Adaptive sliding mode control with gravity compensation: Application to an upper-limb exoskeleton system

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Abstract. This paper presents a robust control algorithm with gravity compensation in presence of parametric uncertainties. The application deals with an upper limb exoskeleton system, aimed for a rehabilitation application. The treated system is an robot with two degrees of freedom acting on the flexion / extension movement of the shoulder and elbow. An adaptive sliding mode algorithm with gravity compensation has been developed to control the upper limb exoskeleton system. A Stability study is realized. Then, a robustness analysis in the presence of parametric uncertainties using Monte Carlo simulation is developed. To prove the performance of the gravity compensation approach, a comparison study is done. Simulation results are presented to highlight the performances and the effectiveness of the proposed controller using gravity compensation.

I. INTRODUCTION

Exoskeleton is a robotic system which can be placed on the human's arm and acts as amplifiers that augment, reinforce or restore human performances. When operating an exoskeleton system, the torques on the input elements depend on the masses of the moving system, as well as the external forces applied to the system. We can find two types of loads generated by the masses of moving elements: the static load depending on the forces of gravity and the dynamic load which takes into account moreover the effects of inertia. These periodic loads increase the energy expenditure that is necessary to operate the system, cause additional vibrations and parasitic dynamic errors.

To eliminate the negative influence of these loads on each actuator, it is necessary to create an opposite additional torque. This additional torque is often referred to as the balancing torque or the discharge torque. Different control methods have been studied to solve this problem. So, we can find spring mechanisms [4], [8] or cam mechanisms [9] to reduce or obtain a constant load on the actuators of mechanical systems; the optimization of movements [10], [11] to find the optimized trajectories or the optimal redistribution of moving masses [14] to eliminate (or reduce) the influence of inertia on the actuators. Also, we find the gravity compensation approach, which is frequently used in the control of the robot [1], [2], [3].

In this context, we are interested to the control of the exoskeleton-upper limb system using the gravity compensation approach. The main goal of controlling an exoskeleton is to reproduce the movements of a healthy human arm. To achieve this goal, it is necessary to apply an appropriate controller. As the interaction between the human arm and the exoskeleton is characterized by a dynamic complexity, researchers have developed several control laws. Sliding mode is used in reference [15] to control the exoskeleton of the upper limbs. The author in [16] used a mixed force and position controller which mixes, for the same degree of freedom, the force and position information.

The performance and the efficiency of the existing controllers when tracking the desired trajectories and the robustness study in the presence of parametric uncertainties and the gravity effect are not studied in literature.

The contribution of this paper is to develop an adaptive sliding mode algorithm with gravity compensation to control the exoskeleton- upper limb system. As a robustness study in presence of parametric uncertainties, a Monte Carlo simulation is used.

The article is presented as: Section 2 deals with the exoskeleton-upper limb system model. Section 3 describes the control and the stability studies using the adaptive sliding mode without and with gravity compensation. The robustness analysis of the system affected by uncertainties using Monte Carlo simulation is presented by section 4. Simulation results and discussions are given in section 5. Finally, conclusion and future work are described by section 6.

II. MODELLING OF THE UPPER- LIMB EXOSKELETON SYSTEM

In this section, we aim to control an upper- limb exoskeleton. We will start by modeling this system and then we pass to the control of both elbow and shoulder articulations.

Based on the Lagrange method, the dynamic model of the robotized exoskeleton system of the upper limbs with two degrees of freedom (Fig.1), taking into account the contact force and the constraints, is given by the following equation:

$$M(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + G(q) + \underbrace{f_v\,\dot{q} + k_i\,sign(\dot{q}_j)}_{F(\dot{q},\,\dot{q})} = \tau \quad (1)$$

$$M(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + G(q) + F(q,\dot{q}) = \tau \quad (2)$$
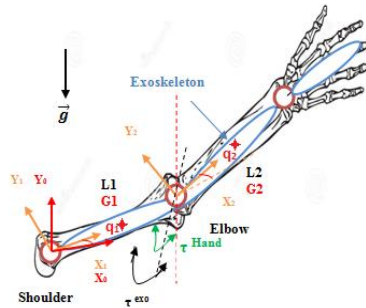

Fig.1. General configuration of a 2 DoF exoskeleton

Where:

- q $\in \mathbb{R}^2$ is the vector of joint positions;

- $\dot{q} \in \mathbb{R}^2$ is the vector of joint velocities;

- $\ddot{q} \in \mathbb{R}^2$ is the vector of joint accelerations;

- M(q)$\in \mathbb{R}^{2*2}$ is the inertia matrix;

- C(q, $\dot{q}$)$\in \mathbb{R}^{2*2}$ is the Coriolis matrix,

- G(q) $\in \mathbb{R}^2$ is the gravitational vector;

- F(q, $\dot{q}$) $\in \mathbb{R}^2$ is the force generated by friction;

- $\tau \in \mathbb{R}^2$ is the control vector;

We pass then to synthesize the algorithm laws used to control the upper- limb exoskeleton system in order to follow the desired trajectories.

III. CONTROL OF THE UPPER -LIMB EXOSKELETON SYSTEM
USING THE ADAPTIVE SLIDING MODE

It is intended to control in this part the flexion / extension movement of the shoulder and the elbow of the upper- limb exoskeleton. The goal of using the adaptive sliding mode is to ensure a dynamic adaptation of the control gain in order to be as small as possible.
The adaptive control is used for its speed and ease of implementation, while the sliding mode for its theoretical foundations reassuring in terms of stability and robustness.

A. Adaptive sliding mode control of an upper-limb exoskeleton

The dynamic of σ is given by:

$$\dot{\sigma} = \frac{\partial \sigma}{\partial x}\dot{x} + \frac{\partial \sigma}{\partial t} = \underbrace{\frac{\partial \sigma}{\partial t} + \frac{\partial \sigma}{\partial x}f(x)}_{\Psi(x,t)} + \underbrace{\frac{\partial \sigma}{\partial x}g(x)}_{\Gamma(x,t)} u \qquad (3)$$

With: $x = f(x) + g(x)u$
The controller may be expressed as follows:

$$u = -k \, sign \, (\sigma \, (x,t)) \qquad (4)$$

With : $\sigma = \lambda^2 (q_d - q) + 2\lambda(\dot{q}_d - \dot{q}) + \ddot{q}_d$
The adaptation law is given by:

$$\dot{K} = \bar{K} \, |\sigma \, (x,t)| \qquad (5)$$

With : $\bar{K} > 0$ and $K(0) > 0$.
Consider the function $E = \frac{1}{2}\|\sigma^2\|$, we get

$$\dot{E} = \frac{1}{2}(\dot{\sigma}^T \sigma + \dot{\sigma}\sigma^T) = \sigma^T \psi - \frac{K}{2\|\sigma\|}\sigma^T (\Gamma + \Gamma^T)$$

We note $\Omega = \Gamma + \Gamma^T$, then

$$\lambda_{min}\|\sigma\|^2 < \sigma^T \Omega \sigma < \lambda_{max}\|\sigma\|^2$$

Where:
- λ_{min} present the minimum eigenvalue of Ω.
- λ_{max} is the maximum. eigenvalue of Ω.

Stability proof: Concerning the stability proof of the upper- limb exoskeleton system with gravity compensation, Lyanunov's stability is considered.
To do this, we choose the following Lyapunov candidate function:

$$v = \frac{1}{2}\|\sigma^2\| + \frac{1}{2\gamma}(k - k^*)^2 \qquad (6)$$

The time derivation of v is given by:

$$\dot{v} = \frac{1}{2}(\dot{\sigma}^T \sigma + \dot{\sigma}\sigma^T) + \frac{1}{\gamma}(k - k^*)\dot{k} \qquad (7)$$

$$\dot{v} = \sigma^T \psi - \frac{K}{2\|\sigma\|}\sigma^T \Omega \sigma + \frac{1}{\gamma}(k - k^*)\dot{k} \qquad (8)$$

$$\dot{v} < \psi_M \|\sigma\| - k\frac{\lambda min}{2}\|\sigma\| + \frac{\lambda min}{2} k^*\|\sigma\| - \frac{\lambda min}{2} k^*\|\sigma\| + \frac{1}{\gamma}(k - k^*)\dot{k} \qquad (9)$$

We introduce the parameter $\beta_k > 0$, $\dot{v}$ satisfies:

$$\dot{v} < (\psi_M - k^*\frac{\lambda min}{2})\|\sigma\| + |k - k^*| (-\frac{\lambda min}{2}\|\sigma\| + \frac{1}{\gamma}\dot{k}) + \beta_k|k - k^*| - \beta_k|k - k^*| \qquad (10)$$

Or $k(t) - k^* < 0$ for $t > 0$, we obtain the following inequality:

$$\dot{v} < - (\psi_M + k^* \tfrac{\lambda min}{2}) \, ||\sigma|| - \beta_k| \, k - k^*| - |k - k^*|(- \tfrac{\lambda min}{2} \, ||\sigma|| + \tfrac{1}{\gamma} \dot{k} - \beta_k) \qquad (11)$$

$$\underbrace{}_{\beta_\sigma} \qquad \underbrace{}_{\xi}$$

The derivative of the Lyapunov function is given by:

$$\dot{v} < \beta_\sigma ||\sigma|| - \beta_k |k - k^*| - \xi \qquad (12)$$

$$\dot{v} = - \beta_\sigma \sqrt{2} \tfrac{||\sigma||}{\sqrt{2}} - \beta_k \sqrt{2\gamma} \tfrac{|k-k\bullet|}{\sqrt{2\gamma}} - \xi \qquad (13)$$

$$\dot{v} < - \min \{\beta_\sigma\sqrt{2}, \beta_k \sqrt{2\gamma}\} \, (\tfrac{|k-k\bullet|}{\sqrt{2\gamma}} + \tfrac{||\sigma||}{\sqrt{2}}) - \xi < - \beta \, v^{\frac{1}{2}} - \xi \qquad (14)$$

With $\beta = \sqrt{2} \min \{\beta_\sigma, \beta_k \sqrt{\gamma}\}$.

Thus, the convergence of finite time to a domain $\sigma = 0$ is guaranteed from any initial condition $|\sigma(0)| > 0$.

The reach time t_r can easily be estimated by:

$$t_r < \frac{2 \, v \, (0)^{\frac{1}{2}}}{\beta}$$

Referring to section III, the sliding variable is expressed as:

$$\sigma = \lambda^2 (q_d - q) + 2 \lambda \, (\dot{q}_d - \dot{q}) + \ddot{q}_d \qquad (15)$$

B. Control of the system with an adaptive sliding mode with gravity compensation

The Adaptive sliding mode control with gravity compensation is given by:

$$\tau = - K \, sign \, (\sigma) + \hat{G} \, (q) \qquad (16)$$

We get :

$$M \, (q) \, \ddot{q} + C(q, \dot{q}) \, \dot{q} + G \, (q) + F(q, \dot{q}) = - K \, sign \, [\lambda^2(q_d - q) + 2\lambda \, (\dot{q}_d - \dot{q}) + \ddot{q}_d] + \hat{G} \, (q) \qquad (17)$$

B.2. Stability study

To study the stability of our system when controlling with gravity compensation, a Lyanunov stability is done:

$$v = \tfrac{1}{2} \, \dot{q}^T M(q) \, \dot{q} + \tfrac{1}{2} \, S^2 \qquad (18)$$

We prove that the derivative of v is negative. So $\dot{v}$ is written:

$$\dot{v} = \dot{q}^T M(q) \, \ddot{q} + \tfrac{1}{2} \, \dot{q}^T \dot{M} \, (q) \, \dot{q} + S \dot{S} \qquad (19)$$

We have:

$$M(q) \, \ddot{q} = \tau - C(q, \dot{q}) \, \dot{q} - G(q) - F(q, \dot{q}) \qquad (20)$$

When ignoring the friction forces, $\dot{v}$ can be written in the following form:

$$\dot{v} = \dot{q}^T [\tau - C (q, \dot{q}) \, \dot{q} - G(q)] + \tfrac{1}{2} \, \dot{q}^T \dot{M} \, (q) \, \dot{q} + S \dot{S} \qquad (21)$$

With $\dot{S} = - K \, sign(S)$.

By replacing τ by its value, we will have:

$$\dot{v} = \dot{q}^T [\hat{G} - K \, sign \, (S) - C \, (q, \dot{q}) \, \dot{q} - G(q)] + \tfrac{1}{2} \, \dot{q}^T \dot{M} \, (q) \, \dot{q} - SK \, sign(S)$$

$$\dot{v} = - SK \, sign(S) + \dot{q}^T [\hat{G} - K \, sign \, (S) - G(q)] + \tfrac{1}{2} \, \dot{q}^T [\dot{M} \, (q) - 2 \, C \, (q, \dot{q})] \, \dot{q}$$

With $\hat{G}$ is an approximated value of G.

We consider the error of the approximation $\tilde{G} = \hat{G} - G$ is delimited as:

$$\| \, \tilde{G} \, \| \leq G$$

We get:

$$\dot{v} = - SK \, sign(S) + \dot{q}^T [\tilde{G} - K \, sign \, (S)] + \tfrac{1}{2} \dot{q}^T [\dot{M} \, (q) - 2 \, C \, (q, \dot{q}] \, \dot{q} \qquad (22)$$

Like $|\tilde{q}|^T = \tilde{q}^T$ sign (S), we get :

$$\dot{v} \leq - S\,K\,sign\,(S) + |\dot{\mathbf{q}}|^T\,G - |\dot{\mathbf{q}}|^T K \qquad (23)$$

With G < K, so we get:

$$\dot{v} \leq - S\,K\,sign\,(S) \qquad (24)$$

Since -K S sign (S) is negative because: K ≥ 0 and since the sign function is constant in pieces so S sign (S) = +1, ∀S so $\dot{v}$ is semi-negative.
Since $v \geq 0$ et $\dot{v} \leq 0$, then the system is asymptotically stable.

IV. ROBUSTNESS ANALYSIS: MONTE CARLO SIMULATION

In this section, a robustness test was done to prove the performance of the tested controller and to obtain the most robust control. To do this, a Monte Carlo simulation was done when applying parametric uncertainties to the system. The Monte Carlo method [13] treats the system incorporating uncertain parameters modeled by random variables [12]. It is a powerful and very general mathematical tool which has earned it a wide range of applications.
In our case, by applying the uncertainties, the dynamic model of the system can be rewritten as follow:

$$\ddot{q} = (f\,(q,\,\dot{q}\,,\,t) + \Delta_f) + (g\,(q) + \Delta_g)\,u(t) \qquad (25)$$

Some statistics of the tracking recorded errors are developed to study the robustness of the proposed controller by calculating of the Root-Mean-Square (RMS), the mean (Mean) and the standard deviation (Std).
We use the following expression to calculate the RMS:

$$X_{RMS} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}|X_n|^2} \qquad (26)$$

The Std can be expressed by:

$$\sigma_x = \sqrt{E\left[x - E[x]^2\right]} = \sqrt{E\left[x^2\right] - E[x]^2} \qquad (27)$$

And the sample mean is defined as:

$$\overline{\theta} = \frac{1}{m}\sum_{i=1}^{m}\theta_i \qquad (28)$$

V. RESULTS AND DISCUSSIONS

To present the robustness and the performance of the treated adaptive sliding mode controller, a comparison between the case of using gravity compensation with the simple law is done. The comparison results (Table.II) are given when controlling the upper- limb exoskeleton in the presence of parametric uncertainties.

The desired trajectories are given by $q_1 = \sin(2\pi t)$ and $q_2 = \sin(2\pi t)$. The initial conditions of the real trajectories are q (0) = [0; pi/4]^T and $\dot{q}$ (0) = [0; 0]^T.

Table II shows the values of the RMS, the Std and the mean calculated during system control with the two tested cases. In this case, the uncertainties applied to the upper- limb exoskeleton are uniform random distributions with Δ_f and Δ_g ε [0; 0.005] at t= 0.2s. Figs. 2 and 3 present the measured and the desired trajectories of the treated cases as well as the errors of tracking the references trajectories.

Table I.   Simulation Parameters

Joint	1	2
Masses of the exoskeleton (Kg)	4.5	3.5
Lengths of the exoskeleton (m)	0.35	0.20
Solid friction coefficient of the exoskeleton (N.m)	0.03	0.02
Masses of the upper limb (Kg)	3.95	3.10
Lengths of the upper limb (m)	0.30	0.25
Solid friction coefficient of the upper limb (N.m)	0.005	0.0041

From these simulations, we can clearly note that the good tracking of the desired trajectories in position as well as in velocities are given when using the gravity compensation in presence of uncertainties. Figures 4 and 5 shows the RMS histograms of each articulation q_1 and q_2 when tracking the desired trajectories in position and velocity. They illustrate the comparison between the case of the control with and without gravity compensation using the adaptive sliding mode. According to these figure, it can clearly be seen that the control with gravity compensation gives more robust results.

In the case of gravity compensation control, we notice that the adaptive sliding mode control is efficient and robust when tracking the desired trajectories and in the presence of parametric uncertainties (RMS between 0.0085 and 0.0087 in position and between 0.0055 and 0.0059 in velocity).
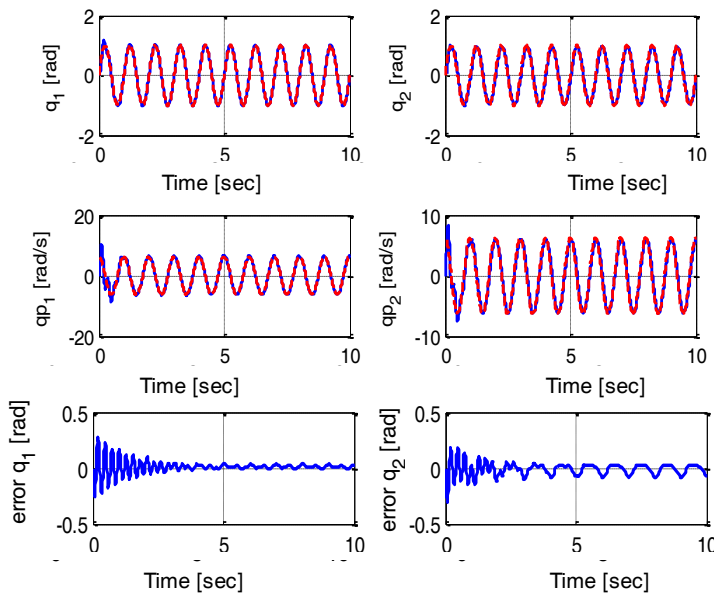

Fig.2. Simulation results of the joints q_1 and q_2 using the adaptive sliding mode controller without gravity compensation
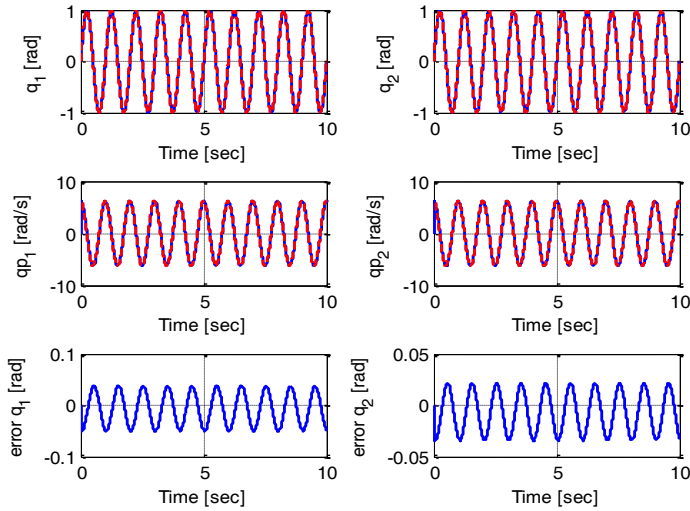
Fig.3. Simulation results of the joints q_1 and q_2 using the adaptive sliding mode controller with gravity compensation

TABLE II. Summary Results Of The Monte Carlo Simulation. Calculation Of The Rms, The Mean Error Value And The Standard Deviation For Each Articulation Q_1 And Q_2 Using The Adaptive Sliding Mode With And Without Gravity Compensation In The Case Of Tracking The Desired Trajectories In Positions And Velocity

|		RMS [rad]		Mean [rad]		Std [rad]	
		q_1	q_2	q_1	q_2	q_1	q_2
Adaptive sliding mode : position simulation	**Without gravity compensation**	0.026	0.028	0.018	0.021	0.0095	0.012
	With gravity compensation	0.0085	0.0087	0.0084	0.0086	0.0061	0.0063
		RMS [rad/s]		Mean [rad/s]		Std [rad/s]	
		q_1	q_2	q_1	q_2	q_1	q_2
Adaptive sliding mode : velocity simulation	**Without gravity compensation**	0.021	0.023	0.015	0.017	0.0088	0.0097
	With gravity compensation	0.0055	0.0059	0.0039	0.0049	0.0028	0.0033
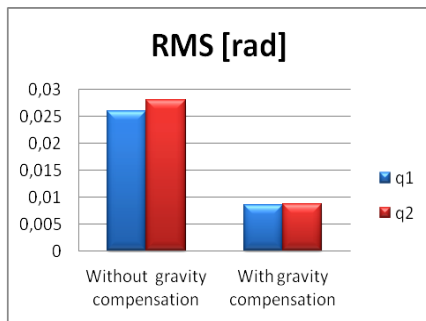
Fig. 4. The RMS calculation of the joints q_1 and q_2 respectively when tracking the desired trajectories in positions using the tested algorithm
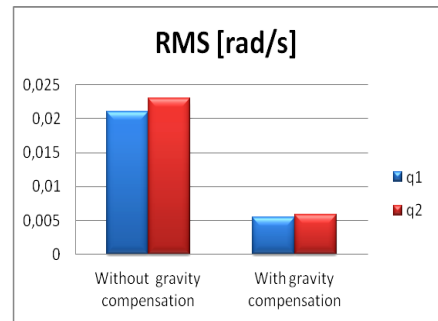

Fig. 5. The RMS calculation of the joints q_1 and q_2 respectively when tracking the desired trajectories in velocity using the tested algorithm

VI. CONCLUSION

This paper deals with the control with gravity compensation of a two degrees of freedom exoskeleton- upper limb system, used for rehabilitation, in presence of parametric uncertainties. A dynamical model of the robot was developed. Then, an adaptive sliding mode algorithm with and without gravity compensation was used to control the system. A Stability and a robustness studies were done to analyse the performance of the upper- limb exoskeleton system in presence of uncertainties. Referring to the simulation results, a comparison between the two tested controller cases, when controlling with and without gravity compensation, was done in order to prove the one the most performing when tracking the desired trajectories and the efficiency of the control with gravity compensation. As a future work, a robustness study when applying matched and unmatched disturbances will be done. A control of the exoskeleton system worn by a human upper limb will be developed.

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