TOPSIS Method for MADM based on Interval Trapezoidal Neutrosophic Number

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Abstract: TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is a very common method for Multiple Attribute Decision Making (MADM) problem in crisp as well as uncertain environment. The interval trapezoidal neutrosophic number can handle incomplete, indeterminate and inconsistent information which are generally occurred in uncertain environment. In this paper, we propose TOPSIS method for MADM, where the rating values of the attributes are interval trapezoidal neutrosophic numbers and the weight information of the attributes are known or partially known or completely unknown. We develop optimization models to obtain weights of the attributes with the help of maximum deviation strategy for partially known and completely unknown cases. Finally, we provide a numerical example to illustrate the proposed approach and make a comparative analysis.

Keywords: Interval trapezoidal neutrosophic number, Multi-attribute decision making, TOPSIS, Unknown weight information.

1 Introduction

Multi-attribute decision making (MADM) is a popular field of study in decision analysis. MADM refers to making choice of the best alternative from a finite set of decision alternatives in terms of multiple, usually conflicting criteria. The decision maker uses the rating value of the attribute in terms of fuzzy sets [1], intuitionistic fuzzy sets [2], hesitant fuzzy sets [3], and neutrosophic sets [4].

In classical MADM methods, the ratings and weights of the criteria are known precisely. TOPSIS [5] is one of the classical methods among many MADM techniques like Preference Ranking Organization METHod for Enrichment of Evaluations (PROMETHEE) [6], Vlse Kriterijuska OptimizacijaI Komoromisno Resenje (VIKOR) [7], ELimination Et Choix Traduisant la REalit (ELECTRE) [8], Analytic Hierarchy Process (AHP) [9], etc. MADM problem has also been studied in fuzzy environment [10–14] and intuitionistic fuzzy environment [15–18]. Researchers have extended the TOPSIS method to deal with MADM problems in different environment. Chen [19] extended the concept of TOPSIS method to develop a methodology for MADM problem in fuzzy environment. Boran et al. [20] extended the TOPSIS method for MADM in intutionistic fuzzy sets. Zhao [21] proposed TOPSIS method under interval intutionistic fuzzy number. Liu [22] proposed TOPSIS method for MADM under trapezoidal intuitionistic fuzzy environment with partial and unknown attribute weight information.

Compared to fuzzy set and intutionistic fuzzy set, neutrosophic set [4] has the potential to deal with MADM problem because it can effectively handle indeterminate and incomplete information. Hybrids of neutrosophic set have been applied in MADM and multi-attribute group decision making (MAGDM) problems [23–30]. Biswas et al. [31] developed TOPSIS strategy for MAGDM with single valued neutrosophic sets [32]. Chi and Liu [33] extended TOPSIS method for MADM problem based on interval neutrosophic set [4]. Zhang and Wu [34] developed TOPSIS strategy in single valued neutrosophic set where weights of the attributes are completely unknown. Biswas et al. [35] introduced Neutrosophic TOPSIS with group decision making. Pramanik et al. [36] extended TOPSIS approach for MAGDM in refined neutrosophic environment. Dey et al. [37] introduced TOPSIS for solving MADM problems under bi-polar neutrosophic environment. Mondal et al. [38] developed Rough neutrosophic TOPSIS for MAGDM problem. Ye [39] proposed TOPSIS method for MAGDM problem based on single valued neutrosophic linguistic number. Dey et al. [40] developed generalized neutrosophic soft MADM based on TOPSIS. Pramanik et al. [41] extended TOPSIS method for MADM based on soft expert set environment. Biswas et al. [42] developed TOPSIS strategy for MADM with trapezoidal neutrosophic numbers.

Interval trapezoidal neutrosophic number (ITrNN) [43] is a generalization of single valued trapezoidal neutrosophic number (SVTrNN). Ye [44] and Subhaş [45] introduced the SVTrNN where each element is expressed by trapezoidal number that has truth, indeterminacy and falsity membership degrees which are single valued. However, decision makers may face difficulties to express their opinions in terms of single valued truth, indeterminacy and falsity membership degrees. In interval trapezoidal neutrosophic number truth, indeterminacy and falsity membership degrees are interval valued. Therefore, decision makers can express their opinion throughout this number in a flexible way to face such difficulties.

The above literature review reflects that the TOPSIS method has not been studied earlier based on interval trapezoidal neutrosophic number, even though this number can play effective role with indeterminate and uncertain information in MADM problem. To fill this research gap, our objectives in this paper are as follows:

- To propose TOPSIS method for MADM problem based on interval valued trapezoidal neutrosophic number.
- To develop the model where the rating values of the attributes are ITrNN and weight information is known, partially known and completely unknown.

We organise the paper as follows: Section 2 describes the preliminaries of fuzzy sets, trapezoidal fuzzy number, neutrosophic sets, SVTrNN, ITrNN, and Hamming distance between ITrNNs. Section 3 briefly presents classical TOPSIS method. Section 4 presents TOPSIS method for MADM based on ITrNN. An application example with comparative analysis is given in Section 5. Finally, Section 6 presents some conclusions and future scopes of research.

2 Preliminaries

In this section, we briefly review the definition of fuzzy sets, single-valued neutrosophic sets, single-valued trapezoidal neutrosophic number, and interval trapezoidal neutrosophic numbers.

Definition 1. [1] Let X be a universe of discourse. Then a fuzzy set A is defined by

$$A = \{\langle x, \mu_A(x)\rangle |\; x \in X\}, \tag{1}$$

which is characterized by a membership function $\mu_A : X \to [0, 1]$, where $\mu_A(x)$ is the degree of membership of the element x to the set A.

Definition 2. [46,47] A generalized trapezoidal fuzzy number A denoted by $A = (a, b, c, d; w)$ is described as a fuzzy subset of a real number $\mathbb{R}$ with membership function μ_A which is defined by

$$\mu_A(x) = \begin{cases} \dfrac{(x-a)w}{b-a}, & a \leq x < b \\ w, & b \leq x \leq c \\ \dfrac{(d-x)w}{d-c}, & c < x \leq d \\ 0, & otherwise. \end{cases}$$

where $a, b, c, d \in \mathbb{R}$ and w is a membership degree.

Definition 3. [32] Let X be universe of discourse. Then a single-valued neutrosophic set A is defined as $A = \{< x, T_A(x), F_A(x), I_A(x) >: x \in X\}$ which is characterized by a truth-membership function $T_A(x) : X \to [0,1]$, falsity membership function $F_A : X \to [0,1]$, and an indeterminacy membership function $I_A : X \to [0,1]$ of the element x to the set A, and the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3 \; \forall \, x \in X$.

Definition 4. [44,45] Let α be a single-valued neutrosophic trapezoidal number (SVNTrN). Then its membership functions are given by

$$T_\alpha(x) = \begin{cases} \dfrac{(x-a)t_\alpha}{b-a}, & a \leq x < b \\ t_\alpha, & b \leq x \leq c \\ \dfrac{(d-x)t_\alpha}{d-c}, & c < x \leq d \\ 0, & otherwise. \end{cases}$$

$$I_\alpha(x) = \begin{cases} \dfrac{b-x+(x-a)i_\alpha}{b-a}, & a \leq x < b \\ i_\alpha, & b \leq x \leq c \\ \dfrac{x-c+(d-x)i_\alpha}{d-c}, & c < x \leq d \\ 0, & otherwise. \end{cases}$$

$$F_\alpha(x) = \begin{cases} \dfrac{b-x+(x-a)f_\alpha}{b-a}, & a \leq x < b \\ f_\alpha, & b \leq x \leq c \\ \dfrac{x-c+(d-x)f_\alpha}{d-c}, & c < x \leq d \\ 0, & otherwise. \end{cases}$$

where T_α is truth membership function, I_α is indeterminancy membership function and F_α is falsity membership function, and they all lie between 0 and 1 and satisfy the condition $0 \leq T_\alpha(x) + I_\alpha(x) + F_\alpha(x) \leq 3$ where a, b, c, d are real numbers. Then $\alpha = ([a, b, c, d]; t_\alpha, i_\alpha, f_\alpha)$ is called a neutrosophic trapezoidal number.

Definition 5. [43] Let $\tilde{\alpha}$ be trapezoidal neutrosophic number. Then its membership functions are given by

$$T_{\tilde{\alpha}}(x)=\begin{cases}\dfrac{(x-a)t_{\tilde{\alpha}}}{b-a}, & a\le x<b\\ t_{\tilde{\alpha}}, & b\le x\le c\\ \dfrac{(d-x)t_{\tilde{\alpha}}}{d-c}, & c<x\le d\\ 0, & otherwise.\end{cases}$$

$$I_{\tilde{\alpha}}(x)=\begin{cases}\dfrac{b-x+(x-a)i_{\tilde{\alpha}}}{b-a}, & a\le x<b\\ i_{\tilde{\alpha}}, & b\le x\le c\\ \dfrac{x-c+(d-x)i_{\tilde{\alpha}}}{d-c}, & c<x\le d\\ 0, & otherwise.\end{cases}$$

$$F_{\tilde{\alpha}}(x)=\begin{cases}\dfrac{b-x+(x-a)f_{\tilde{\alpha}}}{b-a}, & a\le x<b\\ f_{\tilde{\alpha}}, & b\le x\le c\\ \dfrac{x-c+(d-x)f_{\tilde{\alpha}}}{d-c}, & c<x\le d\\ 0, & otherwise.\end{cases}$$

where $T_{\tilde{\alpha}}$ is truth membership function, $I_{\tilde{\alpha}}$ is indeterminancy membership function and $F_{\tilde{\alpha}}$ is falsity membership function and $t_{\tilde{\alpha}}, i_{\tilde{\alpha}}, f_{\tilde{\alpha}}$ are subsets of [0,1] and $0\le \sup(t_{\tilde{\alpha}})+\sup(i_{\tilde{\alpha}})+\sup(f_{\tilde{\alpha}})\le 3$. Then α is called an interval trapezoidal neutrosophic number and it is denoted by $\tilde{\alpha}=([a,b,c,d];t_{\tilde{\alpha}},i_{\tilde{\alpha}},f_{\tilde{\alpha}})$. We take $t_{\tilde{\alpha}}=[\underline{t},\bar{t}], i_{\tilde{\alpha}}=[\underline{i},\bar{i}]$ and $f_{\tilde{\alpha}}=[\underline{f},\bar{f}]$

Definition 6. [43] An interval trapezoidal neutrosophic number (ITrNN) $\tilde{\alpha}=([a,b,c,d];[\underline{t},\bar{t}],[\underline{i},\bar{i}],[\underline{f},\overline{f}])$ is said to be positive ITrNN if $a\ge 0$ and one of the four values of a,b,c,d is not equal to zero.

Definition 7. Let $\tilde{\alpha}=([a_1,b_1,c_1,d_1];[\underline{t}_1,\bar{t}_1],[\underline{i}_1,\bar{i}_1],[\underline{f}_1,\bar{f}_1])$ and $\tilde{\beta}=([a_2,b_2,c_2,d_2];[\underline{t}_2,\bar{t}_2],[\underline{i}_2,\bar{i}_2],[\underline{f}_2,\bar{f}_2])$ be two ITrNNs. Then the following operations are valid:

1. $\tilde{\alpha}\bigoplus\tilde{\beta}=\left(\begin{array}{c}[a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2];\\ [\underline{t}_1+\underline{t}_2-\underline{t}_1\underline{t}_2,\bar{t}_1+\bar{t}_2-\bar{t}_1\bar{t}_2],[\underline{i}_1\underline{i}_2,\bar{i}_1\bar{i}_2],[\underline{f}_1\underline{f}_2,\bar{f}_1\bar{f}_2]\end{array}\right);$

2. $\tilde{\alpha}\bigotimes\tilde{\beta}=\left(\begin{array}{c}([a_1a_2,b_1b_2,c_1c_2,d_1d_2];[\underline{t}_1\underline{t}_2,\bar{t}_1\bar{t}_2],\\ [\underline{i}_1+\underline{i}_2-\underline{i}_1\underline{i}_2,\bar{i}_1+\bar{i}_2-\bar{i}_1\bar{i}_2],\\ [\underline{f}_1+\underline{f}_2-\underline{f}_1\underline{f}_2,\bar{f}_1+\bar{f}_2-\bar{f}_1\bar{f}_2]\end{array}\right);$

3. $\lambda\tilde{\alpha}=\left(\begin{array}{c}\left[\lambda a_1,\lambda b_1,\lambda c_1,\lambda d_1\right];\left[1-(1-\underline{t}_1)^\lambda,1-(1-\bar{t}_1)^\lambda\right]\\ \left[(\underline{i}_1)^\lambda,(\bar{i}_1)^\lambda\right],\left[(\underline{f}_1)^\lambda,(\bar{f}_1)^\lambda\right]\end{array}\right),\lambda\ge 0;$

4. $(\tilde{\alpha})^\lambda=\left(\begin{array}{c}\left[(a_1)^\lambda,(b_1)^\lambda,(c_1)^\lambda,(d_1)^\lambda\right];\left[(\underline{t}_1)^\lambda,(\bar{t}_1)^\lambda\right],\\ \left[1-(1-\underline{i}_1)^\lambda,1-(1-\bar{i}_1)^\lambda\right],\\ \left[1-(1-\underline{f}_1)^\lambda,1-(1-\bar{f}_1)^\lambda\right]\end{array}\right),\lambda\ge 0.$

Definition 8. [43] Let $\tilde{\alpha} = ([a_1, b_1, c_1, d_1]; [\bar{t}_1, \underline{t}_1], [\bar{i}_1, \underline{i}_1], [\bar{f}_1, \underline{f}_1])$ and $\tilde{\beta} = ([a_2, b_2, c_2, d_2]; [\bar{t}_2, \underline{t}_2], [\bar{i}_2, \underline{i}_2], [\bar{f}_2, \underline{f}_2])$ be two ITrNNs. Then the distance between two numbers is defined as

$$\begin{aligned} d(\tilde{\alpha}, \tilde{\beta}) =& \frac{1}{24}\left(\left| \begin{array}{l} a_1(2 + \underline{t}_1 - \underline{i}_1 - \underline{f}_1) + a_1(2 + \bar{t}_1 - \bar{i}_1 - \bar{f}_1) \\ - a_2(2 + \underline{t}_2 - \underline{i}_2 - \underline{f}_2) - a_2(2 + \bar{t}_2 - \bar{i}_2 - \bar{f}_2) \end{array} \right| \right. \\ &+ \left| \begin{array}{l} b_1(2 + \underline{t}_1 - \underline{i}_1 - \underline{f}_1) + b_1(2 + \bar{t}_1 - \bar{i}_1 - \bar{f}_1) \\ - b_2(2 + \underline{t}_2 - \underline{i}_2 - \underline{f}_2) - b_2(2 + \bar{t}_2 - \bar{i}_2 - \bar{f}_2) \end{array} \right| \\ &+ \left| \begin{array}{l} c_1(2 + \underline{t}_1 - \underline{i}_1 - \underline{f}_1) + c_1(2 + \bar{t}_1 - \bar{i}_1 - \bar{f}_1) \\ - c_2(2 + \underline{t}_2 - \underline{i}_2 - \underline{f}_2) - c_2(2 + \bar{t}_2 - \bar{i}_2 - \bar{f}_2) \end{array} \right| \\ &+ \left. \left| \begin{array}{l} d_1(2 + \underline{t}_1 - \underline{i}_1 - \underline{f}_1) + d_1(2 + \bar{t}_1 - \bar{i}_1 - \bar{f}_1) \\ - d_2(2 + \underline{t}_2 - \underline{i}_2 - \underline{f}_2) - d_2(2 + \bar{t}_2 - \bar{i}_2 - \bar{f}_2) \end{array} \right| \right) \end{aligned} \tag{2}$$

This distance is called normalized Hamming distance.

3 TOPSIS method for MADM

TOPSIS [5] method is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution and the longest geometric distance from the negative ideal solution.

Let A= $\{A_i | i = 1, 2, \ldots, m\}$ be the set of alternatives, C= $\{C_j | j = 1, 2, \ldots, n\}$ be the set of criteria and D= $\{d_{ij} | i = 1, 2, \ldots, m : j = 1, 2, \ldots, n\}$ be the performance ratings with the criteria weight vector W= $\{w_j | j = 1, 2, \ldots, n\}$. The idea of classical TOPSIS method can be expressed in a series of following steps:

Step 1. Normalize the decision matrix.

The normalized value $\bar{d}_{ij}$ is calculated as follows:

$$\bar{d}_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{m} (d_{ij})^2}},\ i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n.$$

Step 2. Calculate the weighted normalized decision matrix.

In the weighted normalized decision matrix, the modified ratings are calculated as given below:

$$v_{ij} = w_j \times \bar{d}_{ij} \text{ for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n. \tag{3}$$

where w_j is the weight of the j-th attribute such that $w_j \geq 0$ for $j = 1, 2, \ldots, n$ and $\sum_{j=1}^{n} w_j = 1$.

Step 3. Determine the positive and the negative ideal solutions.

The positive ideal solution (PIS) and the negative ideal solution (NIS) are determined as follows:

$$\begin{aligned} \text{PIS} = A^{+} &= \left\{v_1^{+}, v_2^{+}, \ldots v_n^{+}, \right\} \\ &= \left\{ \left(\max_j v_{ij} | j \in J_1 \right), \left(\min_j v_{ij} | j \in J_2 \right) | j = 1, 2, \ldots, n \right\}; \\ \text{NIS} = A^{-} &= \left\{v_1^{-}, v_2^{-}, \ldots v_n^{-}, \right\} \\ &= \left\{ \left(\min_j v_{ij} | j \in J_1 \right), \left(\max_j v_{ij} | j \in J_2 \right) | j = 1, 2, \ldots, n \right\}, \end{aligned} \tag{4}$$

where J_1 and J_2 are the benefit type and the cost type attributes, respectively.

Step 4. Calculate the separation measures for each alternative from the PIS and the NIS.

The separation values for the PIS can be measured using the n-dimensional Euclidean distance measure as follows:

$$D_i^{+} = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^{+}\right)^2} \quad i = 1, 2, \ldots m. \tag{5}$$

Similarly, separation values for the NIS can be measured as

$$D_i^{-} = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^{-}\right)^2} \quad i = 1, 2, \ldots m. \tag{6}$$

Step 5. Calculate the relative closeness coefficient to the positive ideal solution.

The relative closeness coefficient for the alternative A_i with respect to A^{+} is calculated as

$$C_i = \frac{D_i^{-}}{D_i^{+} + D_i^{-}} \text{ for } \quad i = 1, 2, \ldots m. \tag{7}$$

Step 6. Rank the alternatives.

According to relative closeness coefficient to the ideal alternative, the larger value of C_i reflects the better alternative A_i.

4 TOPSIS for multi-attribute decision making based on ITrNN

In this section, we put forward a framework for determining the attribute weights and the ranking orders for all the alternatives with incomplete weight information under neutrosophic environment.

For a multi-attribute decision making problem, let $A = (A_1, A_2, ..., A_n)$ be a discrete set of alternatives and $C = (C_1, C_2, ..., C_n)$ be a discrete set of attributes. Suppose that $D = [\tilde{a}_{ij}]$ is the decision matrix, where $\tilde{a}_{ij} = ([a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4]; \tilde{t}_{ij}, \tilde{i}_{ij}, \tilde{f}_{ij})$ is ITrNN for alternative A_i with respect to attribute C_j and $\tilde{t}_{ij}, \tilde{i}_{ij}$ and $\tilde{f}_{ij}$ are subsets of $[0, 1]$ and $0 \leq \sup \tilde{t}_{ij} + \sup \tilde{i}_{ij} + \sup \tilde{f}_{ij} \leq 3$ for $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$. Here $\tilde{t}_{ij}$ denotes interval truth membership function, $\tilde{i}_{ij}$ denotes interval indeterminate membership function, $\tilde{f}_{ij}$

denotes interval falsity membership function. Then we have the following decision matrix:

$$D = (\tilde{a}_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \dots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \dots & \tilde{a}_{mn} \end{pmatrix} \end{array} \tag{8}$$

Now, we develop this method when attribute weights are completely known, partially known and completely unknown. The steps of the ranking are as follows:

Step 1: Standardize the decision matrix.

This step transforms various attribute dimensions into non-dimensional attributes which allow comparison across criteria because various criteria are usually measured in various units. In general, there are two types of attribute. One is benefit type attribute and another one is cost type attribute. Let $D = (a_{ij})_{m\times n}$ be a decision matrix where the ITrNN $\tilde{a}_{ij} = ([a^1_{ij}, a^2_{ij}, a^3_{ij}, a^4_{ij}]; \tilde{t}_{ij}, \tilde{i}_{ij}, \tilde{f}_{ij})$ is the rating value of the alternative A_i with respect to the attribute C_j.

In order to eliminate the influence of attribute type, we consider the following technique and obtain the standardize matrix $R = (\tilde{r}_{ij})_{m\times n}$, where $\tilde{r}_{ij} = ([r^1_{ij}, r^2_{ij}, r^3_{ij}, r^4_{ij}]; [\underline{t}_{ij}, \bar{t}_{ij}], [\underline{f}_{ij}, \bar{f}_{ij}], [\underline{f}_{ij}, \bar{f}_{ij}])$ is ITrNN. Then we have

$$\tilde{r}_{ij} = \Big(\Big[\frac{a^1_{ij}}{u^+_j}, \frac{a^2_{ij}}{u^+_j}, \frac{a^3_{ij}}{u^+_j}, \frac{a^4_{ij}}{u^+_j} \Big]; [\underline{t}_{ij}, \bar{t}_{ij}], [\underline{f}_{ij}, \bar{f}_{ij}], [\underline{f}_{ij}, \bar{f}_{ij}] \Big), \quad \text{for benefit type attribute} \tag{9}$$

$$\tilde{r}_{ij} = \Big(\Big[\frac{u^-_j}{a^4_{ij}}, \frac{u^-_j}{a^3_{ij}}, \frac{u^-_j}{a^2_{ij}}, \frac{u^-_j}{a^1_{ij}} \Big]; [\underline{t}_{ij}, \bar{t}_{ij}], [\underline{f}_{ij}, \bar{f}_{ij}], [\underline{f}_{ij}, \bar{f}_{ij}] \Big), \quad \text{for cost type attribute} \tag{10}$$

where $u^+_j = max\{a^4_{ij} : i = 1, 2, ..., m\}$ and $u^-_j = min\{a^1_{ij} : i = 1, 2, ..., m\}$ for $j = 1, 2, ..., n$.

Step 2: Calculate the attribute weight.

The attribute weights may be completely known, partially known or completely unknown. So we need to determine the attribute weights by maximum deviation method which is proposed by Wang [48]. If the attributes have larger deviation, smaller deviation and no deviation then we assign larger weight, smaller weight and zero weight, respectively.

For MADM problem, the deviation values of alternative A_i to the other alternatives under the attribute C_j can be defined as follows:

$$d_{ij}(w) = \sum_{k=1}^{m} d(\tilde{a}_{ij}, \tilde{a}_{kj}) w_j, \; i = 1, 2, ..., m; \;\; j = 1, 2, ..., n, \text{where}$$

$$\begin{aligned}
d(\tilde{a}_{ij},\tilde{a}_{kj}) =& \frac{1}{24}\Bigg(\left|\begin{array}{l} a^1_{ij}(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+a^1_{ij}(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -a^1_{kj}(2+\underline{t}_{kj}-\underline{i}_{kj}-\underline{f}_{kj})-a^1_{kj}(2+\bar{t}_{kj}-\bar{i}_{kj}-\bar{f}_{kj})\end{array}\right| \\
&+\left|\begin{array}{l} a^2_{ij}(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+a^2_{ij}(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -a^2_{kj}(2+\underline{t}_{kj}-\underline{i}_{kj}-\underline{f}_{kj})-a^2_{kj}(2+\bar{t}_{kj}-\bar{i}_{kj}-\bar{f}_{kj})\end{array}\right| \\
&+\left|\begin{array}{l} a^3_{ij}(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+a^3_{ij}(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -a^3_{kj}(2+\underline{t}_{kj}-\underline{i}_{kj}-\underline{f}_{kj})-a^3_{kj}(2+\bar{t}_{kj}-\bar{i}_{kj}-\bar{f}_{kj})\end{array}\right| \\
&+\left|\begin{array}{l} a^4_{ij}(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+a^4_{ij}(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -a^4_{kj}(2+\underline{t}_{kj}-\underline{i}_{kj}-\underline{f}_{kj})-a^4_{kj}(2+\bar{t}_{kj}-\bar{i}_{kj}-\bar{f}_{kj})\end{array}\right|\Bigg) \\
=& \frac{1}{24}\sum_{p=1}^{4}\Bigg(\left|\begin{array}{l} a^p_{ij}(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+a^p_{ij}(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -a^p_{kj}(2+\underline{t}_{kj}-\underline{i}_{kj}-\underline{f}_{kj})-a^p_{kj}(2+\bar{t}_{kj}-\bar{i}_{kj}-\bar{f}_{kj})\end{array}\right|\Bigg)
\end{aligned}$$

The deviation values of all the alternatives to other alternatives for the attributes C_j can be defined as

$$\begin{aligned}
D_j(w) &= \sum_{i=1}^{m} d_{ij}(w) = \sum_{i=1}^{m}\sum_{k=1}^{m} d(\tilde{a}_{ij},\tilde{a}_{kj})w_j \\
&= \sum_{i=1}^{m}\sum_{k=1}^{m}\Big(\frac{1}{24}\sum_{p=1}^{4}\left|\begin{array}{l} a^p_{ij}(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+a^p_{ij}(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -a^p_{kj}(2+\underline{t}_{kj}-\underline{i}_{kj}-\underline{f}_{kj})-a^p_{kj}(2+\bar{t}_{kj}-\bar{i}_{kj}-\bar{f}_{kj})\end{array}\right|\Big)w_j
\end{aligned}$$

Therefore, the total deviation value $D(w) = \sum_{j=1}^{n} D_j(w)$.
In the following, we develop three cases:
Case 1. When the attribute weights are completely known.
In this case, the attribute weights $w_1, w_2, ..., w_n$ are known in advance and $\sum_{j=1}^{n} w_j = 1, w_j \geq 0,\ for\ j = 1, 2, ..., n.$
Case 2. When attributes weights are partially known.
In this case, we assume a non-linear programming model. This model maximizes all deviation values of the attributes.

$$\textbf{Model 1}\begin{cases} max\ D(w) \\ = \dfrac{1}{24}\displaystyle\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\Bigg(\left|\begin{array}{l} a^p_{ij}(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+a^p_{ij}(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -a^p_{kj}(2+\underline{t}_{kj}-\underline{i}_{kj}-\underline{f}_{kj})-a^p_{kj}(2+\bar{t}_{kj}-\bar{i}_{kj}-\bar{f}_{kj})\end{array}\right|\Bigg)w_j \\ \text{subject to } w \in \Delta,\quad \displaystyle\sum_{j=1}^{n} w = 1, w_j \geq 0,\ \text{for } j = 1, 2, ..., n. \end{cases}$$

Here, the incomplete attribute weight information Δ is taken in the following form ([49, 50]):

1. A weak ranking:$\{w_i \geq w_j\}, i \neq j;$
2. A strict ranking:$\{w_i - w_j \geq \epsilon_i(> 0)\}, i \neq j;$
3. A ranking of difference:$\{w_i - w_j \geq w_k - w_p\}, i \neq j \neq k \neq p;$
4. A ranking with multiples:$\{w_i \geq \alpha_i w_j\}, 0 \leq \alpha_i \leq 1, i \neq j;$

5. An interval form: $\{\beta_i \leq w_i \leq \beta_i + \epsilon_i (> 0)\}, 0 \leq \beta_i \leq \beta_i + \epsilon_i \leq 1$.

Solving this model, we get the optimal solution which is to be used as the weight vector.
Case 3. When attribute weights are completely unknown:
In this case, we can establish the following programming model:

$$\textbf{Model 2}\begin{cases} max\ D(w) \\ = \dfrac{1}{24}\sum\limits_{j=1}^{n}\sum\limits_{i=1}^{m}\sum\limits_{k=1}^{m}\sum\limits_{p=1}^{4}\left(\left|\begin{array}{l} a_{ij}^p(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+a_{ij}^p(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -a_{kj}^p(2+\underline{t}_{kj}-\underline{i}_{kj}-\underline{f}_{kj})-a_{kj}^p(2+\bar{t}_{kj}-\bar{i}_{kj}-\bar{f}_{kj}) \end{array}\right|\right)w_j \\ \text{subject to } w \in \Delta, \quad \sum\limits_{j=1}^{n} w_j^2 = 1, w_j \geq 0, \text{ for } j = 1,2,......,n. \end{cases}$$

To solve this model, we construct the Lagrangian function:

$$L(w,\xi) = \frac{1}{24}\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l} a_{ij}^p(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+a_{ij}^p(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -a_{kj}^p(2+\underline{t}_{kj}-\underline{i}_{kj}-\underline{f}_{kj})-a_{kj}^p(2+\bar{t}_{kj}-\bar{i}_{kj}-\bar{f}_{kj}) \end{array}\right|\right)w_j + \frac{\xi}{48}\Big(\sum_{j=1}^{n} w_j^2 - 1\Big) \tag{11}$$

where $\xi \in \mathbb{R}$ is Lagrange multiplier.

Now, we calculate the partial derivatives of L with respect to $w_j (j = 1, 2, ...n)$ and ξ:

$$\frac{\partial L}{\partial w_j} = \sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l} a_{ij}^p(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+a_{ij}^p(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -a_{kj}^p(2+\underline{t}_{kj}-\underline{i}_{kj}-\underline{f}_{kj})-a_{kj}^p(2+\bar{t}_{kj}-\bar{i}_{kj}-\bar{f}_{kj}) \end{array}\right|\right) + \xi w_j = 0 \tag{12}$$

$$\frac{\partial L}{\partial \xi} = \sum_{j=1}^{n} w_j^2 - 1 = 0 \tag{13}$$

From Eq. (12), we get

$$w_j = \frac{-\sum\limits_{i=1}^{m}\sum\limits_{k=1}^{m}\sum\limits_{p=1}^{4}\left(\left|\begin{array}{l} a_{ij}^p(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+a_{ij}^p(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -a_{kj}^p(2+\underline{t}_{kj}-\underline{i}_{kj}-\underline{f}_{kj})-a_{kj}^p(2+\bar{t}_{kj}-\bar{i}_{kj}-\bar{f}_{kj}) \end{array}\right|\right)}{\xi}, \quad j = 1,2,.....,n \tag{14}$$

Putting this value in Eq.(13), we get

$$\xi^2 = \sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{m}\left(\left|\begin{array}{l} a_{ij}^p(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+a_{ij}^p(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -a_{kj}^p(2+\underline{t}_{kj}-\underline{i}_{kj}-\underline{f}_{kj})-a_{kj}^p(2+\bar{t}_{kj}-\bar{i}_{kj}-\bar{f}_{kj}) \end{array}\right|\right)\right)^2 \tag{15}$$

$$\Rightarrow \xi = -\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{m}\left(\left|\begin{array}{l} a_{ij}^p(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+a_{ij}^p(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -a_{kj}^p(2+\underline{t}_{kj}-\underline{i}_{kj}-\underline{f}_{kj})-a_{kj}^p(2+\bar{t}_{kj}-\bar{i}_{kj}-\bar{f}_{kj}) \end{array}\right|\right)\right)^2} \text{ for } \xi < 0 \tag{16}$$

From Eq. (14) and Eq. (16), we get the formula for determining attribute weights for $C_j(j = 1, 2, ..., n)$:

$$w_j = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{m}\left(\left|\begin{array}{l} a_{ij}^p(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+a_{ij}^p(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -a_{kj}^p(2+\underline{t}_{kj}-\underline{i}_{kj}-\underline{f}_{kj})-a_{kj}^p(2+\bar{t}_{kj}-\bar{i}_{kj}-\bar{f}_{kj})\end{array}\right|\right)}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{m}\left(\left|\begin{array}{l} a_{ij}^p(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+a_{ij}^p(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -a_{kj}^p(2+\underline{t}_{kj}-\underline{i}_{kj}-\underline{f}_{kj})-a_{kj}^p(2+\bar{t}_{kj}-\bar{i}_{kj}-\bar{f}_{kj})\end{array}\right|\right)\right)^2}} \tag{17}$$

Now, we can get the normalized attribute weight as

$$\bar{w}_j = \frac{w_j}{\sum_{j=1}^{n} w_j} = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{m}\left(\left|\begin{array}{l} a_{ij}^p(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+a_{ij}^p(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -a_{kj}^p(2+\underline{t}_{kj}-\underline{i}_{kj}-\underline{f}_{kj})-a_{kj}^p(2+\bar{t}_{kj}-\bar{i}_{kj}-\bar{f}_{kj})\end{array}\right|\right)}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{m}\left(\left|\begin{array}{l} a_{ij}^p(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+a_{ij}^p(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -a_{kj}^p(2+\underline{t}_{kj}-\underline{i}_{kj}-\underline{f}_{kj})-a_{kj}^p(2+\bar{t}_{kj}-\bar{i}_{kj}-\bar{f}_{kj})\end{array}\right|\right)} \tag{18}$$

Therefore, we get the normalized weight vector $\bar{w} = \{\bar{w}_1, \bar{w}_2, \ldots, \bar{w}_n\}$.

Step 3: Determine the positive and negative ideal solutions.

The normalized decision matrix $R = (\tilde{r}_{ij})_{m\times n}$ in the interval trapezoidal neutroshopic number, the positive and negative ideal solutions are defined as follows:

$\tilde{r}^+ = (\tilde{r}_1^+, \tilde{r}_2^+, \ldots, \tilde{r}_n^+)$ and $\tilde{r}^- = (\tilde{r}_1^-, \tilde{r}_2^-, \ldots, \tilde{r}_n^-)$ where,

$$\begin{aligned}\tilde{r}_j^+ =&([r_j^{1+}, r_j^{2+}, r_j^{3+}, r_j^{4+}]; [\underline{t}_j^+, \bar{t}_j^+], [\underline{i}_j^+, \bar{i}_j^+], [\underline{f}_j^+, \bar{f}_j^+]) \\ =&\Big([\max_i(r_{ij}^1), \max_i(r_{ij}^2), \max_i(r_{ij}^3), \max_i(r_{ij}^4)]; \\ &[\max_i(\underline{t}_{ij}), \max_i(\bar{t}_{ij})][\min_i(\underline{i}_{ij}), \min_i(\bar{i}_{ij})], [\min_i(\underline{f}_{ij}), \min_i(\bar{f}_{ij})]\Big)\end{aligned} \tag{19}$$

$$\begin{aligned}\tilde{r}_j^- =&([r_j^{1-}, r_j^{2-}, r_j^{3-}, r_j^{4-}]; [\underline{t}_j^-, \bar{t}_j^-], [\underline{i}_j^-, \bar{i}_j^-], [\underline{f}_j^-, \bar{f}_j^-]) \\ =&\Big([\min_i(r_{ij}^1), \min_i(r_{ij}^2), \min_i(r_{ij}^3), \min_i(r_{ij}^4)]; \\ &[\min_i(\underline{t}_{ij}), \min_i(\bar{t}_{ij})][\max_i(\underline{i}_{ij}), \max_i(\bar{i}_{ij})], [\max_i(\underline{f}_{ij}), \max_i(\bar{f}_{ij})]\Big)\end{aligned} \tag{20}$$

The global positive and negative ideal solutions for ITrNN can be considered as $\tilde{r}_j^+ = ([1, 1, 1, 1]; [1, 1], [0, 0], [0, 0])$ and $\tilde{r}_j^- = ([0, 0, 0, 0]; [0, 0], [1, 1], [1, 1])$.

Step 4: Calculate the separation measure from ideal solutions.

Now, using Eqs.(2), (19) and (20), we calculate separation measure d_i^+ from positive ideal solution and d_i^- from negative ideal solution as

$$\begin{aligned}d_i^+ =&\sum_{j=1}^{n} w_j d(\tilde{r}_{ij}, \tilde{r}_j^+) \\ =&\frac{1}{24}\sum_{j=1}^{n} w_j \sum_{p=1}^{4}\left(\left|\begin{array}{l} r_{ij}^p(2+\underline{t}_{ij}-\underline{i}_{ij}-\underline{f}_{ij})+r_{ij}^p(2+\bar{t}_{ij}-\bar{i}_{ij}-\bar{f}_{ij}) \\ -r_j^{p+}(2+\underline{t}_j^+-\underline{i}_j^+-\underline{f}_j^+)-r_j^{p+}(2+\bar{t}_j^+-\bar{i}_j^+-\bar{f}_j^+)\end{array}\right|\right), \quad i = 1, 2, ..., m.\end{aligned} \tag{21}$$

$$
\begin{aligned}
d_i^- =& \sum_{j=1}^{n} w_j d(\tilde{r}_{ij}, \tilde{r}_j^-) \\
=& \frac{1}{24} \sum_{j=1}^{n} w_j \sum_{p=1}^{4} \left(\left| \begin{matrix} r_{ij}^p(2 + \underline{t}_{ij} - \underline{i}_{ij} - \underline{f}_{ij}) + r_{ij}^p(2 + \bar{t}_{ij} - \bar{i}_{ij} - \bar{f}_{ij}) \\ - r_j^{p-}(2 + \underline{t}_j^- - \underline{i}_j^- - \underline{f}_j^-) - r_j^{p-}(2 + \bar{t}_j^- - \bar{i}_j^- - \bar{f}_j^-) \end{matrix} \right| \right), \quad i = 1, 2, ..., m. \quad (22)
\end{aligned}
$$

Step 5: Calculate the relative closeness co-efficient.
We calculate the relative closeness co-efficient of an alternative A_i with respect to the ideal alternative A^+ as

$$
RCC(A_i) = \frac{d_i^-}{d_i^+ + d_i^-}, \ for\ i = 1, 2, ..., n, \quad (23)
$$

where $0 \leq RCC(A_i) \leq 1$. We then rank the best alternative according to RCC.
Step 6: End.

5 An illustrative example

In order to demonstrate the proposed method, we consider the following MADM problem. Suppose that a person wants to buy a laptop. Let there be four companies A_1, A_2, A_3, A_4 and laptop of each company has three attributes such as cost, warranty, and quality. We consider C_1 for cost, C_2 for warranty and C_3 for quality type of attribute.
The person evaluates the rating values of the alternatives A_i $(i = 1, 2, 3, 4)$ with respect to attributes $C_j(j = 1, 2, 3)$. Then we get the neutrosophic decision matrix $D = (\tilde{a}_{ij})_{4\times3} =$

	C_1
A_1	$([50, 60, 70, 80]; [0.1, 0.2], [0.2, 0.3], [0.4, 0.5])$
A_2	$([30, 40, 50, 60]; [0.3, 0.4], [0.2, 0.3], [0.1, 0.2])$
A_3	$([70, 80, 90, 100]; [0.6, 0.7], [0.2, 0.3], [0.4, 0.5])$
A_4	$([40, 50, 60, 70]; [0.4, 0.5], [0.6, 0.7], [0.2, 0.3])$
	C_2
A_1	$([30, 40, 50, 60]; [0.2, 0.3], [0.4, 0.5], [0.6, 0.7])$
A_2	$([10, 20, 30, 40]; [0.1, .2], [0.3, 0.4], [0.6, 0.7])$
A_3	$([50, 60, 70, 80]; [0.1, 0.2], [0.3, 0.4], [0.6, 0.7])$
A_4	$([70, 80, 90, 100]; [0.2, 0.3], [0.4, 0.5], [0.6, 0.8])$
	C_3
A_1	$([40, 50, 60, 70]; [0.4, 0.5], [0.6, 0.7], [0.7, 0.8])$
A_2	$([20, 30, 40, 50]; [0.1, 0.2], [0.3, 0.4], [0.8, 0.9])$
A_3	$([70, 80, 90, 100]; [0.3, 0.5], [0.4, 0.6], [0.7, 0.8])$
A_4	$([30, 40, 50, 60]; [0.4, 0.5], [0.6, 0.7], [0.7, 0.8])$

Now, with the help of the proposed method, we find the best alternative following the steps given below:
Step 1: Standardize the decision matrix.
In the decision matrix, the first column represents the cost type attribute, and the second and the third columns represent benefit type attribute. Then, using Eqs. (9) and (10), we get the following standardize decision

matrix R_{ij} =

	C_1
A_1	$([0.38, 0.43, 0.50, 0.60]; [0.1, 0.2], [0.2, 0.3], [0.4, 0.5])$
A_2	$([0.50, 0.60, 0.75, 1.0]; [0.3, 0.4], [0.2, 0.3], [0.1, 0.2])$
A_3	$([0.30, 0.33, 0.38, 0.43]; [0.6, 0.7], [0.2, 0.3], [0.4, 0.5])$
A_4	$([0.43, 0.50, 0.60, 0.75]; [0.4, 0.5], [0.6, 0.7], [0.2, 0.3])$
	C_2
A_1	$([0.30, 0.40, 0.50, 0.60]; [0.2, .3], [0.4, 0.5], [0.6, 0.7])$
A_2	$([0.10, 0.20, 0.30, 0.40]; [0.1, 0.2], [0.3, 0.4], [0.6, 0.7])$
A_3	$([0.50, 0.60, 0.70, 0.80]; [0.1, 0.2], [0.3, 0.4], [0.6, 0.7])$
A_4	$([0.70, 0.80, 0.90, 1.0]; [0.2, 0.3], [0.4, .5], [0.6, 0.8])$
	C_3
A_1	$([0.40, 0.50, 0.60, 0.70]; [0.4, 0.5], [0.6, 0.7], [0.7, 0.8])$
A_2	$([0.20, 0.30, 0.40, 0.50]; [0.1, 0.2], [0.3, 0.4], [0.8, 0.9])$
A_3	$([0.70, 0.80, 0.90, 1.0]; [0.3, 0.5], [0.4, 0.6], [0.7, 0.8])$
A_4	$([0.30, 0.40, 0.50, 0.60]; [0.4, .5], [0.6, 0.7], [0.7, 0.8])$

Step 2: Calculate the attribute weight.

Here we assume three cases for the attribute weight.

Case 1 : When the attribute weights are completely known, let the weight vector be $\bar{w} = (0.25, 0.55, 0.20)$.

Case 2 : When the attribute weights are partially known, we select the weight information as follows:

$$\Delta = \begin{cases} 0.35 \leq w_1 \leq 0.75 \\ 0.25 \leq w_2 \leq 0.60 \\ 0.30 \leq w_3 \leq 0.45 \\ \text{and } w_1 + w_2 + w_3 = 1 \end{cases}$$

Using Model 1, we develop the single objective programming problem as

$$\begin{cases} max(D) = 45.92w_1 + 109.56w_2 + 98.20w_3 \\ \text{subject to } w \in \Delta \text{ and } \sum_{j=1}^{3} w_j = 1,\ w_j > 0\ for\ j = 1, 2, 3. \end{cases}$$

Solving this problem with optimization software LINGO 11, we get the optimal weight vector as

$\bar{w} = (0.35, 0.35, 0.30)$.

Case 3 : When the attribute weights are completely unknown, we use Model 2 and Eqn. (18) and obtain the following weight vector:

$\bar{w} = (0.18, 0.43, 0.39)$.

Step 3: Determine the positive and negative ideal solutions.

Since the cost of the laptop is cost type attribute, and warranty and quality are benefit type attributes, therefore, using Eqs.(19) and (20), we get the following neutrosophic positive and negative ideal solutions:

$$A^+ = \begin{pmatrix} \left([0.30,0.33,0.38,0.43];[0.10,0.20],[0.20,0.30],[0.20,0.30]\right) \\ \left([0.70,0.80,0.90,1.0];[0.20,0.30],[0.40,0.50],[0.60,0.70]\right) \\ \left([0.70,0.80,0.90,1.0];[0.40,0.50],[0.60,0.70],[0.80,0.90]\right) \end{pmatrix}$$

$$A^- = \begin{pmatrix} ([0.50,0.60,0.75,1.0];[0.60,0.70],[0.40,0.50],[0.40,0.50]) \\ ([0.10,0.20,0.30,0.40];[0.10,0.20],[0.30,0.40],[0.60,0.70]) \\ ([0.20,0.30,0.40,0.50];[0.10,0.20],[0.30,0.40],[0.70,0.80]) \end{pmatrix}$$

Step 4 : Calculate the separation measure from ideal solutions.

Case 1 : From Eq. (21), we get the separation measure d_i^+ of each A_i from A^+:
$d_1^+ = d(A_1, A^+) = 0.179, d_2^+ = d(A_2, A^+) = 0.425, d_3^+ = d(A_3, A^+) = 0.106, d_4^+ = d(A_4, A^+) = 0.325$
From Eq. (22), we get the separation measure d_i^- of each A_i from A^-:
$d_1^- = d(A_1, A^-) = 0.304, d_2^- = d(A_2, A^-) = 0.083, d_3^- = d(A_3, A^-) = 0.485, d_4^- = d(A_4, A^-) = 0.503$
Case 2 : From Eq. (21), we get the separation measure d_i^+ of each A_i from A^+:
$d_1^+ = d(A_1, A^+) = 0.185, d_2^+ = d(A_2, A^+) = 0.434, d_3^+ = d(A_3, A^+) = 0.141, d_4^+ = d(A_4, A^+) = 0.335$
From Eq. (22), we get the separation measure d_i^- of each A_i from A^-:
$d_1^- = d(A_1, A^-) = 0.299, d_2^- = d(A_2, A^-) = 0.084, d_3^- = d(A_3, A^-) = 0.479, d_4^- = d(A_4, A^-) = 0.381$
Case 3 : From Eq. (21), we get the separation measure d_i^+ of each A_i from A^+:
$d_1^+ = d(A_1, A^+) = 0.167, d_2^+ = d(A_2, A^+) = 0.429, d_3^+ = d(A_3, A^+) = 0.126, d_4^+ = d(A_4, A^+) = 0.307$
From Eq. (22), we get the separation measure d_i^- of each A_i from A^-:
$d_1^- = d(A_1, A^-) = 0.604, d_2^- = d(A_2, A^-) = 0.094, d_3^- = d(A_3, A^-) = 0.554, d_4^- = d(A_4, A^-) = 0.467$

Step 5: Calculate the relative closeness co-efficient.

In this step, using Eq. (22), we calculate the relative closeness coefficient of the alternatives A_1, A_2, A_3, A_4 and obtain the following results (see Table 1):

Table 1: Relative closeness co-efficient

$RCC(A_i)$	Case 1	Case 2	Case 3
$RCC(A_1)$	0.629	0.618	0.783
$RCC(A_2)$	0.163	0.162	0.180
$RCC(A_3)$	0.819	0.773	0.814
$RCC(A_4)$	0.607	0.532	0.603

From the above table, we see that $RCC(A_3) \geq RCC(A_1) \geq RCC(A_4) \geq RCC(A_2)$ in all cases. Therefore, we conclude that

$$A_3 \succ A_1 \succ A_4 \succ A_2$$

where A_3 is the best alternative.
Step 6: End.

5.1 Comparative analysis

The study made by Liu [22] presents TOPSIS method for MADM based on trapezoidal intuitionistic fuzzy number and does not include indeterminate type information in the decision making process. The preference value considered in our paper is interval trapezoidal neutrosophic number, which deals with indeterminate type information effectively along with truth and falsity type information. The method presented by Ye [44] and

Subaş [45] discusses some aggregation operators of trapezoidal neutrosophic number and the decision making method proposed by Biswas et al. [42] presents trapezoidal neutrosophic number based TOPSIS method for MADM with partially known, and completely unknown weight information. We know that interval trapezoidal neutrosophic number is a generalization of trapezoidal neutrosophic number. The approach provided by Biswas et al. [43] discusses ITrNN based MADM with known weight information, whereas our proposed model develops ITrNN based MADM model with known, partially known, and completely unknown weight information. Furthermore, the methods suggested by Biswas et al. [42], Ye [44], and Subaş [45] are not suitable for the decision making problem in this paper. In Table 2, we compare our results with those obtained by the method given by Biswas et al. [43].

Table 2: A comparison of the results

Method	Type of weight information	Ranking result
Biswas et al.'s method [43]	Partially known	Not Applicable
	Completely unknown	Not Applicable
Proposed method	Partially known	$A_3 \succ A_1 \succ A_4 \succ A_2$
	Completely unknown	$A_3 \succ A_1 \succ A_4 \succ A_2$

Therefore, our proposed method is more general than the existing methods because the existing methods cannot deal with ITrNN based MADM with partially known, and completely unknown weight information.

6 Conclusions

TOPSIS method is a very popular method for MADM problem and this method has been extended under different environments like fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets. In this paper, we have extended TOPSIS method based on ITrNN. First, we have developed an optimization model to calculate the attribute weight with the help of maximum deviation strategy when the weight information is partially known. We have also developed another model by using Lagrangian function to determine attributes' weights for unknown weight information case. With these weights we have solved MADM problem by TOPSIS method. Finally we have provided a numerical example of MADM problem and compared with existing methods. The proposed strategy can be extended to multi-attribute group decision making problem with ITrNN. This model can be used in various selection problems like weaver selection problem [51, 52], data mining [53], teacher selection problem [54], brick field selection problem [55], center location selection problem [56,57], etc. under ITrNN environment.

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References

[1] L.A. Zadeh. Fuzzy sets. Information and control, 8(3) (1965), 338-353.

[2] K. Atanassov. More on intutionistic fuzzy sets. Fuzzy Sets and System, 33 (1989), 37-46.

[3] V. Torra. Hesitant fuzzy sets. International Journal of Intelligence System, 25 (2010), 529-539.

[4] F. Samarandache. A unifying field in logics. Neutrosophy: neutrosophic probability, set and logic. American Research Press, Rehoboth 1998.

[5] C.L. Hwang and K. Yoon. Multiple attribute decision making, methods and applications. Springer-Verlag, New York 1981.

[6] J.P. Brans, P. Vincke, and B. Mareschal. How to select and how to rank projects: The PROMETHEE method. European Journal of Operation Research, 24 (1986), 228-238.

[7] S. Opricovic and G. H. Tzeng. Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. European Journal of Operation Research, 156 (2004), 445-455.

[8] B. Roy. The outranking approach and the foundations of ELECTRE methods. Theory and Decision, 31 (1991), 49-73.

[9] T.L. Saaty. The analytic hierarchy process. McGraw-Hill, New York 1980.

[10] S.J. Chen and C.L. Hwang. Fuzzy multiple attribute decision making. Springer, Berlin, Heidelberg 1992.

[11] Y.J. Wang and H.S. Lee. Generalizing TOPSIS for fuzzy multiple-criteria group decision-making. Computers and Mathematics with Applications, 53 (2007), 1762-1772.

[12] S. Pramanik and K. Mondal. Weighted fuzzy similarity measure based on tangent function and its application to medical diagnosis. International Journal of Innovative Research in Science, Engineering and Technology, 4(2) (2015), 158-164.

[13] T.Y. Chen. An ELECTRE-based outranking method for multiple criteria group decision making using interval type-2 fuzzy sets. Information Sciences, 263 (2014), 1-21.

[14] W. Wang, X. Liu, and Y. Qin. Multi-attribute group decision making models under interval type-2 fuzzy environment. Knowledge-Based Systems, 30 (2012), 121-128.

[15] G.W. Wei. Maximizing deviation method for multiple attribute decision making in intuitionistic fuzzy setting. Knowledge-Based Systems, 21 (2008), 833-836.

[16] S. Pramanik and D. Mukhopadhyaya. Grey relational analysis based intuitionistic fuzzy multi-criteria group decision making approach for teacher selection in higher education. International Journal of Computer Applications, 34 (2011), 21-29.

[17] P. P. Dey, S. Pramanik and B. C. Giri. Multi-criteria group decision making in intuitionistic fuzzy environment based on grey relational analysis for weaver selection in Khadi institution. Journal of Applied and Quantitative Methods, 10 (4) (2015), 1-14.

[18] K. Mondal and S. Pramanik. Intuitionistic fuzzy similarity measure based on tangent function and its application to multi-attribute decision making. Global Journal of Advanced Research, 2(2) (2015), 464-471.

[19] C.T. Chen. Extensions of the TOPSIS for group decision making under fuzzy environment. Fuzzy Sets and Systems, 114 (2000), 1-9.

[20] F.E. Boran, S. Gen, M. Kurt and D. Akay. A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method. Expert Systems with Applications, 36 (2009), 11363-11368.

[21] X. Zhao. TOPSIS method for interval-valued intuitionistic fuzzy multiple attribute decision making and its application to teaching quality evaluation. Journal of Intelligent and Fuzzy Systems, 26 (2014), 3049-3055.

[22] A. F. Liu. TOPSIS method for multiple attribute decision making under trapezoidal intuitionistic fuzzy environment. Journal of Intelligent & Fuzzy Systems, 26 (2014), 2403–2409.

[23] Ru-xia Liang, Jian-qiang Wang, Hong-yu Zhang. A multi-criteria decision-making method based on single-valued trapezoidal neutrosophic preference relations with complete weight information. Neural Computing and Applications. (2017) DOI 10.1007/s00521-017-2925-8.

[24] S. Pramanik and K. Mondal. Interval neutrosophic multi attribute decision-making based on grey relational analysis. Neutrosophic Sets and Systems, 9(2015) 13-22.

[25] K. Mondal, S. Pramanik and B. C. Giri. Single valued neutrosophic hyperbolic sine similarity measure based strategy for MADM problems. Neutrosophic Sets and Systems, 20 (2018), 4-12.

[26] K. Mondal, S. Pramanik and B. C. Giri. Interval neutrosophic tangent similarity measure and its application to MADM problems. Neutrosophic Sets and Systems, 19 (2018), 47-57.

[27] K. Mondal, S. Pramanik and B. C. Giri. Hybrid binary logarithm similarity measure for MAGDM problems under SVNS assessments. Neutrosophic Sets and Systems, 20 (2018), 13-26.

[28] S. Pramanik, R. Mallick and A. Dasgupta. Contributions of selected Indian researchers to multi-attribute decision making in neutrosophic environment. Neutrosophic Sets and Systems, 20 (2018), 108-131.

[29] S. Dalapati, S. Pramanik. A revisit to NC-VIKOR based MAGDM strategy in neutrosophic cubic set environment, Neutrosophic Sets and Systems, 21 (2018), 131-141.

[30] S. Pramanik, S. Dalapati, S. Alam, F. Smarandache and T.K. Roy. NS–cross entropy based MAGDM under single valued neutrosophic set environment. Information, 9(2)(2018), 37; doi: 10.3390/info9020037

[31] P. Biswas, S. Pramanik, and B.C. Giri. TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment. Neural Computing and Applications, 27 (2016), 727-737.

[32] H. Wang, F. Smarandache, R. Sunderraman and Y.Q. Zhang. Single valued neutrosophic sets. Multi-space and Multi-structure, 4 (2010), 410-413.

[33] P. Chi and P. Liu. An extended TOPSIS method for the multiple attribute decision making problems based on interval neutrosophic set. Neutrosophic Sets and Systems, 1(1)(2013), 63–70.

[34] Z. Zhang, and C. Wu. A novel method for single-valued neutrosophic multi-criteria decision making with incomplete weight information. Neutrosophic Sets and Systems, 4 (2014), 35-49.

[35] P. Biswas, S. Pramanik and B.C. Giri. Neutrosophic TOPSIS with group decision making. In C. Kahraman and I. Otay (Eds.): C. Kahraman and I. Otay (eds.), Fuzzy Multi-criteria Decision Making Using Neutrosophic Sets, Studies in Fuzziness and Soft Computing 369 (2018), 543-585.

[36] S. Pramanik, D. Banerjee and B.C. Giri. TOPSIS approach for multi attribute group decision making in refined neutrosophic environment. In F. Smarandache, and S. Pramanik (Eds.), New trends in neutrosophic theory and applications (2016) 79-91. Brussels: Pons Editions.

[37] P.P. Dey, S. Pramanik and B.C. Giri. TOPSIS for solving multi-attribute decision making problems under bi-polar neutrosophic environment. In F. Smarandache, and S. Pramanik (Eds), New trends in neutrosophic theory and applications (2016) 65-77. Brussels: Pons Editions.

[38] K. Mondal, S. Pramanik and F. Smarandache. Rough neutrosophic TOPSIS for multi-attribute group decision making. Neutrosophic Sets and Systems, 13(2016), 105-117.

[39] J. Ye. An extended TOPSIS method for multiple attribute group decision making based on single valued neutrosophic linguistic numbers. Journal of Intelligent and Fuzzy Systems, 28 (2015), 247-255.

[40] P.P. Dey, S. Pramanik and B.C. Giri. Generalized neutrosophic soft multi attribute group decision making based on TOPSIS. Critical Review, 11(2015), 41-55.

[41] S. Pramanik, P.P. Dey and B.C. Giri. TOPSIS for single valued neutrosophic soft expert set based multi-attribute decision making problems. Neutrosophic Sets and Systems, 10 (2015), 88-95.

[42] P. Biswas, S. Pramanik and B.C. Giri, TOPSIS strategy for multi-attribute decision making with trapezoidal neutrosophic numbers. Neutrosophic Sets and Systems, 19 (2018), 40-46.

[43] P. Biswas, S. Pramanik and B.C. Giri, Distance measure based MADM strategy with interval trapezoidal neutrosophic numbers. Neutrosophic Sets and Systems. 19 (2018) 40-46.

[44] J. Ye. Some weighted aggregation operators of trapezoidal neutrosophic numbers and their multiple attribute decision making method. Informatica, 28 (2017), 387-402.

[45] Y. Subas. Neutrosophic numbers and their application to multi-attribute decision making problems (in Turkish). (Master Thesis, Kilis 7 Aralik University, Graduate School of Natural and Applied Science) (2015).

[46] D. Dubois and H. Prade. Ranking fuzzy numbers in the setting of possibility theory. Information Sciences, 30(3) (1983), 183-224.

[47] S. Heilpern. The expected value of a fuzzy number. Fuzzy Sets and Systems, 47(1) (1992), 81-86.

[48] Y. Wang. Using the method of maximizing deviation to make decision for multiindices. Journal of Systems Engineering and Electronics, 8 (1997), 21-26.

[49] K. S. Park, S.H. Kim and W.C. Yoon. Establishing strict dominance between alternatives with special type of incomplete information. European Journal of Operational Research, 96(1997), 398–406.

[50] J.H. Park, I.Y. Park, Y.C. Kwun and X. Tan. Extension of the TOPSIS method for decision making problems under interval-valued intuitionistic fuzzy environment. Applied Mathematical Modelling, 35(2011), 2544–2556.

[51] P. P. Dey, S. Pramanik and B. C. Giri. Multi-criteria group decision making in intuitionistic fuzzy environment based on grey relational analysis for weaver selection in khadi institution. Journal of Applied and Quantitative Methods, 10 (4) (2015), 1-14.

[52] P. P. Dey, S. Pramanik and B. C. Giri. An extended grey relational analysis based interval neutrosophic multi-attribute decision making for weaver selection. Journal of New Theory,(9) (2015), 82-93.

[53] K. Mondal, S. Pramanik and F. Smarandache. Role of neutrosophic logic in data mining. In F. Smarandache, and S.Pramanik (Eds.), New Trends in Neutrosophic Theory and Application. Pons Editions, Brussels, 2016, 15-23.

[54] K. Mondal and S. Pramanik. Multi-criteria group decision making approach for teacher recruitment in higher education under simplified neutrosophic environment. Neutrosophic Sets and Systems, 6 (2014), 28-34.

[55] K. Mondal and S. Pramanik. Neutrosophic decision making model for clay-brick selection in construction field based on grey relational analysis. Neutrosophic Sets and Systems, 9 (2015), 72-79.

[56] S. Pramanik, S. Dalapati and T. K. Roy. Logistics center location selection approach based on neutrosophic multi-criteria decision making, In F. Smarandache, and S. Pramanik (Eds.), New Trends in Neutrosophic Theory and Application. Pons Editions, Brussels, 2016, 161-174.

[57] S. Pramanik, S. Dalapati and T. K. Roy. Neutrosophic multiattribute group decision making strategy for logistic center location selection. In F. Smarandache, M. A. Basset and V. Chang (Eds.), Neutrosophic Operational Research, Vol. III. Pons Asbl, Brussels, 2018, 13-32.

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