today economic world changing, the leadership in change is allways in transport aviation. The transport aviation must set up example of efforts to minimization costs as a part of global economy.

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TURBOSRAIGTINIO TRANSPORTO LĖKTUVO OPTIMALAUS KILIMO PLANO POVEIKIS ORO BENDROVEI

P. Mirosavljević , S. Gvozdenovic, O. Čokorilo

Santrauka

Šiandien oro transporte svarbūs du tikslai: skrydžio metu sunaudojamo kuro mažinimas ir skrydžio laiko trumpinimas. Žinoma kainos struktūra ir kainos komponentės yra reikšmingos išankstinės sąlygos, padedančios sudaryti kainos optimizavimo strategiją oro transporto bendrovėje. Oro transporto bendrovės valdytojai turi ribotą sąnaudų valdymo priemonių komplektą – tai „Priežiūros inžinieriaus vadovas" ir „Orlaivio skrydžio vadovas". Straipsnyje aptariama turbosraigtinio lėktuvo vertikalios skrydžio krypties problema, parodomas pasirinktos kilimo technikos poveikis bendrai skrydžio kainai. Dabartiniame lėktuvo ruošimosi skrydžiui procese nėra pabrėžiama kilimo fazės įtaka bendrų skrydžio sąnaudų mažinimui. Straipsnyje pateikiama procedūra, kaip nustatyti optimalų orlaivio kilimą mažiausiomis sąnaudomis, taip pat optimalumo funkcijos korekciją kilimo plane. Optimalumo funkcijos aproksimavimas ir jos naudojimas leidžia taikyti mažiausių sąnaudų kilimo techniką lėktuve. Trumpalaikiuose skrydžiuose kilimo fazė gali sumažinti kruizinio skrydžio trukmę iki 60 proc. Straipsnyje rodoma, kaip lėktuvo kilimo režimas daro poveikį turbosraigtinio lėktuvo skrydžiui laiko, kuro ir sąnaudų požiūriu remiantis duomenimis, paimtais iš „Priežiūros inžinieriaus vadovo". Skaičiai rodo, kaip minimalių sąnaudų kilimo režimas sumažina bendrąsias skrydžio sąnaudas. Straipsnyje taip pat atskleidžiamos sąlygos, kurios turi būti įvykdytos norint pritaikyti minimalių sąnaudų kilimo techniką. Ją naudojant nustatytas lėktuvo skrydžio greitis ir pakilimo aukštis. Minimalių sąnaudų kilimo technikos sąlygos yra logaritminio diferencialo sprendiniai. Optimaliems skaitmeniniams rezultatams gauti panaudota Niutono ir Rapsono formulė.

Reikšminiai žodžiai: oro bendrovė, ekonomija, transporto lėktuvas, kilimas, techniniai duomenys, optimizavimas, sąnaudos.

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