## "Modelling Plant Capacity and Productivity: The Multi-Machine Case"

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# Modelling Plant Capacity and Productivity: The Multi-Machine Case 

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#### Abstract

This study deals with systems (lines, departments or production units) made up of multiple machines, and it analyzes the meaning and assessment of productivity, utilization, efficiency and so forth at system level. Special emphasis is placed on calculating indices for the whole system starting from parameters referred to a single machine. Two basic system types are discussed, i.e. pure parallel systems and pure serial systems. The latter ones are further split into serial systems with tightly interconnected machines (i.e. without buffers), serial systems with loosely interconnected machines (i.e. provided with some decoupling buffers among stages), and independent departments systems.


Key words: Capacity, utilisation, efficiency, productivity, performance measurement. JEL Classification: M11.

## 1. Introduction

The objective of this paper consists in highlighting the most relevant features of a production system that are more likely to determine system's productivity. This due to that system-wide productivity depends on various throughput loss causes acting on single machines and on the way these losses spread throughout the system. The extent and patterns of such combination and propagation are linked to the nature of the losses and patterns also depend on the system configuration. So, according to this aim, different production systems made up of more than one machine are analysed and their specific features and application limits are outlined. The usefulness of a calculation model allowing to move from machine-level indices to system-level indices is quite relevant during the production system design phase, since the system's performance resulting from different machine layouts and coordination patterns can be estimated.
Before going deeper into the discussion, notice that the definition of different status conditions (Nakajima, 1988; Dal et al., 2000) is only significant for stand-alone machines, whereas it can be somehow ambiguous and even meaningless when a whole system is considered, since - at any given time - each machine may be in a different status. Moreover, to give reliable, system-level working definitions to the indices and parameters developed for single machines, the specific system's configuration has to be clearly determined, i.e. the specific production type must be well defined (Hopp and Spearman, 2000, Freiheit et al., 2004). Three system features are particularly significant for the purpose of this paper:

1) the system layout;
2) the decoupling level among different system elements;
3) the variance of production statuses over time.

These features have an impact on:

1) the methods used and the hurdles found when drawing system-level parameters from single-machine parameters;
2) the significance of the various system-level parameters and their relationship with single-machine
parameters;

[^0]3) the possibility to separate the effects of individual causes for productivity losses.

As a general rule, the higher the decoupling level among system elements is, the lower is the possibility to separate effects and the smaller is the direct relationship between machine performance and system performance. As the variance of production statuses increases, the calculation difficulties grow, the significance of the various system-wide parameters decreases and the relationship between machine parameters and system parameters tends to disappear.
The discussion that follows outlines two basic system types which are regarded as ideal models, i.e. pure parallel systems and pure serial systems. The latter systems are further divided into:

1) systems (lines) made up of tightly interconnected machines (or stations), with no buffers in between;
2) systems made up of loosely interconnected machines, with some decoupling points among different stages;
3) systems made up of completely independent sequenced machines, with a very high decoupling level.

However, these system types are limited to production processes characterized by a high flow consistency over time. The reason for this lies in that within systems with cross-linked flows and variable cycles (e.g. job-shops), the evaluation of different production capacity levels is not significant, because:

1) the machines can be technologically very different, which makes it usually impossible to add up their throughputs;
2) the utilization rates can be remarkably different due to long waiting times;
3) the stability of the production mix over time can be extremely low;
4) production cycles and their associated sequences can be extremely variable.

For these reasons such systems are not taken into account in this study ${ }^{1}$.

## 2. Background

The research area of this paper has been treated according to multiple perspectives in several studies (Grando and Turco, 2005). In particular, management researchers early focused on the single factors to productivity and their interactions (Eilon et al., 1976; Bernolak, 1997, Sgrell and West, 2001; Jeong and Phillips, 2001), while the Japanese research line in the area of industrial engineering investigated the system's single structural elements and the productivity process (Shingo, 1981; Monden, 1983; Nakajima, 1988).

This research path has been later followed by Spencer (1997) with reference to JIT, by Buitenhek et al. (2002) and Liberopulos (2002) with reference to FMSs and by Iwata et al. (2003) with reference to VLSI manufacturing. Moreover, Eilon's approach has been renewed by Tempelmeier (1997), Hee-Don Jung (1999) and Hannula (2002), while both Knox Lovell (2003) and Maniadakis, Thanassoulis (2004) focused on applying the Malmquist productivity index in the operationsrelated field. This paper has become part of this research stream and it is focused on the productivity performance measurement in specific production models.

[^1]Deployment of productivity indices and time-based calculus
(adapted from Grando and Turco, 2005)

|  | Index | Symbol | Formula |
| :---: | :---: | :---: | :---: |
| 1 | Productivity | P | P = Un $\cdot \mathrm{Ef}$ |
| 2 | Net utilization | Un | Un = Ul $\cdot \mathrm{Ui}$ |
| 2.1 | Gross utilization | UI | $U I=T u l / T a$ |
| 2.2 | Internal utilization | Ui | Ui = Tun / Tul |
| 3 | Working efficiency | Ef | $\mathrm{Ef}=\mathrm{D} \cdot \mathrm{S} \cdot \mathrm{Ge}$ |
| 3.1 | Working utilization | Uf | Uf = D $\cdot \mathrm{S}$ |
| 3.1.1 | Availability | D | $D=T p / / T u n$ |
| 3.1.2 | Saturation | S | $S=T p n / T p l$ |
| 3.2 | Global efficiency | R | $\mathrm{R}=\mathrm{Rv} \cdot \mathrm{Rq} \cdot \mathrm{Rc}$ |
| 3.2.1 | Speed efficiency | Rv | $R v=(T p n-t r v) / T p n$ |
| 3.2.2 | Actual yield (actual volume) | Rq | $R q=(T p n-t r v-t r q) /(T p n-t r v)$ |
| 3.2.3 | Conformance efficiency | Rc | $R c=(T p n-t r v-t r q-t r c) /(T p n-t r v-t r q)$ |

As shown in Table 1 (see left side), productivity can be split into its basic drivers, i.e. net utilization (defined as the product of gross utilization times internal utilization) and working efficiency. In turn, working efficiency can be divided into a utilization index, called working utilization and an efficiency index, called global efficiency. The nature of these indices is discussed below together with their basic components, i.e. availability and saturation for the former one; speed efficiency, actual yield and conformance efficiency for the latter one.

Referring to Table 1 (see right side), all the indices (italicized) can be calculated on the basis of the times linked to the different plant statuses. Their values are obtained by subtracting from ordinary calendar times, specific times to be measured in field. The plant calendar time (Ta), i.e. the longest amount of time for which production can be planned, is obtained by subtracting all idle times (tna) resulting from the need to comply with law regulations and corporate decisions (e.g. summer vacations, holidays) from the calendar year (Ts).
The gross (theoretical) utilization time (Tul) is the period during which the plant could theoretically be used, since the technical conditions required for use are fulfilled. It is found by subtracting all unplanned times (tnu) resulting from external reasons from the plant calendar time calculated above. External reasons include lack of customer orders (tmo), lack of materials or information (tmmo), lack of manpower (tsc), time losses caused by technical tests (tpr), etc.

The net (actual) utilization time (Tun) is the time interval during which the plant can be used and its utilization is actually required. It can be calculated by subtracting the overall stand-by time (tsb) from the theoretical utilization time. Stand-by times can be caused either by the system structure e.g. shift changes, man-machine interactions - by poor materials quality (unsuitable properties) and internal logistic constraints (e.g. missing components for a single machine, missing service vehicles, tmmi) or by stoppages following failures to other machines (tam).
The gross (theoretical) operation time ( Tpl ) is the time interval during which utilization of the plant is required and the plant is actually available. It results from the net utilization time minus the overall failure and maintenance downtime (tgm). The net (actual) operation time (Tpn) results from the gross operation time minus the time spent for indirect production tasks (tsu) such as setups, tooling, cleaning, etc. and the overall minor stoppages time (tfm) resulting from resets, adjustments, etc. The marketable production time ( Tpv ) is the time during which the plant manufactures acceptable products; it results from the net operation time minus all time losses caused by speed reductions (trv), yield reductions ( $\operatorname{trq}$ ) and defective products ( $\operatorname{trc}$ ).

Still talking about the measurement phase, even if all values are expressed in time, time losses due to defective products as well as yield and speed reductions should be determined during the data collection phase through the measurement of quantities (Grando and Turco, 2005).

Before going deeper in the analysis and modelling of systems, some additional remarks about the concept of production capacity are advisable. The production capacity of a system can be determined by applying the throughput value to a clearly defined time interval (Boehmer, 1982). The different capacity values are required to provide a suitable answer to the typical questions about how to manage the throughput of a production environment during the operation phase rather than during the design phase (see Table 2, Olhager et al., 2001). The presence of various measurements of the installed capacity also allows to gain some knowledge about existing throughput loss areas and consequently to start targeted improvement plans.

Table 2
Production capacity measures

| Index | Symbol | Formula |
| :--- | :--- | :--- |
| Theoretical capacity | Ct | $\mathrm{pmix} \cdot \mathrm{Ta}$ |
| Gross usable capacity | Cul | $\mathrm{pmix} \cdot \mathrm{Ta} \cdot \mathrm{UI}$ |
| Net usable capacity | Cun | $\mathrm{pmix} \cdot \mathrm{Ta} \cdot \mathrm{Un}$ |
| Available capacity | Cd | $\mathrm{pmix} \cdot \mathrm{Ta} \cdot \mathrm{Un} \cdot \mathrm{D}$ |
| Actual capacity | Ce | $\mathrm{pmix} \cdot \mathrm{Ta} \cdot \mathrm{Un} \cdot \mathrm{D} \cdot \mathrm{S}$ |
| Marketable capacity | Cv | $\mathrm{pmix} \cdot \mathrm{Ta} \cdot \mathrm{Un} \cdot \mathrm{D} \cdot \mathrm{S} \cdot \mathrm{Rv} \cdot \mathrm{Rq} \cdot \mathrm{Rc}$ |
| Productivity | P | $\mathrm{Cv} / \mathrm{Ct}=\mathrm{Un} \cdot \mathrm{D} \cdot \mathrm{S} \cdot \mathrm{R}$ |

An initial reference value is the design (nominal) capacity, which can be found by multiplying the nominal throughput by the plant calendar time. This is the nominal (or peak) value defined during start-up plant tests or after every plant-upgrading investment; it can be achieved under ideal working conditions and coincides with the highest pace the system can theoretically stand. The theoretical capacity $(\mathrm{Ct})$ is found by multiplying the plant calendar time by the standard mix throughput, i.e. the throughput determined by process engineering based on plant working conditions that differ from the design ones, due to e.g. lack of manpower, defective raw materials, etc. Despite being quite rough, this parameter can prove useful for management purposes to determine the actually available gross production capacity.

A more important role for interpretation purposes is played by the gross usable capacity ( Cul ) and net usable capacity (Cun), which quantify the exploitable production capacity that is left after subtracting all losses due to machine-external causes and resulting from organizational and logistic problems found both inside and outside of the social and technical system under study. Even if these parameters can be useful when planning gross capacities, they obviously do not show the actually available capacity, since further allowance has to be made for losses caused by poor machine operation. This task is fulfilled by the available capacity ( Cd ), i.e. the capacity that is left after all time losses caused by failures and maintenance operations have been subtracted. This value expresses the production capacity which is reasonably available, assuming that the events that affected machine operation in the past might to some extent occur again in the future, given the same conditions. The available capacity (or the net usable capacity, according to the circumstances) can be usefully used in the development of production plan.

The need to highlight and separately calculate time losses due to tooling and minor stoppages has led to the definition of one more capacity parameter called actual capacity (Ce), which is found by subtracting the saturation index from the available capacity to make setup times visible. However, this capacity still includes some global efficiency differences and therefore tends to overestimate the production expected over the reference time span. To solve this problem the marketable capacity $(\mathrm{Cv})$ can be calculated by adjusting the actual capacity through efficiency indexes (speed effi-
ciency, actual yield and conformance efficiency). These parameters make it possible to calculate productivity $(\mathrm{P})$, which is the ratio between marketable capacity and theoretical capacity.

## 3. Modelling parallel systems

A parallel system is made up of several machines carrying out the same technical function: these machines are often identical (though not necessarily so) and manufacture the same product or product mix. They are independent from one another from technological and organizational viewpoints, i.e. each of them performs a complete production cycle without any interaction with other system components. Examples of this type of parallel system are the production or assembly lines manufacturing the same product family, the metal sheet production lines, the rubber extrusion lines, the plastic moulding departments, the electric wire and cable production lines, the natural and man-made fibre spinning lines, the weaving looms, the curing departments for tyre production lines, the pumping stations scattered over several production units.

Production throughputs can be added up in these systems, since their products are assumed to be similar or identical and their production volumes can be compared through suitable coefficients. Therefore the system-level use of several single-machine parameters and indices does not raise any special problem. The basic rule states that, for each different level of a parallel system, production capacity is found by adding up the production capacity (at that level) of all the machines of the system. The concept of production capacity at a given level refers to the production volume achievable over the plant calendar time based on the machine throughputs and allowing for different reduction factors (e.g. speed efficiency, actual yield, conformance efficiency). The additive property of production capacities applies to all parallel systems and allows to ignore any difference among machine throughputs. Not even differences in the individual machine calendar times (e.g. the number of shifts) raise any additional problem when calculating system indices.

The calculation rules apply even if an overcapacity is present. However the meaning of the indices has to be carefully interpreted in the light of design and organizational choices: e.g. one of the parallel units might be used as a back-up in the case of a failure to one of the main operating units. In this case (i.e. redundant system with stand-by back-ups) the capacity used up by the back-up machine might be extremely low and it might contribute to a low value of the net system utilization. An additional example is provided by systems where manpower is purposely sized in such a way as not to allow all machines to operate simultaneously: this could cause a low machine gross utilization value or a low internal utilization value ${ }^{1}$.

The calculation of plant productivity factors (i.e. Ul, Ui, Un, Ef, D, S, Rv, Rq, Rc) is based on this general rule: the system-level value of any index is the ratio between a production capacity which is immediately lower (numerator) and a production capacity which is immediately higher (denominator) than that referred to in the index under study. Hence equation (1) holds:

$$
\begin{equation*}
I_{S}=\frac{C_{\text {low }}}{C_{\text {high }}} \tag{1}
\end{equation*}
$$

where $I_{S}$ is the value of the general system-level index, and $C_{\text {low }}$ and $C_{h i g h}$ are the two capacity values (respectively lower and higher than the value referred to in the index).
For example the system gross utilization (Ul) is given by the ratio between theoretical system capacity - which in turn is found by adding up the various machine-level theoretical capacities - and system gross usable capacity, found by adding up the machine-level gross usable capacities. Similarly, the availability value is the ratio between available capacity and net usable capacity; the sys-

[^2]tem working efficiency value is the ratio between system marketable capacity and net usable capacity, and so on.
The value of system-level factors also corresponds to the weighted average of the individual values of those indices for each machine. The weighs are given by the ratio between the production capacity of each machine and the overall capacity of all parallel machines. The capacities to be used in the coefficient-weighting process are at a level immediately higher than that referred to by the index under study: theoretical capacity for gross utilization (Ul), gross usable capacity for internal utilization (Ui), net usable capacity for availability (D) and so on.

For example, the system-level working efficiency $\left(E f_{S}\right)$ is indicated by the weighted average of the working efficiencies of the individual machines $\left(E f_{i}\right)$, with the weights being determined by the ratio between the machine net usable capacity $\left(C u n_{i}\right)$ and the sum of the net usable capacities of all machines:
$E f_{S}=\frac{\sum_{i} E f_{i} \cdot \text { Cun }_{i}}{\sum_{i} \text { Cun }_{i}}$.
The capacities to be included in the coefficient-weighting process can usually be obtained by multiplying the machine throughput by the time the machine remains in the stage immediately higher than that associated to the performance under study, e.g. plant calendar time (Ta) for gross utilization, net utilization time (Tun) for Ef, net operation time (Tpn) for the overall efficiency and so on. The criterion mentioned above takes on the following general form:
$I_{S}=\frac{\sum_{i} I_{i} \cdot \text { pmix }_{i} \cdot T S_{i}}{\sum_{i} p^{2} x_{i} \cdot T S_{i}}$,
where $I_{i}$ is a general, machine-level performance index, pmix $_{i}$ is the standard mix throughput for the $i^{\text {th }}$ machine $(\mathrm{i}=1,2 \ldots \mathrm{n})$ and $T S_{i}$ is the time the machine remains in the stage immediately higher than that associated to the performance under study.
When calculating the overall system productivity index $\left(P_{S}\right)$ the plant calendar time is used as a reference value:
$P_{S}=\frac{\sum_{i} P_{i} \cdot \text { pmix }_{i} \cdot T a_{i}}{\sum_{i} \text { pmix }_{i} \cdot T a_{i}}$.
Table 3 shows a calculation example for a system made up of 3 parallel machines which work 2 shifts per day, 5 days per week. Machine C acts as a partial capacity back-up for demand peaks and its gross utilization value is low due to lack of orders. Should the low utilization be due to lack of manpower, the factor affected would be internal utilization.

Table 3
Example of a parallel system made up from 3 machines where production mixes are the same

| Index | Unit of measure | Notation | Machine A | Machine B | Machine C | Overall <br> system |
| :--- | :---: | :---: | ---: | ---: | ---: | ---: |
| Plant calendar time | hours/year | Ta | 3,680 | 3,680 | 3,680 | N.A. |
| Standard mix potential | pieces/hour | pmix,i | 5.51 | 6.03 | 4.70 | N.A. |
| Theoretical capacity | pieces/year | Ct | 20,277 | 22,190 | 17,296 | 59,763 |
| Gross utilization |  | Ul | 0.975 | 0.981 | 0.605 | 0.870 |
| Gross usable capacity | pieces/year | Cul | $19,769.9$ | $21,768.8$ | $10,464.1$ | $52,002.7$ |
| Internal utilization |  | Ui | 0.950 | 0.950 | 0.960 | 0.952 |
|  | pieces/year | pmix, ${ }^{*}$ Tun | $18,781.4$ | $20,680.3$ | $10,045.5$ | $49,507.2$ |

Table 3 (continued)

| Index | Unit of measure | Notation | Machine A | Machine B | Machine C | Overall <br> system |
| :--- | :---: | :---: | ---: | ---: | ---: | ---: |
| Net utilization |  | Un | 0.926 | 0.932 | 0.581 | 0.859 |
| Net usable capacity | pieces/year | Cun | $18,781.4$ | $20,680.3$ | $10,045.5$ | $49,507.2$ |
| Working efficiency |  | Ef | 0.786 | 0.848 | 0.668 | 0.788 |
| Availability |  | D | 0.930 | 0.970 | 0.880 | 0.937 |
| Available capacity | pieces/year | Cd | $17,466.7$ | $20,059.9$ | $8,840.1$ | $46,366.7$ |
| Saturation |  | S | 0.880 | 0.920 | 0.870 | 0.895 |
| Actual capacity | pieces/year | Ce | $15,370.7$ | $18,455.1$ | $7,690.8$ | $41,516.7$ |
| Global efficiency |  | R | 0.960 | 0.951 | 0.873 | 0.940 |
| Speed efficiency |  | Rv | 1.000 | 0.970 | 0.940 | 0.976 |
| Actual yield |  | Rq | 0.990 | 1.000 | 0.970 | 0.991 |
| Conformance efficiency |  | Rc | 0.970 | 0.980 | 0.957 | 0.972 |
| Defect rate | d | 0.030 | 0.020 | 0.043 | 0.028 |  |
| Marketable capacity | pieces/year | Cv | $14,760.5$ | $17,543.5$ | $6,711.0$ | $39,014.9$ |
| Overall productivity |  | P | 0.7279 | 0.7906 | 0.3880 | 0.6528 |

## 4. Modelling serial systems

A serial system is made up of sequenced machines (stations or departments) arranged in such a way as to generate a one-way manufacturing flow, i.e. production always flows in the same way through the various machines. Re-entrant lines (see e.g. Cigolini et al., 1996), i.e. systems in which downstream products flow back upstream in a loop, are not discussed here. A serial system can be designed as a line made up of tightly interconnected machines or stations. In this case the machines (stations) performing the various manufacturing operations must necessarily work at the same pace, which is determined by the line bottleneck.
Alternatively, the system can be designed as a loosely interconnected line, with its looseness level being determined by the capacity of the decoupling buffers. In this case the buffers allow to recover time losses due to failures, minor stoppages or setups: e.g. through suitably sized buffers the set-up of a non-bottleneck station can be carried out 'hiddenly'. However the line average throughput is still affected by the bottleneck, i.e. by the machine which shows the lowest average throughput after all causes for capacity losses have been removed.

Finally, a serial system can be designed as a sequence of independent departments crossed by the same production flow, but so much decoupled from one another to be regarded as totally independent. Each department can have its own utilization and working parameters and, since a very high decoupling level is assumed, short-term production speeds can also be regarded as independent, which requires large stocks. However the medium-term working throughput of such systems is also limited by the throughput of the bottleneck department.

### 4.1. Tightly interconnected lines

A serial system made up of tightly interconnected machines can be regarded as a single machine (a black box): this applies e.g. to transfer production lines, packaging lines, automatic assembly lines, filling lines, pharmaceutical product lines. Given the close dependence constraint, general choices of production organization must be assumed for all machines or stations making up the line: therefore only one value for plant calendar time (Ta), gross utilization (Ul) and internal utilization (Ui) is used.
As far as availability is concerned, the following formula can theoretically be used to find overall availability $\left(D_{S}\right)$ as a function of the availability of the individual $\left(D_{i}\right)$ machines of the serial system:
$D_{S}=\prod_{i} D_{i}$.
However, this calculation method is acceptable only if action is taken on one machine at a time, without any preventive or corrective measures being applied to the others. In the case of a tightly interconnected serial system, failure stops are usually regarded as a good opportunities to maintain other parts of the system as well as that which has failed (this is called 'opportunistic' maintenance). Moreover, preventive maintenance stops are very likely to involve the whole line rather than a single machine. In other words, it is not always realistic to assume a multiplying interaction among basic availability's as in (5). Rather, an aggregate availability value for the whole system should be used.

Therefore system availability has to be calculated based not on the mean time between failures (MTBF), mean time between maintenances (MTBM), mean time to repair (MTTR) and mean down time (MDT) parameters for the single machines, but on the direct measurement of time losses for the whole line. Assuming $t f g$ is the duration of a failure stop and tmp is the duration of a stop for the scheduled maintenance of the whole line, (6) holds:

$$
\begin{equation*}
D_{S}=1-\frac{t f g+t m p}{T u n} . \tag{6}
\end{equation*}
$$

The same remarks pointed out about utilization indices also apply to saturation, since setup and changeover operations are performed on the whole line in tightly interconnected systems. Therefore the same saturation index value applies to all the machines or stations making up the system.
Moving now to the three different efficiency types, broadly speaking, it is not possible to calculate the system-level production losses caused by a higher defect rate, a lower actual yield and a lower line speed or pace from single-machine values. This is due to that the line bottleneck, which affects the line actual throughput, is located in different positions along the line, based on the working losses taken into account.

In tightly interconnected lines, the line standard throughput equals the smallest standard throughput for a single serial machine or station, i.e. the bottleneck throughput. Therefore the 'design' line bottleneck coincides with the machine or station with the highest standard cycle time (Tcmf) and the lowest standard throughput ( pm ) ; this corresponds to the line 'design' standard cycle time $T c_{S}$ and standard throughput $p_{S}$ :

$$
\begin{align*}
& T c_{S}=\max _{i}\left\{T c m f_{i}\right\}  \tag{7}\\
& p_{S}=\min _{i}\left\{p m_{i}\right\} . \tag{8}
\end{align*}
$$

In (7) and (8), machine cycle times and throughputs refer to the product unit (either good or defective) which represents the line output under design conditions assuming a speed and conformance efficiency of 1 and technical standard values for the actual yields. The need to calculate the line design throughput taking into account technical standards for the actual yields of all the machines of the line, stems from that those yields closely depend on the technology used and therefore they are linked to system design choices. Thus the unit standard cycle times of the machines of the line is usually longer than the unit standard cycle times required by those machines when working in stand-alone mode. For example, if the standard cycle time of a continuous paper-processing machine is $0.6 \mathrm{sec} . / \mathrm{kg}$ of machine output and the standard yield of the downstream process steps (cutting and packaging) is 0.97 , the standard machine cycle time per kg of final product will be 0.618 sec. (i.e. 0.6/0.97).
Moving now to machine actual cycle times and throughputs, they do not depend only on machineexternal system parameters ( $\mathrm{Ul}, \mathrm{Ui}, \mathrm{D}, \mathrm{S}$ ) and on single machine cycle times ( $T c m f_{i}$ ), but also on the speed efficiency, actual yield and conformance efficiency of the machines making up the line. Assuming the yield is smaller than 1, the actual cycle time for each machine included in the line is increased - vis-à-vis the theoretical cycle time - to an extent which usually differs from one ma-
chine to the next. Therefore there is an operating bottleneck determined by the station with the longest actual cycle time and the lowest actual throughput, and the 'operating' bottleneck may differ from the 'design' bottleneck. Thus the reduction of the line throughput can be measured by the ratio between the operating and the design bottleneck throughput. However, two different scenarios should be distinguished when analyzing the dependence relationship between cycle times and yields, based on whether the non-conforming production units are detected at the end of the line or at the end of each step.

Now let consider the former scenario, where defects are detected at the end of the line. A real-life example of this scenario can be found in several forced-cycle continuous processes such as those of the paper, tyre and pharmaceutical industry. The operating cycle time of a machine in the line is defined as the machine standard cycle time (still referred to the good or defective product unit which is the line output) plus all time losses due to poor speed efficiencies and actual yields. The line operating cycle time is the shortest among all machine operating times, which correspond for each machine to the standard cycle time plus time losses due to the actual yield of all downstream machines. As a matter of fact, when moving upstream along the line, the production flow to be processed by each station has to be increased (and the machine throughput, in terms of end-line output, has to be decreased) in such a way as to take into account the yield of all downstream stations (see Figure 1).


Fig. 1. Upstream increase in the production flow to be processed due to poor yields in a tightly interconnected line, with defective products detected at the end of the line

Hence, assuming $\operatorname{Tomf}_{i}$ is the operating cycle time of the $i$-th machine in the line:
$\operatorname{Tomf}_{i}=\frac{T c m f_{i}}{R v_{i} \cdot R q_{i} \cdot \prod_{j} R q_{j}}$,
where the product is extended to the $(j)$ machines located downstream of the $i$-th machine under study. Similarly, assuming pom $_{i}$ is the operating throughput of a machine in the line:

$$
\begin{equation*}
\operatorname{pom}_{i}=p m_{i} \cdot R v_{i} \cdot R q_{i} \cdot \prod_{j} R q_{j} \tag{10}
\end{equation*}
$$

where the product in the right hand side of $(10)$ is extended to the $(j)$ machines located downstream of the machine under study $(i)$. Therefore, assuming $T o_{S}$ and $p o_{S}$ are the system operating cycle time and operating throughput, respectively:
$T o_{S}=\max _{i}\left\{\operatorname{Tomf}_{i}\right\} ;$
$p o_{S}=\min _{i}\left\{\right.$ pom $\left._{i}\right\}$.
The $T c_{S} / T o_{S}$ (or $p o_{S} / p_{S}$ ) ratio is the throughput reduction factor which takes into account the losses caused by actual yields lower than the design ones as well as the losses due to a poor speed efficiency. The value of this aggregate factor corresponds to the product of the line speed efficiency multiplied by the line actual yield, but it does not allow to distinguish between the impact of speed efficiency and that of the actual yield, i.e.:
$R v_{S} \cdot R q_{S}=T c_{S} / T o_{S}$.
Table 4 and Figure 2 apply the concepts mentioned above to a 5 -station system. Table 4 lists the standard cycle time $T c m f_{i}$, the actual yield $R q_{i}$ and the speed efficiency $R v_{i}$ for each station and calculates the operating cycle time $\operatorname{Tomf}_{i}$ according to (9). This shows that, when the design operation is replaced with the actual operation, the line bottleneck shifts from step 4 (which has the longest standard cycle time) to step 2 , which has the longest operating cycle time.


Fig. 2. Design standard and operating cycle time of a 5 -station line
Table 4
Standard cycle time, actual yield and speed efficiency for a 5-station system

| Index | Notation | Station |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 |
| Standard cycle time |  | 4.570 | 4.700 | 4.600 | 4.720 | 4.610 |
| Actual yield |  | 0.970 | 1.000 | 0.990 | 0.990 | 0.980 |
| Speed efficiency |  | 1.000 | 0.970 | 0.960 | 0.980 | 1.000 |
| Operating cycle time |  | 4.905 | 5.045 | 4.988 | 4.964 | 4.704 |
| System standard cycle time | Tcs |  |  |  | 4.720 |  |
| System standard cycle time | Tos |  | 5.045 |  |  |  |

As far as conformance efficiency is concerned, it preserves its meaning as the ratio between marketable product units and the overall number of units produced (good and defective units), and it can be calculated as the product of the conformance efficiencies of all individual stations, i.e.:

$$
\begin{equation*}
R c_{S}=\prod_{i} R c_{i} \tag{14}
\end{equation*}
$$

Finally, the system efficiency is found by multiplying the $T c_{S} / T o_{S}$ factor by the conformance efficiency, i.e.:
$R_{S}=R v_{S} \cdot R q_{S} \cdot R c_{S}=\frac{T c_{S}}{T o_{S}} \cdot R c_{S}$.

To calculate the operating cycle time for the individual machines mentioned in (9) and consequently the system operating cycle time, reference must be made to the efficiency of each machine when working in stand-alone mode, not as part of the line. Appendix 1 shows an example of a serial system made up of 4 tightly interconnected machines.

Now let consider the latter scenario, where defective products are detected and removed step by step. A practical case of this scenario could be a durable goods assembly line with very small buffers, where a basic product core flows through the whole line and is gradually equipped with more and more components assembled according to a fixed sequence. Several examples of this kind can be found in the real-life industrial world: car final assembly lines, white goods assembly lines, TV sets, VRCs, Hi-Fi products, etc., assuming however that an in-process quality control is performed step by step on each single unit.

In this case conformance efficiencies have an impact on the operating cycle time of each machine, which is regarded as a part of the overall line. Unlike the former scenario, this operating cycle time only refers to the good product units manufactured by the line, while defective units are excluded; therefore it equals the standard cycle time plus time losses due to the actual speed efficiency, yield and conformance efficiency of the machine under study, plus time losses due to the actual yield and conformance efficiency of all the downstream machines.


Fig. 3. Increase in the production flow to be processed due to poor conformance efficiencies in a tightly interconnected line, with defective products detected at the end of each step
(The process is part-based, with all actual yields equal to 1 )
Conformance efficiencies (Rc) act very similarly to actual yields (Rq), since they increase the operating cycle time of a machine when included in the line. Figure 3 shows the increase in the product flow to be processed by each upstream machine or station as a result of the conformance efficiency constraint. Thus it follows that:

$$
\begin{align*}
& \operatorname{Tomf}_{i}=\frac{\operatorname{Tcmf}_{i}}{R v_{i} \cdot R q_{i} \cdot R c_{i} \cdot \prod_{j} R q_{j} \cdot \prod_{j} R c_{j}}  \tag{16}\\
& \operatorname{pom}_{i}=p m_{i} \cdot R v_{i} \cdot R q_{i} \cdot R c_{i} \cdot \prod_{j} R q_{j} \cdot \prod_{j} R c_{j} \tag{17}
\end{align*}
$$

where the products at the right-hand side of (16) and (17) are extended to the $(j)$ machines located downstream of the machine under study. Therefore, as in the previous case (but with a different meaning for times $\operatorname{Tomf}_{i}$ ):
$T o_{S}=\max _{i}\left\{T o m f_{i}\right\}$,
$p o_{S}=\min _{i}\left\{\right.$ pom $\left._{i}\right\}$.
See again appendix 1 for an example. Two remarks should be made before concluding this section about tightly interconnected lines.

First: in general, it is not possible to determine separately the time losses caused by a poor conformance efficiency (tsc), by a reduction in actual yields (tmr) or by a decrease in speed efficiency (tsl). These 3 values determine the line operating throughput (operating bottleneck) jointly and their influences cannot easily be separated. However, it is at least still possible to calculate the global efficiency, which can be determined as follows:

1) starting from machine data, as the ratio between the line standard and operating cycle time (keep in mind that in general these two times refer to different machines or stations along the line);
2) alternatively, starting from actual production volumes (at the end of the process), as the ratio between marketable production time (Tpv) and net production time (Tpn):
$R_{S}=\frac{T p v}{T p n}=\frac{T c s \cdot Q B}{T a \cdot U l \cdot U i \cdot D \cdot S}$.
In both cases, the ratios mentioned above are the outcome of the losses caused by the three parameters (speed efficiency, yield and conformance efficiency) starting from the design standard cycle time for the line $T c_{S}$. The line productivity $(P)$ can be calculated according to the scheme as above, but it is not possible to break down the global efficiency into its components (speed efficiency, yield and conformance efficiency):
$P=U l_{S} \cdot U i_{S} \cdot E f_{S}=U l_{S} \cdot U i_{S} \cdot D_{S} \cdot S_{S} \cdot R_{S}$.
Second: several combinations of the 2 scenarios analyzed above are possible in the real-life industrial plants which can be regarded as tightly interconnected lines. In such cases, defective products are not actually detected step by step but rather at the end of a multiple-step sequence; this applies both to forced-cycle processes and to part-based manufacturing processes, which can be approached by combining the 2 methods described above.

### 4.2. Loosely interconnected lines

The stations of loosely interconnected lines are decoupled up to a certain extent due to buffers which (depending on their size) allow a partial or full recovery of the time losses caused by failures, minor stoppages and setups. Buffers obviously contribute to an increase in the line throughput.
Figure 4 shows the throughput of a line made up of 2 stations with a known availability and standard throughput, which can be interconnected either tightly or loosely (in the latter case with very large buffers). The throughput of the tightly coupled line is lower than the lowest operating throughput of the 2 machines due to the multiplication of the 2 availability values. In the latter case, since the 2 stations are totally decoupled, the line throughput equals the lowest, singlemachine operating throughput. The capacity increase made possible by the buffer can be remarkable, as the example shows.
In this type of lines, calendar time ( Ta ) is the same for all machines or stations; the same applies to gross utilization (Ul), whereas the value of internal utilization (Ui) may vary from one machine to the next. The impact of failure or maintenance stops on the line capacity decreases as the size of the decoupling buffers increases; in non-bottleneck steps, it disappears when the buffer is extremely large (infinite). Therefore the system availability cannot be regarded as the product of sin-gle-machine availabilities. The same applies to saturation (i.e. tsu and minor stoppages times).


Fig. 4. Throughput of a 2-station line with either tightly or loosely interconnected stations
However, buffers are only designed to make up (according to their size) for the short-term variability of an operating production pace or for temporary stops and restarts. Consequently they only prevent or reduce operation abnormalities in the short term, whereas in the medium term the line operating speed (throughput) is still limited by the speed of the slowest station. Therefore the bottleneck location depends not just on speed efficiencies, yields and conformance efficiencies (as under tightly interconnected lines) but also on availability and saturation (setup times). In other words, availability and saturation are very similar to yields ( Rq ) or to conformance efficiencies in that they decrease the operating throughput and determine the station with the lowest average operating throughput (bottleneck).
It follows that the line operating cycle time and throughput are only meaningful as average med (and long) term values. They are calculated in the same way as in tightly interconnected lines, but they also depend on the availability and saturation of the individual station $j$; e.g. supposing that defective products are detected and removed step by step:
$T o_{S}=\max _{i}\left\{\frac{T c m f_{i}}{R c_{i} \cdot R q_{i} \cdot R v_{i} \cdot D_{i} \cdot S_{i} \cdot \prod_{j} R q_{j} \cdot \prod_{j} R c_{j}}\right\}$,
$p o_{S}=\min _{i}\left\{p m_{i} \cdot R c_{i} \cdot R q_{i} \cdot R v_{i} \cdot D_{i} \cdot S_{i} \cdot \prod_{j} R q_{j} \cdot \prod_{j} R c_{j}\right\}$.
Therefore, besides the impact of efficiencies and yields as in the previous case, the impacts of availability and saturation apart from each other cannot be easily predicted: a good method to separate impacts in such a case is the use of simulation. It is nevertheless possible to calculate the system working efficiency $E f_{S}$, resulting from the $T c_{S} / T o_{S}$ (or $p o_{S} / p_{S}$ ) ratio. Productivity is calculated as follows:

$$
\begin{equation*}
P=U l_{S} \cdot U i_{S} \cdot T c s \cdot \frac{Q B}{T u n}=U l_{S} \cdot U i_{S} \cdot \frac{T c s}{T o s}=U l_{S} \cdot U i_{S} \cdot \frac{p o s}{p s}=U l_{S} \cdot U i_{S} \cdot E f_{S} . \tag{24}
\end{equation*}
$$

### 4.3. Sequenced and independent departments

This system type refers to sequenced departments crossed by the same production flow but provided with a decoupling level so high as to be regarded as fully independent. Each department can have its own internal organization, provided with the production data required for our purposes that are measurable and significant. In this case all parameters might differ from one department to the next one, including calendar time (Ta), e.g. following the number of shifts worked by each department. The values of gross utilization (Ul) and internal utilization (Ui) are also usually unique for each department.
Given the high decoupling level, short-term production rates can be regarded as independent, which requires a high inventory level. However, even in this case the system medium-term operating throughput is limited by the operating throughput of the bottleneck department, i.e. by the lowest department operating throughput ${ }^{1}$. In turn, department operating throughputs must account for all capacity losses, including external and internal logistic and organizational losses as well as ma-chine-specific working losses.

Even in this case 2 different scenarios should be distinguished, based on whether defective products are detected and removed at the end of the process or, which is more likely in such a context, on a step-by-step basis. In the former scenario the department operating throughput is given by the standard throughput multiplied by the department productivity. In the latter scenario the operating throughput of each department must be reduced to allow for the department own losses and for the conformance efficiency of downstream departments (see appendix 2 for an example of a system made up of 4 sequenced and independent departments).

## 5. Concluding remarks

This paper highlighted the most relevant features of a production system that determine system's productivity. The usefulness of a calculation model allowing to move from machine-level to sys-tem-level indices is relevant both during the production system design and during the performance measurement phase (Bititci et al. 2000, Bourne et al. 2000, Kennerly and Neely 2002, Lillis 2002). To give reliable, system-level working definitions to the indices and parameters developed for single machines, the specific system's configuration has been firstly determined, namely parallel system and serial system, referring to both tightly and loosely interconnected machines.

For each considered model, starting from plant calendar time, several indices have been calculated, e.g. gross utilisation, net utilisation, working efficiency, availability and saturation. These indices allow to appropriately determine both overall system productivity and capacity according to the looseness of the interconnections among machines that constitute the production system itself.

The presented framework has been applied in some manufacturing plants and has provided useful results in order to:

1) improve analysis, and make proper diagnosis of causes, of productivity loss and to track their evolution over the time;
2) design and implement specific improvement projects, aimed at removing losses causes and, thus, increasing efficiency and productivity;
3) establish proper production capacities, by focusing on bottleneck machines, to keep under control the actual system throughput;
4) make comparisons and internal benchmarking aimed at defining machine management and productive maintenance best practices;
5) design plant performance reporting systems and build appropriate tools to collect data from the field.
[^3]Nevertheless, the system types observed are limited to production processes characterized by a high flow consistency over time. This is due to that in systems with cross-linked flows and variable cycles (e.g. job-shops), basic parameters (e.g. the mix standard throughput) cannot be uniquely defined with an acceptable accuracy for the purpose of this paper.

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## Appendix 1. A serial system made up of 4 tightly interconnected machines

Table 5 refers to a lead oxide production where the detection of defective products is assumed to take place only at the end of the line. The plant is made up of the following steps:

1) smelting of the lead metal;
2) pre-oxidation of the molten lead mass inside stirring tanks;
3) final oxidation in a continuous reactor;
4) packaging.

This is a typical continuous flow process where the actual yields of the individual steps are usually different from one another. The standard cycle times per product unit provided for each of the 4 stations refer to the end product units manufactured by the line under design conditions (rather than to the output units for each station), and take the technical standards of theoretical yields into account. Therefore the design standard cycle time for the line is assumed to be equal to the longest single-station cycle time.

The 'target operation parameters' section provides target values for the line calendar time as well as for gross utilization, internal utilization and saturation; following the considerations made above, these values refer to the system as a whole. As far as availability is concerned, maintenance operations are assumed to be carried out on a machine only when a failure occurs on that machine; therefore the system availability is calculated as the product of all single-machine availabilities. Standard yield values are also listed in the table: speed efficiencies are assumed to be equal to 1 for simplicity's sake; this means that the operation speeds achievable under normal conditions do not differ from the theoretical ones. The same applies to actual yields, though they are not to be regarded as absolute values but rather as ratios between target and technical standard yields (the latter having being dealt with during the calculation of design standard cycle times); therefore a target yield of one means technical standard yields are used as target standard values; finally, the system-level conformance efficiency is the product of the standard conformance efficiencies of all machines.

Production capacities at different levels are calculated as well and correspond to the lowest singlemachine values up to the net usable capacity. The available and actual capacities differ from the corresponding minimum machine values due to the impact of availability. The system marketable capacity differs from its minimum values due to the joint impact of availability and conformance efficiencies.

The section entitled 'actual operation parameters' shows that the availability of actual operation values measured on single machines allows to find system-level parameters. The remarks made for target parameters apply here as well, but in this case the speed efficiencies and yields of the various stations differ from standard ones. This requires to recalculate the operating cycle time for the individual stations through (9) and to determine the line operating cycle time, which is the longest single-station cycle time. In the example, the line bottleneck moves from station $C$ (the design bottleneck) to station B (the operating bottleneck) due to the joint influence of yields and speed efficiencies. This means that the line throughput reduction factor due to these two causes equals the ratio between the operating bottleneck throughput and the design bottleneck throughput. As stated above, in this case it is impossible to distinguish between the impact of yields and the impact of speed efficiencies on the system productivity and marketable production capacity. The line efficiency (Rs) is found by multiplying the line conformance efficiency ( $R c_{S}$, i.e. the product of single-machine conformance efficiencies) by the $T c_{S} / T o_{S}$ ratio, according to (13).

Table 5
Example of a serial system (with continuous flow production) made up from 4 tightly interconnected machines where defects are detected only at the end of the line

| Index area | Index | Unit of measure | Notation | Machine A | Machine B | Machine C | Machine D | Overall line | Overall system |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Machine standard cycle time per end product unit (with technical standard yields) | sec./kg | Tcmf | 3.157 | 3.420 | 3.520 | 3.315 | N.A. | N.A. |
|  | Design standard cycle time for the line | sec./kg | Tcs | N.A. | N.A. | N.A. | N.A. | 3.520 | N.A. |
|  | Machine standard throughput | kg/h | pm | 1,140.32 | 1,052.63 | 1,022.73 | 1,085.97 | N.A. | N.A. |
|  | Line standard throughput | kg/h | ps | N.A. | N.A. | N.A. | N.A. | 1,022.73 | N.A. |
|  | Plant calendar time | h/year | Ta | N.A. | N.A. | N.A. | N.A. | 7,200 | N.A. |
|  | Gross utilization |  | UI | N.A. | N.A. | N.A. | N.A. | 0.980 | N.A. |
|  | Internal utilization |  | Ui | N.A. | N.A. | N.A. | N.A. | 0.960 | N.A. |
|  | Net utilization |  | Un | N.A. | N.A. | N.A. | N.A. | 0.941 | N.A. |
|  | Availability |  | D | 0.980 | 0.960 | 0.980 | 1.000 | 0.922 | N.A. |
|  | Saturation |  | S | N.A. | N.A. | N.A. | N.A. | 0.920 | N.A. |
|  | Speed efficiency |  | Rv | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | N.A. |
|  | Actual yield |  | Rq | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | N.A. |
|  | Conformance efficiency |  | Rc | 0.990 | 0.990 | 0.990 | 1.000 | 0.970 | N.A. |
|  | Global efficiency |  | R | 0.990 | 0.990 | 0.990 | 1.000 | 0.970 | N.A. |
|  | Working efficiency |  | Ef | N.A. | N.A. | N.A. | N.A. | 0.823 | N.A. |
|  | Overall productivity |  | P | N.A. | N.A. | N.A. | N.A. | 0.774 | N.A. |
|  | Theoretical capacity | ton/year | Ct | 8,210.33 | 7,578.95 | 7,363.64 | 7,819.00 | 7,363.64 | N.A. |
|  | Gross usable capacity | ton/year | Cul | 8,046.12 | 7,427.37 | 7,216.36 | 7,662.62 | 7,216.36 | N.A. |
|  | Net usable capacity | ton/year | Cun | 7,724.27 | 7,130.27 | 6,927.71 | 7,356.12 | 6,927.71 | N.A. |
|  | Available capacity | ton/year | Cd | 7,569.79 | 6,845.06 | 6,789.15 | 7,356.12 | 6,387.24 | N.A. |
|  | Actual capacity | ton/year | Ce | 6,964.21 | 6,297.46 | 6,246.02 | 6,767.63 | 5,876.26 | N.A. |
|  | Marketable capacity | ton/year | Cv | 6,894.56 | 6,234.48 | 6,183.56 | 6,767.63 | 5,701.73 | N.A. |
|  | Plant calendar time | h/year |  |  |  |  |  |  | 7,015 |
|  | Gross utilization |  | UI | N.A. | N.A. | N.A. | N.A. | N.A. | 0.940 |
|  | Internal utilization |  | Ui | N.A. | N.A. | N.A. | N.A. | 0.910 | 0.935 |
|  | Net utilization |  | Un | N.A. | N.A. | N.A. | N.A. | 0.865 | 0.879 |
|  | Availability |  | D | 0.970 | 0.950 | 0.980 | 1.000 | 0.903 | 0.894 |
|  | Saturation |  | S | N.A. | N.A. | N.A. | N.A. | 0.920 | 0.911 |
|  | Speed efficiency |  | Rv | 0.940 | 0.930 | 0.980 | 0.990 | 0.938 | 0.945 |
|  | Actual yield |  | Rq | 0.980 | 0.990 | 0.990 | 1.000 |  |  |
|  | Conformance efficiency |  | Rc | 0.990 | 0.990 | 0.990 | 1.000 | 0.970 | 0.959 |
|  | Global efficiency |  | R | 0.912 | 0.911 | 0.960 | 0.990 | 0.910 | 0.906 |
|  | Working efficiency |  | Ef | 0.814 | 0.797 | 0.866 | 0.911 | 0.756 | 0.738 |
|  | Machine operating cycle time per end product unit (with actual yields) | s/kg | Tomf.i | 3.497 | 3.752 | 3.628 | 3.348 | N.A. | N.A. |

Table 5 (continued)

|  | Line operating cycle time | s/kg | Tos | N.A. | N.A. | N.A. | N.A. | 3.752 | N.A. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Machine operating throughput in the line | kg/h | pom.i | 929.56 | 959.47 | 992.25 | 1,075.11 | N.A. | N.A. |
|  | Line operating throughput | kg/h | pos | N.A. | N.A. | N.A. | N.A. | 959.47 | N.A. |
|  | Overall productivity |  | P | 0.704 | 0.689 | 0.749 | 0.787 | 0.654 | 0.649 |
|  | Theoretical capacity | ton/year | Ct | 8,210.33 | 7,578.95 | 7,363.64 | 7,819.00 | 7,363.64 | 7,174.43 |
|  | Gross usable capacity | ton/year | Cul | 7,799.81 | 7,200.00 | 6,995.45 | 7,428.05 | 6,995.45 | 6,743.97 |
|  | Net usable capacity | ton/year | Cun | 7,097.83 | 6,552.00 | 6,365.86 | 6,759.53 | 6,365.86 | 6,305.61 |
|  | Available capacity | ton/year | Cd | 6,884.89 | 6,224.40 | 6,238.55 | 6,759.53 | 5,748.82 | 5,637.21 |
|  | Actual capacity | ton/year | Ce | 6,334.10 | 5,726.45 | 5,739.46 | 6,218.77 | 5,288.91 | 5,135.50 |
|  | Marketable capacity | ton/year | Cv | 5,776.62 | 5,219.62 | 5,512.74 | 6,156.58 | 4,814.40 | 4,654.00 |

Finally, the last column of this section shows that, starting from the values of time-based parameters measured on the whole system (gross utilization, internal utilization, availability, saturation) and from the amount of good products $(Q B)$ and bad products $(Q S)$ measured at the end of the line, the values of the productivity and capacity indices can be found. In this case the value of $R c_{S}$ results from the $Q B /(Q B+Q S)$ ratio. The global efficiency Rs results from the $T c_{S} \cdot Q B / T p n$ ratio, where Tpn can be calculated by multiplying Ta by the reduction factors Ul, Ui, D, S. Once Rs has been determined, the Rvs $\cdot$ Rqs product ( $=\mathrm{Rs} / \mathrm{Rcs}$ ), Efs and the other parameters can be calculated as well.

Table 6 shows an example of a serial line made up of 4 tightly interconnected machines; defective products are assumed to be detected and removed step by step through an on-line control procedure. The simplified process shown here refers to the production of shafts for low-power, short circuit, rotor-based electric engines. The following production steps can be identified:

1) parting;
2) turning of pulley and rotor housings;
3) rolling of pulley and rotor housings;
4) grinding.

This is a typical fabrication-assembly production process. In this type of production the value of yields usually equals 1 by definition, since a quantitative reduction in product volumes in a given step could only be determined by a lack of conformance.

Table 6
Example of a serial system (parts fabrication assembly process) made up from 4 tightly interconnected machines where defects are detected only at the end of the line

| Index area | Index | Unit of measure | Notation | Machine A | Machine B | Machine C | Machine D | Overall line | Overall system |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Machine standard cycle time per end product unit (with technical standard yields) | $\mathrm{min} / \mathrm{p}$ | Tcmf | 0.147 | 0.158 | 0.174 | 0.162 | N.A. | N.A. |
|  | Design standard cycle time for the line | $\mathrm{min} / \mathrm{p}$ | Tcs | N.A. | N.A. | N.A. | N.A. | 0.174 | N.A. |
|  | Machine standard throughput | $\mathrm{p} / \mathrm{h}$ | pm | 408.16 | 379.75 | 344.83 | 370.37 | N.A. | N.A. |
|  | Line standard throughput | $\mathrm{p} / \mathrm{h}$ | ps | N.A. | N.A. | N.A. | N.A. | 344.828 | N.A. |


| Index area | Index | Unit of measure | Notation | Machine A | Machine B | Machine C | Machine D | Overall line | Overall system |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Plant calendar time | h/year |  | N.A. | N.A. | N.A. | N.A. | 3,450 | N.A. |
|  | Gross utilization |  | UI | N.A. | N.A. | N.A. | N.A. | 0.960 | N.A. |
|  | Internal utilization |  | Ui | N.A. | N.A. | N.A. | N.A. | 0.930 | N.A. |
|  | Net utilization |  | Un | N.A. | N.A. | N.A. | N.A. | 0.893 | N.A. |
|  | Availability |  | D | 0.960 | 0.980 | 0.980 | 0.990 | 0.913 | N.A. |
|  | Saturation |  | S | N.A. | N.A. | N.A. | N.A. | 0.900 | N.A. |
|  | Speed efficiency |  | Rv | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | N.A. |
|  | Actual yield |  | Rq | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | N.A. |
|  | Conformance efficiency |  | Rc | 0.990 | 0.990 | 0.990 | 1.000 | 0.970 | N.A. |
|  | Global efficiency |  | R | 0.990 | 0.990 | 0.990 | 1.000 | 0.970 | N.A. |
|  | Working efficiency |  | Ef | 0.855 | 0.873 | 0.873 | 0.891 | 0.797 | N.A. |
|  | Overall productivity |  | P | 0.764 | 0.780 | 0.780 | 0.795 | 0.712 | N.A. |
|  | Theoretical capacity (thousands) | p/year | Ct | 1,408.16 | 1,310.13 | 1,189.66 | 1,277.78 | 1,189.66 | N.A. |
|  | Gross usable capacity (thousands) | p/year | Cul | 1,351.84 | 1,257.72 | 1,142.07 | 1,226.67 | 1,142.07 | N.A. |
|  | Net usable capacity (thousands) | p/year | Cun | 1,257.21 | 1,169.68 | 1,062.12 | 1,140.80 | 1,062.12 | N.A. |
|  | Available capacity (thousands) | p/year | Cd | 1,206.92 | 1,146.29 | 1,040.88 | 1,129.39 | 969.47 | N.A. |
|  | Actual capacity | p/year | Ce | 1,086.23 | 1,031.66 | 936.79 | 1,016.45 | 872.52 | N.A. |
|  | Marketable capacity | p/year | Cv | 1,075.37 | 1,021.34 | 927.43 | 1,016.45 | 846.61 | N.A. |
|  | Plant calendar time | h/year |  | N.A. | N.A. | N.A. | N.A. | 3,450 | 3,425 |
|  | Gross utilization |  | UI | N.A. | N.A. | N.A. | N.A. | 0.950 | 0.941 |
|  | Internal utilization |  | Ui | N.A. | N.A. | N.A. | N.A. | 0.930 | 0.928 |
|  | Net utilization |  | Un | N.A. | N.A. | N.A. | N.A. | 0.884 | 0.873 |
|  | Availability |  | D | 0.945 | 0.978 | 0.976 | 0.985 | 0.888 | 0.876 |
|  | Saturation |  | S | N.A. | N.A. | N.A. | N.A. | 0.908 | 0.905 |
|  | Speed efficiency |  | Rv | 0.940 | 0.925 | 0.990 | 0.990 | 0.949 | 0.933 |
|  | Actual yield |  | Rq | 1.000 | 1.000 | 1.000 | 1.000 |  |  |
|  | Conformance efficiency |  | Rc | 0.990 | 0.960 | 0.980 | 0.990 |  |  |
|  | Global efficiency |  | R | 0.931 | 0.888 | 0.970 | 0.980 | 0.949 | 0.933 |
|  | Working efficiency |  | Ef | 0.804 | 0.783 | 0.856 | 0.873 | 0.765 | 0.739 |
|  | Machine operating cycle time per end product unit (with actual yields) | min/p | Tomf, | 0.170 | 0.183 | 0.181 | 0.165 | N.A. | N.A. |
|  | Line operating cycle time | $\mathrm{min} / \mathrm{p}$ | Tos | N.A. | N.A. | N.A. | N.A. | 0.183 | N.A. |
|  | Machine operating throughput in the line | $\mathrm{p} / \mathrm{h}$ | pom,i | 353.78 | 327.17 | 331.21 | 363.00 | N.A. | N.A. |
|  | Line operating throughput | $\mathrm{p} / \mathrm{h}$ | pos | N.A. | N.A. | N.A. | N.A. | 327.17 | N.A. |
|  | Overall productivity |  | P | 0.718 | 0.699 | 0.764 | 0.780 | 0.676 | 0.646 |
|  | Theoretical capacity (thousands) | p/year | Ct | 1,408.16 | 1,310.13 | 1,189.66 | 1,277.78 | 1,189.66 | $\begin{gathered} \hline 1,181.0 \\ 3 \\ \hline \end{gathered}$ |
|  | Gross usable capacity (thousands) | p/year | Cul | 1,337.76 | 1,244.62 | 1,130.17 | 1,213.89 | 1,130.17 | $\begin{array}{\|c} \hline 1,111.3 \\ 5 \\ \hline \end{array}$ |
|  | Net usable capacity (thousands) | p/year | Cun | 1,244.11 | 1,157.50 | 1,051.06 | 1,128.92 | 1,051.06 | $\begin{array}{\|c} \hline 1,031.3 \\ 4 \\ \hline \end{array}$ |
|  | Available capacity (thousands) | p/year | Cd | 1,175.69 | 1,132.03 | 1,025.83 | 1,111.98 | 933.87 | 903.45 |
|  | Actual capacity | p/year | Ce | 1,067.52 | 1,027.88 | 931.46 | 1,009.68 | 847.95 | 817.62 |
|  | Marketable capacity | p/year | Cv | 993.44 | 912.76 | 903.70 | 989.59 | 804.52 | 762.65 |

Only in batch production steps such as thermal treatments, drying and painting might yields be higher than 1 due to the saturation of the machine performing the operation. What said about the process-based production (see the previous example) also applies to the 'design data' section of this example, but in this case standard cycle times are not influenced by yields, which are equal to 1 by definition. The 'target operation parameters' section shows no major differences vis-à-vis the previous example.

In the section entitled 'actual operation parameters' the difference lies in that operating cycle times are calculated by taking into account actual conformance and speed efficiency values (since yields are equal to 1 by definition) and using (11) and (12): consequently, the influence of speed efficiency on the system productivity and marketable production capacity cannot be told apart from the influence of conformance efficiency. The global line efficiency (Rs) results from the $T c_{S} / T o_{S}$ ratio and usually corresponds to the Rvs $\cdot \mathrm{Rqs} \cdot \mathrm{Rcs}$ product. The line bottleneck shifts from station C (design bottleneck) to station B (operating bottleneck). Finally, the last column of this section shows that efficiency, productivity and capacity indices can be calculated starting from the values of time-based parameters measured for the whole system (gross utilization, internal utilization, availability, saturation) and from the amount of defective products (QS) measured step by step in compliance with the basic assumption. In this case the value of the global system efficiency (Rs) equals $T c_{S} \cdot \mathrm{QB} / \mathrm{Tpn} ; E f_{S}$ and the other parameters can be calculated accordingly. The global efficiency value does not allow to tell the influence of conformance efficiency, actual yield and speed efficiency apart from one another. Also in this case the system-level global efficiency is higher than the mere conformance efficiency. The sum of the QS amounts only serves the purpose to calculate the overall number of units rejected by any line station. The system conformance efficiency $R c_{S}$, which is the product of the conformance efficiencies of the individual stations (or the ratio between good units and the overall number of output units measured at the end of the line) still has a quantitative meaning, but it cannot be used to calculate time and capacity losses due to lack of conformance.

## Appendix 2. A system made up of $\mathbf{4}$ sequenced and independent departments

Table 7 shows a system made up of 4 sequenced and independent departments working only 1 shift, 5 days a week. The number of identical machines making up each department varies from one department to the next. The layout scheme is indicated in Figure 5. For simplicity's sake defective products were assumed to be detected and removed only at the end of the process.

The first part of Table 7 provides standard production efficiency values for the machines in each department. These standard values include the cycle time increases regarded as unavoidable during normal operation, such as those caused by actually acceptable speeds, materials quality, physiological stops, etc. Therefore the standard department productivity throughput (Pr) results from the standard throughput of each machine multiplied by the number of machines located in the department. The system average standard throughput corresponds to the lowest department throughput (department D).

Table 7
Example of a system (parts fabrication assembly process) made up from 4
sequenced and independent departments where defects are detected only at the end of the line

| Index area | Index | Unit of measure | Notation | Machine A | Machine B | Machine C | Machine D | Overall system |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Machine standard cycle time | $\mathrm{min} / \mathrm{p}$ | Tcm | 7.500 | 3.800 | 5.910 | 12.300 |  |
|  | Machine number |  |  | 4 | 2 | 3 | 6 | 15 |
|  | Department standard cycle time | $\mathrm{min} / \mathrm{p}$ | Tcr | 1.875 | 1.900 | 1.970 | 2.050 |  |
|  | System standard cycle time | $\mathrm{min} / \mathrm{p}$ | Tcs |  |  |  |  | 2.050 |
|  | Department standard throughput | $\mathrm{p} / \mathrm{h}$ | pr | 32.00 | 31.58 | 30.46 | 29.27 |  |
|  | Plant calendar time | h/year |  |  |  |  |  | 1840 |
|  | Gross utilization |  | UI | 0.960 | 0.980 | 0.950 | 0.950 | 0.950 |
|  | Internal utilization |  | Ui | 0.930 | 0.890 | 0.920 | 0.970 | 0.957 |
|  | Net utilization |  | Un | 0.893 | 0.872 | 0.874 | 0.922 | 0.909 |
|  | Availability |  | D | 0.960 | 0.910 | 0.980 | 0.990 | 0.942 |
|  | Saturation |  | S | 0.900 | 0.950 | 0.930 | 0.890 | 0.948 |
|  | Speed efficiency |  | Rv | 0.900 | 0.850 | 0.980 | 0.960 |  |
|  | Actual yield |  | Rq | 1.000 | 1.000 | 1.000 | 1.000 |  |
|  | Conformance efficiency |  | Rc | 0.980 | 0.990 | 0.910 | 0.990 |  |
|  | Global efficiency |  | R | 0.882 | 0.842 | 0.892 | 0.950 | 0.843 |
|  | Working efficiency |  | Ef | 0.762 | 0.727 | 0.813 | 0.837 | 0.753 |
|  | Overall productivity |  | P | 0.680 | 0.635 | 0.710 | 0.772 | 0.685 |
|  | Department operating throughput | $\mathrm{p} / \mathrm{h}$ | por | 21.771 | 20.037 | 21.636 | 22.585 | 20.037 |
|  | Theoretical capacity (thousands) | p/year | Ct | 58,880 | 58,105 | 56,041 | 53,854 | 53,854 |
|  | Gross usable capacity (thousands) | p/year | Cul | 56,525 | 56,943 | 53,239 | 51,161 | 51,161 |
|  | Net usable capacity (thousands) | p/year | Cun | 52,568 | 50,679 | 48,979 | 49,626 | 48,979 |
|  | Available capacity (thousands) | p/year | Cd | 50,465 | 46,118 | 48,000 | 49,130 | 46,118 |
|  | Actual capacity | p/year | Ce | 45,419 | 43,812 | 44,640 | 43,726 | 43,726 |
|  | Marketable capacity | p/year | Cv | 40,059 | 36,868 | 39,810 | 41,557 | 36,868 |
| $\begin{gathered} \frac{y}{0} \\ \stackrel{0}{0} \\ \stackrel{0}{\omega} \\ \stackrel{1}{2} \end{gathered}$ | Marketable capacity target | p/year | Cv |  |  |  |  | 44,000 |
|  | Operating throughput target at the system-level | $\mathrm{p} / \mathrm{h}$ | pos |  |  |  |  | 23.913 |
|  | Overall productivity target |  | P | 0.747 | 0.757 | 0.785 | 0.817 |  |

Table 7 (continued)

| Index area | Index | Unit of measure | Notation | Machine A | $\begin{gathered} \text { Machine } \\ \text { B } \\ \hline \end{gathered}$ | Machine C | Machine D | Overall system |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Action \# 1: dept. B |  |  |  |  |  |  |  |
|  | - increase Rv to 0,96 | $\mathrm{p} / \mathrm{h}$ | por | 21.771 | 24.156 | 21.636 | 22.585 |  |
|  | - increase availability to 0,94 | $\mathrm{p} / \mathrm{h}$ | pos |  |  |  |  | 21.636 |
|  | - increase Ui to 0,90 |  | P | 0.680 | 0.765 | 0.710 | 0.772 |  |
|  |  | p/year | Cv | 40,059 | 44,446 | 39,810 | 41,557 | 39,810 |
|  | Action \# 2: dept. C |  |  |  |  |  |  |  |
|  | - increase Rc to 0,96 | $\mathrm{p} / \mathrm{h}$ | por | 21.771 | 24.156 | 24.076 | 22.585 |  |
|  | - increase saturation to 0,95 | $\mathrm{p} / \mathrm{h}$ | pos |  |  |  |  | 21.771 |
|  | - increase Ui to 0,95 |  | P | 0.680 | 0.765 | 0.790 | 0.772 |  |
|  |  | p/year | Cv | 40,059 | 44,446 | 44,299 | 41,557 | 40,059 |
|  | Action \# 3: dept. A |  |  |  |  |  |  |  |
|  | - increase Rv to 0,95 | $\mathrm{p} / \mathrm{h}$ | por | 24.002 | 24.156 | 24.076 | 22.585 |  |
|  | - increase saturation to 0,94 | $\mathrm{p} / \mathrm{h}$ | pos |  |  |  |  | 22.585 |
|  |  |  | P | 0.750 | 0.765 | 0.790 | 0.772 |  |
|  |  | p/year | Cv | 44,164 | 44,446 | 44,299 | 41,557 | 41,557 |
|  | Action \# 4: dept. D |  |  |  |  |  |  |  |
|  | - increase Ul to 0,98 | $\mathrm{p} / \mathrm{h}$ | por | 24.002 | 24.156 | 24.076 | 24.027 |  |
|  | - increase Rv to 0,99 | p/h | pos |  |  |  |  | 24.002 |
|  |  |  | P | 0.750 | 0.765 | 0.790 | 0.821 |  |
|  |  | p/year | Cv | 44,164 | 44,446 | 44,299 | 44,209 | 44,164 |

The 'actual operation parameters' section highlights the actual reduction factors for each department. These factors allow to recalculate the department operating throughput (Por) which includes all throughput losses. Notice that the line bottleneck shifts from department D (design bottleneck) to department B (operating bottleneck). Department production capacities on different levels are also calculated; the resulting system production capacity is the lowest value found among departments operating on the same level. The values of utilization and system productivity indices can be calculated as the ratios between the related capacities. These parameters should only be used as an indication, since it would not be significant to infer time losses due to different causes from them.


Fig. 5. Layout of a system made up of 4 sequenced and independent departments
The 'targets and improvement actions' section shows an example of an improvement plan which aims at increasing the system throughput while rebalancing the role played by its steps or departments. The improvement target is expressed in terms of annual product volumes (marketable capacity); the target system operating throughput and the productivity values to be achieved in the individual departments are drawn from it. The priority actions are determined by the initial operating throughput of the various departments; hence: departments B, C, A, D. Some possible combined actions are indicated for each department which might meet the minimum system operating throughput constraint while at the same time balancing the working pace of the various departments. Obviously, the actions to be performed in practice would also require a cost-benefit evaluation.


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    ${ }^{* *}$ Politecnico di Milano, Italy.

[^1]:    ${ }^{1}$ For the sake of clarity, each time a machine (or system) throughput is mentioned hereinafter, no distinction is made between nominal and mix throughput.

[^2]:    ${ }^{1}$ If the machine stand-by for lack of manpower is to be regarded as a structural event to be included among the systemexternal causes ascribed to design choices the parameter to be considered is the gross utilization. If the wait for manpower is due to organizational causes such as a man-machine interaction, the parameter to be considered is the internal utilization.

[^3]:    ${ }^{1}$ In this case the term cycle time could be inappropriate, since it usually refers to the unit production time when the machine is operating. Therefore its conditions are standardized or at best only include increases due to machine-specific causes and operation mode.

