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NONLINEAR ANALYSIS OF STATICALLY INDETERMINATE WOODEN STRUCTURES AND OPTIMIZATION OF CROSS SECTION DIMENSIONS OF DOME RIBS

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Abstract: The analysis of the behaviour of natural structures of laminated wood domes and the numerous preliminary calculations have shown the possibility of saving materials by reducing the height of cross sections of meridional ribs. This is especially effective when you include in design of skins, performing a role of building shell, the collaboration with frame elements (annular and longitudinal ribs). Multiple static indeterminacy of such structure allows its non-linear work and the redistribution of forces under nonuniform loads. At the same time, the skin carries a significant part of the forces appearing in the shell and the ribs are underloaded. The stress-strain states of all elements are investigated. For the frame analysis the calculation is performed by the method of integral module that allows controlling strength resistance of a structure at any moment of its operation. The design recommendations for section dimensions of a shell are developed.

Keywords: laminated wood domes, plastic deformations, complex stress state, numerical calculations, integral module method, rational dimensions of element cross-sections

НЕЛИНЕЙНЫЙ РАСЧЕТ СТАТИЧЕСКИ НЕОПРЕДЕЛИМЫХ ДЕРЕВЯННЫХ КОНСТРУКЦИЙ И ОПТИМИЗАЦИЯ РАЗМЕРОВ СЕЧЕНИЙ РЕБЕР КУПОЛОВ

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Аннотация: Анализ работы натурных конструкций куполов из клееной древесины и многочисленные предварительные расчеты показали возможность экономии материалов за счет уменьшения высоты сечения меридиональных ребер. Особенно это эффективно при включении при проектировании обшивок, выполняющих роль ограждающих конструкций, в совместную работу с элементами каркаса (кольцевыми и меридиональными ребрами). Многократная статическая неопределимость такой конструкции допускает нелинейную её работу и перераспределение усилий при неравномерных нагрузках. При этом обшивки воспринимают значительную часть усилий, возникающих в оболочке, а ребра оказываются недогруженными. Выполнены численные расчеты оболочки в режиме реального времени при ступенчато изменяющейся односторонней (на половине покрытия) нагрузке с учетом сезонных ее колебаний в течение 5,4 лет. Исследовано НДС всех элементов. Для анализа работы каркаса расчет выполняется методом интегрального модуля, позволяющего контролировать силовое сопротивление конструкции в любой момент её эксплуатации. Разработаны рекомендации по назначению размеров сечений оболочки при проектировании.

Ключевые слова: купола из клееной древесины, пластические деформации, сложное напряженное состояние, численные расчеты, метод интегрального модуля, рациональные размеры сечений элементов

During the last two decades the production of large glued wooden structures has been formed. So it has become possible to design and build large-span buildings and structures, in particular, domes. There are already hundreds of such buildings [1]. Nowadays all stages of construction are being developed, the possibilities of new types of connections and protection of structure elements are being studied, etc.

Dome structures with a diameter of up to 100 m are usually made from frames in the form of arches, bars and fencing elements – slabs or floorings. Here slabs and floorings are fixed to a frame and further are engaged in its joint work. However, for a number of reasons this joint work is not taken into account while designing, and a part of system strength resistance is lost. The main ribs of a dome frame are meridian and traditionally with a section height of 1/40 span. They appear to be underloaded in the design taking into account the operation of slabs filling the cells between the ribs. The paper shows [2] that the meridional ribs of a conical dome can have a cross-section height of 1/70 span, what saves rib materials up to 25%. While the design and construction of the indoor skating rink in Moscow, the ribs between the diaphragms of short cylindrical shells were made with the cross-section height of 1/52,5 span equal to 42 m.

The facility has been successfully operated for 35 years (it was the first prototype [3]). The model experiments and the calculation by the integral deformation modulus [3] have showed that plastic creep deformations can be allowed under long-term loads.

The present work proposes the numerical studies of a dome with the diameter of 60 m and the cross-section height of ribs of 1/60 the span under long-term loads for up to 50 years. The reduction of loads under the absence of snow in summer time was taken into account. According to the previous publications the non-linear calculation determines the strength resistance of the structure taking into account the linear creep (the first stage by A.R.Rzhanitsyn) and the steady state (the second stage) [3]. It turns out that in the range of design loads the structures usually operate at the first stage.

The method of nonlinear calculation of a planar frame of laminated wood together with some thin sheathing is published in detail in numerous articles relating to various computational situations as well as in the monograph [3].

The calculation method is based on the method of integral module of deformations developed by V.M. Bondarenko in relation to reinforced concrete structures and it was adapted by the authors for the calculation of wooden structures. Taking into account the specific properties and structure work of wood the developed method is original and can be considered as a new theory.

The application of this method helps by iterative process to trace the changes in stress-stain states of structures under nonlinear and non-equilibrium long-term deformation, to take into account the process of forces' redistribution in individual cross sections and along the length of elements. This method makes it possible to linearize the calculation process and apply at each stage of successive approximations Betty's theorem on reciprocity of works, Maxwell's theorem on reciprocity of displacements, Moore's formula for displacements. In this case, the linearization keeps the relationship between the stiffness characteristics and the loading level. Temporary fixation takes into account the mode influence and duration of loading.

The diagram of wood operation, which was got experimentally, is shown in Fig. 1.

As the approximating function for the nonlinear relation

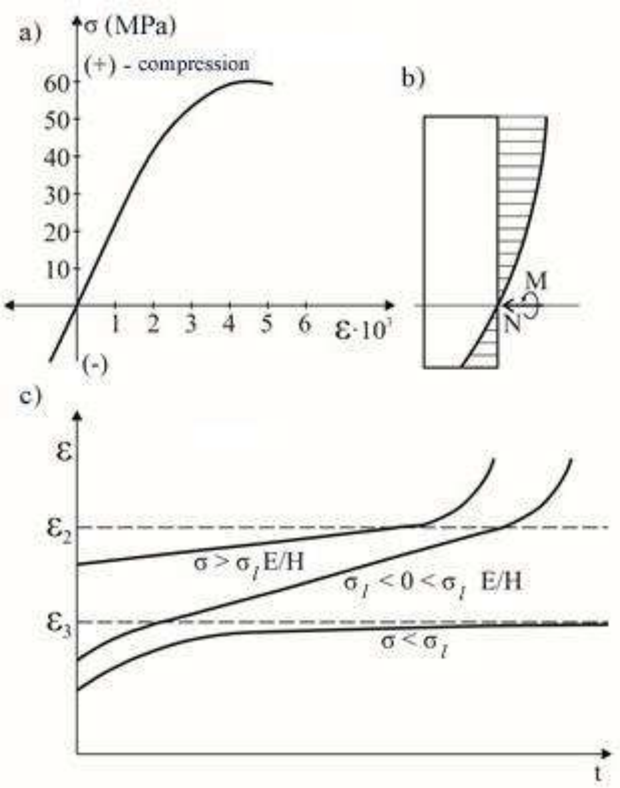
$$\sigma = f(\varepsilon)$$

the following equation is accepted:

$$\sigma = E_0\,\varepsilon - \frac{E_0^2}{4\sigma_{pp}}\varepsilon^2 \qquad (1)$$

The mechanical state equations are drawn up in relation to the three stages of creep according to Rzhanitsyn [4].

Numerous calculations have shown that in the range of design stresses the structures usually work at the first stage - linear creep, although the assumption does not reduce the generality of the solution, and it is easily possible to move to the second stage – steady creep. There is a transition time and the equation of mechanical state for this case.



<u>Figure 1.</u> Diagram of wood deformation: a – compression-tension; b – short-term bending; c –under the long-term action of a constant load.

In the method of integral estimations the process of successive approximations is a way of integral refinement of internal forces and stresses that transform in time due to their redistribution from more loaded areas to less loaded ones.

In statically indeterminate structures, where the distribution of forces is due to the nature of the change in stiffness, in addition to the process of internal iterations required to clarify the stiffness it is necessary to combine with it the process of external iterations clarifying the rule of forces distribution according to the stiffness data. The combination of internal and external iteration processes in solving the problem of stress and strain states is the following:

1) in an usual elastic-linear formulation a given statically indeterminate system is calculated and diagrams of internal forces are designed (zero approximation);
2) cross-sections are assigned, where the estimated stiffness is refined according the data of the forces of zero approximation (reduced modulus of deformation);
3) according to the new law of stiffness distribution the static calculation of the system is repeated taking into account the variability along the spans of the calculated stiffness. This clarifies the diagrams of force distribution along the axes of the system (first approximation);
4) for the forces of the first approximation, the calculated stiffness is again specified, and then force diagrams in the second approximation are found, etc. to a stable convergence with a given degree of precision.

To take into account the variability of rod stiffness, each element of the statically indeterminate system is divided into several sections, where the stiffness is considered constant and equal to the average stiffness of the boundary zones of the site. The given deformation modulus is defined as the arithmetic mean of several intermediate sites.

Changes in the external load over time in the calculation are reduced to a step scheme in such a way that within each of the intervals the load and all the characteristics of stress stain states (SSS) are considered constant. The same applies to the variability of physical and mechanical properties of materials.

In the nonlinear phase of creep instead of solving the differential equation of A.R. Rzhanitsyn the empirical dependence of Yu.M. Ivanov is assumed to be

$$\varepsilon(t) = \varepsilon(t_0)(1 + bt^{0,21}), \qquad (2)$$

where

$$b = \frac{10^{-2}}{0,735 - 0,02086\ W}, \qquad (3)$$

W – wood moisture, %.

The equation of prolonged deformation of wood when $\sigma < \sigma_l$ for a given step change of stresses can be written as following:

$$\varepsilon(t) = \varepsilon(t_0)\left[1 + b(t - t_0)^{0,21}\right] + \\ + \sum_{i=1}^{k} \frac{\Delta\sigma_i}{E_0 - \frac{E_0^2}{4\sigma_{pp}}\varepsilon_{i-1}^a}\left[1 + b(t - t_i)^{0,21}\right], \quad (4)$$

where

$$\varepsilon_{i-1}^a = \varepsilon(t_0) + \sum_{i=1}^{k} \frac{\Delta\sigma_i}{E_0 - \frac{E_0^2}{4\sigma_{pp}}(\varepsilon_{i-2}^a + \Delta\varepsilon_{i-1}^a)}, \quad (5)$$

ε_{i-1}^a – the total value of instantaneous (short-term) increments of relative deformations.

The equation (3) takes into account the influence of humidity W, but in the form of (4) the constancy of humidity at all stages of deformation is assumed.

Long-term strength depends on the wood moisture. Taking into account humidity E.N. Kvasnikov [7] obtained dependences of long-term strength on the duration of load action, on the humidity and on the type of stress state in the form of:

$$\sigma_l = a - b\lg t. \quad (6)$$

Here this value is taken to be 22 MPa, t – days, the coefficients a and b are determined experimentally.

The direct use of a nonlinear equation of material mechanical states in solving problems of structural mechanics is very bulky. So the authors use S.E. Frayfeld's proposal to introduce a time deformation modulus

$$E_l(t_0, t) = \frac{\sigma(t)}{\varepsilon(t_0, t)}, \quad (7)$$

where $\sigma(t)$ – stresses at the time of observation t; $\varepsilon(t_0,t)$ – relative deformations at the time of observation t, which are established taking into account the influence of the age of the material, its aging properties, the mode and duration of loading.

For the dependence corresponding to the linear creep the authors use the expression for the long modulus of deformation in the following form:

$$E_l(t_0, t) = \left[\frac{\varepsilon(t_0, t)}{\sigma(t)}\right]^{-1} \quad (8)$$

and then

$$E_l(t_0, t) = \left[\frac{\varepsilon(t_0)\left(1 + b(t - t_0)^{0.21}\right)}{\sigma(t)} + \right. \\ \left. + \sum_{i=1}^{k} \frac{\Delta\varepsilon_i}{\sigma(t)}\left(1 + b(t - t_i)^{0.21}\right)\right]^{-1}. \quad (9)$$

For a given step change of stresses $E_l(t_0,t)$ is to be:

$$E_l(t_0, t) = \frac{E_0 - \frac{E_0^2}{4\sigma_{pp}}\varepsilon(t_0)}{1 + b(t - t_0)^{0.21}} \cdot \left[1 + \right. \\ \left. + \sum_{i=1}^{k} \frac{1 + b(t - t_i)^{0.21}}{1 + b(t - t_0)^{0.21}} \cdot \frac{\Delta\sigma_i}{4\sigma_{pp}\left(1 - \frac{E_0}{4\sigma_{pp}}\varepsilon_{i-1}^a\right)^2}\right]^{-1} \quad (10)$$

where ε_i^a – active deformation.

There would be less mathematical difficulties associated with the use of equations of mechanical state to describe SSS of elements whose materials are deformed nonlinearly and with delay, if you apply the method of integral estimations, which is based on the use of the integral deformation module.

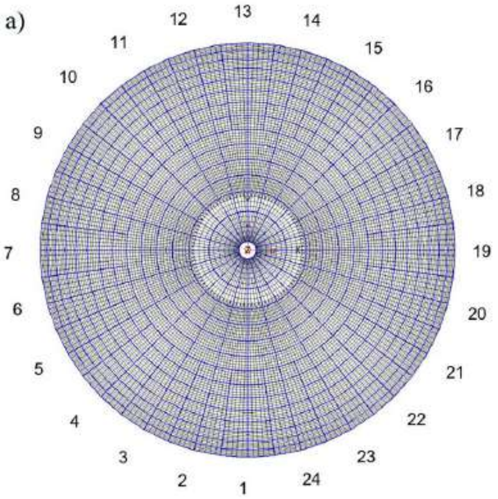
The following approach can be considered for the most common compressed-bent wooden element in constructions analogically with the derivation of equations for determining the long-term modulus of deformation. Estimating the real deformability of elements and at the same time not operating with different deformation modules in each discrete layer, it is possible to write the deviation of the values of real deformations ε and deformations ε_{in}, identified by $\varepsilon_{in}(x, t)$. The essence of the integral estimate is to minimize the deviation, which is carried out for the section as a whole, and after performing a series of transformations of the expression for the desired deformation module, the following expression for the first stage of deformation is to be (linear creep):

$$E_{in}(x,t) = \Phi(\varepsilon_\Phi^A, b, a)\frac{E_0 - \dfrac{E_0^2\varepsilon_\Phi^A}{4\sigma_{pp}}}{1+bt^{0,21}} \cdot \left[1 + \sum_{i=1}^{k}\frac{\Delta\sigma_i\left(1+b(t-t_i)^{0,21}\right)}{4\sigma_{mn}(1-\dfrac{E_0}{4\sigma_{pp}}\varepsilon_{i-1}^a)^2(1+bt^{0,21})}\right]^{-1}. \quad (11)$$

To validate the adopted design provisions the experimental studies have been carried out: the studies of the basic types of the coating shells of engineering constructions and residential buildings under asymmetric loadings when the most apparent redistribution of forces and nonlinear deformation take place.

THE STRUCTURES UNDER STUDY

The scheme of the ribbed ring dome is shown in Fig.2. The dome diameter is 60 m, the height - 20 m. The meridional ribs of laminated wood are located in increments of 3,926 m along the reinforced concrete support ring. The ribs are attached at the top of the dome to a metal lantern ring. The ribs via one are shortened due to the less stress because of ribs condensation. The height of the ribs is equal to 1/60 of the diameter, i.e. 1000 mm, the width – 140 mm.
The annular ribs of the cross section $b \times h$ 140x200 mm are located in increments of 2,464 m orthogonally to the meridional ribs.
The cells between the ribs are filled with plank-nail slabs resting on cranial bars. In the corner areas the gaps between the slabs and ribs are filled with the polymer solution (Fig. 3). The filling slabs are made from two layers of planks with the thickness of 15 mm. The thickness of plates was brought up to 40 mm. (Calculations were carried out also for the reduced thickness of plates of 20 mm).
The real-time calculations of the structures were carried out using the finite element method by MicroFe software complex with the control of the SSS of the shells using G.A.Geniev's strength criteria.



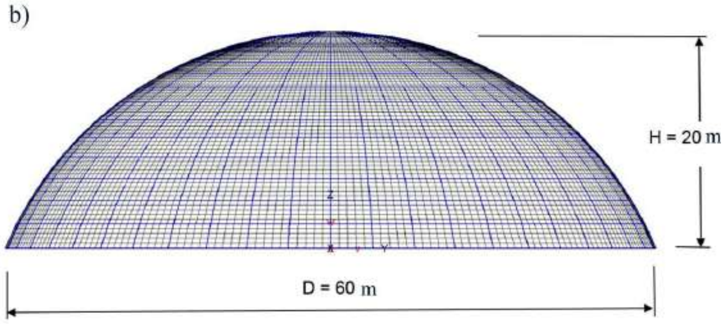


Figure 2. Scheme of ribbed ring dome: a) plan; b) facade.

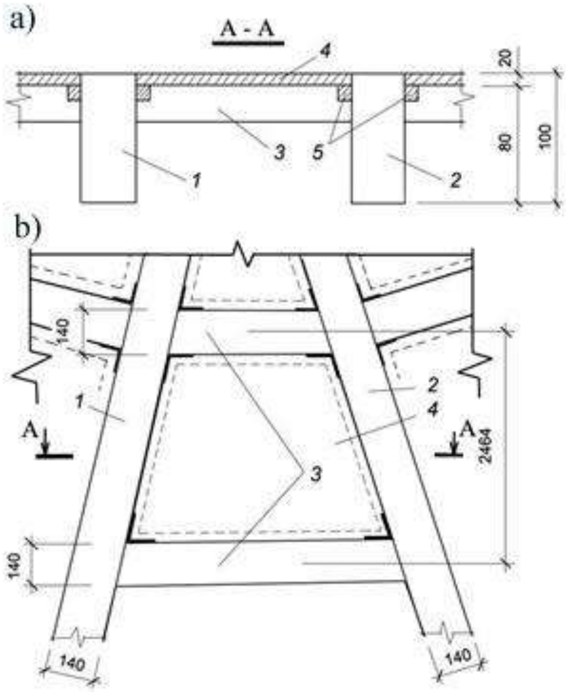


Figure 3. Scheme of plates on the dome frame: a) section; b) plan.
1, 2 – meridional ribs; 3 – annular ribs; 4 – panel; 5 – timber.

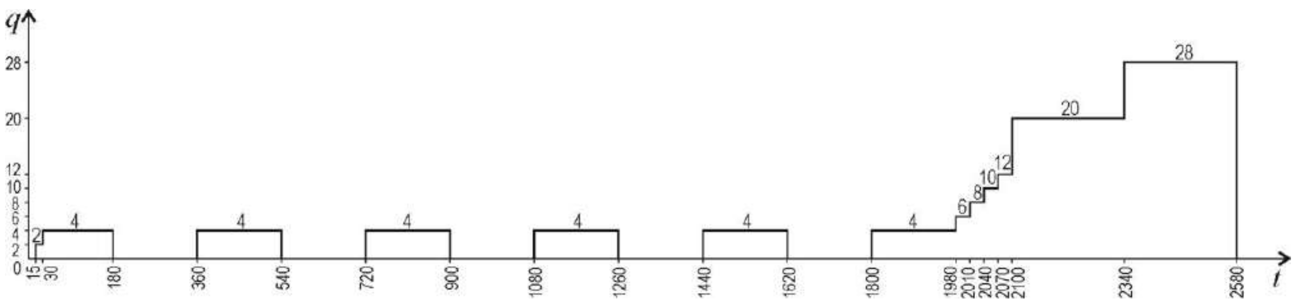


Figure 4. The mode of load application.

The program of loading at numerical calculations is shown in Fig. 4. The load is taken evenly distributed on the left half of the dome. The mode of load application mainly reflects seasonal changes in the snow load and the possibility of its uneven distribution over the surface.

Processing of the results of numerical calculations was carried out in accordance with the program determining the effect of nonlinearity of the deformations caused by prolonged action of load, seasonal change in the intensity of loads, the features of the joint work of the frame, the characteristics of redistribution of forces under uneven loads.

The main characteristics at the results processing are meridional and annular forces in the frame elements, shear forces and stresses in the skins, deflections of the entire structure and changes in the deformation modules depending on the magnitude and duration of loads.

The SSS characteristics were determined sequentially at 39 stages of loading. The maximum time of application of seasonal loads was taken to be 1980 days or 5.42 years. Further, the load was taken as incrementally increasing to assess the possibility of beyond design conditions for the structure.

The results of calculations are given in Table. 1.

Table 1. Stress-strain state of the dome with the covering of 40 mm thickness.

Stages of loading	*q, kPa*	Time exposure *t*	Stress in meridional ribs σ, *MPa*	Shear stress τ, *MPa*	Max deflections u_z, *mm*	Integral module E_{in}, *MPa*
1	Dead weight	1 min	0.0255	0.011	0	14120
3	2	15 days	0.873	0.503	-3.5	14109
5	4	180	1.45	0.97	-6.35	13785
9	4	360	1.474	0.972	-6.54	13715
13	4	720	1.46	0.972	-6.57	13685
17	4	1080	1.453	0.972	-6.58	13526
22	4	1620	1.451	0.971	-6.62	13465
27	6	1980	2.02	1.446	-9.57	13227
32	10	2070	3.20	2.378	-15.7	13212
34	12	2100	3.776	2.847	-18.7	13204
36	20	2340	6.126	4.66	-30.75	13157
38	28	2580	8.46	6.519	-43.15	13116
39	36	2580	10.67	8.37	-55.26	12457

THE CALCULATION RESULTS ACCORDING TO THE TABLE 1.

The behaviour of the dome under one-way load with intensity changing in time is investigated. It was simulated that there was no snow load in summer, and in winter the snow load was taken of higher intensity than its usual level relative to the middle European part of Russia. Calculation by the method of integral module of deformations allows defining of strength resistance of structures at any time of its operation at any changing loading.

The particular attention is paid to the effectiveness of joint work of the frame of laminated wood and wood panels.

The main indicators according to which the analysis of SSS of the investigated structure is carried out are the following: the value of the applied loads by stages and the duration of the structure exposure under this load, the stresses in meridional ribs, the shear stresses in the joints of skin elements to ribs, the maximum deflections and the value of integral module of the ribs deformations varied depending on the stress magnitude.

It was found that at the reduced thickness of skin equal to 40 mm the strength resistance of the shell is very large, and even at the maximum load exceeding the design of 3.2 kPa in 10 times, the stresses in the sections of meridional ribs did not reach the calculated value. The maximum stress under load of 36 kPa was 10.67 MPa.

Similarly, deflections throughout the loading process of 7.07 years vary by several mm and at the considered maximum load were only 55.26 mm, i.e. 1/1086 of the dome diameter. With a load of 4 kPa, the deflection was 6.35 mm, and when the shell was held under this load for 1440 days (almost 4 years), it increased up to 6.62 mm (or 1/9050 of diameter *D*), i.e. by 0.28 mm.

Similarly, the maximum value of normal and shear stresses for this period has not changed (Table 1).

Meridional ribs of the frame were taken with a reduced cross-section height to 1/60 of the span. The maximum normal stress in the supporting part of meridional ribs is 172.5 kN/m^2 ≈0.2 MPa.

So the shell thickness of 40 mm almost does not require ribs (sustainability is not considered here). At the same time, it should be noted that the value of integral modulus of deformation at

a load of 4 kPa decreased from $1.4 \cdot 10^4$ to $1.35 \cdot 10^4$ MPa - by 3.3%.
The data of the numerical experiment of this shell with a thickness of 20 mm at a load of 3.2 kPa was taken for comparison (Table 2). As a result of the shell exposure for 50 years the deflection of the shell increased from 13.3 mm to 14.9 mm (to 1/4027 diameter). Stresses in the meridional ribs increased from 1.53 MPa to 1.6 MPa (4.1%). The absolute stress value was less than the design resistance of 13 MPa. The shear stresses at exposure under load have not increased (details of changes in shear stresses will be discussed in another article).

Table 2. Stress-strain state of the dome at a constant one-sided load of 3.2 kPa (The thickness of shell is 20 mm).

Number of stage	Time exposure, days	Max bending moment in meridional ribs, *kNm*	Max normal stress in meridional ribs, *MPa*	Max shear stresses, *MPa*	Deflection, *mm*	Integral module of deformations, *MPa*
1	0	1380.0	1.527	2.03	13.3	14399
2	1	1417.0	1.581	2.03	13.66	14200
3	180	1428.0	1.582	2.04	13.72	14027
4	730	1443.0	1.585	2.02	13.92	13884
5	18250	1462.0	1.590	2.03	14.27	13186
				1.980	14.9	

The decrease of the deformation modulus in 2 years is: 14399/13186 = 1.09 times, i.e. less than 10%.
The calculation of the frame from dome ribs without sheathing showed that at the same load of 3.2 kPa the deflection in the middle of the loaded part was 293 mm, which is 1/205 diameter, and the maximum stresses were equal to 10.95 MPa, i.e. it is quite close to the calculated resistance of 13.0 MPa. These results were obtained at a relative height of ribs' cross-section *h/D*=1/60 – reduced if you compare with the recommended norms for planar structures of 1/40 diameter.
Rib bend on opposite side is 1/206 D, i.e. it is the same with the deflection and for a planar construction (beams of attic floors and girders, trusses) and it is within the permissible limits (<1/200 D). However, for beams and trusses, deflection of covering is limited to 1/300 D.
The maximum moment is M_s = 309.386 kN m.
The corresponding longitudinal force but around support area is N = 333,3 kN when $b \times h$ =0.2·1m.

$$\sigma = \frac{N}{A} + \frac{M}{W} = \frac{333.3}{0.2} + \frac{309.4}{0.033333} = 10.95 \text{ MPa} < R_u = 13 \text{ MPa}$$

It is quite close to the calculated resistance, but it should be taken into account that this is for h/D = 1/60 < 1/40, which already indicates the spatial work of the dome frame.
The maximum stress in the annular ribs is when N = 109,88 kN; M = 4,69 kNm; $b \times h$ = 10·20 cm:

$$\sigma = \frac{109.9}{0.02} + \frac{4.7}{0.000667} = 12.5 \text{ MPa} < 13 \text{ MPa}.$$

SUMMARY

The joint work of ribs of a dome frame with elements filling the cells between them has a great influence on increasing the strength resistance of a structure as a whole.
Nonlinear calculations using the method of integral estimates allow us to analyse the strength resistance of complex modern wooden structures and possibly others, taking into

account the long-term loading of any uneven loads in time and magnitude.
The calculations of domes with a reduced height of ribs' cross-section by 20% in comparison with the accepted design guidelines have been carried out. The state of a structure and its strength resistance under different loading conditions and with different stiffness of the cell filling panels is analysed.
The possibility of reducing the section height of meridional ribs and other possibilities of saving materials of these common structures are considered.
In general, the new effective calculation method and design recommendations can provide material savings up to 25%.
Thus, despite the available effective mathematical apparatus for calculating structures taking into account the nonlinear operation of wood, there are no recommendations for its application in the norms and standards and there is no indication of the need to design taking into account the joint work of elements to ensure the structural safety while reducing the consumption of materials.

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